

Signal & Linear system

Chapter 2 Time Domain Analysis of CT System

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2.1 Introduction

Systems described by Linear, Constant-Coefficient Differential Equations are Continuous-Time, Linear Time-Invariant (LTI) Systems

Differential equations like this are LTI

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + \dots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

- coefficients (a 's & b 's) are constants $\Rightarrow$ TI
- No nonlinear terms $\Rightarrow$ Linear

2.1 Introduction

Many physical systems are modeled w/ **Differential Eqs**—
Because physics shows that electrical (& mechanical!) components often have “V-I Rules” that depend on derivatives

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

Given: Input $x(t)$
Find: Output $y(t)$

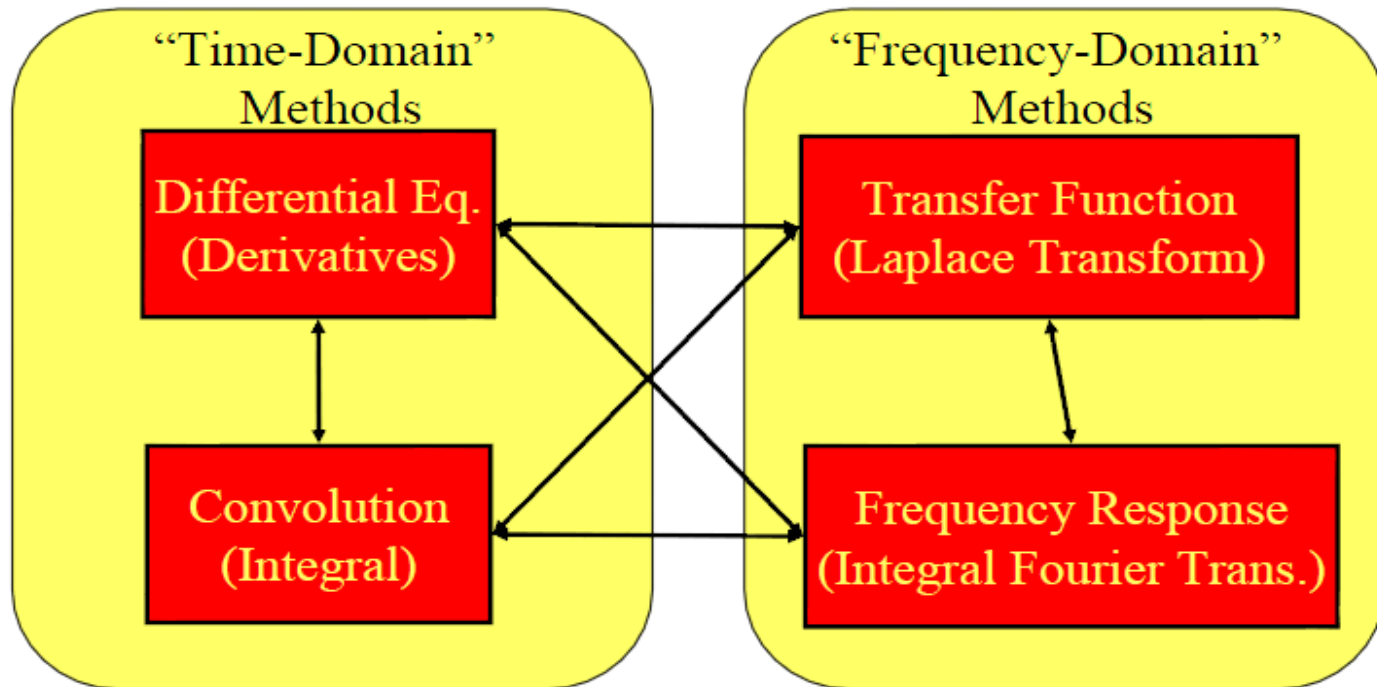
This is what it means to “solve” a differential equation!!

However, engineers use **Other Math Models** to help solve and analyze differential eqs—The concept of “**Frequency Response**” and the related concept of “**Transfer Function**” are the most widely used such math models>“**Fourier Transform**” is the math tool underlying Frequency Response—Another helpful math model is called “**Convolution**”

2.1 Introduction

Relationships Between System Models

- These 4 models all are equivalent
- ...but one or another may be easier to apply to a given problem



2.1 Introduction

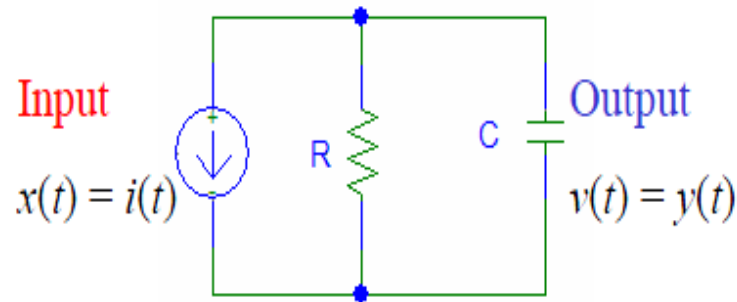
Example System: RC Circuit

System You've seen in Circuits Class that R , L , C circuits are modeled by Differential Equations:

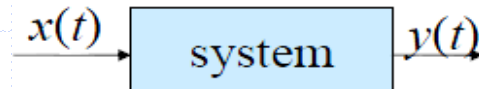
- From Physical Circuit...get schematic
- From Schematic write circuit equations...get Differential Equation
- Solve Differential Equation for specific input...get specific output

2.1 Introduction

“Schematic View”:



“System View”:



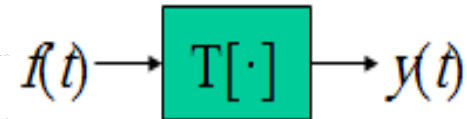
Circuits class showed how to model this physical system mathematically:

$$\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{C} x(t)$$

Given input $x(t)$, the output $y(t)$ is the solution to the differential equation.

2.2 Continuous-Time Domain Analysis

For a linear system,



$$\text{Total response} = \underbrace{\text{zero - input response}}_{\substack{\text{when } f(t)=0 \\ \text{results from internal system conditions only} \\ \text{independent of } f(t)}} + \underbrace{\text{zero - state response}}_{\substack{\text{response to non-zero } f(t) \\ \text{all initial conditions are zero} \\ \text{dependent on } f(t)}}$$

The two components are independent of each other. Each component can be computed independently of the other.

2.2 Continuous-Time Domain Analysis

- **Zero-input response**

- Response when $f(t) = 0$
- Results from internal system conditions only
- Independent of $f(t)$
- For most filtering applications (e.g. your stereo system), we want no zero-input response.

- **Zero-state response**

- Response to non-zero $f(t)$ when system is relaxed
- A system in zero state cannot generate any response for zero input.
- Zero state corresponds to initial conditions being zero.

2.2 Continuous-Time Domain Analysis

- Consider that the input “starts at $t = t_0$ ”: (i.e. $x(t) = 0$ for $t < t_0$)
- Let $y(t_0)$ be the output voltage when the input is first applied (initial condition)
- Then, the solution of the differential equation gives the output as:

$$y(t) = \underbrace{y(t_0)e^{-(1/RC)(t-t_0)}}_{\text{Part due to Initial Condition}} + \underbrace{\int_{t_0}^t \frac{1}{C} e^{-(1/RC)(t-\lambda)} x(\lambda) d\lambda}_{\text{Part due to Input}}$$

Part due to Initial Condition
 (“Zero Input Response”)

Part due to Input
 (“Zero State Response”)

Recall : This part is the solution to the “Homogeneous Differential Equation”

1. Set input $x(t) = 0$
2. Find characteristic polynomial (Here it is $\lambda + 1/RC$)
3. Find all roots of characteristic polynomial: λ_i (Here there is only one)
4. Form homogeneous solution from linear combination of the exp $\{\lambda_i (t-t_0)\}$
5. Find constants that satisfy the initial conditions (Here it is $y(t_0)$)

2.2 Continuous-Time Domain Analysis

In this course we focus on finding the zero-state response (I.C.'s = 0)

$$y_{zs}(t) = \int_{t_0}^t \underbrace{\frac{1}{C} e^{-\frac{1}{RC}(t-\lambda)}}_{\triangleq h(t-\lambda)} x(\lambda) d\lambda$$



$$y_{zs}(t) = \int_{t_0}^t h(t-\lambda) x(\lambda) d\lambda$$

General form for so-called “linear, constant-coefficient differential equations”

Later will look at this general form...It's called “convolution”

2.2 Linear Constant-Coefficient Differential Equations

General Form: (N^{th} -order)

$$y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{i=0}^M b_i x^{(i)}(t)$$

Indicates
 i^{th} order
derivative

Input: $x(t)$

Output: $y(t)$

Solution of the Differential Equation

Recall: Two parts to the solution

“Homogeneous Solution”

(i) one part due to ICs with zero-input (“zero-input response”)

(ii) one part due to input with zero ICs (“zero-state response”)

2.2 Linear Constant-Coefficient Differential Equations

Characteristic Polynomial: $\lambda^N + a_{n-1}\lambda^{N-1} + \dots + a_1\lambda + a_0$

N roots: $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_N$

N “modes”: Assuming distinct roots... $e^{\lambda_1 t}, e^{\lambda_2 t}, \dots, e^{\lambda_N t}$

$$\Rightarrow y_{ZI}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \dots + C_N e^{\lambda_N t}$$

Then: $y(t) = y_{ZI}(t) + y_{zs}(t)$

($y_{zs}(t)$ is our focus, so we will often say ICs = 0)

2.2 Linear Constant-Coefficient Differential Equations

So how do we find $y_{zs}(t)$?

