

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3310 Signals & Systems
Med-Term Exam
November 25, 2004

Dr. Basil Hamed

Exam Time 11:00-12:30

Name:

Solution

ID#

Grade:

GOOD LUCK

1. a- Write the signal shown in Figure 1 in terms of elementary functions. (3 Points)

$$x_4(t) = r(t) - 2u(t-1) - r(t-2)$$

OR

$$x_4(t) = u(1-t)r(t) - u(t-1) + r(t-1) - r(t-2)$$

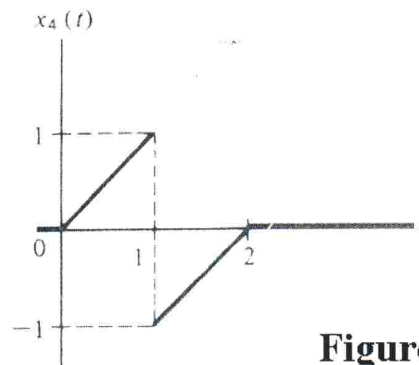


Figure 1

- b- Determine whether the above signal is power or energy signal justify your answer (2 Points)

$$E = \lim_{L \rightarrow \infty} \int_{-L}^L |x(t)|^2 dt$$

$$|x(t)| = \begin{cases} t & 0 < t < 1 \\ t-2 & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \int_0^1 t^2 dt + \int_1^2 (t-2)^2 dt$$

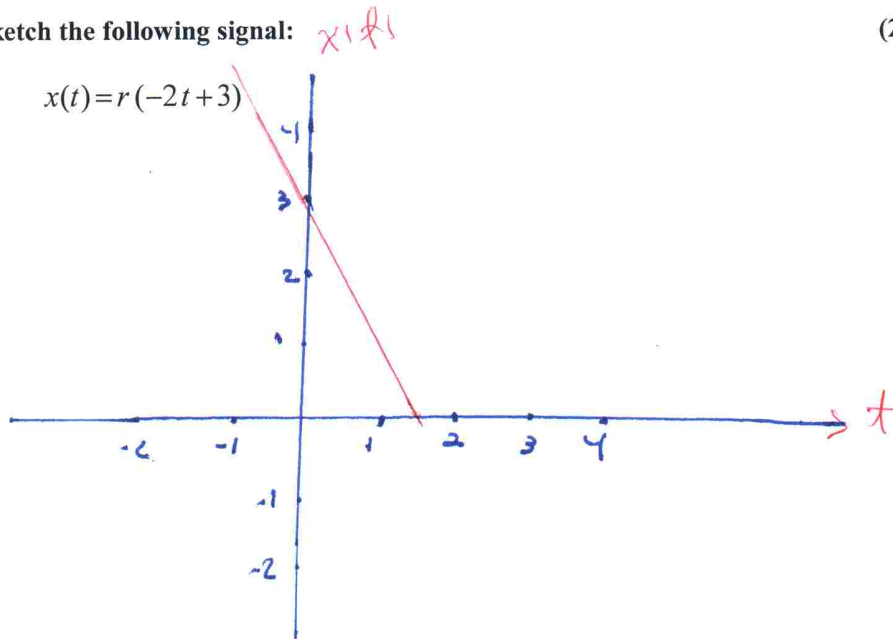
$$E = \left. \frac{t^3}{3} \right|_0^1 + \left. \frac{(t-2)^3}{3} \right|_1^2 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$P=0$$

$\therefore$ it is energy signal

c- Sketch the following signal:

(2 Points)



2. a) Determine whether the system described by the following differential equation is linear or nonlinear, causal or noncausal, time variant, or invariant. Also give its order (justify your answer) (5 Points)

$$\frac{dy}{dt} + 3y + 2 \int_{-\infty}^t y(t) dt = x(t)$$

* $\frac{d^2y}{dt^2} + 3 \frac{dy}{dt} + 2y(t) = \frac{dx}{dt}$

2nd order

* Let $x = ax_1 + bx_2 \Rightarrow \frac{dx}{dt} = a \frac{dx_1}{dt} + b \frac{dx_2}{dt}$

