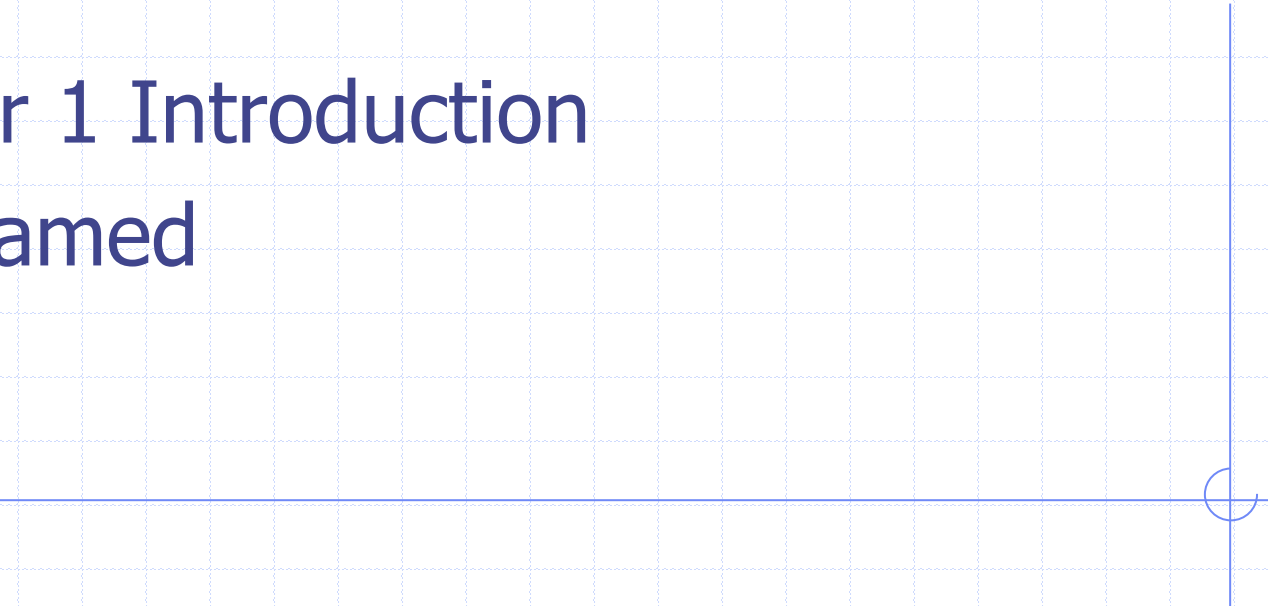


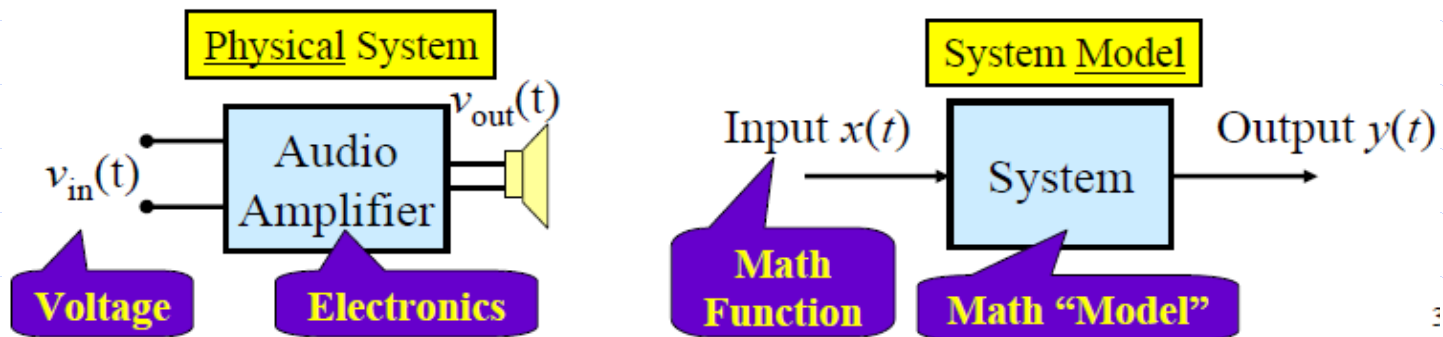
Signal & Linear system

Chapter 1 Introduction
Basil Hamed



Signals & Systems

- Because most “systems” are driven by “signals” EEs& CEs study what is called “Signals & Systems”
- **“Signal”** = a time-varying voltage (or other quantity) that generally carries some information
- The job of the **“System”** is often to extract, modify, transform, or manipulate that carried information
- So...a big part of **“Signals & Systems”** is using **math models** to see what a system “does” to a signal



Some Application Areas

Application Area	Specific Uses of Signals & Systems
Telecommunications	Answering machines, modems, fax machines, cell phones, speaker phones
Speech and Audio	Voice mail, speaker verification, synthetic speech, audio compression (e.g., mp3)
Automotive	Engine control, antilock braking systems, active suspension, airbag control, system diagnosis
Medical	Hearing aids, remote monitoring, ultrasound imaging, magnetic resonance imaging (MRI)
Image Processing	3D animation, image enhancement, image compression (JPEG), video compression (MPEG), high-definition TV
Control Systems	Head positioning in disk drives, laser control (e.g., printers, CD/DVD drives), engine & motor control, robots
Military & Aerospace	Radar & sonar, navigation systems (e.g., GPS), secure communications, missile guidance, battlefield sensors

In each of these areas you can't build the electronics until your math models tell you what you need to build

What is a signal ?

The concept of signal refers to the space or time variations in the physical state of an object.

SIGNALS

Signals are functions of independent variables that carry information. For example:

- **Electrical signals** --- voltages and currents in a circuit
- **Acoustic signals** --- audio or speech signals (analog or digital)
- **Video signals** --- intensity variations in an image (e.g. a CAT scan)
- **Biological signals** --- sequence of bases in a gene.

THE INDEPENDENT VARIABLES

- Can be continuous—Trajectory of a space shuttle—Mass density in a cross-section of a brain
- Can be discrete—DNA base sequence—Digital image pixels
- Can be 1-D, 2-D, ... N-D
- For this course: Focus on a single (1-D) independent variable which we call “time”. Continuous-Time (CT)

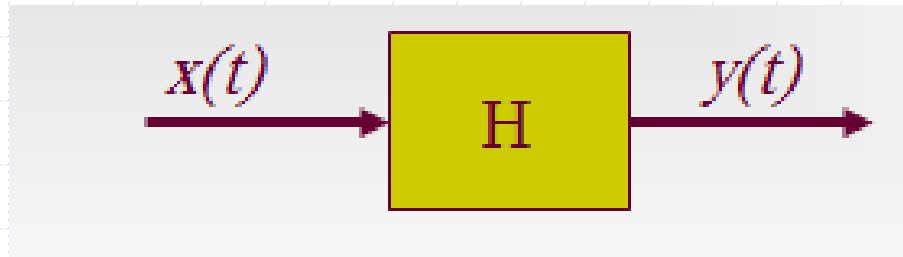
signals: $x(t)$, t —continuous values Discrete-Time (DT)

signals: $x[n]$, n —integer values only

System

Signals may be processed further by systems, which may modify them or extract additional information from them.

System is a black box that transforms input signals to output signals



1.1 Size of a signal

Power and Energy of Signals

- Energy: accumulation of absolute of the signal

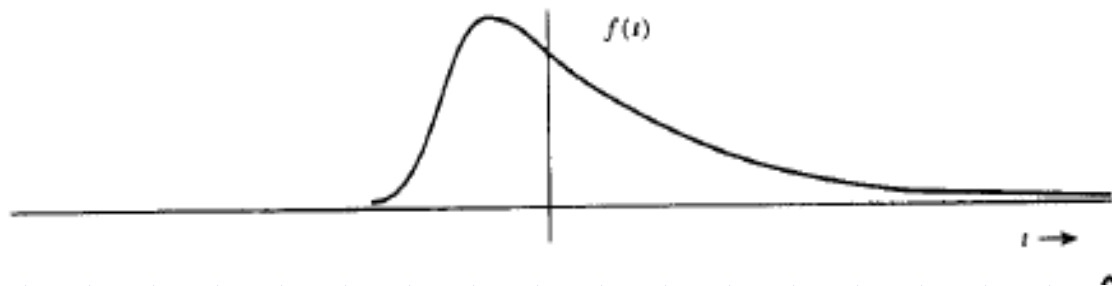
$$E_{\infty} \triangleq \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- Power: average of absolute of the signal

$$P_{\infty} \triangleq \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{E_{\infty}}{2T}$$

1.1 Size of a signal

Signal Energy



Signal Power



Power and Energy of Signals

Energy signal iff $0 < E < \infty$, and so $P = 0$.

EX.

$$x(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & t \geq 0 \end{cases}$$

Power signal iff $0 < P < \infty$, and so $E = \infty$.

$$\{x[n]\} = \{\dots -1, 1, -1, 1, -1, 1 \dots\}$$

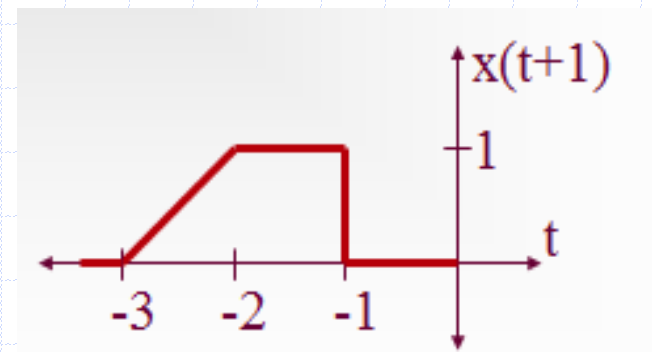
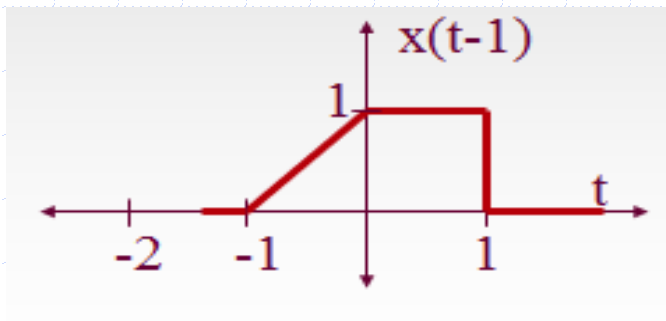
1.2 Some Useful Signal Operations (Transformation)

Three possible *time* transformations:

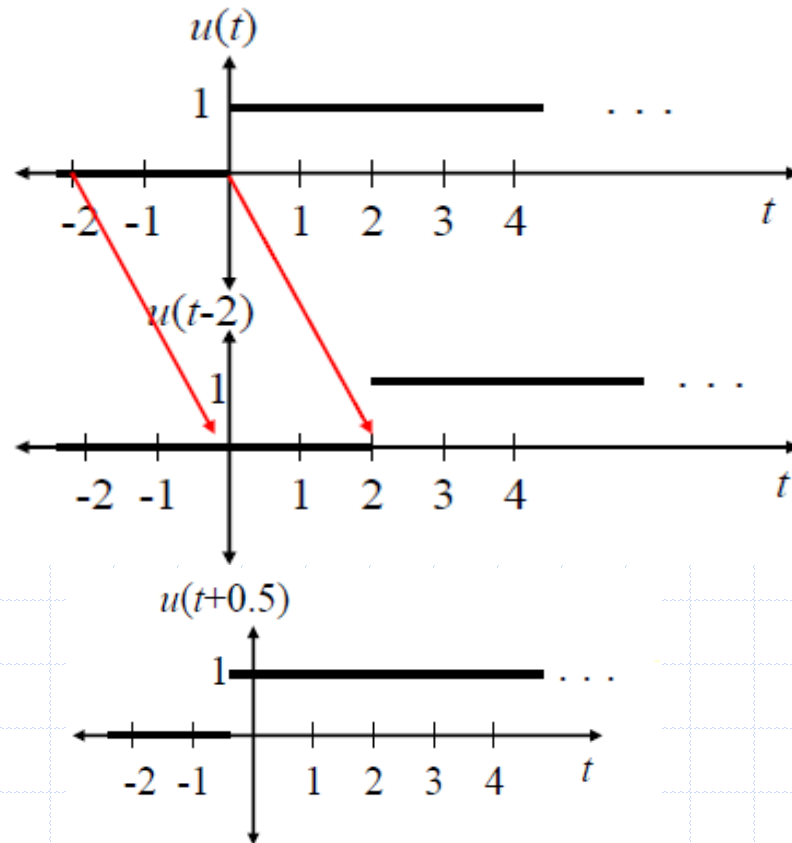
- Time Shifting
- Time Scaling
- Time Reversal