If you examine the zero-state part for all the example solutions of differential equations we have seen you'll see that they all look like this:

$$y_{zs}(t) = \int_{t_0}^t h(t-\lambda)x(\lambda)d\lambda$$

Input

Output when
"in zero state"

This is called "**Convolution**"

So we need to find out:

Given a differential equation, what is $h(t-\lambda)$

Really just need to know $h(t)$... it is called the system's "Impulse Response"

2.2 Linear Constant-Coefficient Differential Equations

Ex. Find $y_0(t)$ the zero-input component of the response, for a LTI system described by the following differential equation:

$$(D^2 + 3D + 2)y(t) = Dx(t)$$

when the initial conditions are

$$y_0(0) = 0, \dot{y}_0(0) = -5.$$

For zero-input response, we want to find the solution to:

- The **characteristic equation** for this system is therefore:

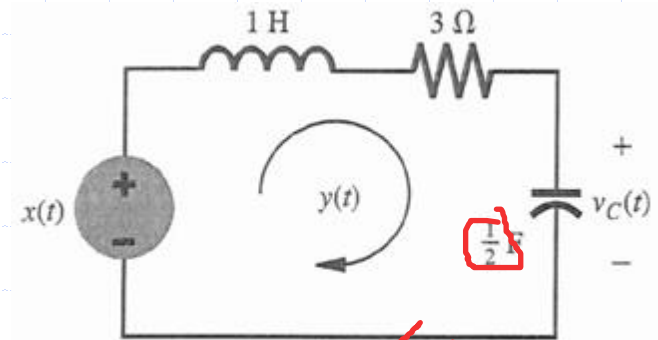
$$(D^2 + 3D + 2)y_0(t) = 0$$

- The **characteristic roots** are therefore. $\lambda_1 = -1$ $\lambda_2 = -2$.

$$\lambda^2 + 3\lambda + 2 = (\lambda + 1)(\lambda + 2) = 0$$

- The **zero-input response** is

$$y_0(t) = c_1 e^{-t} + c_2 e^{-2t}$$



0,29

2.2 Linear Constant-Coefficient Differential Equations

To find the two unknowns c_1 and c_2 , we use the **initial conditions**

$$y_0(0) = 0, \quad \dot{y}_0(0) = -5.$$

فصل

- This yields to two simultaneous equations:

$$0 = c_1 + c_2$$

$$-5 = -c_1 - 2c_2$$

معطيني معادلتين وحدة مشتقة والتانية عادية

معاني

- Solving this gives:

$$c_1 = -5 \quad c_2 = 5$$

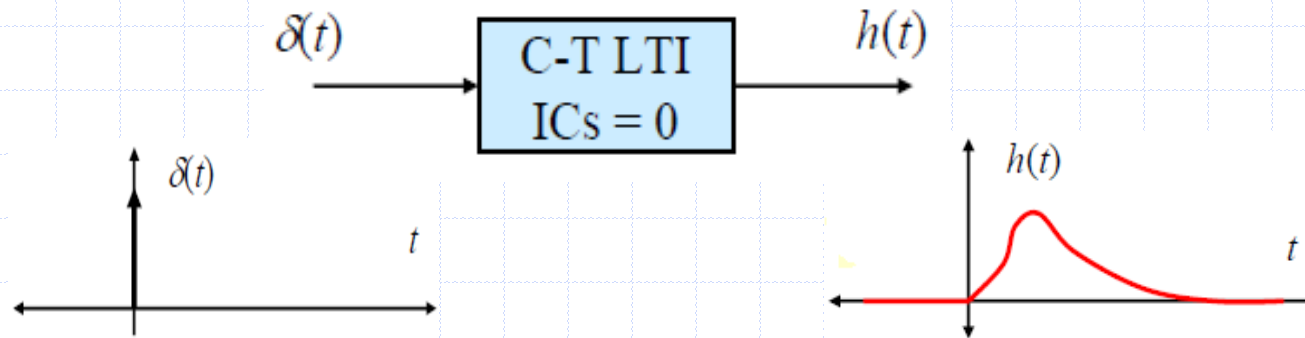
- Therefore, the zero-input response of $y(t)$ is given by:

$$y_0(t) = -5e^{-t} + 5e^{-2t}$$

2.3 Unit Impulse response $h(t)$

Impulse response of a system is response of the system to an input that is a unit impulse

Impulse Response: $h(t)$ is what “comes out” when $\delta(t)$ “goes in”

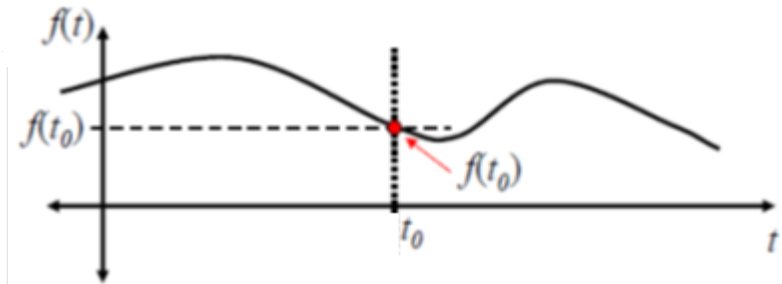


Use $h(t)$ to find the zero-state response of the system for an input

2.4 System Response to external Input: Zero-State Response

The Sifting Property of $\delta(t)$:

$$\int_{t_0 - \varepsilon}^{t_0 + \varepsilon} f(t) \delta(t - t_0) dt = f(t_0) \quad \forall \varepsilon > 0$$

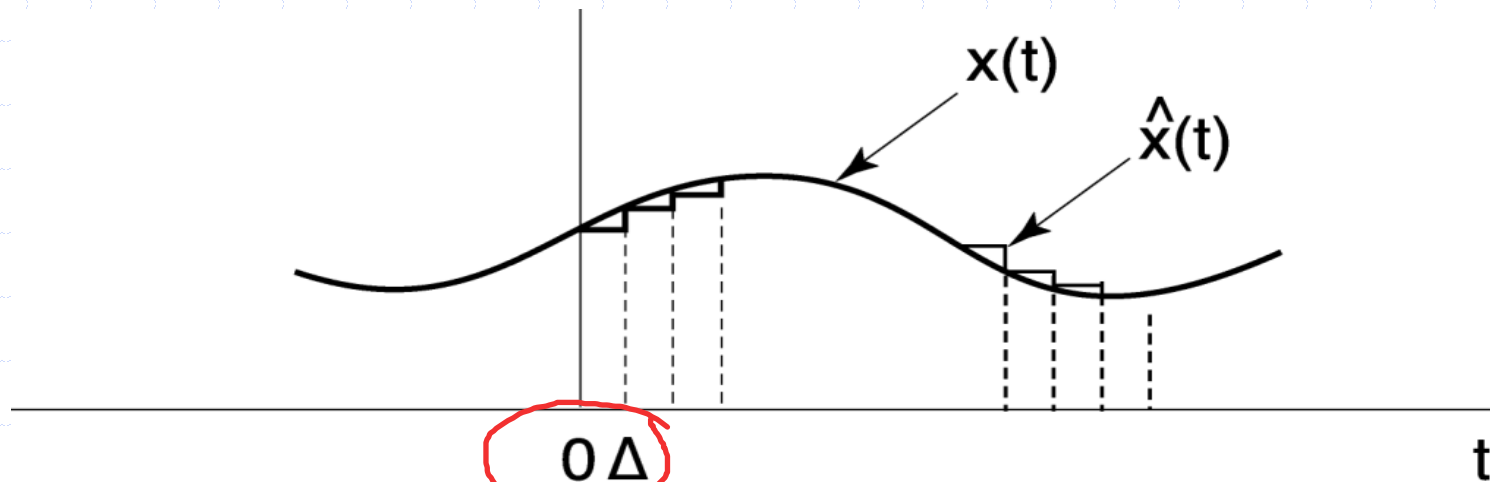


Above eq. shows that the impulse function sifts out everything except the value of $x(t)$ at $t=t_0$

just at t_0

2.4 System Response to external Input: Zero-State Response

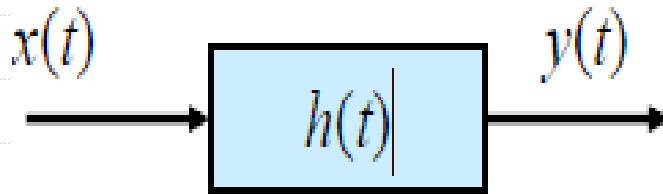
Approximate any input $x(t)$ as a sum of shifted, scaled pulses



Above figure shows that any signal $x(t)$ can be expressed as a continuum of weighted impulses

$f(t-\tau), \dots$

2.4-1 The Convolution Integral



C-T LTI
ICs = 0

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

CONVOLUTION

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

Properties of the Convolution:

1. Commutative: $x(t) * h(t) = h(t) * x(t)$

2. Associative: $x(t) * h_1(t) * h_2(t) = [x(t) * h_1(t)] * h_2(t)$
 $[x(t) * h_2(t)] * h_1(t)$

3. Distributive:

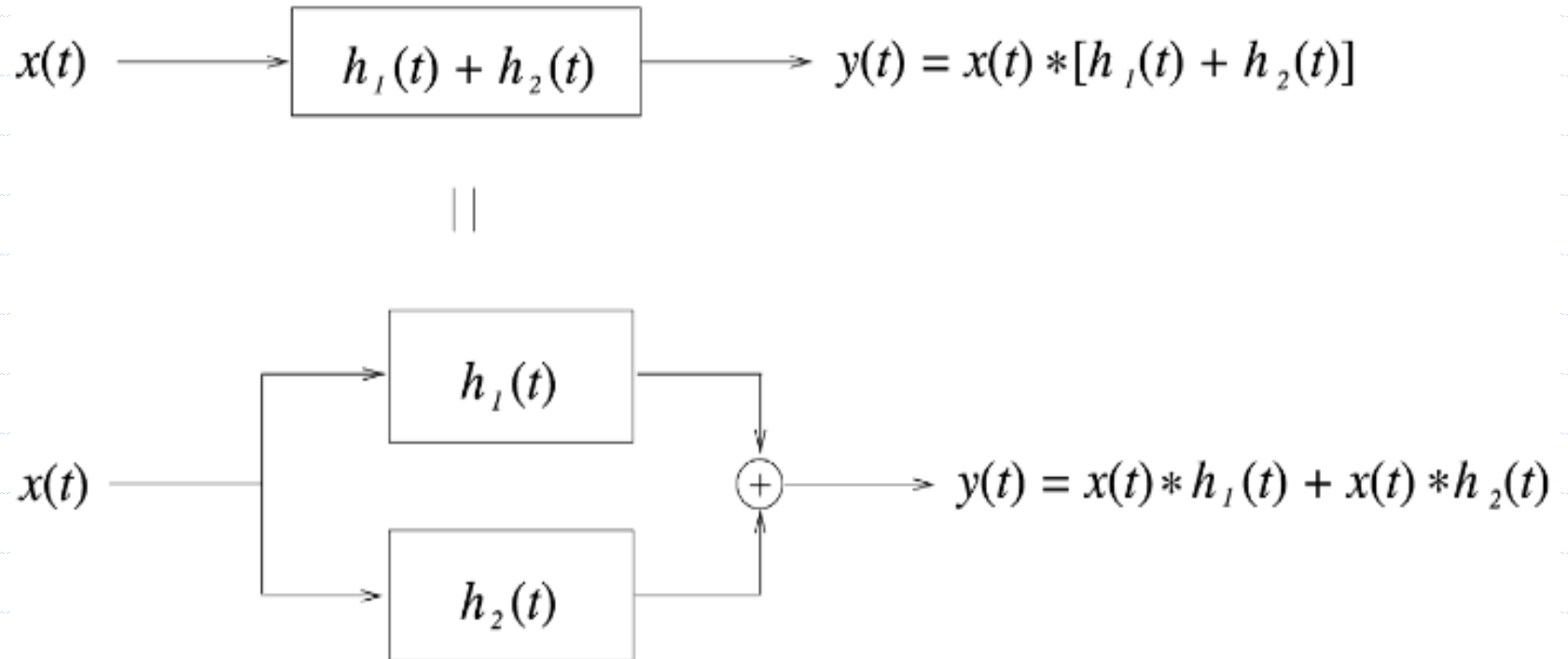
$$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$$

4. Convolution with an impulse: $x(t) * \delta(t) = x(t)$

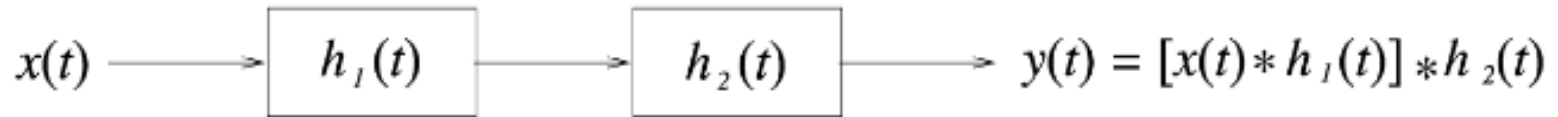
5. Convolution with Unit step:

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau) u(t - \tau) d\tau = \int_{-\infty}^t x(\tau) d\tau$$

DISTRIBUTIVITY



Associative

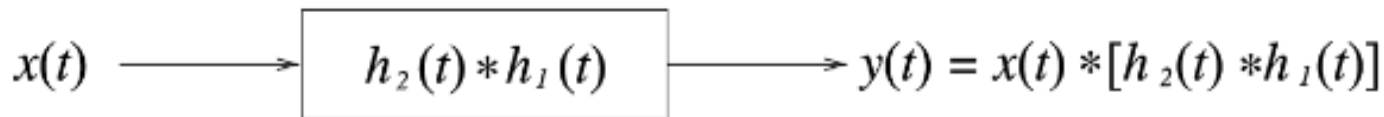


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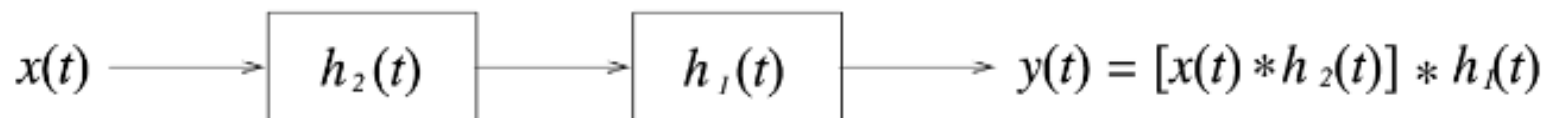


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← Commutativity



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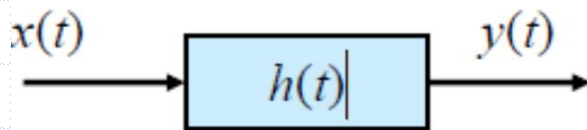


Examples of Convolution

Find $u(t) * u(t)$

$$y(t) = u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau) u(t - \tau) d\tau = \int_{-\infty}^t u(\tau) d(\tau) = r(t)$$

Ex 2. P 175



Find $y(t)$

$$x(t) = e^{-t}u(t)$$

$$h(t) = e^{-2t}u(t)$$

Examples of Convolution

$$y(t) = x(t) * h(t) =$$

$$\int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-2(t-\tau)} u(t - \tau) d\tau$$

Note that

$$\begin{aligned} u(\tau) u(t - \tau) &= 1 & 0 < \tau < t \\ &= 0 & \text{Otherwise} \end{aligned}$$

$$y(t) = \int_0^t e^{-\tau} e^{-2(t-\tau)} d\tau = e^{-2t} \int_0^t e^{\tau} d\tau = e^{-2t} [e^t - 1]$$

$$y(t) = (e^{-t} - e^{-2t}) u(t)$$

Examples of Convolution

Ex. Use direct integration, find the expression for

$$y(t) = e^{-at}u(t) * e^{-bt}u(t)$$

Solution:

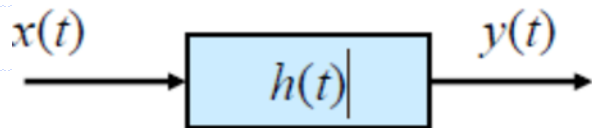
$$\begin{aligned} & e^{-at}u(t) * e^{-bt}u(t) \\ &= \int_0^t e^{-a\tau} e^{-b(t-\tau)} d\tau = e^{-bt} \int_0^t e^{(b-a)\tau} d\tau \\ &= \frac{e^{-bt}}{b-a} e^{(b-a)\tau} \Big|_0^t = \frac{e^{-bt}}{b-a} [e^{(b-a)t} - 1] \\ &= \frac{e^{-at} - e^{-bt}}{a-b} \end{aligned}$$

Because both functions are casual, their convolution is zero for $t < 0$

$$y(t) = e^{-at}u(t) * e^{-bt}u(t) = \left(\frac{e^{-at} - e^{-bt}}{a-b} \right) u(t)$$

Examples of Convolution

Ex 2.6 P 178



$$x(t) = 10e^{-3t}u(t)$$

$$h(t) = [2e^{-2t} - e^{-t}]u(t)$$

Find $y(t)$

Solution:

$$y(t) = x(t) * h(t)$$

$$= [10e^{-3t}u(t) * 2e^{-2t}u(t)] - [10e^{-3t}u(t) * e^{-t}u(t)]$$

$$y(t) = \int_{-\infty}^{\infty} 10e^{-3\tau}u(\tau)2e^{-2(t-\tau)}u(t-\tau)d\tau -$$
$$\int_{-\infty}^{\infty} 10e^{-\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau$$

Note that $u(\tau)u(t-\tau) = 1 \quad 0 < \tau < t$
 $= 0 \quad \text{Otherwise}$

Examples of Convolution

$$y(t) = \int_0^t 20e^{-3\tau} e^{-2(t-\tau)} d\tau - \int_0^t 10e^{-3\tau} e^{-(t-\tau)} d\tau$$

$$y(t) = [20e^{-2t} - 15e^{-3t} - 5e^{-t}]u(t)$$

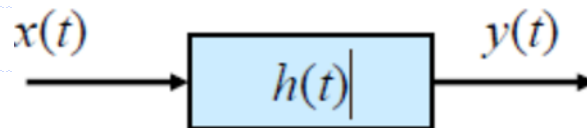
Examples of Convolution

Ex. A first-order lowpass filter impulse response is given by

$$h(t) = -\delta(t) + 2e^{-t}u(t)$$

Find the zero-state response of this filter for the input $e^t u(-t)$

Solution:



$$h(t) = -\delta(t) + 2e^{-t}u(t)$$

$$x(t) = e^t u(-t)$$

$$y(t) = h(t) * x(t)$$

$$= [-\delta(t) + 2e^{-t}u(t)] * e^t u(-t)$$

$$= -\delta(t) * e^t u(-t) + 2e^{-t}u(t) * e^t u(-t)$$

Examples of Convolution

From convolution table on Page 177

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
13	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} u(t) + e^{\lambda_2 t} u(-t)}{\lambda_2 - \lambda_1} \quad \text{Re } \lambda_2 > \text{Re } \lambda_1$

$$2e^{-t}u(t) * e^t u(-t) = 2 \left[\frac{e^{-t}u(t) + e^t u(-t)}{2} \right]$$

$$y(t) = -e^t u(-t) + e^{-t} u(t) + e^t u(-t) = e^{-t} u(t)$$

Graphical Convolution Operation

Graphical convolution allows us to grasp visually or mentally the convolution integral's result. Many signals have no exact mathematical description, so they can be described only graphically.

We'll learn how to perform "Graphical Convolution," which is nothing more than steps that help you use graphical insight to evaluate the convolution integral.

Steps for Graphical Convolution $x(t)*h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

1. Re-Write the signals as functions of τ : $x(\tau)$ and $h(\tau)$
2. Flip just one of the signals around $t = 0$ to get either $x(-\tau)$ or $h(-\tau)$
 - a. It is usually best to flip the signal with shorter duration
 - b. For notational purposes here: we'll flip $h(\tau)$ to get $h(-\tau)$
3. Find Edges of the flipped signal
 - a. Find the left-hand-edge τ -value of $h(-\tau)$
 - b. Find the right-hand-edge τ -value of $h(-\tau)$
4. Shift $h(-\tau)$ by an arbitrary value of t to get $h(t - \tau)$ and get its edges
 - a. Find the left-hand-edge τ -value of $h(t - \tau)$ as a function of t
 - b. Find the right-hand-edge τ -value of $h(t - \tau)$ as a function of t

Steps for Graphical Convolution $x(t)*h(t)$

5. Find Regions of τ -Overlap

- a. What you are trying to do here is find intervals of t over which the product $x(\tau) h(t-\tau)$ has a single mathematical form in terms of τ
- b. In each region find: Interval of t that makes the identified overlap happen
- c. Working examples is the best way to learn how this is done

6. For Each Region: Form the Product $x(\tau) h(t-\tau)$ and Integrate

- a. Form product $x(\tau) h(t-\tau)$
- b. Find the Limits of Integration by finding the interval of τ over which the product is nonzero
- c. Integrate the product $x(\tau) h(t-\tau)$ over the limits found in b

7. "Assemble" the output from the output time-sections for all the regions

Graphically Convolve Two Signals

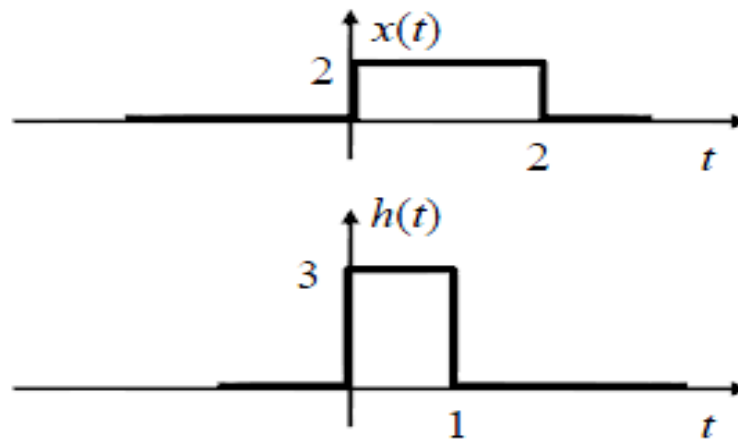
$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

By “Properties of Convolution”...
these two forms are
Equal

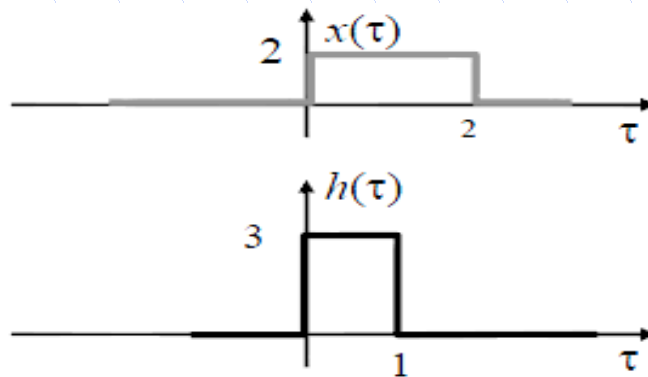
This is why we can
flip either signal

Ex. Convolve these two signals:

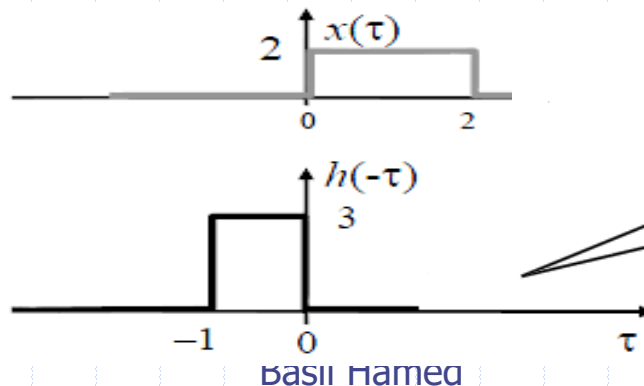


Graphically Convolve Two Signals

Step #1: Write as Function of τ Usually



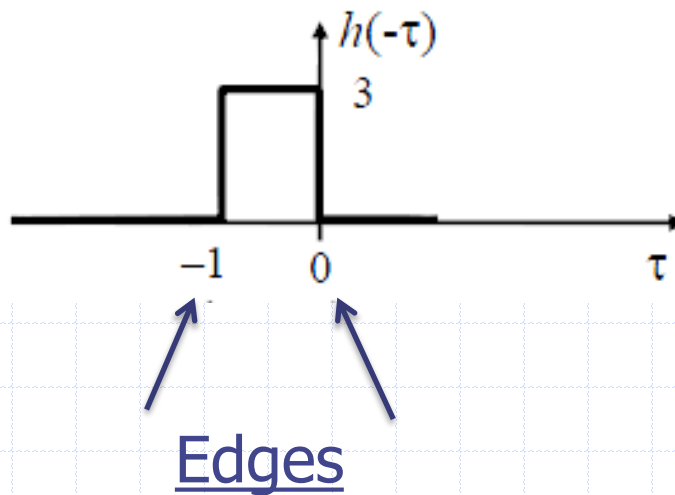
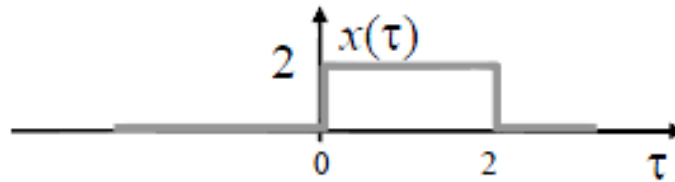
Step #2: Flip $h(\tau)$ to get $h(-\tau)$



Usually Easier
to Flip the
Shorter Signal

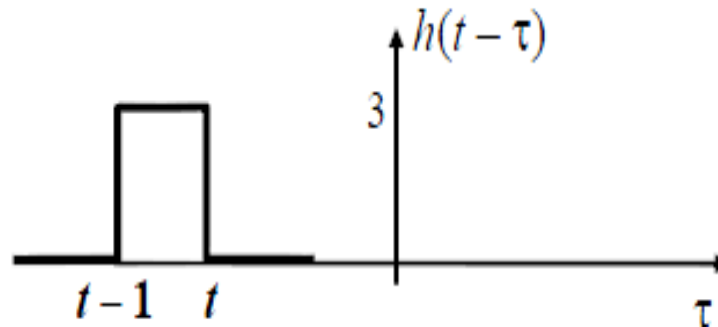
Graphically Convolve Two Signals

Step #3: Find Edges of Flipped Signal

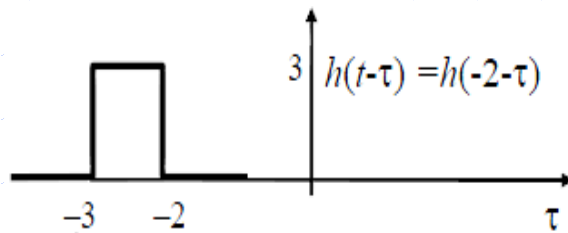


Graphically Convolve Two Signals

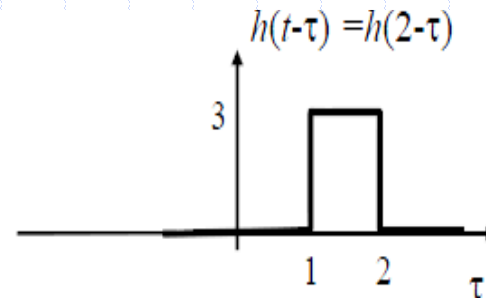
Step #4: Shift by t to get $h(t-\tau)$ & Its Edge



For $t = -2$

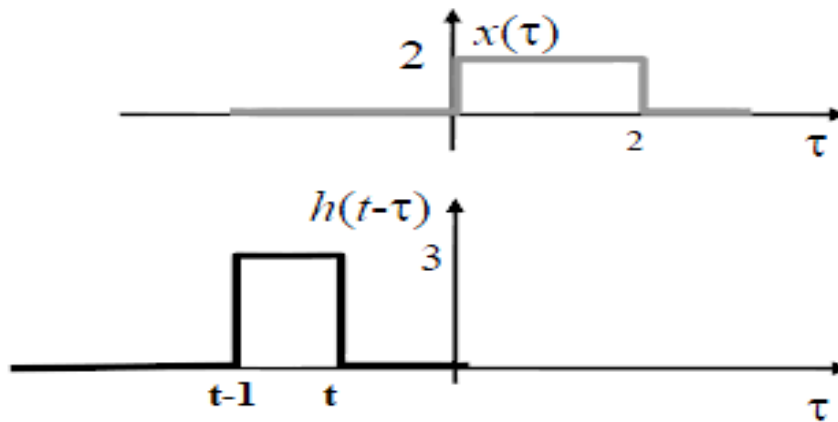


For $t = 2$



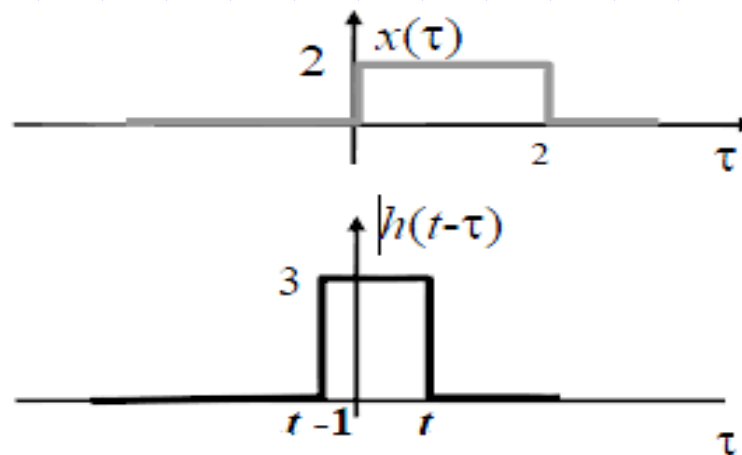
Graphically Convolve Two Signals

Step #5: Find Regions of τ -Overlap



Region I

No τ -Overlap
 $t < 0$

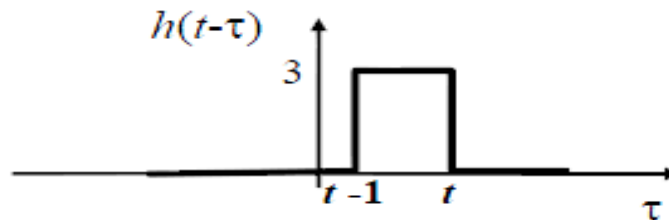
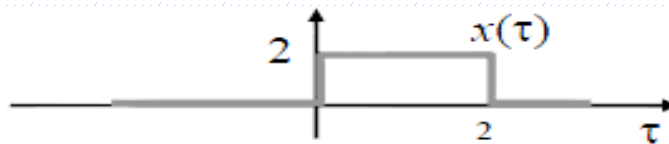


Region II

Partial τ -Overlap
 $0 \leq t \leq 1$

Graphically Convolve Two Signals

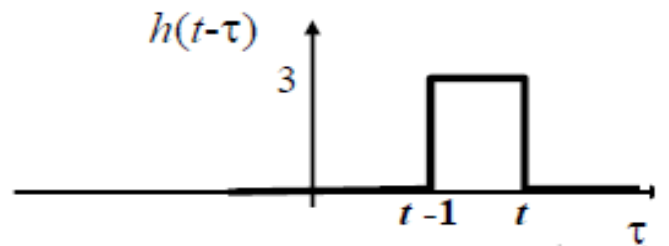
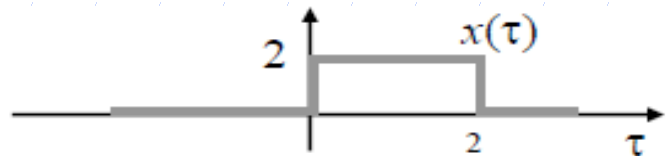
Step #5: Find Regions of τ -Overlap



Region III

Total τ -Overlap

$$1 < t \leq 2$$



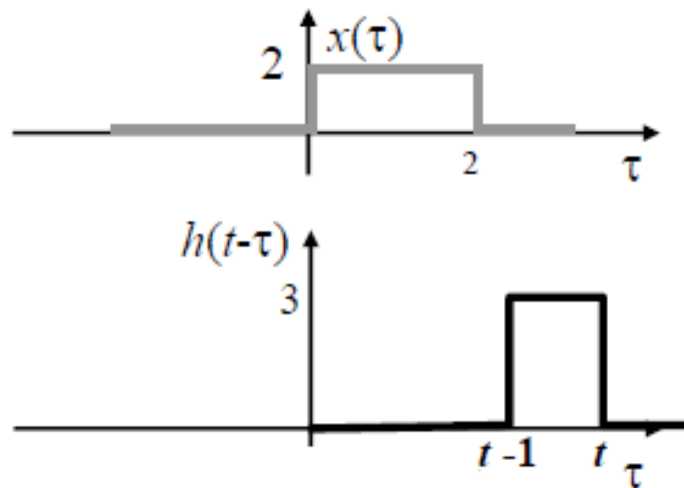
Region IV

Partial τ -Overlap

$$2 < t \leq 3$$

Graphically Convolve Two Signals

Step #5: Find Regions of τ -Overlap

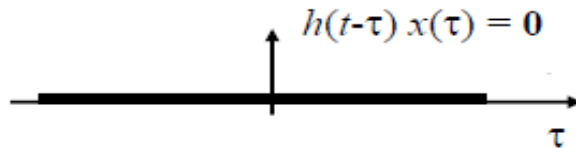
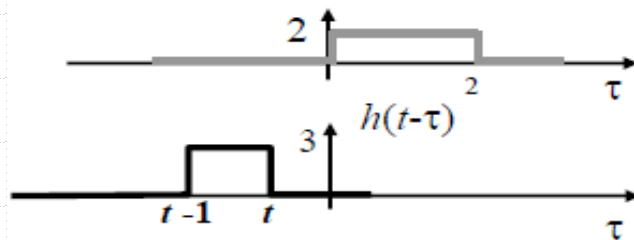


Region V

No τ -Overlap
 $t > 3$

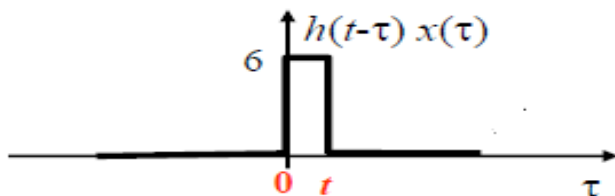
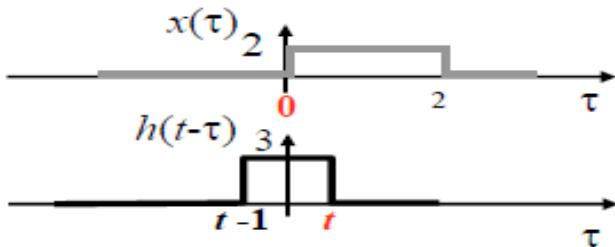
Graphically Convolve Two Signals