$a \frac{d^2y_1}{dt^2} + 3a \frac{dy_1}{dt} + 2ay_1 + b \frac{d^2y_2}{dt^2} + 3b \frac{dy_2}{dt} + 2by_2 = a \frac{dx_1}{dt} + b \frac{dx_2}{dt}$

$\frac{d^2}{dt^2} [ay_1 + by_2] + 3 \frac{d}{dt} [ay_1 + by_2] + 2[ay_1 + by_2] = a \frac{dx_1}{dt} + b \frac{dx_2}{dt}$

$\therefore \frac{d^2y}{dt^2} + 3 \frac{dy}{dt} + 2y = \frac{dx}{dt}$

$\therefore$ system is Linear

* system is Time invariant (no multiplication of t)

* system is causal, it does not depend on the future

b) Given an LTI systems described by

(3 Points)

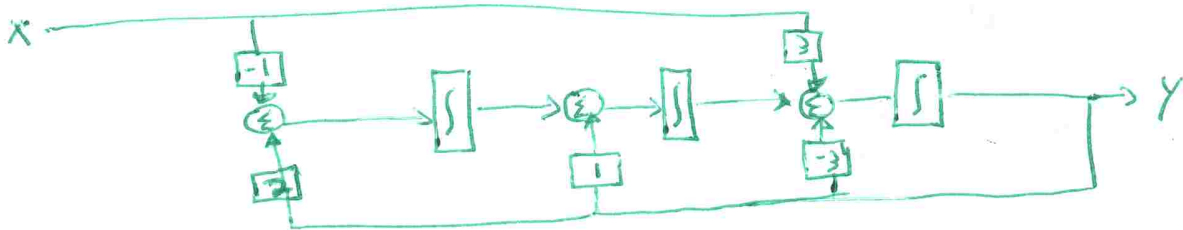
$$y'''(t) + 3y''(t) - y'(t) - 2y(t) = 3x''(t) - x(t)$$

Find the first or second canonical form simulation diagram

$N=3$

$$y = b_N x + \bar{D}^{-1}(b_{N-1}x - a_{N-1}y) + \dots + \bar{D}^{-N}(b_0x - a_0y)$$

$$y = \bar{D}^{-1}(3x - 3y) + \bar{D}^{-2}(0x + 1y) + \bar{D}^{-3}(-x + 2y)$$

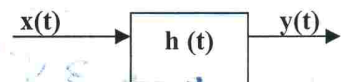
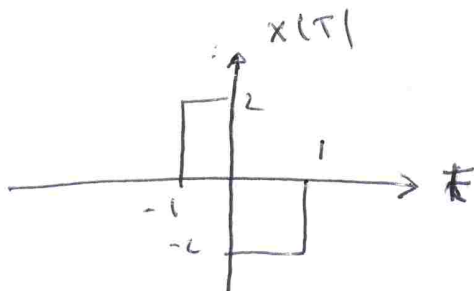
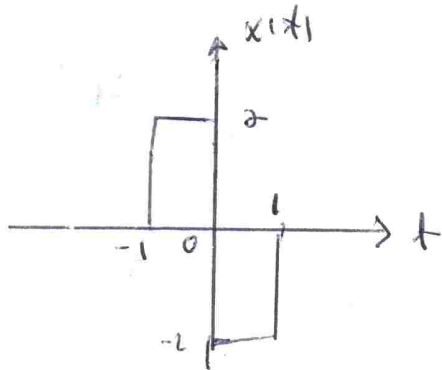