Time Shift

Signal $x(t \pm 1)$ represents a time shifted version of $x(t)$



Time Shift



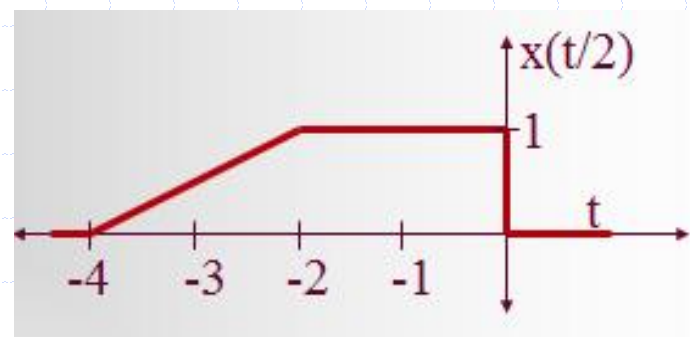
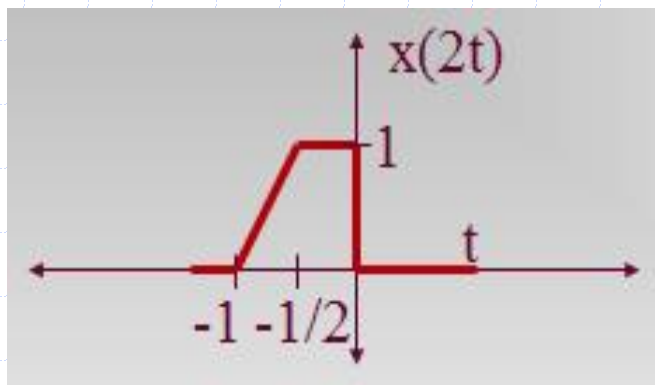
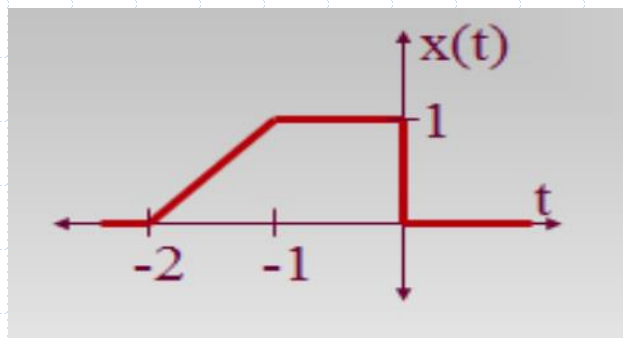
General View:

$$x(t \pm t_0) \quad \text{for } t_0 > 0$$

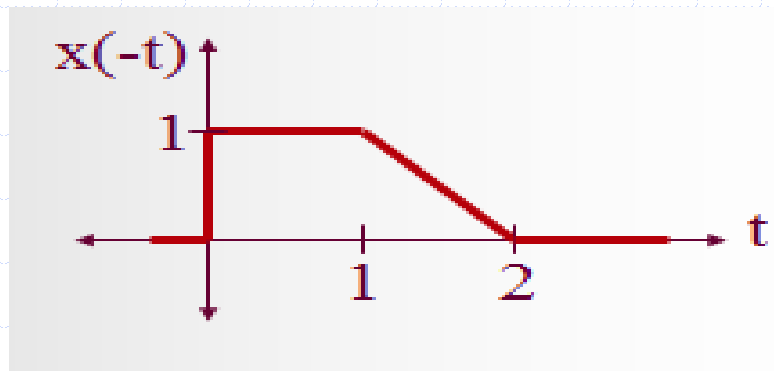
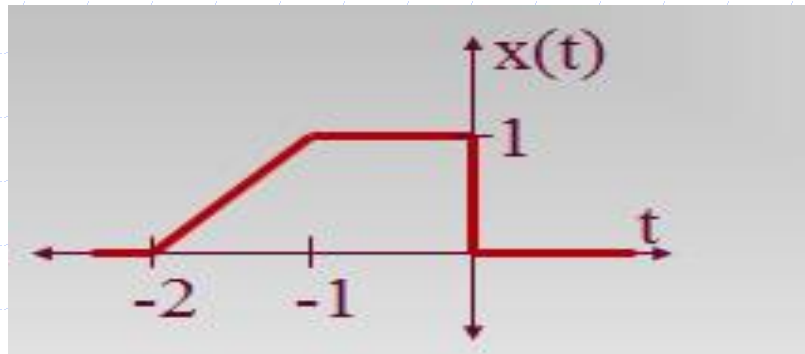
“ $+t_0$ ” gives **Left** shift (**Advance**)

“ $-t_0$ ” gives **Right** shift (**Delay**)

Time-scale



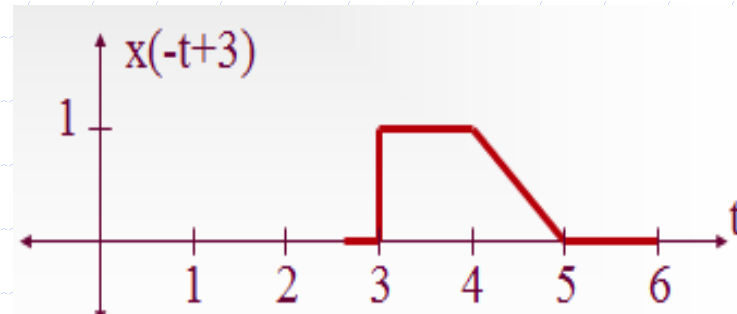
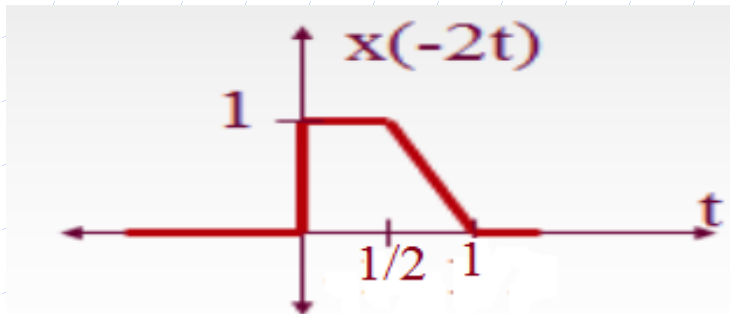
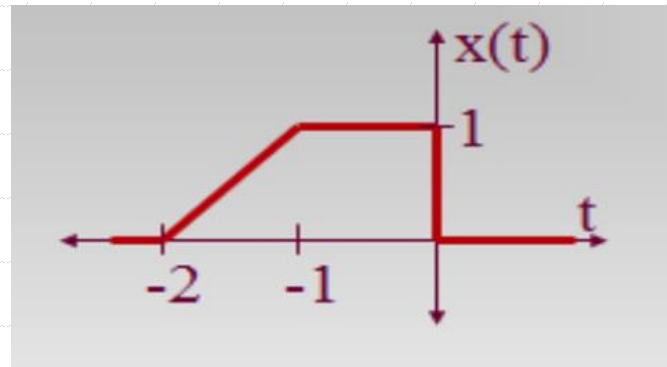
Time- Reversal (Flip)



Combined Operations

Certain complex operations require simultaneous use of more than one of the operations.

EX. Find i. $x(-2t)$
ii. $x(-t+3)$



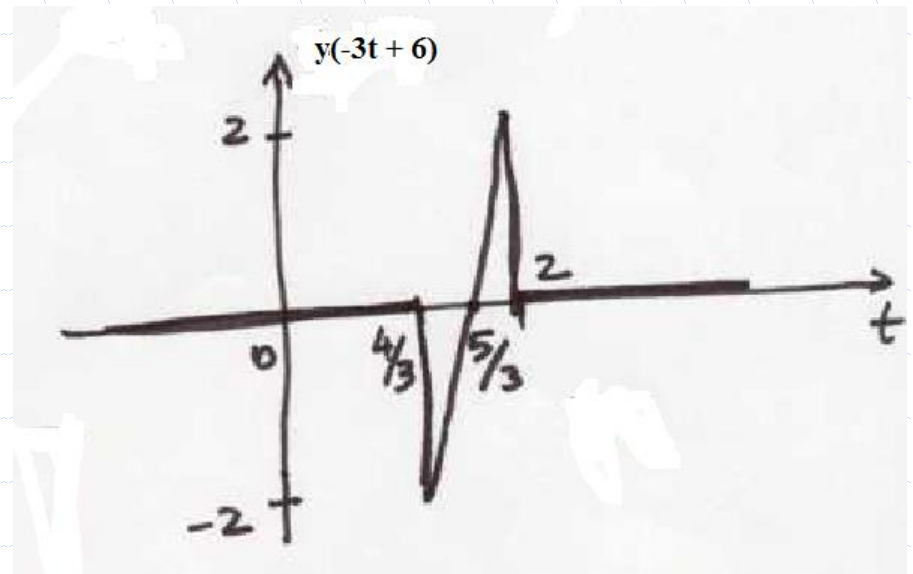
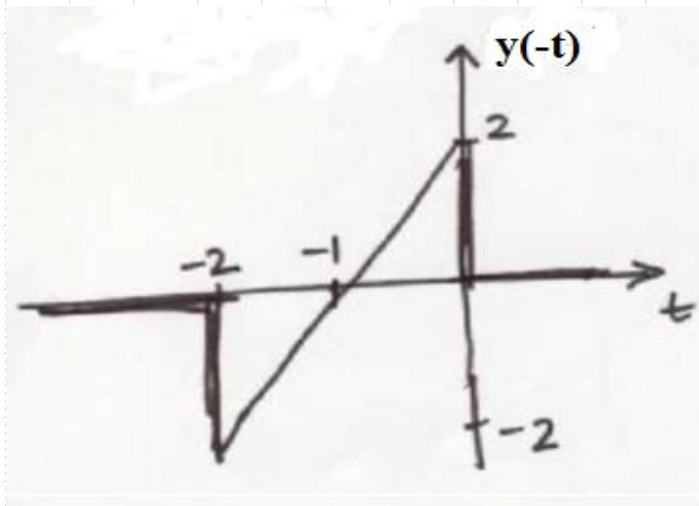
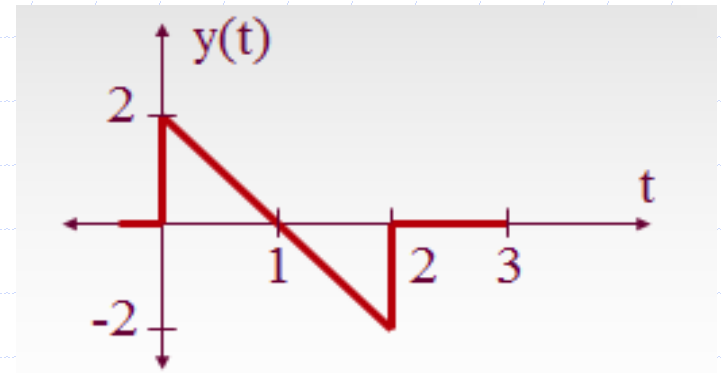
Combined Operations