Step #6: Form Product & Integrate For Each Region



Region I: $t < 0$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} 0d\tau = 0 \\ y(t) &= 0 \quad \text{for all } t < 0 \end{aligned}$$

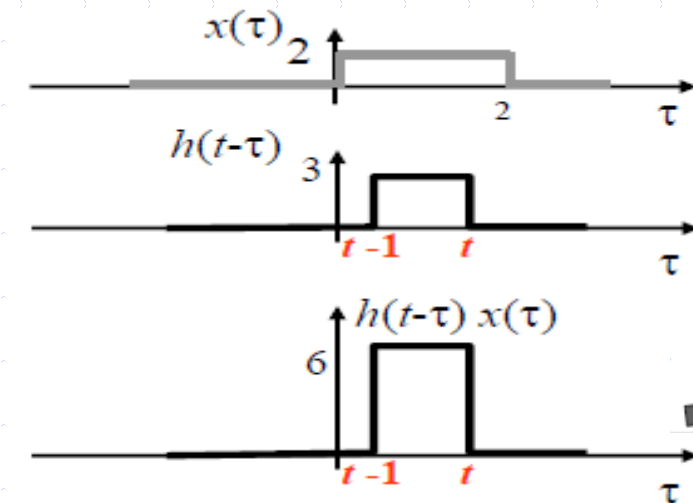


Region II: $0 \leq t \leq 1$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^t 6d\tau = [6\tau]_0^t = 6t - 6 \times 0 = 6t \\ y(t) &= 6t \quad \text{for } 0 \leq t \leq 1 \end{aligned}$$

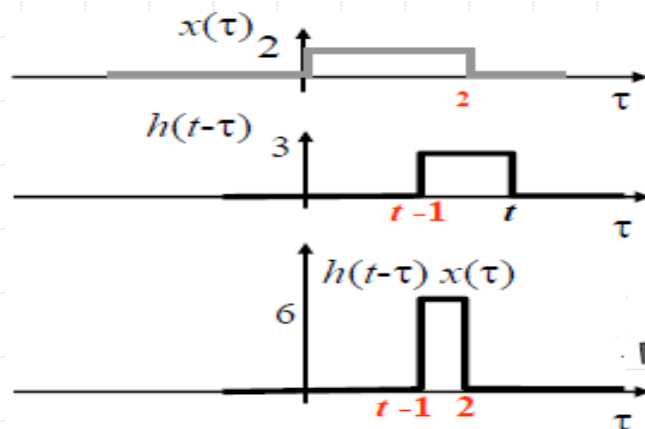
Graphically Convolve Two Signals

Step #6: Form Product & Integrate For Each Region



Region III: $1 < t \leq 2$

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{t-1}^t 6d\tau = [6\tau]_{t-1}^t = 6t - 6(t-1) = 6 \\
 y(t) &= 6 \quad \text{for all } t \text{ such that: } 1 < t \leq 2
 \end{aligned}$$

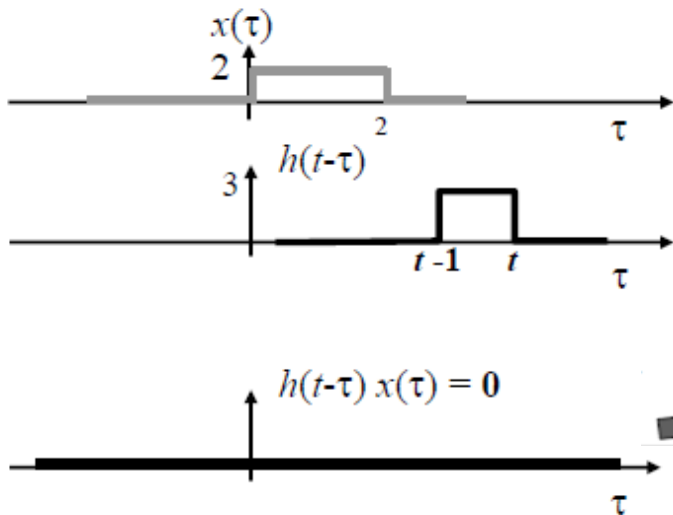


Region IV: $2 < t \leq 3$

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{t-1}^2 6d\tau = [6\tau]_{t-1}^2 = 6 \times 2 - 6(t-1) = -6t + 18 \\
 y(t) &= -6t + 18 \quad \text{for } 2 < t \leq 3
 \end{aligned}$$

Graphically Convolve Two Signals

Step #6: Form Product & Integrate For Each Region

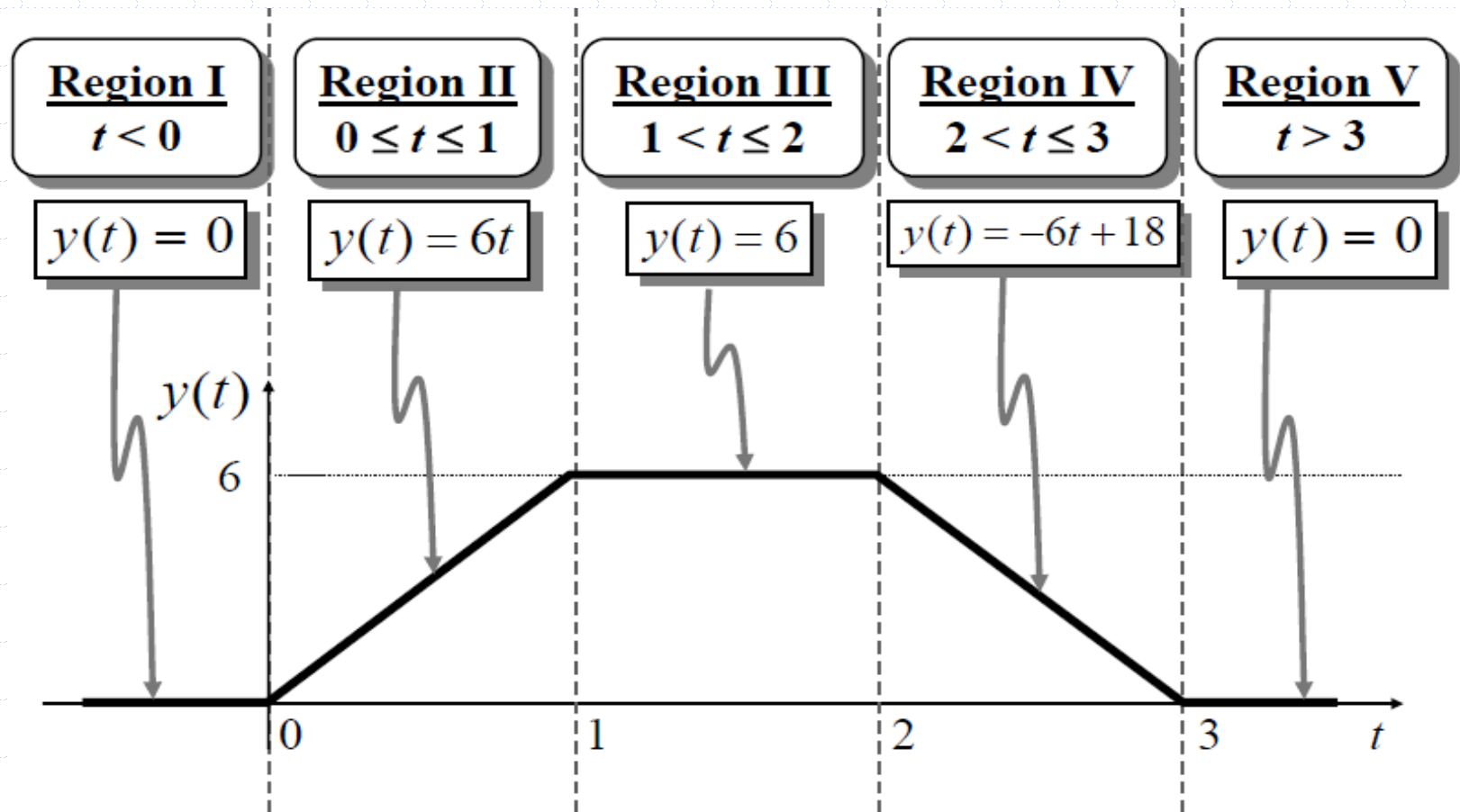


Region V: $t > 3$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} 0d\tau = 0 \\ y(t) &= 0 \quad \text{for all } t > 3 \end{aligned}$$

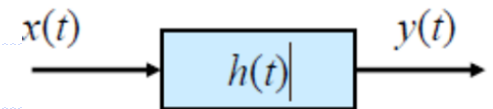
Graphically Convolve Two Signals

Step #7: “Assemble” Output Signal

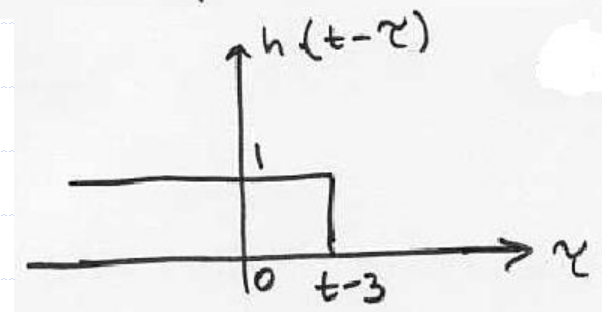
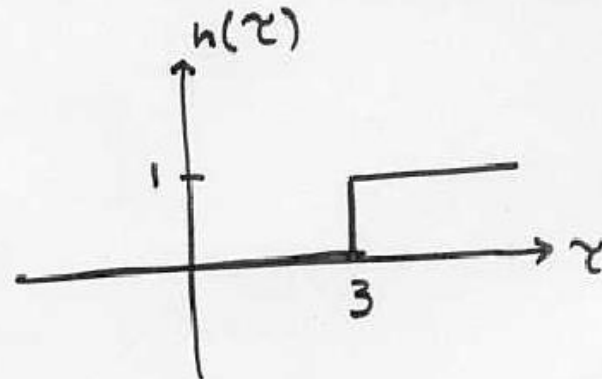
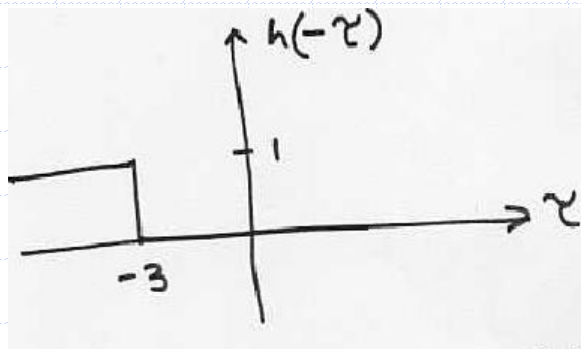
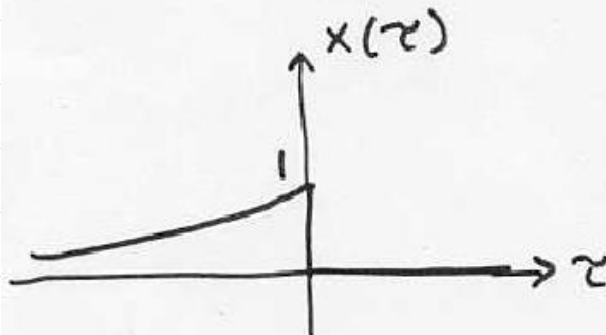