first canonical

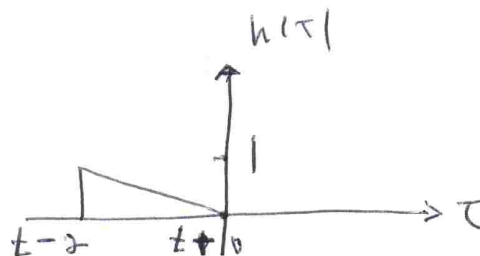
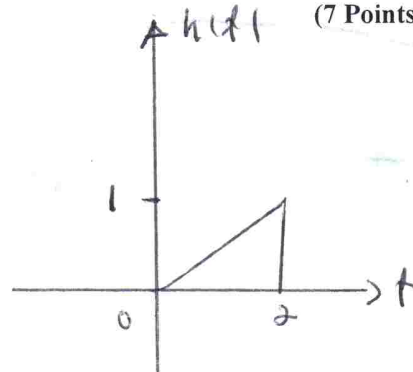
3. Find $y(t)$ where;

$$x(t) = 2u(t+1) - 4u(t) + 2u(t-1)$$

$$h(t) = 0.5r(t)u(2-t)$$



(7 Points)



$$y(t) = x(t) * h(t)$$

$$-1 < t < 3$$

$$(D^N + \sum_{j=0}^{N-1} a_j D^j) v(x) = x(x)$$

$$y(x) = \left(\sum_{i=0}^N b_i D^i \right) v(x)$$

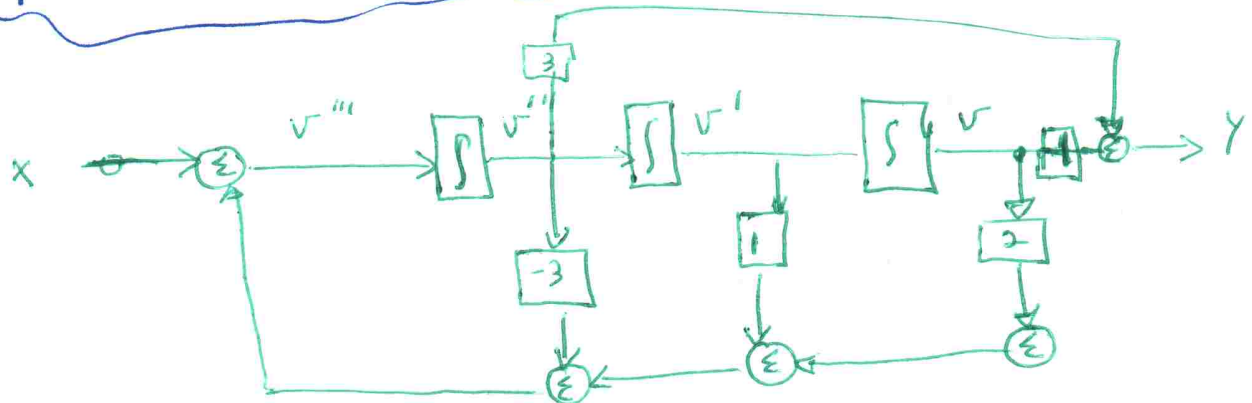
$$(y - a_0 - x_0) \frac{1}{s} + \dots + (y - a_{N-1} - x_{N-1}) \frac{1}{s^N} + x_0 = x$$

$$D^3 v(x) + a_2 D^2 v(x) + a_1 D^1 v(x) + a_0 D^0 v(x) = x(x)$$

$$v''' + 3v'' - 1v' - 2v = x(x)$$

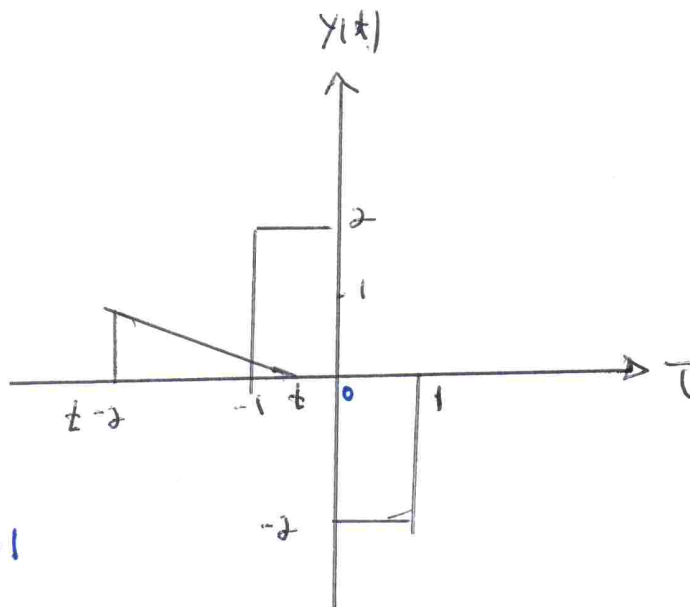
$$y(x) = b_3 D^3 v(x) + b_2 D^2 v(x) + b_1 D^1 v(x) + b_0 D^0 v(x)$$