Example

Given $y(t)$, find $y(-3t+6)$

Solution

Flip/Scale/Shift



1.3 Classification of Signals

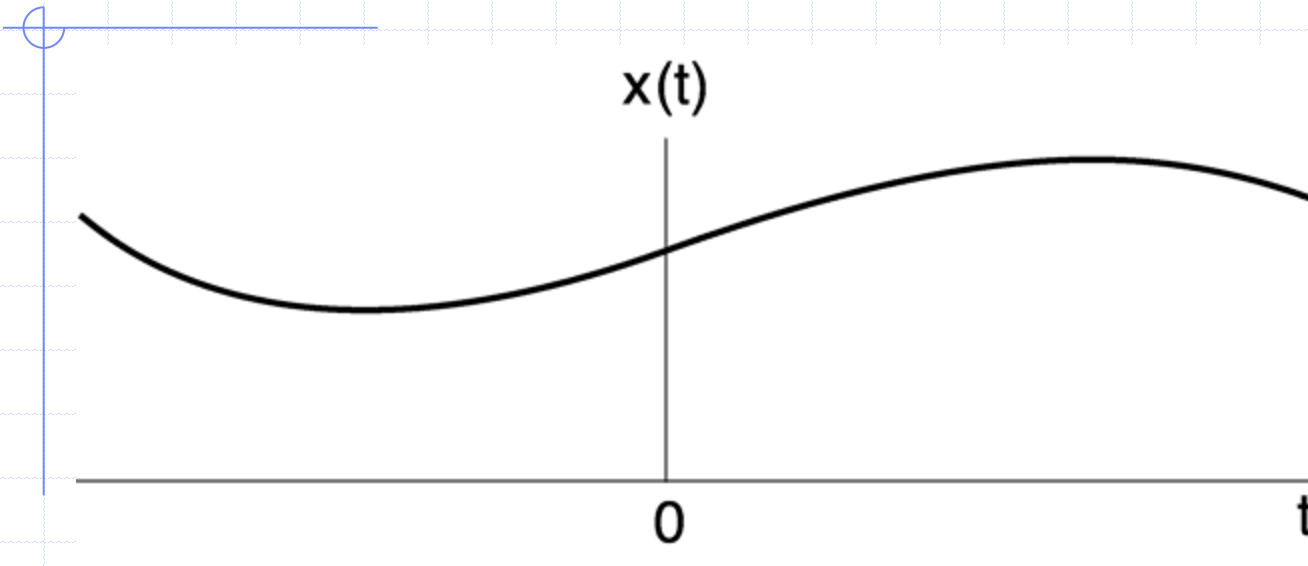
There are several classes of signals:

- 1- Continuous-time and Discrete-time signals
- 2- Periodic and Aperiodic Signals
- 3- Energy and Power Signals
- 4- Deterministic and probabilistic Signals

Continuous-time and Discrete-time Signals

- **Continuous-time signals are functions of a real argument**
 $x(t)$ where t can take any real value
 $x(t)$ may be 0 for a given range of values of t
- **Discrete-time signals are functions of an argument that takes values from a discrete set**
 $x[n]$ where $n \in \{\dots-3,-2,-1,0,1,2,3\dots\}$
We sometimes use “index” instead of “time” when discussing discrete-time signals
- **Values for x may be real or complex**

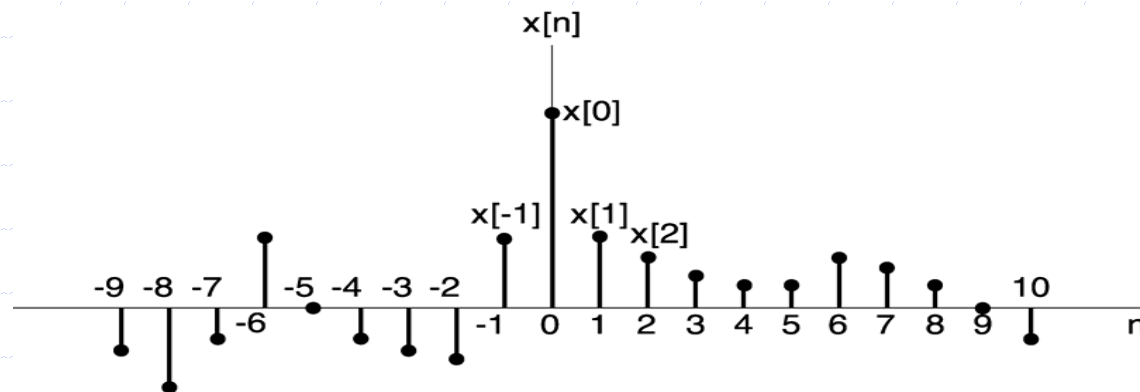
CT Signals



Most of the signals in the physical world are CT signals—E.g. voltage & current, pressure, temperature, velocity, etc.

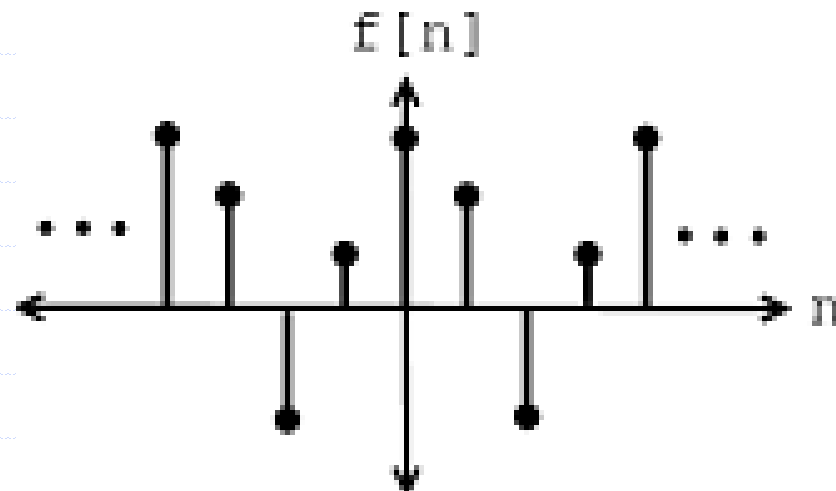
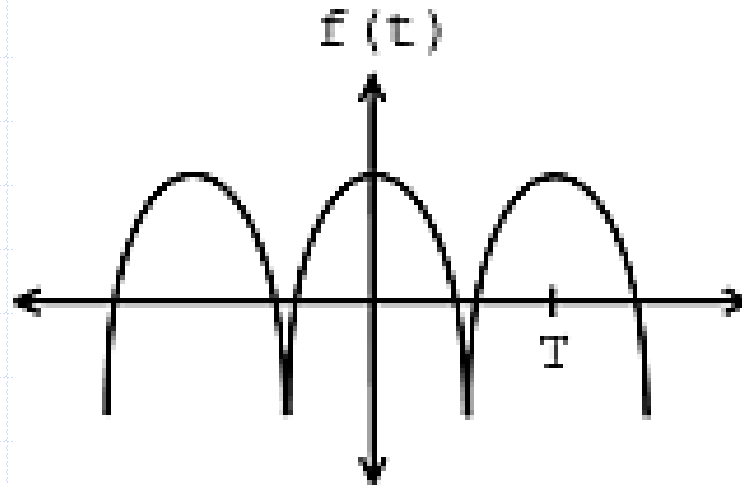
DT Signals

- $x[n]$, n —integer, time varies discretely



- Examples of DT signals in nature:
 - DNA base sequence
 - Population of the n th generation of certain species

Continuous Time-Discrete Time



Many human-made DT Signals

Ex.#1 Weekly Dow-Jones industrial average



Ex.#2 digital image



Why DT? —Can be processed by modern digital computers and digital signal processors (DSPs).

Applications

- **Electrical Engineering**

 - voltages/currents in a circuit

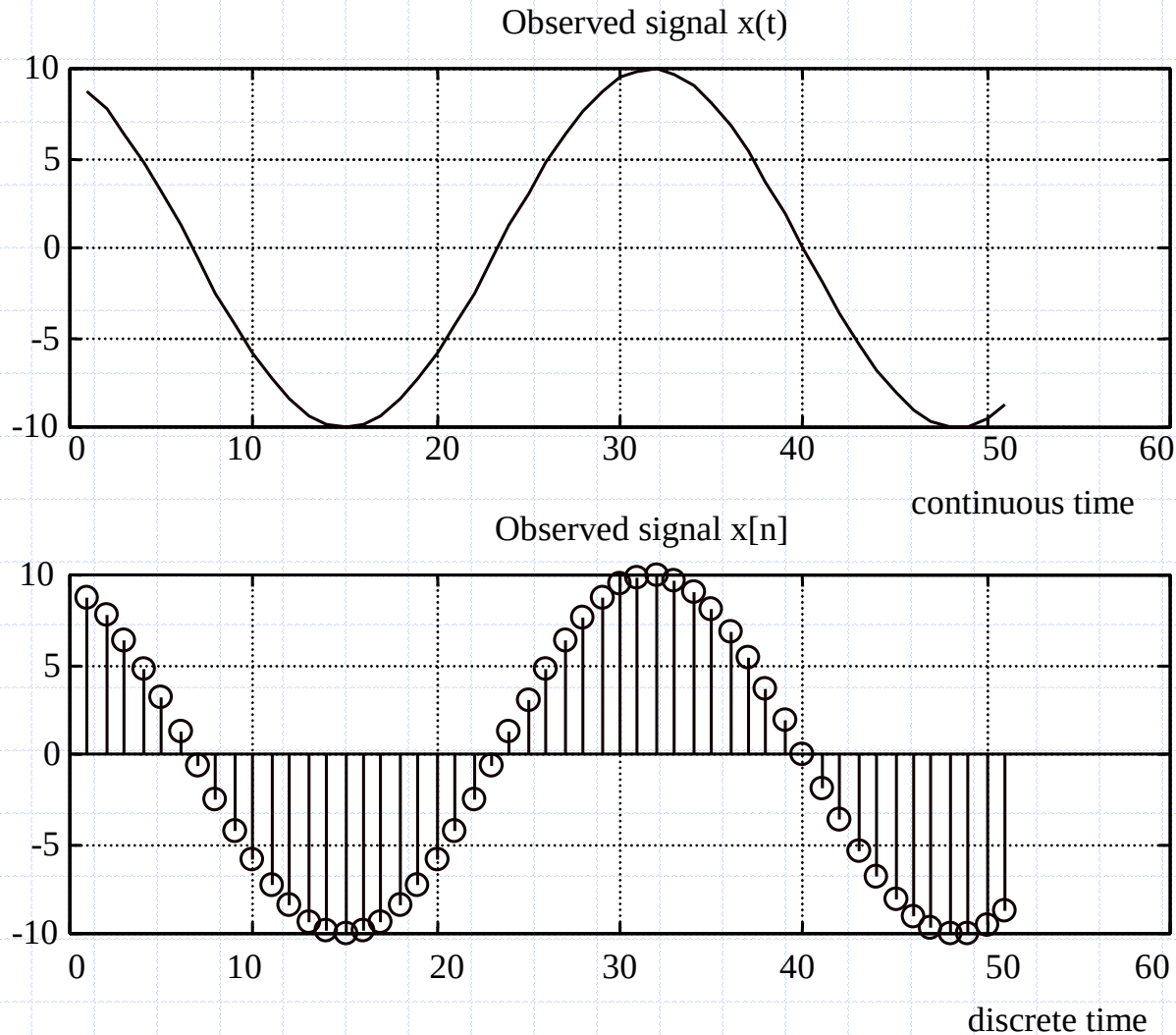
 - speech signals

 - image signals

- **Physics**

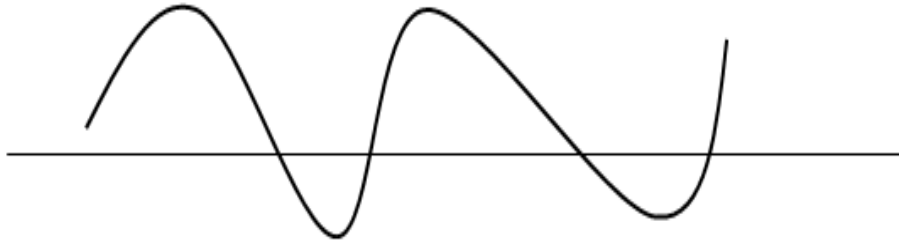
 - radiation

From Continuous to Discrete: Sampling

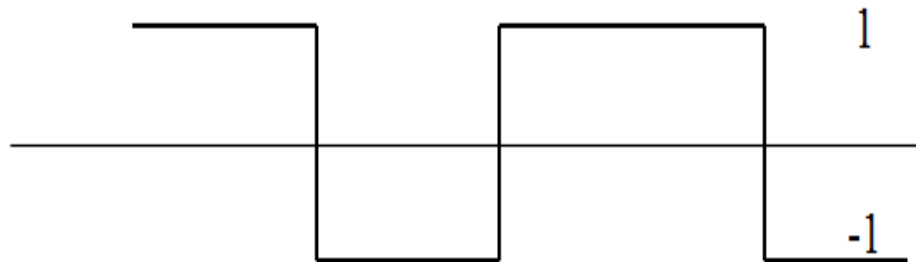


Analog vs. Digital

- The amplitude of an analog signal can take any real or complex value at each time/sample