Examples

Ex. $x(t) = e^{2t}u(-t)$ $h(t) = u(t - 3)$ Find $y(t)$



Solution: $y(t) = x(t) * h(t)$



Examples

Region I: $t < 3$

$$y(t) = \int_{-\infty}^{t-3} 1 e^{2\tau} d\tau = \frac{1}{2} e^{2\tau}$$

$$\int_{-\infty}^{t-3}$$

$$y(t) = \frac{1}{2} (e^{2t-6} - e^{-\infty}) = \frac{e^{2t-6}}{2}$$

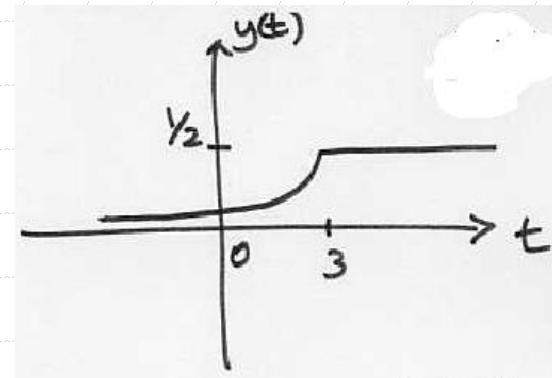
Region II: $t > 3$

$$y(t) = \int_{-\infty}^0 1 e^{2\tau} d\tau = \frac{1}{2} e^{2\tau} = \frac{1}{2} (e^0$$

$$\int_{-\infty}^0$$

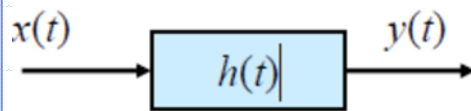
$$- e^{-\infty}) = \frac{1}{2}$$

$$y(t) = \begin{cases} \frac{e^{2t-6}}{2} & t < 3 \\ \frac{1}{2} & t > 3 \end{cases}$$

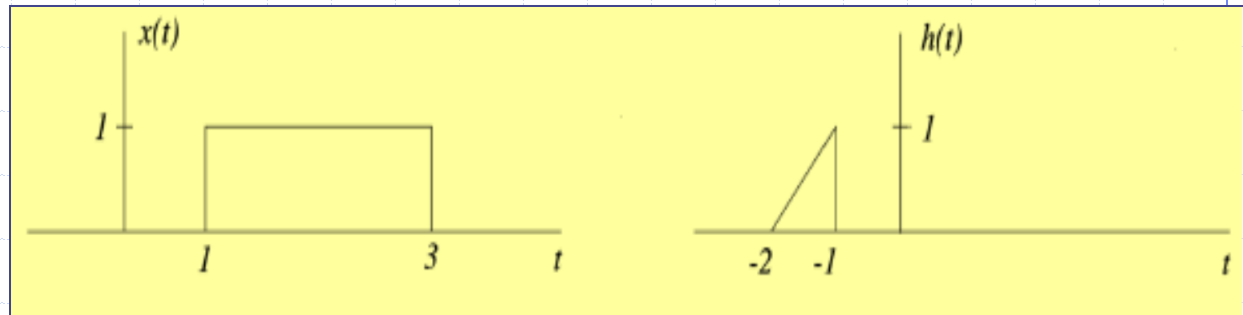


Examples

Ex. given



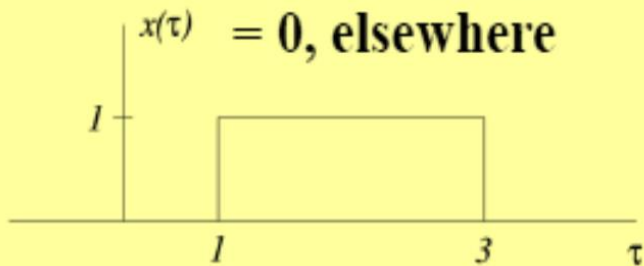
Find $y(t)$



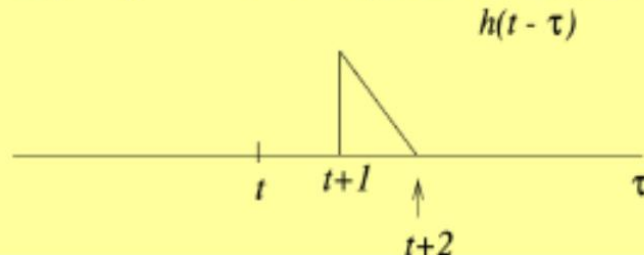
Solution $x(t) = 1$, for $1 \leq t \leq 3$
 $= 0$, elsewhere

$h(t) = t+2$, for $-2 \leq t \leq -1$
 $= 0$, elsewhere

$x(\tau) = 1$, for $1 \leq \tau \leq 3$
 $= 0$, elsewhere



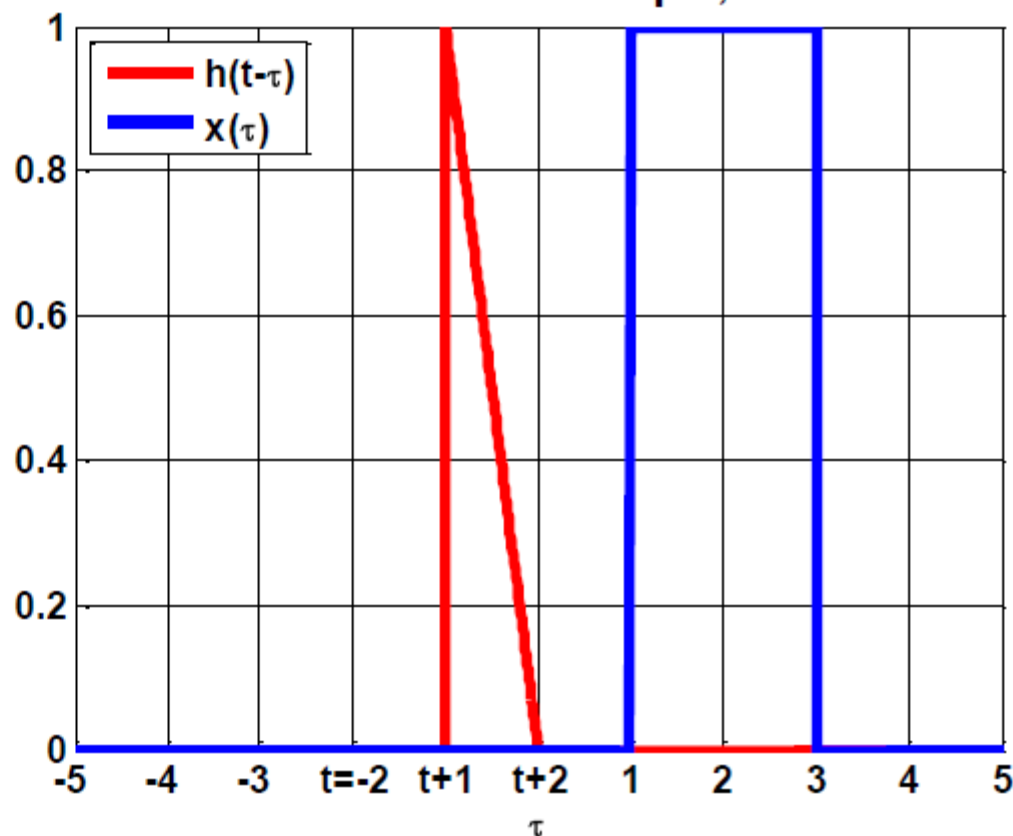
$h(t-\tau) = t-\tau+2$, for $-2 \leq t-\tau \leq -1$
 i.e. $h(t-\tau) = -\tau+t+2$, for $t+1 \leq \tau \leq t+2$



Convolution Example: $t \leq -1$

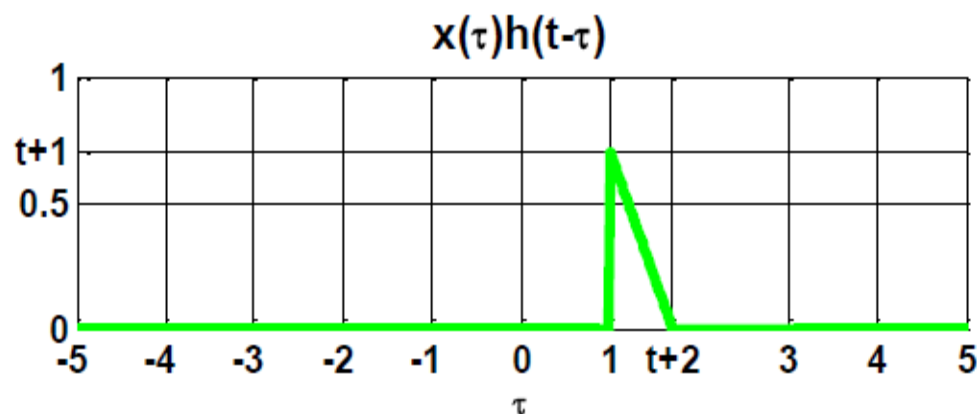
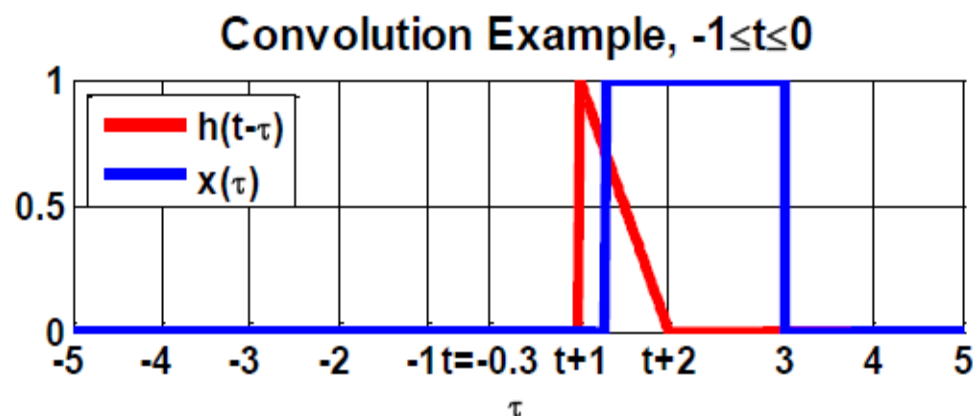
Time Interval	$x(\tau) \cdot h(t-\tau)$	Overlap Interval	$y(t)$
$t \leq -1$	0	None	0