$$y(x) = 0 + 3v'' + 0 + 1v'$$



Second canonical

(13)



$$x(t) = 2 \quad -1 < t < 0$$

$$h(t) = \frac{t}{2} \quad 0 < t < 2$$

$$y(t) = 0 \quad t \leq -1$$

$$y(t) = \int_{-1}^t x(\tau) h(t-\tau) d\tau \quad -1 \leq t < 0$$

$$y(t) = \int_{-1}^t 2 \left(\frac{t-\tau}{2} \right) d\tau = \left. t\tau - \frac{\tau^2}{2} \right|_{-1}^t = \frac{1}{2} (t^2 + 2t + 1)$$

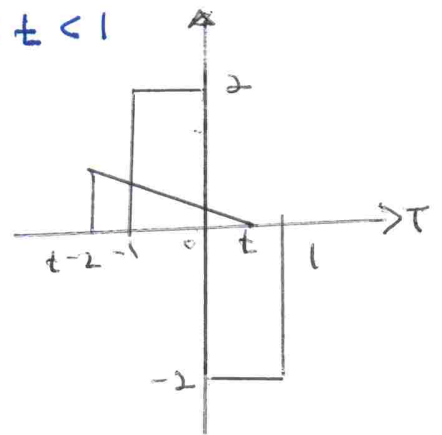
$$\therefore y(t) = \frac{1}{2} (t+1)^2 \quad -1 \leq t < 0$$

$$y(t) = \int_{-1}^0 2 \left(\frac{t-\tau}{2} \right) d\tau + \int_0^t -2 \left(\frac{t-\tau}{2} \right) d\tau$$

$$y(t) = \left. t\tau - \frac{\tau^2}{2} \right|_{-1}^0 - \left. \left(t\tau - \frac{\tau^2}{2} \right) \right|_0^t$$

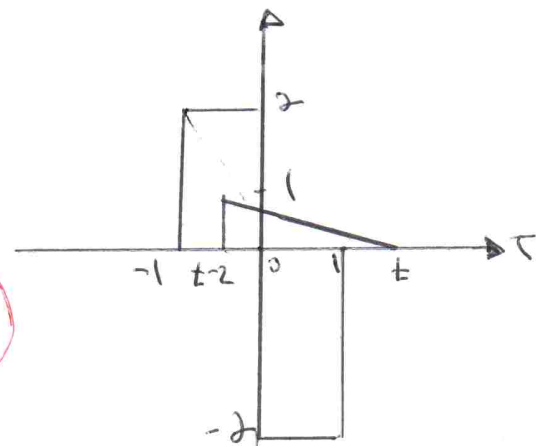
$$y(t) = \frac{1}{2} (-t^2 + 2t + 1) \quad 0 \leq t < 1$$

$$0 \leq t < 1$$



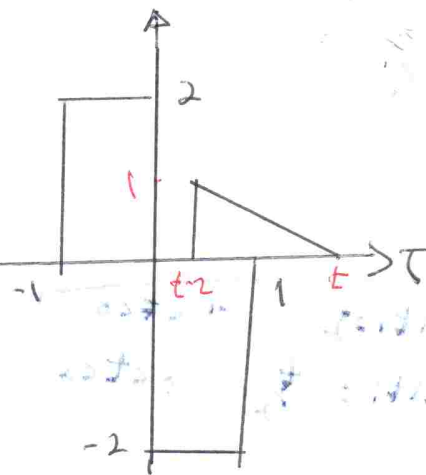
$$y(t) = \int_{t-2}^0 2 \left(\frac{t-\tau}{2} \right) d\tau + \int_0^1 2 \left(\frac{t-\tau}{2} \right) d\tau$$