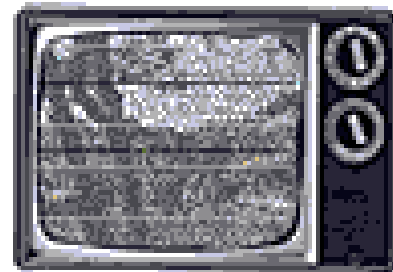


- Amplitude of a digital signal takes values from a discrete set



Digital vs. Analog

Analog
Signal



if signal is weak,
picture is weak,
lots of static

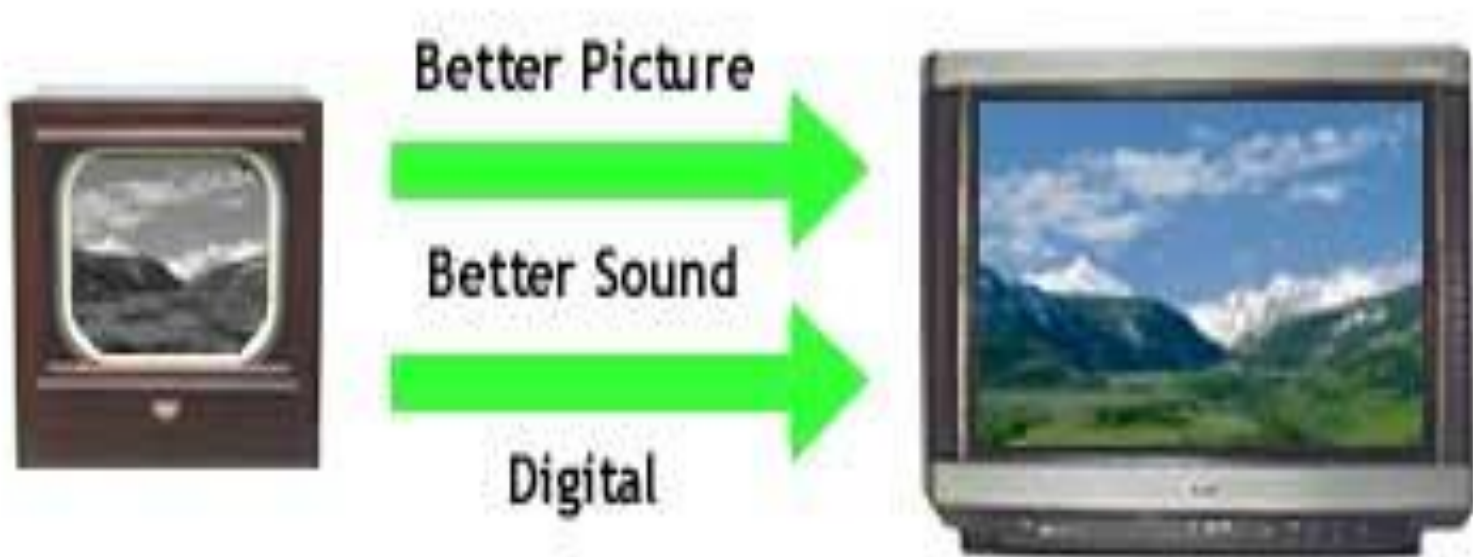
both signals weaken over distance

Digital
Signal



as long as tv
is receiving a
signal, picture
is perfect

Digital vs. Analog



Analog-Digital

Examples of analog technology

- ◆ photocopiers
- ◆ telephones
- ◆ audio tapes
- ◆ televisions (intensity and color info per scan line)

Examples of digital technology

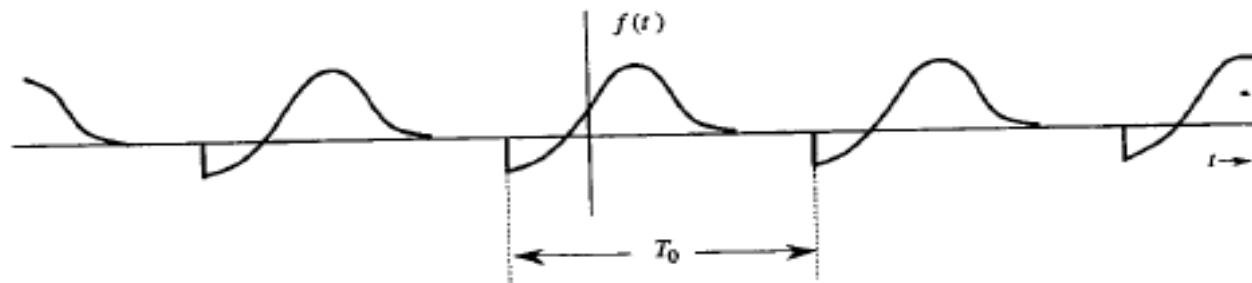
- ◆ Digital computers!

Periodic and Aperiodic Signals

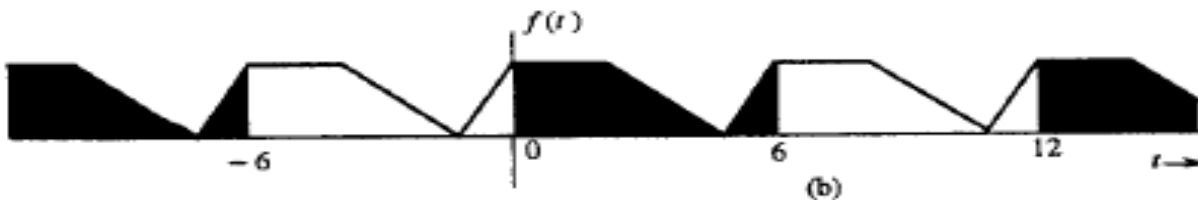
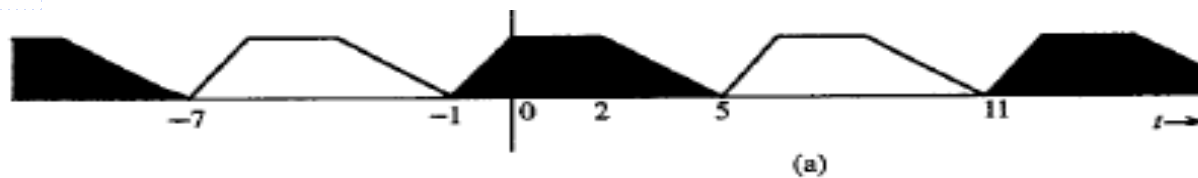
Periodicity condition

$$x(t) = x(t+T)$$

If **T** is period of $x(t)$, then $x(t) = x(t+nT)$ where $n=0,1,2,\dots$



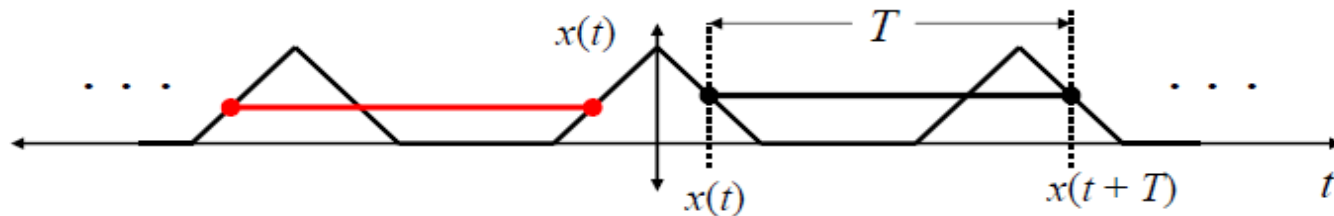
A periodic signal of period T_0 .



Periodic Signals

Periodic signals are important because many human-made signals are periodic. Most test signals used in testing circuits are periodic signals (e.g., sine waves, square waves, etc.)

A Continuous-Time signal $x(t)$ is periodic with period T
if: $x(t + T) = x(t) \quad \forall t$



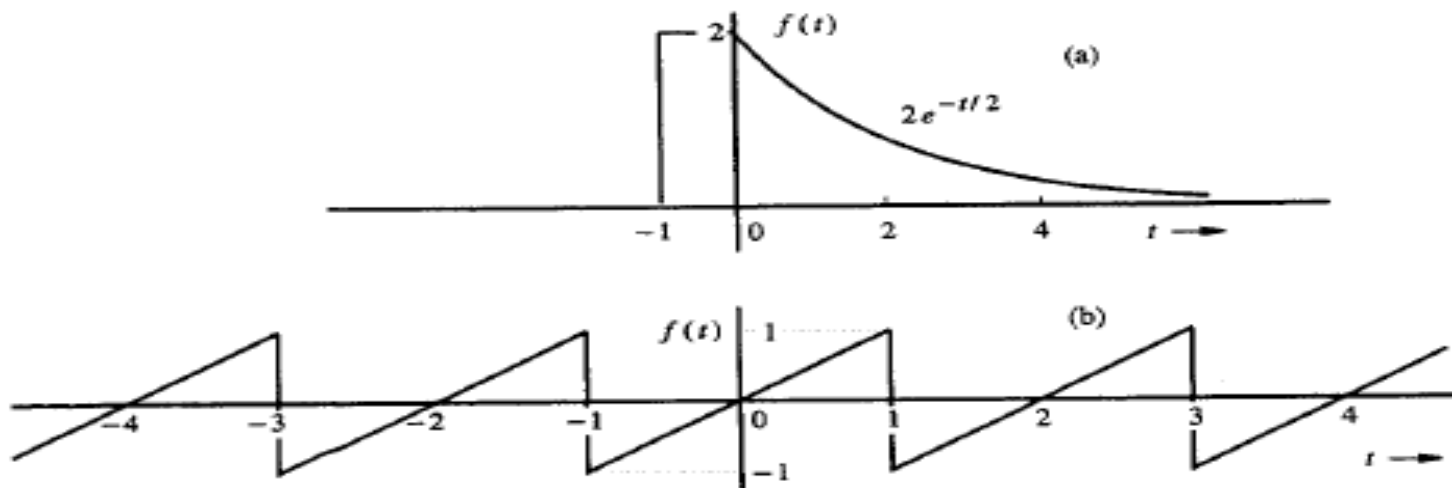
Fundamental period = smallest such T

When we say “Period” we almost always mean “Fundamental Period”

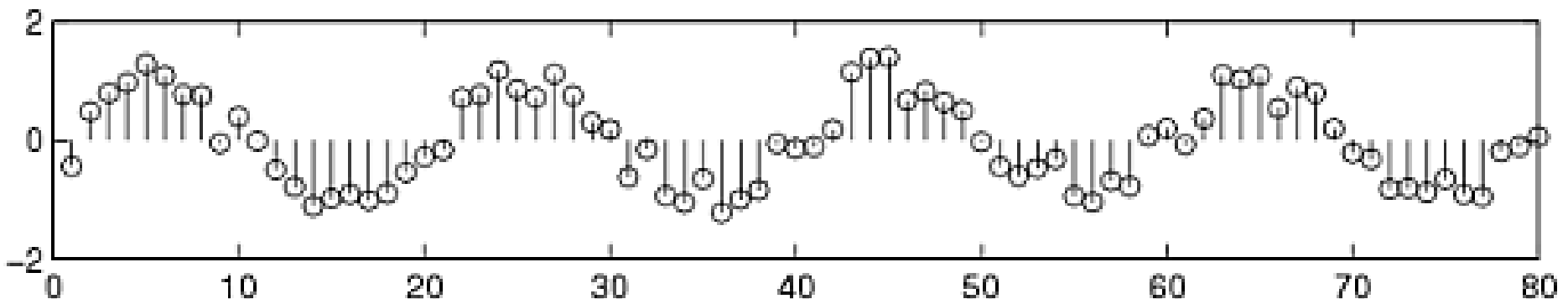
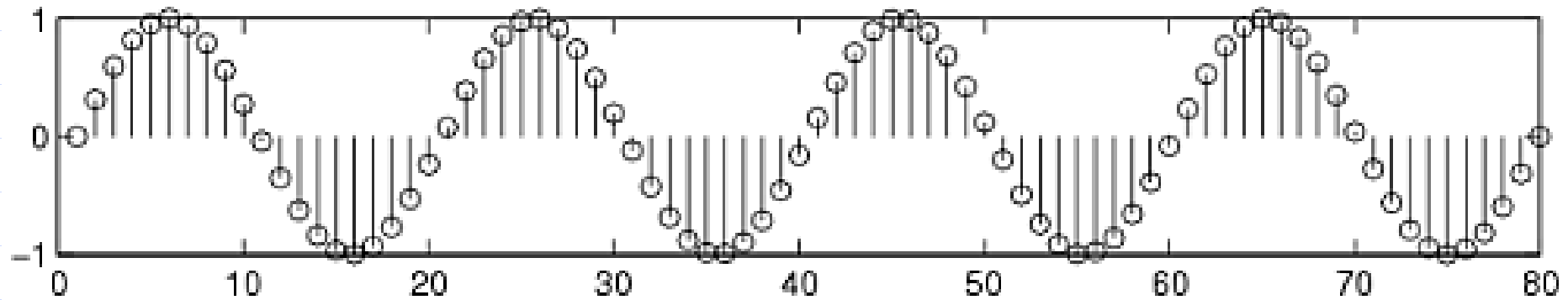
Energy and Power Signals

signal with finite energy is an energy signal, and a signal with a finite and nonzero power is a power signal.