Convolution Example, $t \leq -1$



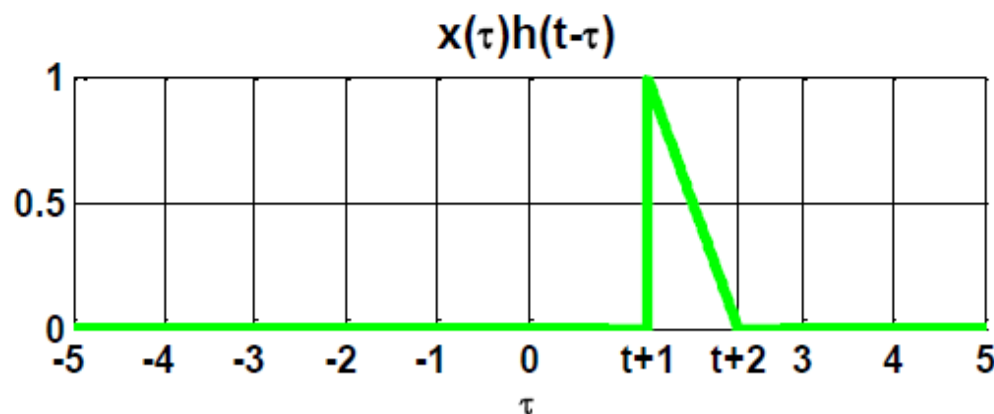
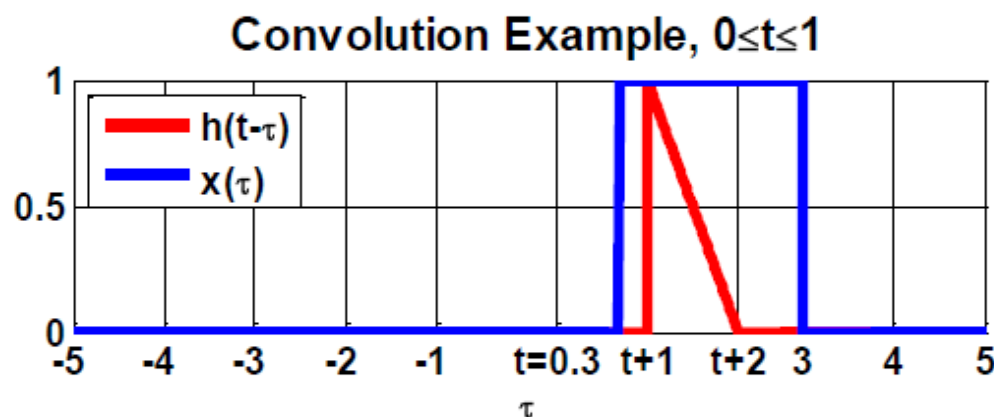
Convolution Example: $-1 \leq t \leq 0$

Time Interval	$x(\tau) \cdot h(t-\tau)$	Overlap Interval	$y(t)$
$-1 \leq t \leq 0$	$-\tau + t + 2$	$1 \leq \tau \leq t+2$	$= \int_{\tau=1}^{t+2} (-\tau + t + 2) d\tau$ $= \frac{(t+1)^2}{2}$



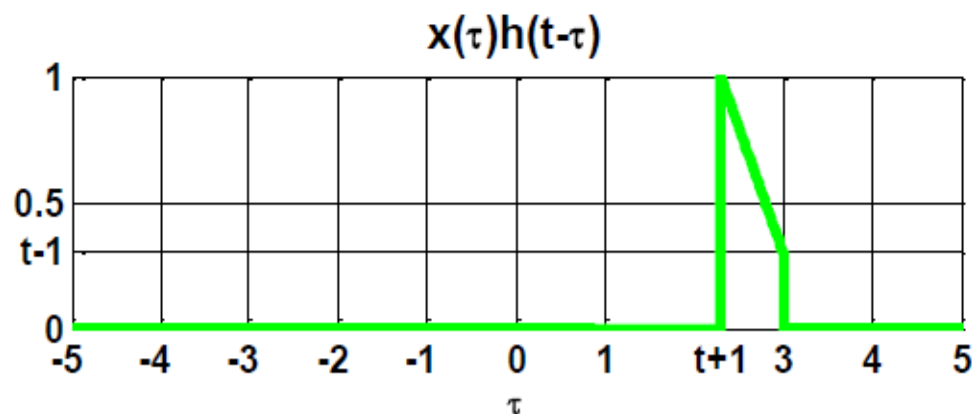
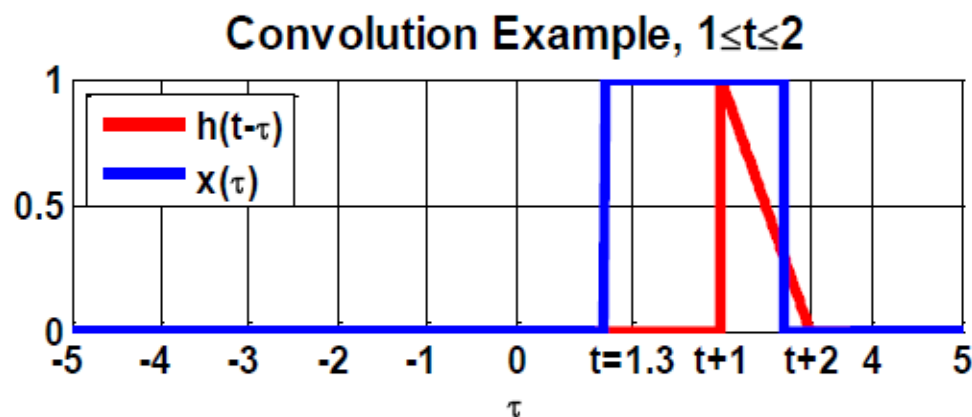
Convolution Example: $0 \leq t \leq 1$

Time Interval	$x(\tau) \cdot h(t-\tau)$	Overlap Interval	$y(t)$
$0 \leq t \leq 1$	$-\tau + t + 2$	$t+1 \leq \tau \leq t+2$	$= \int_{\tau=t+1}^{t+2} (-\tau + t + 2) d\tau$ $= \frac{1}{2}$



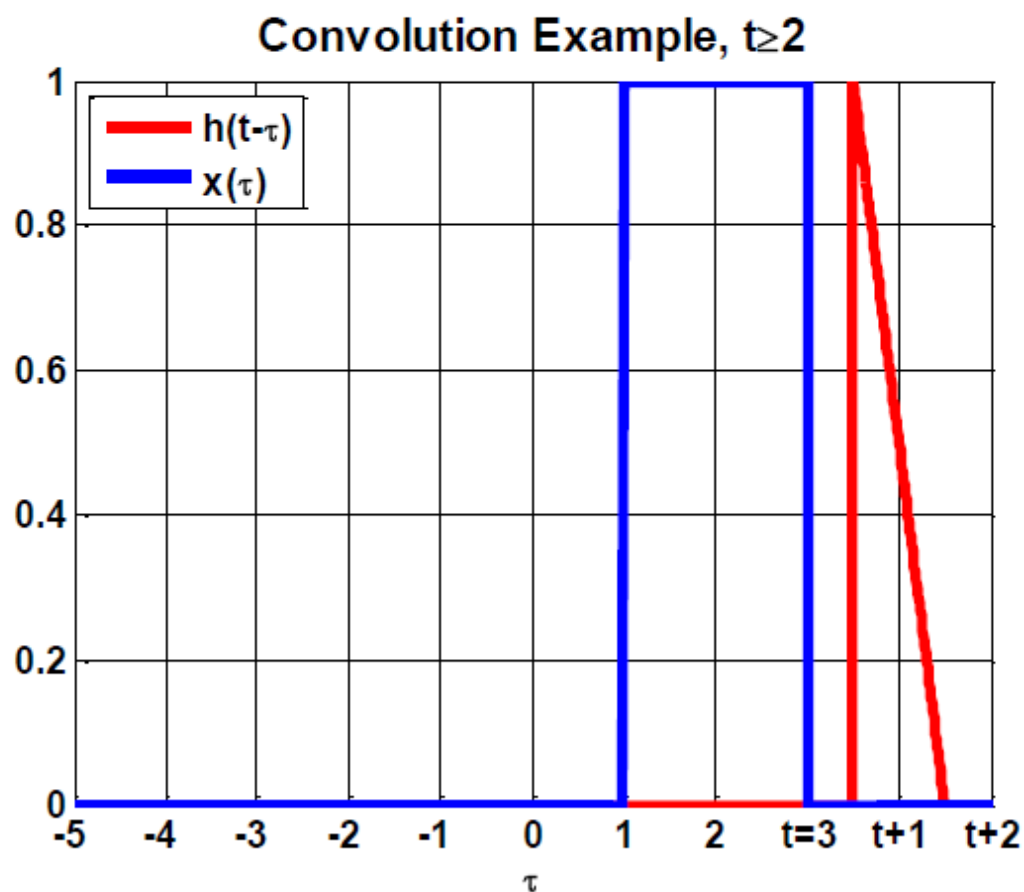
Convolution Example: $1 \leq t \leq 2$

Time Interval	$x(\tau) \cdot h(t-\tau)$	Overlap Interval	$y(t)$
$1 \leq t \leq 2$	$-\tau + t + 2$	$t+1 \leq \tau \leq 3$	$= \int_{\tau=t+1}^3 (-\tau + t + 2) d\tau$ $= \frac{1}{2} - \frac{(t-1)^2}{2}$