$$y(t) = \frac{1}{2} (-t^2 - 2t + 1) + 2 \quad 1 \leq t < 2$$



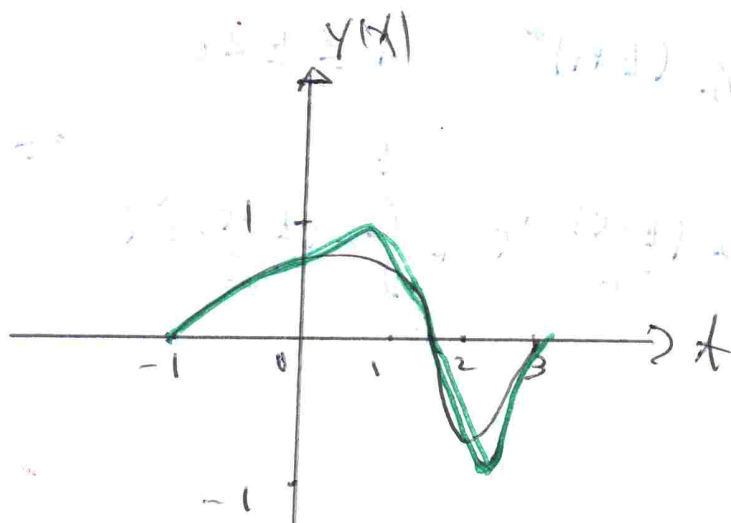
$$y(t) = \int_{t-2}^t -2 \frac{(t-\tau)}{2} d\tau$$

$$2 \leq t < 3$$



$$y(t) = \frac{1}{2} (t^2 - 2t + 1) - 2 \quad 2 \leq t < 3$$

$$\therefore y(t) = \begin{cases} 0 & t \leq -1 \\ \frac{1}{2} (t+1)^2 & -1 \leq t < 0 \\ \frac{1}{2} (-t^2 + 2t + 1) & 0 \leq t < 1 \\ \frac{1}{2} (-t^2 - 2t + 1) + 2 & 1 \leq t < 2 \\ \frac{1}{2} (t^2 - 2t + 1) - 2 & 2 \leq t < 3 \end{cases}$$



4. a-Find the exponential Fourier series representation of the signal shown in Figure 2.
(4 Points)

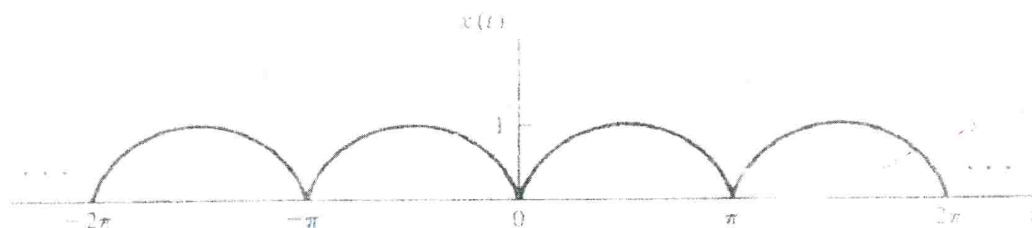


Figure 2

$$x(t) = \sin t \quad 0 < t < \pi$$

$$T = \pi, \omega_0 = 2$$

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{\pi} \int_0^{\pi} \sin t e^{-j2nt} dt = \frac{1}{\pi} \int_0^{\pi} \left(\frac{e^{jt} - e^{-jt}}{2j} \right) e^{-j2nt} dt$$