Signals in Fig below are energy (a) and power (b) signals



Deterministic-Stochastic Signals



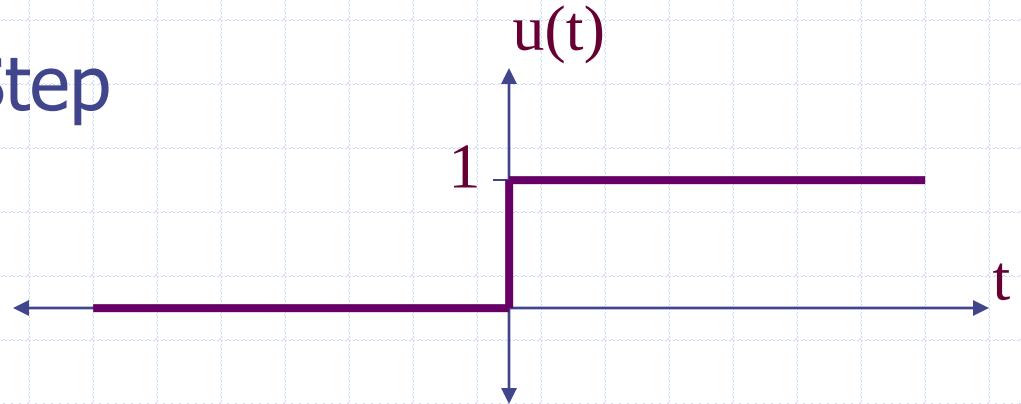
1.4 Some Useful Signal Model

- Step Signal
- Ramp Signal
- Impulse Signal
- Exponential Signal

Unit Step

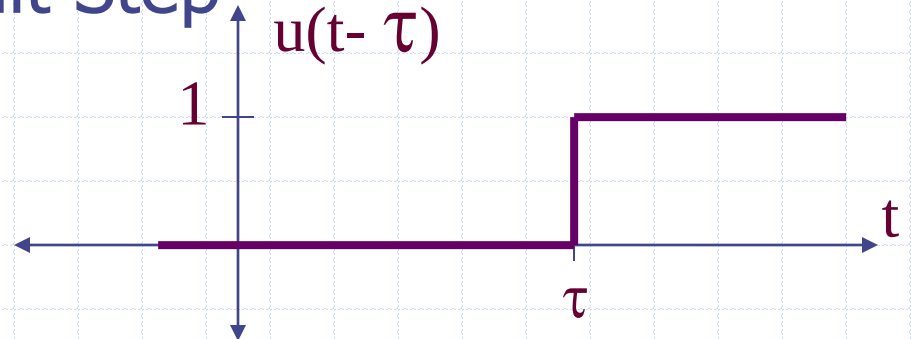
- Continuous Unit Step

$$u(t) = \begin{cases} 1 & , t > 0 \\ 0 & , t < 0 \end{cases}$$



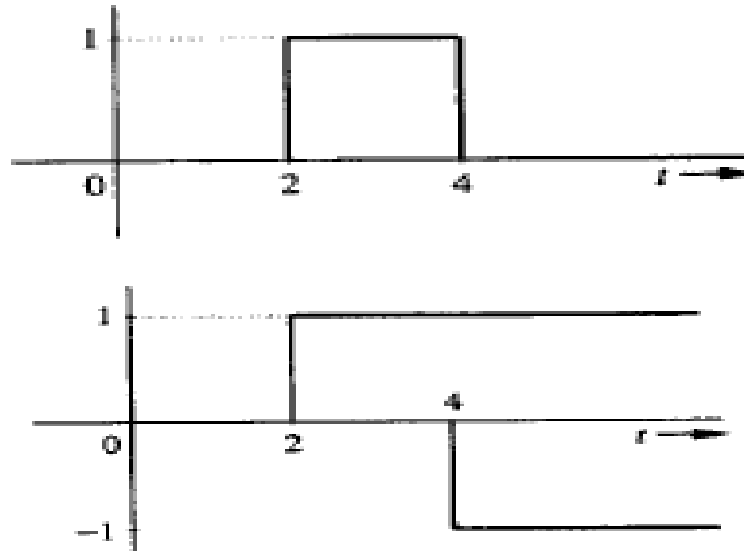
- Continuous Shifted Unit Step

$$u(t-\tau) = \begin{cases} 1 & , t > \tau \\ 0 & , t < \tau \end{cases}$$



Unit Step

Ex. Express the signal showing using step function

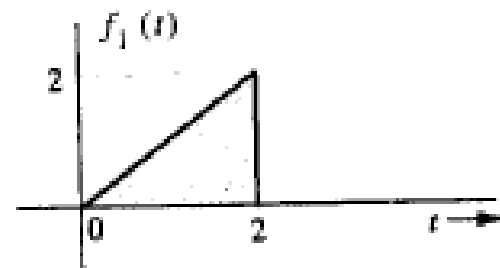
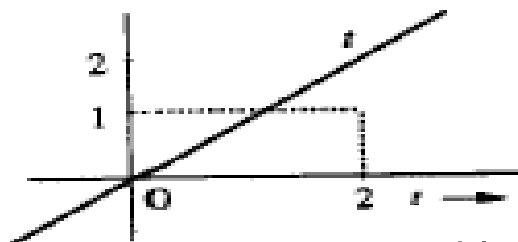
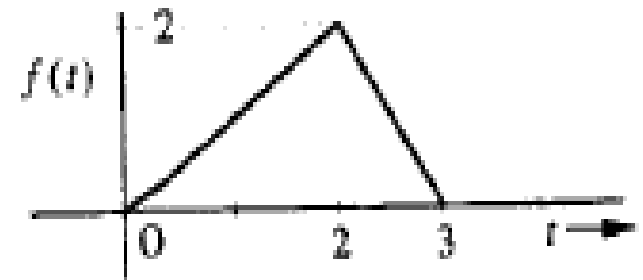


$$X(t) = u(t-2) - u(t-4)$$

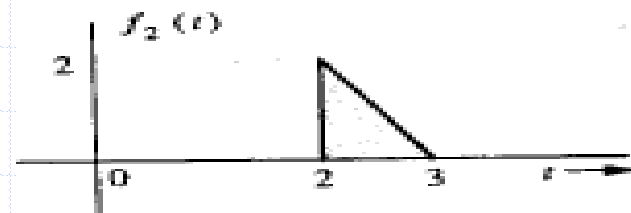
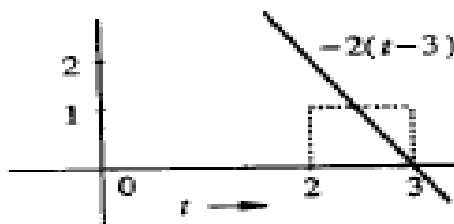
Unit Step

Ex 1.6 P.88

Describe the signal in Figure using step fun



$$f_1(t) = t[u(t) - u(t-2)]$$

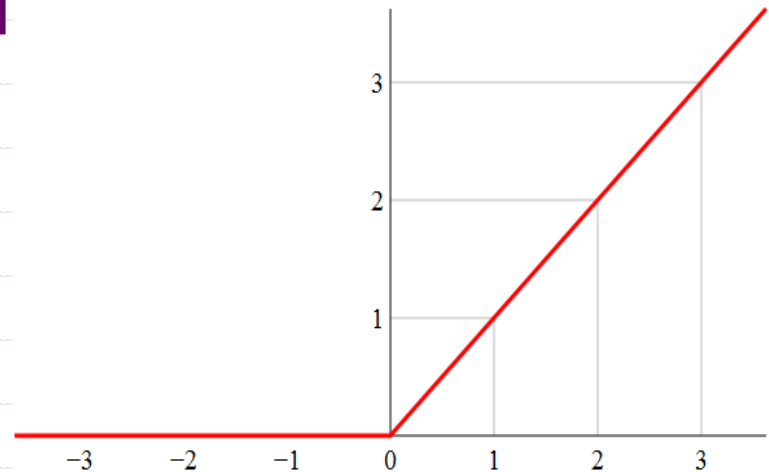


$$f_2(t) = -2(t-3)[u(t-2) - u(t-3)]$$

$$F(t) = f_1 + f_2 = tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3)$$

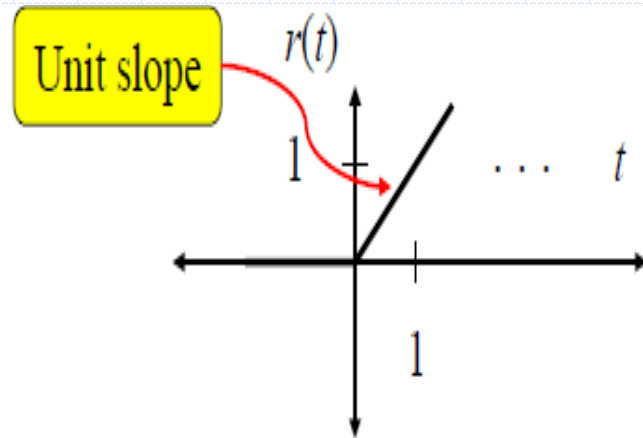
Ramp Function

$$R(t) = \begin{cases} t & , t > 0 \\ 0 & , t < 0 \end{cases}$$



Note: A ramp with slope m can be made from: $mr(t)$

$$mr(t) = \begin{cases} mt, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

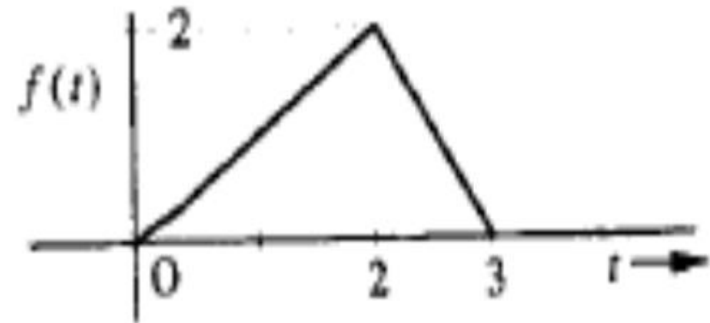


Ramp Function

Ex Describe the signal shown in Fig

Using ramp function

$$F(t) = r(t) - 3r(t-2) + 2r(t-3)$$



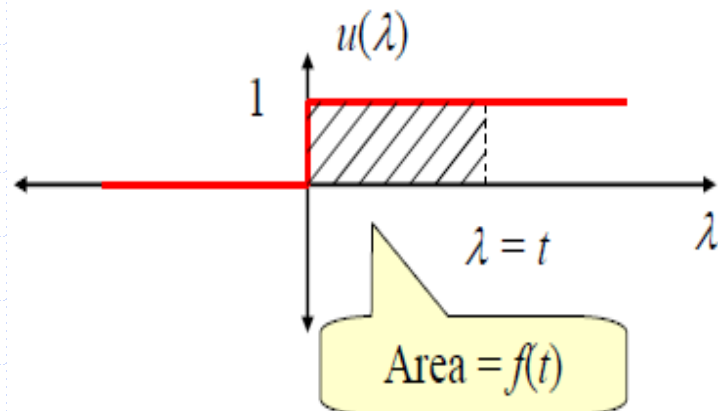
Relationship between $u(t)$ & $r(t)$

$$f(t) = \int_{-\infty}^t u(\lambda) d\lambda \quad \text{What is } f(t)?$$

-Write unit step as a function of λ

-Integrate up to $\lambda = t$ i.e., Find Area

-How does area change as t changes?