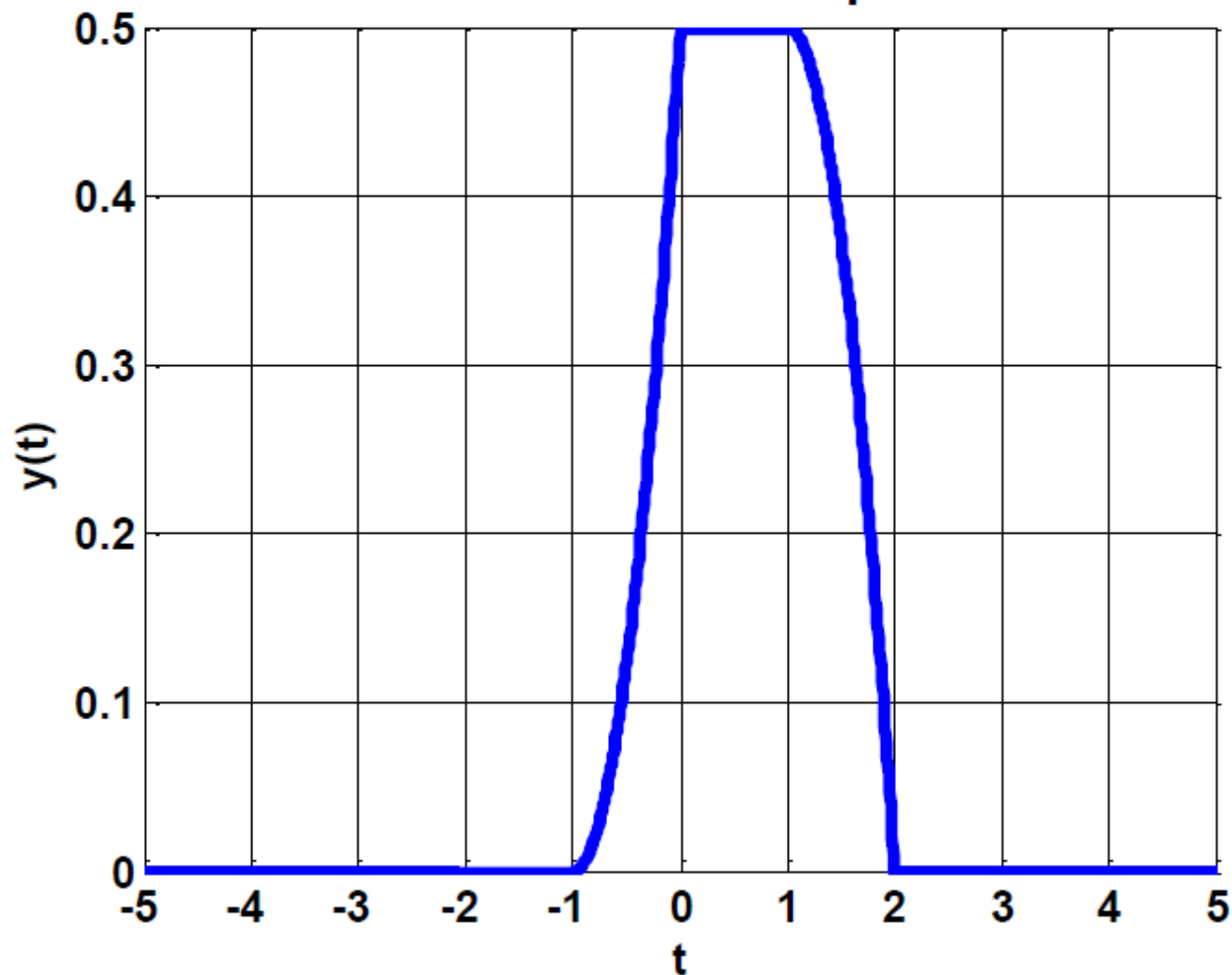
Convolution Example: $t \geq 2$

Time Interval	$x(\tau) \cdot h(t-\tau)$	Overlap Interval	$y(t)$
$t \geq 2$	0	None	0



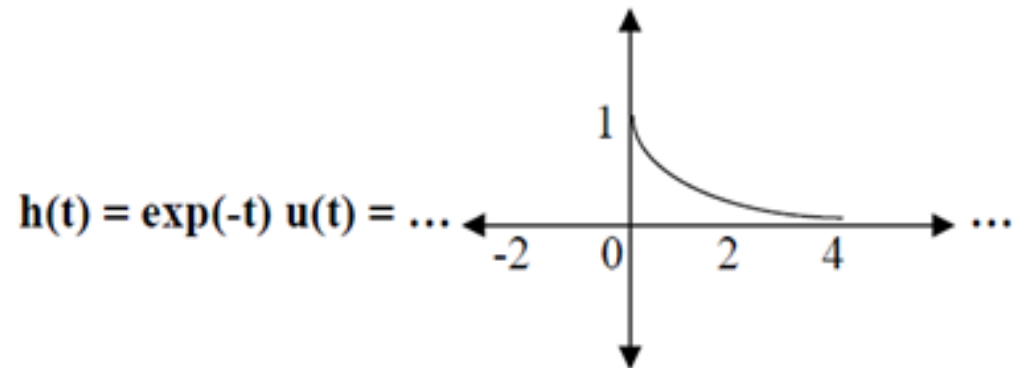
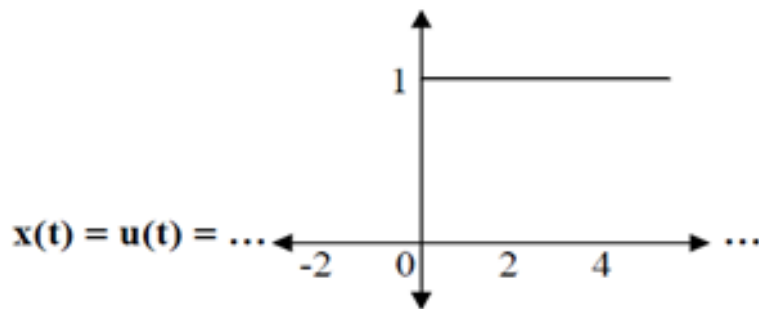
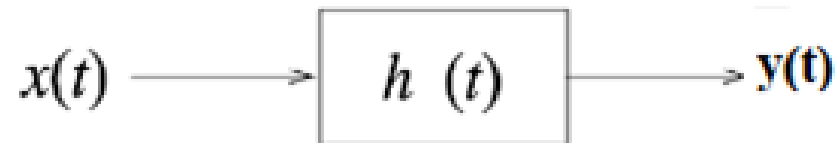
Convolution Example:

Convolution Output



Midterm Exam 2017

Determine $y(t)$ where $x(t)$ and $h(t)$ is as shown in the figure below

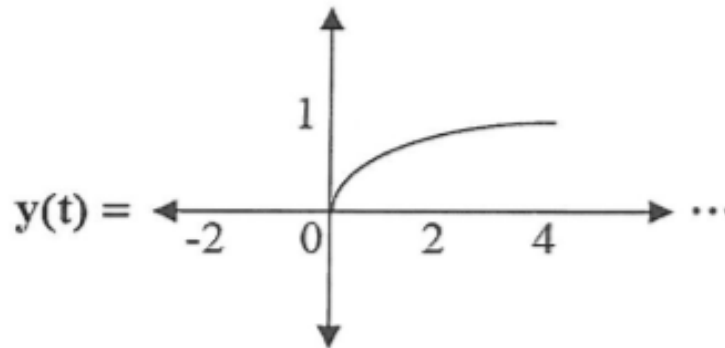


Midterm Exam 2017

Again you can make this problem simpler by switching $x(t)$ and $h(t)$, so that the convolution integral becomes

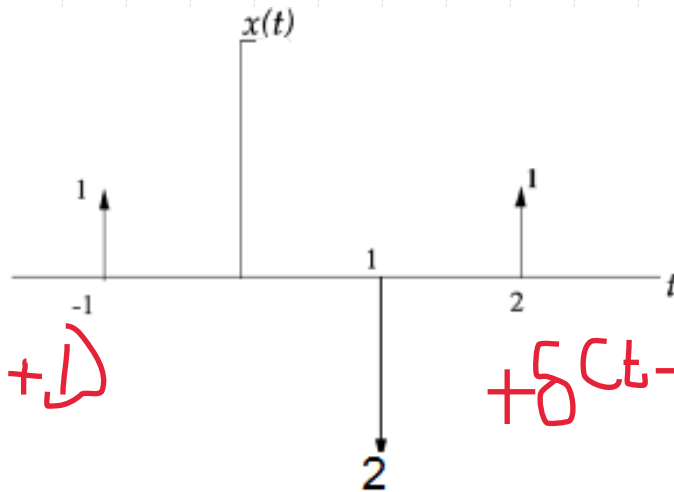
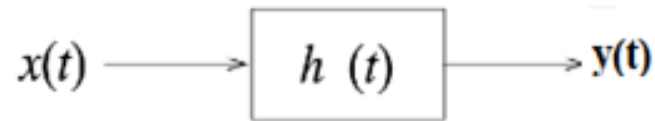
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} \exp(-\tau)u(\tau)u(t - \tau)d\tau = \int_0^t \exp(-\tau)d\tau = [-\exp(-\tau)]_0^t = 1 - \exp(-t)$$



Midterm Exam 2018

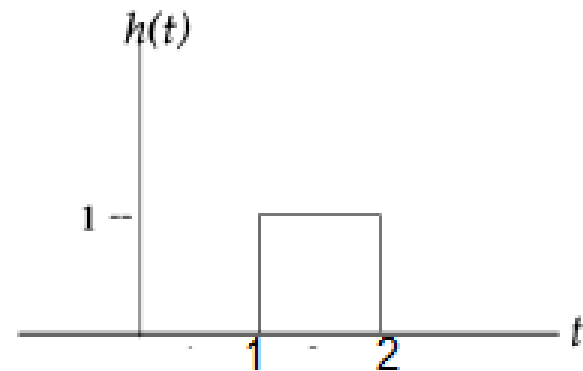
Determine $y(t)$ where $x(t)$ and $h(t)$ is as shown in the figure below



$$+ \delta(t+1)$$

$$+ \delta(t-2)$$

$$- 2 \delta(t-1)$$

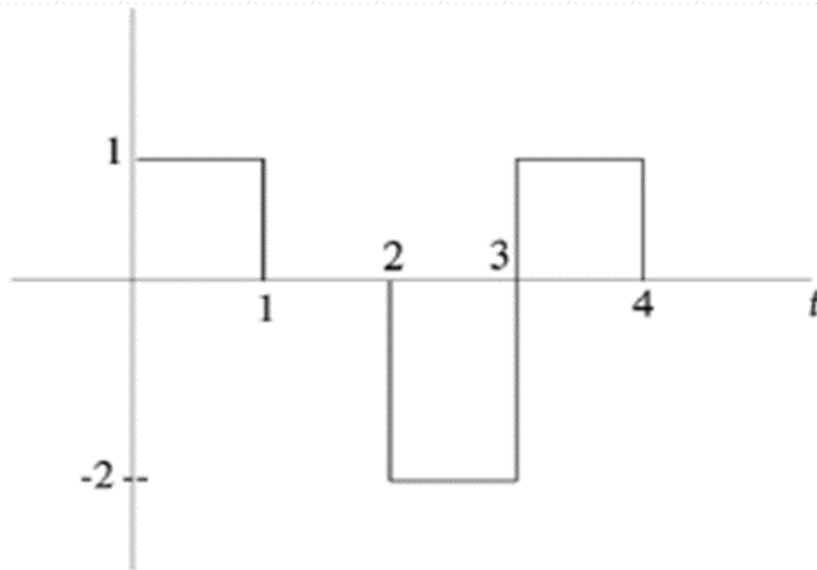


Midterm Exam 2018

Solution

$$x(t) = \delta(t + 1) - 2\delta(t - 1) + \delta(t - 2)$$

$$y(t) = x(t) * h(t) = h(t+1) - 2h(t-1) + h(t-2)$$



2.6 System Stability

Roughly speaking: A “stable” system is one whose output does not keep getting bigger and bigger in response to an input that does not keep getting bigger.

Stability : A system is stable if it results in a bounded output for any bounded input, i.e. bounded-input/bounded-output.

“Bounded-Input, Bounded-Output”(BIBO) stability:

- ◆ A system is said to be BIBO stable if
- ◆ for any bounded input: $|x(t)| \leq C_1 < \infty$
- ◆ the output remains bounded: $|y(t)| \leq C_2 < \infty$

Stability: CT LTI system is stable $\Leftrightarrow \int_{-\infty}^{+\infty} |h(\tau)| d\tau < \infty$

2.6 System Stability

Ex. Determine if the system is stable or unstable:

i. $h_1(t) = -3e^{2t}u(t)$

ii. $h_2(t) = 5\delta(t + 5)$

iii. $h_3(t) = e^{4t}u(-t)$

iv. $y(t) = e^{x(t)}u(t)$

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} f(t)\delta(t-t_0)dt = f(t_0) \quad \forall \varepsilon > 0$$

Solution:

i. $h_1(t) = \int_0^\infty -3e^{2t} = \infty$

ii. $h_2(t) = 5$

iii. $h_3(t) = \int_{-\infty}^0 e^{4t} = 1/4$

iv. If $x(t)=u(t)$ $y(t) = \text{Constant}$

System is unstable

System is stable

System is stable

System is stable