$$C_n = \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right)$$

OR

$$C_n = \frac{2}{\pi} \left(\frac{1}{1-n^2} \right) \begin{matrix} n \\ \text{even} \end{matrix}$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) e^{jn\omega_0 t}$$

$$C_n = \frac{1}{\pi} \left[\int_0^{\pi} \frac{e^{j(1-2n)t}}{2j} dt - \int_0^{\pi} \frac{e^{-j(1+2n)t}}{2j} dt \right]$$

$$C_n = \frac{1}{2\pi} \left[\frac{e^{j(1-2n)t}}{-(1-2n)} \Big|_0^{\pi} - \frac{e^{-j(1+2n)t}}{(1+2n)} \Big|_0^{\pi} \right]$$

$$C_n = \frac{-1}{2\pi} \left[\frac{e^{j(1-2n)\pi} - 1}{(1-2n)} + \frac{e^{-j(1+2n)\pi} - 1}{(1+2n)} \right]$$

$$e^{j(1 \pm 2n)\pi} = \cos(1 \pm 2n)\pi + j \sin(1 \pm 2n)\pi = -(-1)^{2n}$$

$$\therefore C_n = -\frac{1}{2\pi} \left[\frac{-1 - 1}{1-2n} + \frac{-1 - 1}{1+2n} \right]$$

$$\left(C_n = \frac{2}{\pi} \left[\frac{1}{1-4n^2} \right] \right) \quad \text{OR}$$

$$e^{j(1 \pm n)\pi} = \cos(1 \pm n)\pi + j \sin(1 \pm n)\pi = -(-1)^n$$

$(n \Rightarrow \text{even})$

$$C_n = -\frac{1}{2\pi} \left[\frac{-1 - 1}{1-n} + \frac{-1 - 1}{1+n} \right] = \frac{2}{\pi} \left[\frac{1}{1-n^2} \right]$$

for $n = \text{even}$

b- Plot the amplitude and phase spectrum for $n = \pm 1, \pm 2, \pm 3, \pm 4$.

(2 Points)

$$C_n = \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) \Rightarrow \text{OR} \quad C_n = \frac{2}{\pi} \left(\frac{1}{1-n^2} \right) \quad n: \text{even}$$

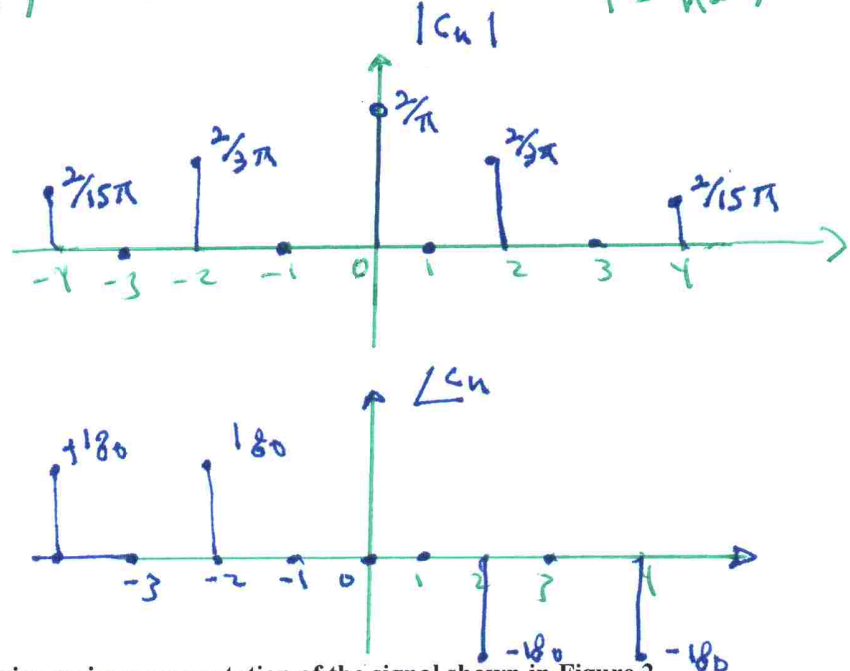
$$C_0 = \frac{2}{\pi} \angle 0$$

$$C_1 = 0$$

$$C_2 = \frac{2}{\pi(-3)} = \frac{2}{3\pi} \angle -180$$

$$C_3 = 0$$

$$C_4 = \frac{2}{\pi(-15)} = \frac{2}{15\pi} \angle -180$$



c- Find the trigonometric Fourier-series representation of the signal shown in Figure 2.