$$\Rightarrow f(t) = \int_{-\infty}^t u(\lambda) d\lambda = 1 \cdot t = t = r(t)$$

$$\Rightarrow r(t) = \int_{-\infty}^t u(\lambda) d\lambda$$

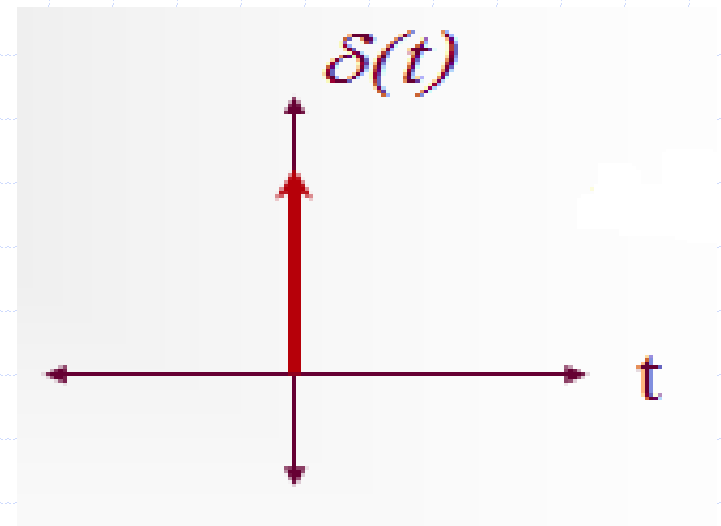
Impulse Signal

Other Names: Delta Function,
Dirac Delta Function

- One of the most important functions for understanding systems!! Ironically...it does not exist in practice!!
- It **is** a theoretical tool used to understand what is important to know about systems! But...it leads to ideas that are used all the time in practice!!

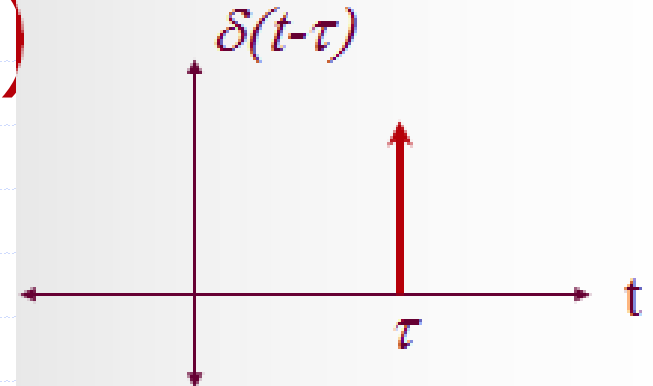
$$\delta(t) = \begin{cases} \infty & , t = 0 \\ 0 & , t \neq 0 \end{cases}$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(t) dt = 1$$



Unit Impulse (cont'd)

- Continuous Shifted Unit Impulse
- Properties of continuous unit impulse



$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$\delta(-t) = \delta(t)$$

$$x(t)\delta(t) = x(0)\delta(t)$$

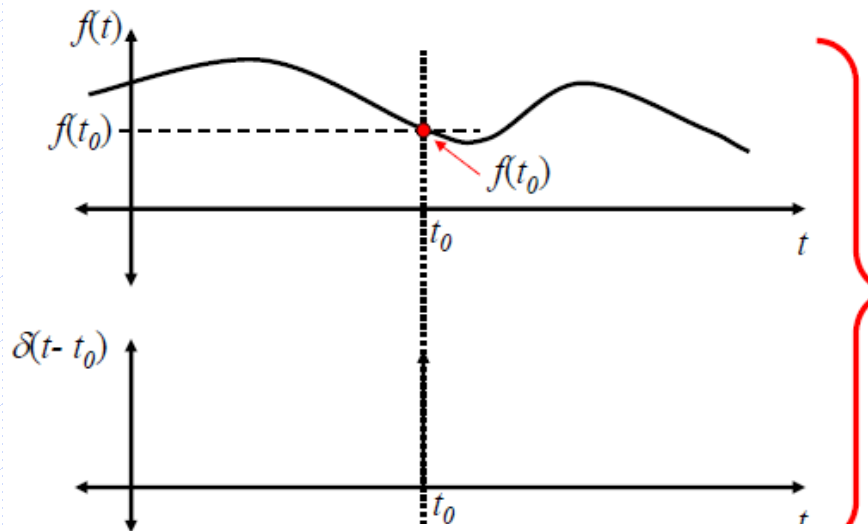
$$x(t)\delta(t-\tau) = x(\tau)\delta(t-\tau)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Unit Impulse (cont'd)

The Sifting Property is the most important property of $\delta(t)$:

$$\int_{t_0 - \varepsilon}^{t_0 + \varepsilon} f(t) \delta(t - t_0) dt = f(t_0) \quad \forall \varepsilon > 0$$



Integrating the product of $f(t)$ and $\delta(t - t_0)$ returns a single number... the value of $f(t)$ at the “location” of the shifted delta function

As long as the integral's limits surround the “location” of the delta... otherwise it returns zero

Euler's Equation

Euler's formulas

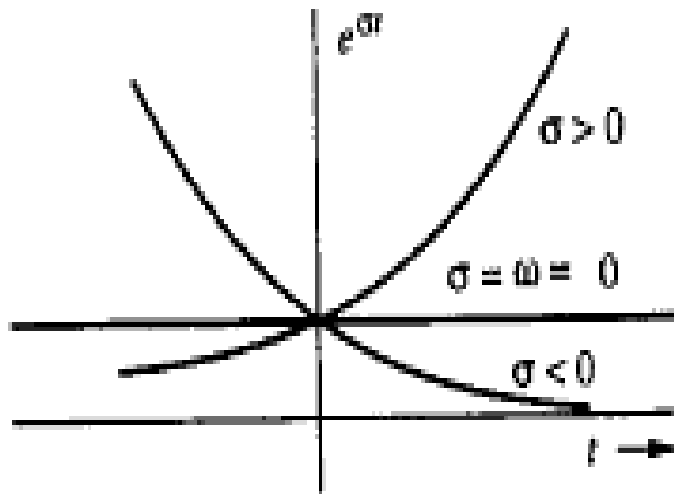
$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\cos \theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$\sin \theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

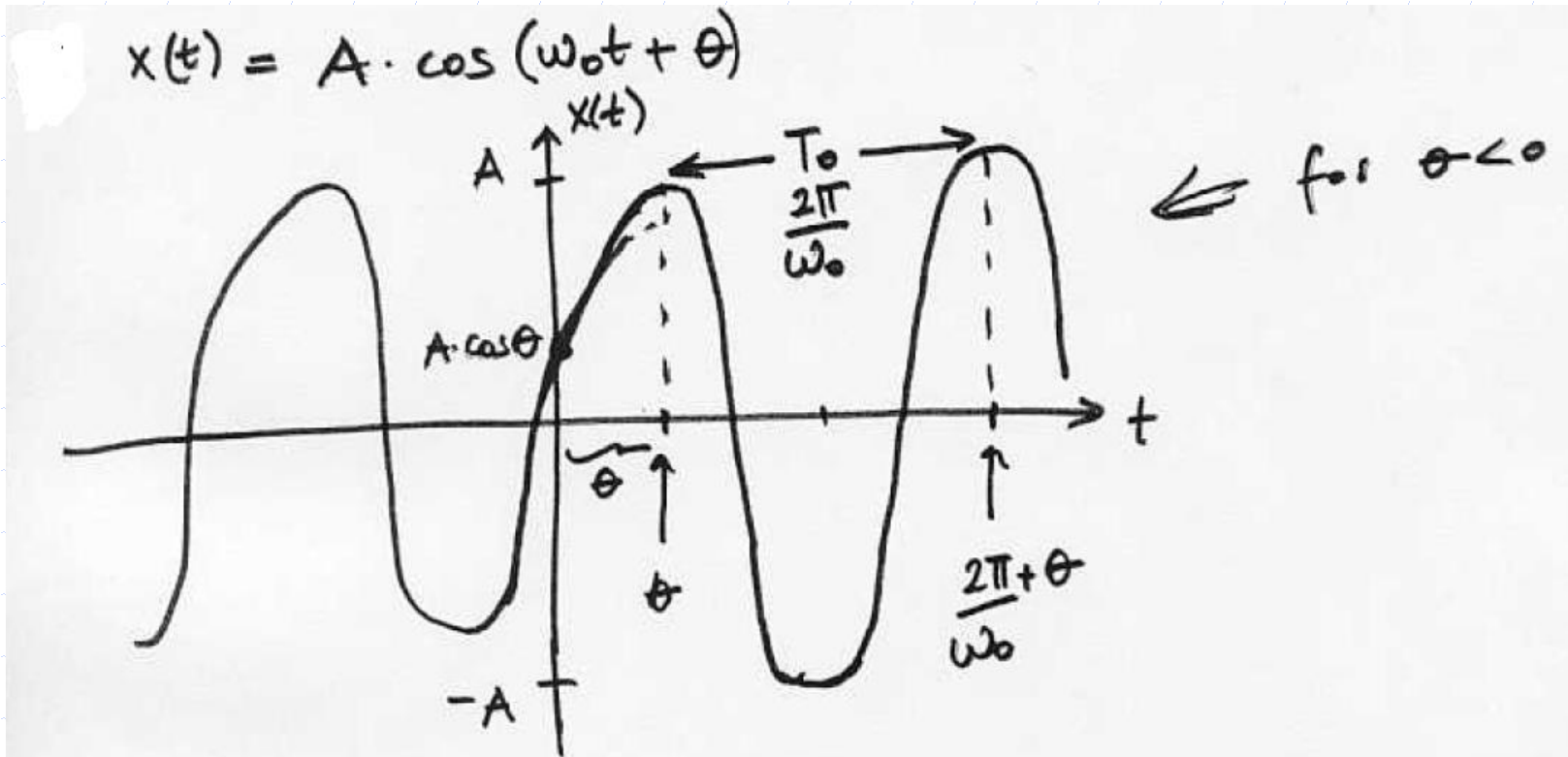
Real Exponential Signals

- $x(t) = C e^{\sigma t}$



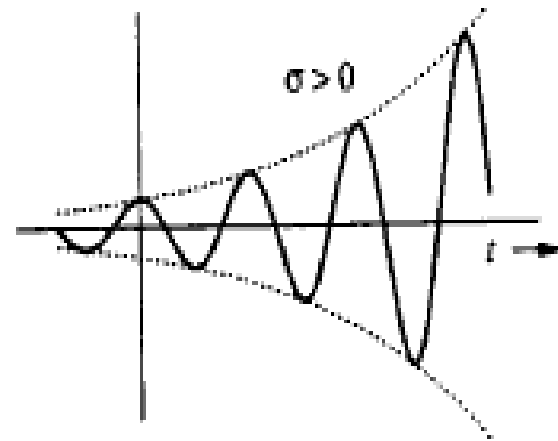
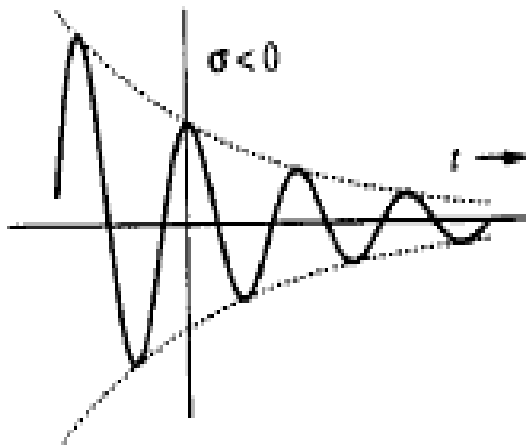
Sinusoidal Signals

- $x(t) = A \cos(\omega_0 t + \theta)$



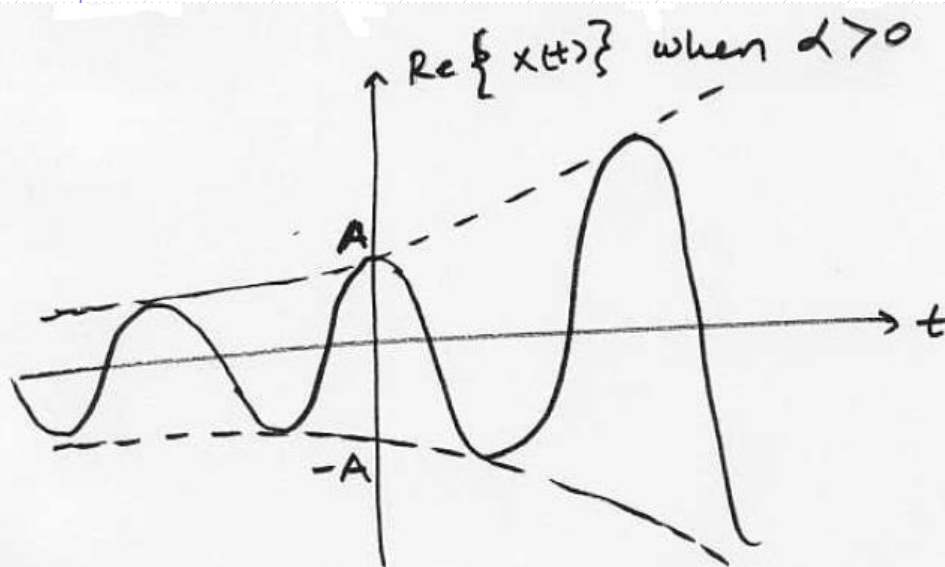
Complex Exponential Signals

■ $x(t) = Ae^{(j\omega + \sigma)t}$

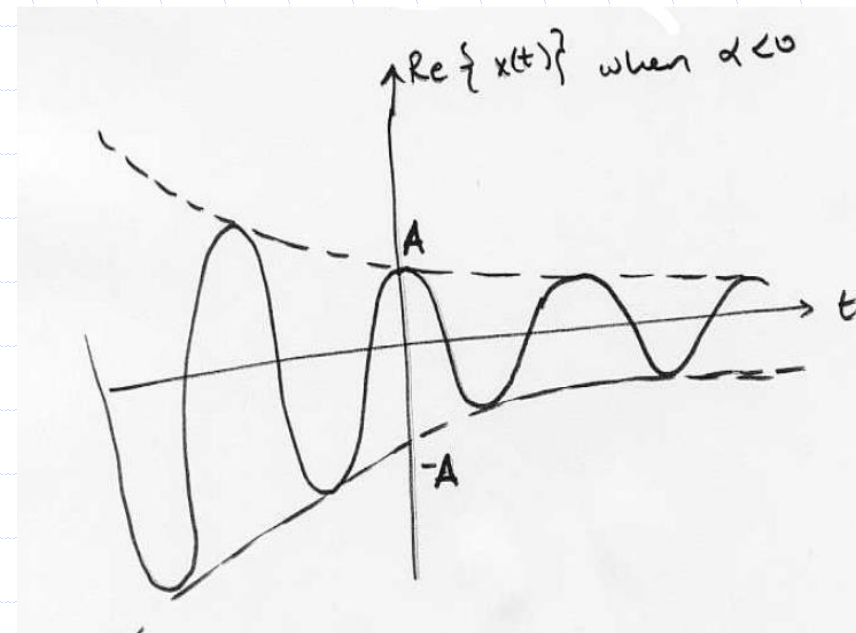


Complex Exponential Signals

$$\begin{aligned}x(t) &= A \cdot e^{(j\omega + \alpha)t} \\&= A \cdot e^{\alpha t} \cdot e^{j\omega t} \\&= A \cdot e^{\alpha t} (\cos(\omega t) + j \cdot \sin(\omega t))\end{aligned}$$



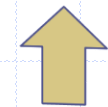
Basil Hamed



TO

1.5 Even and Odd Signals

- $x(t)$ is even, if $x(t)=x(-t)$ *Ex. Cosine*
- $x(t)$ is odd, if $x(-t)=-x(t)$ *Ex. Sine*
- Any signal $x(t)$ can be divided into two parts:
 - $Ev\{x(t)\} = (x(t)+x(-t))/2$
 - $Od\{x(t)\} = (x(t)-x(-t))/2$
 - $x(t)=1/2[x(t)+x(-t)]+1/2[x(t)-x(-t)]$



even



odd

1.5 Even and Odd Signals

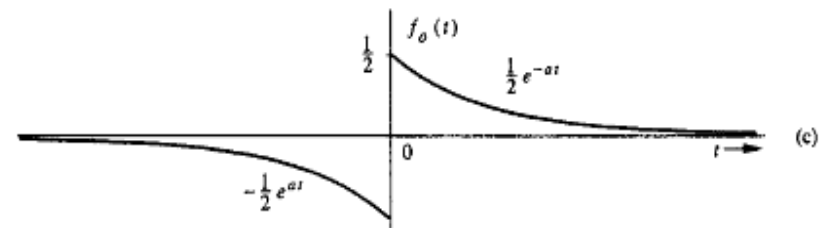
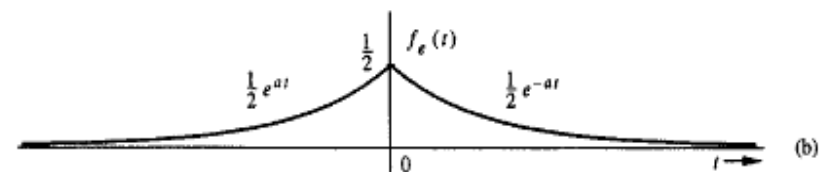
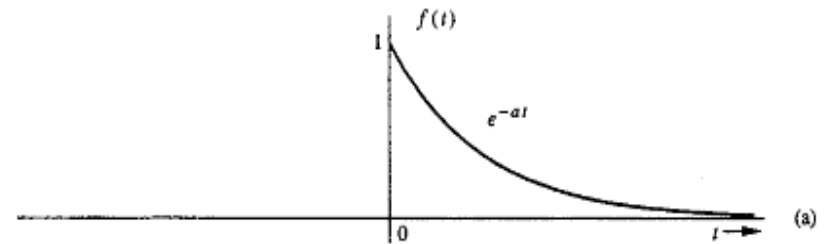
Consider the function $f(t) = e^{-at}u(t)$

Expressing this function as a sum of the even and odd components, we obtain;

$$f(t) = f_e(t) + f_o(t)$$

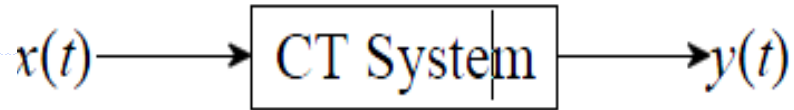
$$f_e(t) = \frac{1}{2} [e^{-at}u(t) + e^{at}u(-t)]$$

$$f_o(t) = \frac{1}{2} [e^{-at}u(t) - e^{at}u(-t)]$$



1.6 Systems

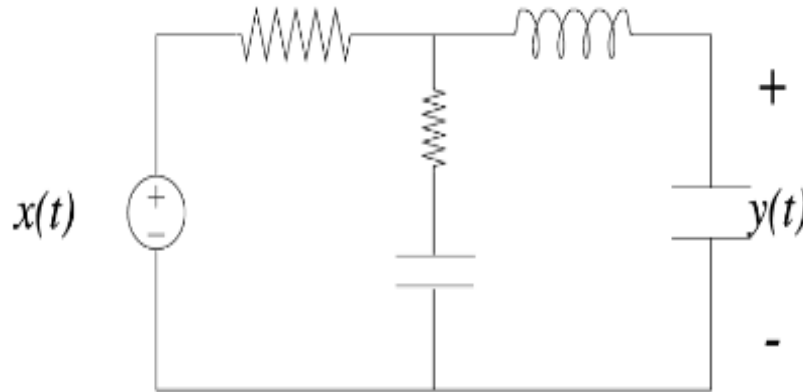
- System are used to process signals in order to modify or to extract additional information from the signal. A system may consists a physical components (hardware realization) or may consist of algorithmic that compute the output signal from the input signal (software realization).
- A system responds to applied input signals, and its response is described in terms of one or more output signals



- A system is characterized by its input, its output, and the rule of operations adequate to describe its behavior.

EXAMPLES OF SYSTEMS

- An RLC circuit



- Dynamics of an aircraft or space vehicle
- An algorithm for analyzing financial and economic factors to predict bond prices
- An algorithm for post-flight analysis of a space launch
- An edge detection algorithm for medical images

1.7 Classification of Systems

- Linear and Nonlinear
- Constant-parameter and Time-Varying parameter Systems
- Instantaneous (Memoryless) and Dynamic (with Memory)
- Casual and Noncasual Systems
- Lumped parameter and distributed parameter System
- Analog and Digital System

Linear and Nonlinear System

- Many systems are nonlinear. For example: many circuit elements (e.g., diodes), dynamics of aircraft, econometric models,...
- However, in this class we focus exclusively on **linear** systems.
- Why?
 - ❖ Linear models represent accurate representations of behavior of many systems (e.g., linear resistors, capacitors, other examples,...)
 - ❖ Can often linearize models to examine “small signal” perturbations around “operating points”
 - ❖ Linear systems are analytically tractable, providing basis for important tools and considerable insight

Linear and Nonlinear System

- A system is linear if it satisfies the properties:
 - It is *additivity*: $x(t) = x_1(t) + x_2(t) \Rightarrow y(t) = y_1(t) + y_2(t)$
 - And it is *homogeneity* (or *scaling*): $x(t) = a x_1(t) \Rightarrow y(t) = a y_1(t)$, for a any complex constant.
- The two properties can be combined into a single property:
 - **Superposition:**
 $x(t) = a x_1(t) + b x_2(t) \Rightarrow y(t) = a y_1(t) + b y_2(t)$

Linear and Nonlinear System

- **Linear system**

Possesses the property of **superposition**:

If the input consists of the weighted sum of several signals, then the output is the weighted sum of the responses of the system to each of those signals.

1. Additivity property

Response to $x_1(t)$ is $y_1(t)$

Response to $x_2(t)$ is $y_2(t)$

Response to the signal $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$.

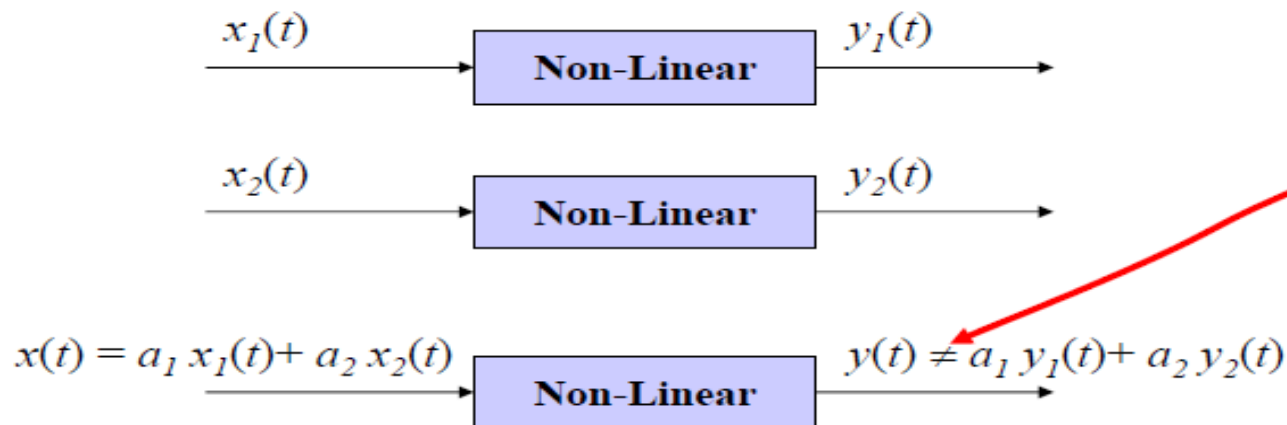
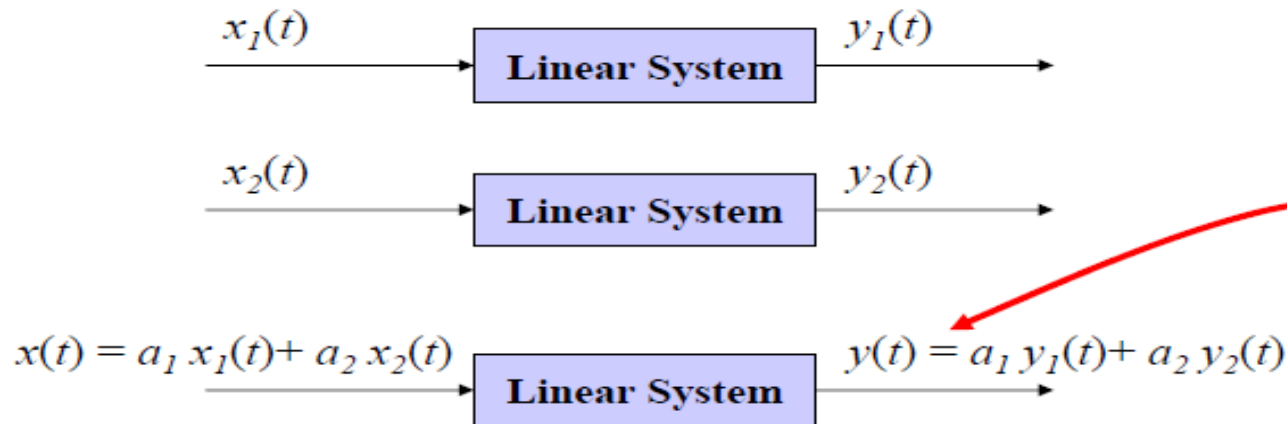
2. Homogeneity property

Response to $x_1(t)$ is $y_1(t)$

Response to the signal $\alpha x_1(t)$ is $\alpha y_1(t)$.