(2 Points)

$$a_0 = C_0 = \frac{2}{\pi}$$

$$a_n = 2 \operatorname{Re} \{ C_n \} = 2 \left[\frac{2}{\pi} \left(\frac{1}{1-n^2} \right) \right] \quad n: \text{even}$$

$$a_n = \frac{4}{\pi} \left(\frac{1}{1-n^2} \right) \quad n: \text{even}$$

$$b_n = -2 \operatorname{Im} \{ C_n \} = 0$$

$$\therefore x(t) = \frac{2}{\pi} + \sum_{\substack{n=1 \\ n: \text{even}}}^{\infty} \frac{4}{\pi} \left(\frac{1}{1-n^2} \right) \cos n\omega_0 t$$

b- Plot the amplitude and phase spectrum for $n = \pm 1, \pm 2, \pm 3, \pm 4$.

(2 Points)

$$C_n = \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right)$$

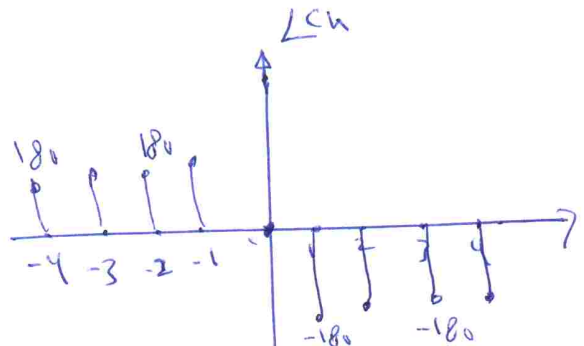
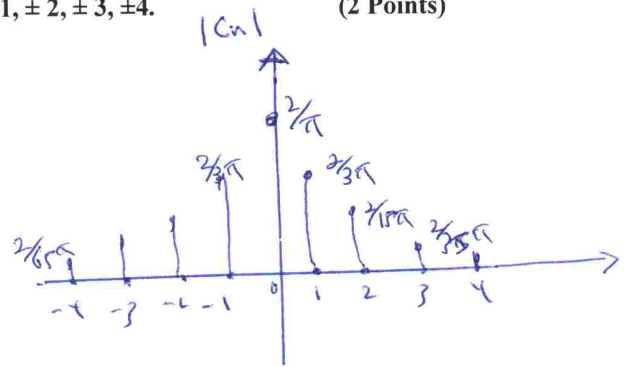
$$C_0 = \frac{2}{\pi} \angle 0$$

$$C_1 = \frac{2}{\pi(-3)} = \frac{2}{3\pi} \angle -180^\circ$$

$$C_2 = \frac{2}{\pi(-15)} = \frac{2}{15\pi} \angle -180^\circ$$

$$C_3 = \frac{2}{\pi(-35)} = \frac{2}{35\pi} \angle -180^\circ$$

$$C_4 = \frac{2}{\pi(-65)} = \frac{2}{65\pi} \angle -180^\circ$$



c- Find the trigonometric Fourier-series representation of the signal shown in Figure 2.
(2 Points)

$$a_0 = C_0 = \frac{2}{\pi}$$

$$a_n = 2 \operatorname{Re} \{ C_n \} = \left[\frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) \right]$$

$$a_n = \frac{4}{\pi} \left(\frac{1}{1-4n^2} \right)$$

$$b_n = -2 \operatorname{Im} \{ C_n \} = 0$$

$$\therefore X(\omega) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(1-4n^2)} \cos n \omega t$$