Linear and Nonlinear System

Linearity: A system is linear if superposition holds:



Linear and Nonlinear System

**When superposition holds, it makes our life easier!
We then can decompose complicated signals into a sum of simpler signals...and then find out how each of these simple signals goes through the system!!**

Systems with only R , L , and C are linear systems! Systems with electronics (diodes, transistors, op-amps, etc.) maybe non-linear, but they could be linear...at least for inputs that do not exceed a certain range of inputs.

Linear and Nonlinear System

Examples

- **Transcendental system** $y(t) = \cos(x(t))$

Answer: Nonlinear (in fact, fails both tests)

- **Squarer** $y(t) = x^2(t)$

Answer: Nonlinear (in fact, fails both tests)

- **Differentiation is linear**

$$y(t) = \frac{d}{dt} x(t) \quad \xrightarrow{x(t)} \quad \boxed{\frac{d}{dt} (\bullet)} \quad \xrightarrow{y(t)}$$

– Homogeneity test:

$$\frac{d}{dt} (a x(t)) = a \frac{d}{dt} x(t)$$

– Additivity test:

$$\frac{d}{dt} (x_1(t) + x_2(t)) = \frac{d}{dt} x_1(t) + \frac{d}{dt} x_2(t)$$

Linear and Nonlinear System

Is the following system linear

$$y(t) = x^2(t)$$

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

$$x(t) = ax_1(t) + bx_2(t) \stackrel{?}{\Rightarrow} y(t) = ay_1(t) + by_2(t)$$

$$\begin{aligned} y(t) &= (ax_1(t) + bx_2(t))^2 \\ &= a^2 x_1^2(t) + b^2 x_2^2(t) + 2ab x_1(t) x_2(t) \end{aligned}$$

$$\begin{aligned} &= a^2 y_1(t) + b^2 y_2(t) + 2ab x_1(t) x_2(t) \\ &\neq ay_1(t) + by_2(t) \end{aligned}$$

So the superposition does not hold system is nonlinear

Linear and Nonlinear System

Ex. 1.9 P 103 Show the system described by the eq. is linear

$$\frac{dy}{dt} + 3y(t) = f(t)$$

Solution:

Let $\frac{dy_1}{dt} + 3y_1(t) = f_1(t)$

$$\frac{dy_2}{dt} + 3y_2(t) = f_2(t)$$

$$\frac{d}{dt} [k_1 y_1(t) + k_2 y_2(t)] + 3 [k_1 y_1(t) + k_2 y_2(t)] = k_1 f_1(t) + k_2 f_2(t)$$

$$f(t) = k_1 f_1(t) + k_2 f_2(t)$$

$$y(t) = k_1 y_1(t) + k_2 y_2(t)$$

System is linear because superposition holds

Time-Invariant and Time varying

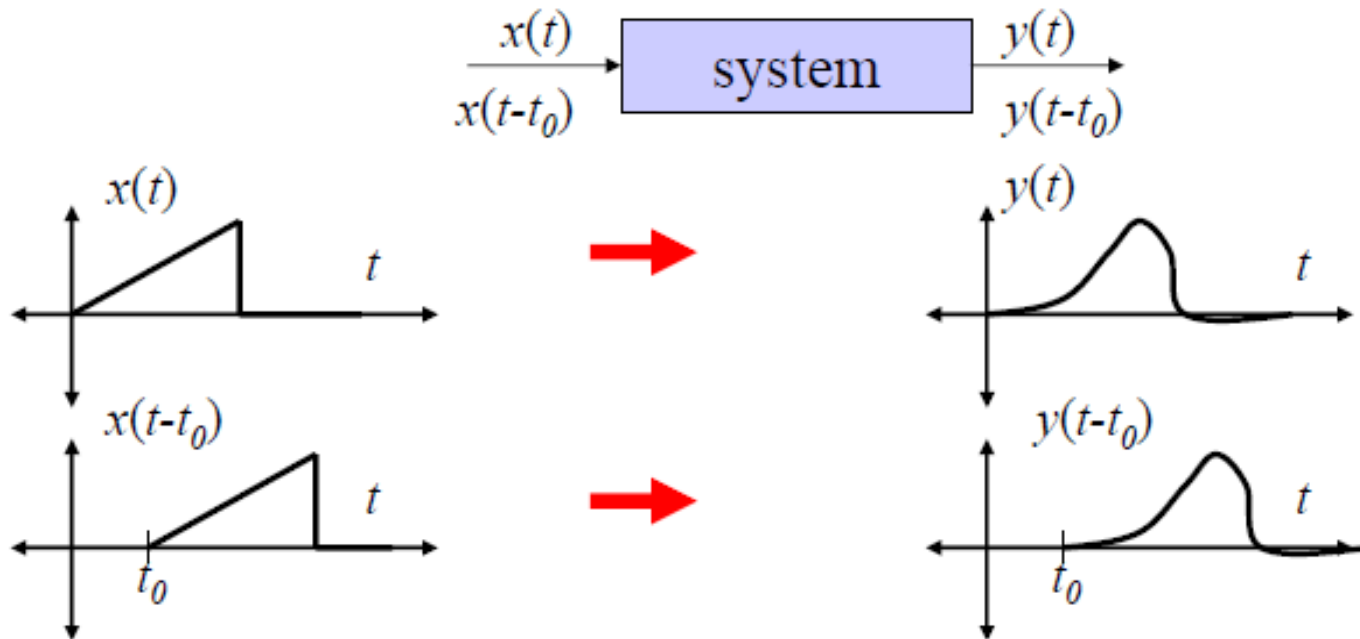
Time-Invariance

Physical View: The system itself does not change with time

Ex. A circuit with fixed R , L , C is time invariant. Actually, R, L, C values change slightly over time due to temperature & aging effects. A circuit with, say, a variable R is time variant

Time-Invariant and Time varying

Technical View: A system is time invariant (TI) if:



Time-Invariant and Time varying

Examples

- Identity system $y(t) = x(t)$
 - Step 1: compute $y_{\text{shifted}}(t) = x(t - t_0)$
 - Step 2: does $y_{\text{shifted}}(t) = y(t - t_0)$? YES.

Answer: Time-invariant

$$y(t) = \cos(x(t))$$

- Transcendental system

Answer: Time-invariant

$$y(t) = x^2(t)$$

- Squarer

Answer: Time-invariant

$$y(t) = \frac{d}{dt} x(t)$$

- Differentiator

Answer: Time-invariant

$$\frac{d}{dt} x(t - \tau) = y(t - \tau)$$

Memory vs. Memoryless Systems

- *Memoryless* (or *static*) Systems: System output $y(t)$ depends only on the input at time t , i.e. $y(t)$ is a function of $x(t)$.
- *Memory* (or *dynamic*) Systems: System output $y(t)$ depends on input at past or future of the current time t , i.e. $y(t)$ is a function of $x(\tau)$ where $-\infty < \tau < \infty$.
- Examples:
 - A resistor: $y(t) = R x(t)$
 - A capacitor: $y(t) = \frac{1}{C} \int_{-\infty}^t x(\tau) d\tau$

Memory vs. Memoryless Systems

$y(t) = \sigma x(t)$  Memoryless

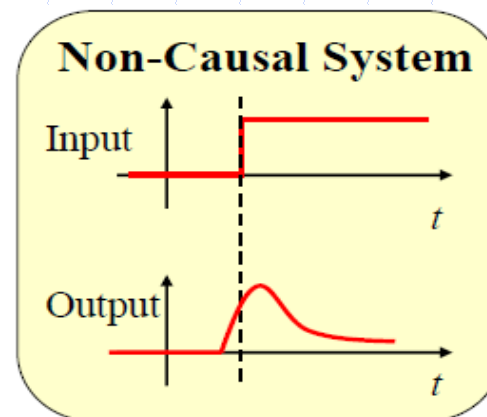
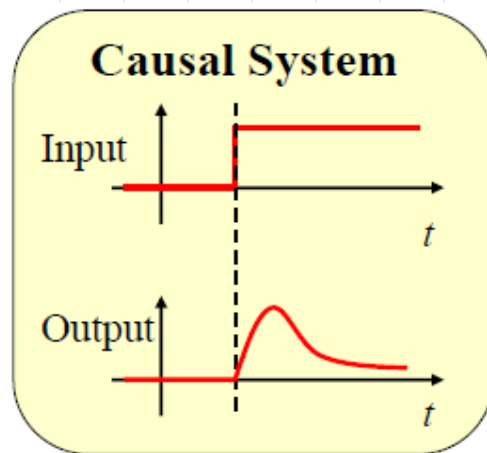
$y(t) = x(t) + x(t-1)$  Memory

Causal and Non Causal System

Causality: A causal (or non-anticipatory) system's output at a time t_1 does not depend on values of the input $x(t)$ for $t > t_1$

The “future input” cannot impact the “now output”

A Causal system (with zero initial conditions) cannot have a non-zero output until a non-zero input is applied.

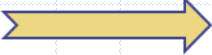


Causal and Non Causal System

Most systems in nature are causal

All real-time physical systems are causal, because time only moves forward. Effect occurs after cause. (Imagine if you own a noncausal system whose output depends on tomorrow's stock price.)

But...we need to understand non-causal systems because theory shows that the “best” systems are non-causal! So we need to find causal systems that are as close to the best non-causal systems!!!

$y(t) = \alpha x(t) + x(t-1)$		Causal
$y(t) = x(t) + x(t+1)$		NonCausal

Causal and Non Causal System

Example

$$y(t) = x^2(t - 1)$$

Causal

$$y(t) = x(t + 1)$$

NonCausal

Examples

Determine if the following systems are TI, Linear, Causal and/or memoryless.

a. $\frac{dy(t)}{dt} + 6y(t) = 4x(t)$

b. $\frac{dy(t)}{dt} + 4ty(t) = 2x(t)$

c. $y(t) = \sin(x(t))$

d. $\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} + 4x(t)$

Examples

- a. This is an ordinary differential equation with constant coefficients, therefore, it is linear and time-invariant. It contains memory and it is causal.
- b. This is an ordinary differential equation. The coefficients of $4t$ and 2 do not depend on y or x , so the system is linear. However, the coefficient $4t$ is not constant, so it is time-varying. The system is also causal and has memory
- c. check linearity:
 $y_1(t) = \sin(x_1(t))$
 $y_2(t) = \sin(x_2(t))$
Solution to an input of $a_1 x_1(t) + a_2 x_2(t)$ is $\sin(a_1 x_1(t) + a_2 x_2(t))$.
This is not equal to $a_1 y_1(t) + a_2 y_2(t)$.

Examples

- c. the system is causal since the output does not depend on future values of time, and it is memoryless the system is time-invariant
- d. This is an ordinary differential equation with constant coefficients, so it is linear and time-invariant. It is also causal and has memory.