

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3310 Signals & Systems
Med-Term Exam
November 17, 2005

Dr. Basil Hamed

Exam Time 9:00-10:30

Name:

Solution

ID#

Grade:

GOOD LUCK

1. a- For the signal $x(t)$ illustrated in Fig. 1 sketch, $x(2-t)$

(3 Points)

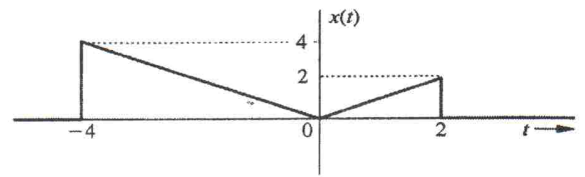
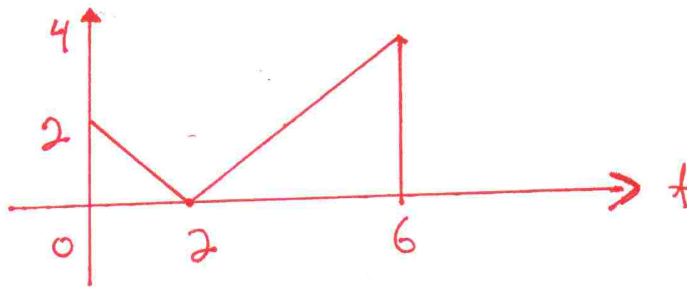


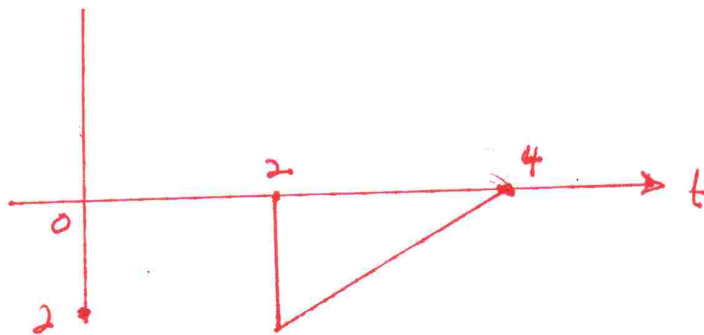
Fig. 1



b- Sketch the following signal:

$$(t-4)[u(t-2) - u(t-4)]$$

(2 Points)



c- An aperiodic signal is defined as $x(t) = \sin(\pi t) u(t)$. Is the odd portion of this signal, $x_o(t)$, periodic? Justify your answer.

(3 Points)

$$\begin{aligned} x_o(t) &= \frac{1}{2} (x(t) - x(-t)) = 0.5 [\sin(\pi t) u(t) - \sin(-\pi t) u(-t)] \\ &= 0.5 \sin(\pi t) (u(t) + u(-t)) \end{aligned}$$

Since $\sin(0) = 0$, this reduce to

$$x_o(t) = 0.5 \sin(\pi t), \text{ which is a } (T=2)$$

$\therefore x_o(t) = 0.5 \sin(\pi t)$ is periodic signal.

2. a) Given the system

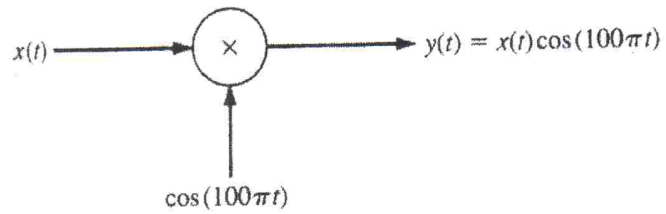


Fig. 2

Determine whether the system shown in Fig. 2 is linear or nonlinear, causal or noncausal, time variant, or invariant. (Justify your answer) (5 Point.)

$$* y_1(t) = x_1(t) \cos(100\pi t)$$

$$y_2(t) = x_2(t) \cos(100\pi t)$$

$$x_1(t) = ax_1(t) + bx_2(t)$$

$$ay_1 + by_2 = [ax_1(t) + bx_2(t)] \cos(100\pi t)$$

$\therefore$ system is Linear

* System is time-varying because multiplying by $\cos(100\pi t)$

$$1. y_1(t) = x_1(t) \cos(100\pi t)$$

$$2. y_2(t) = x_2(t) = x_1(t-t_0) \cos(100\pi t)$$

$$3. y_1(t-t_0) = x_1(t-t_0) \cos[100\pi(t-t_0)]$$

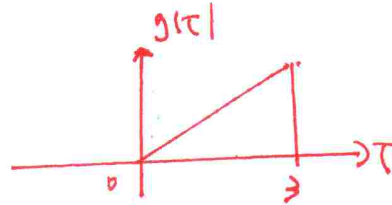
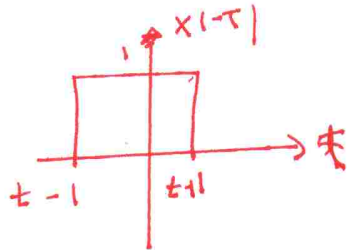
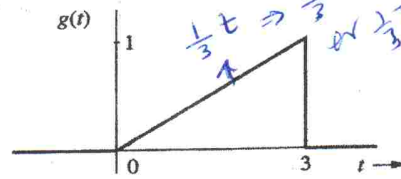
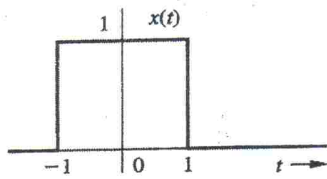
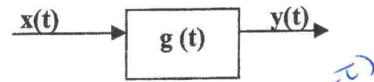
$$4. 2 \neq 3 \quad \therefore \text{system is time-varying.}$$

* It is causal because the output depends on only the input.

$$y(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{6}(t+1)^2 & -1 \leq t \leq 1 \\ \frac{2}{3}t & 1 < t \leq 2 \\ -\frac{1}{6}(t^2 - 2t - 8) & 2 < t \leq 4 \\ 0 & 4 < t \end{cases}$$

b) Find $y(t)$ where;

(7 Points)



$$y(t) = 0$$

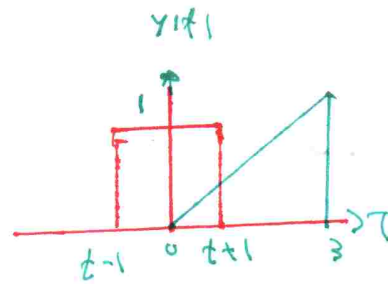
$$t < -1$$

$$x(t) = 1 ; \quad g(t) = \frac{1}{3}t$$

$$y(t) = \int_0^{t+1} (1) \left(\frac{1}{3}\tau\right) d\tau$$

$$y(t) = \frac{1}{6}(t+1)^2$$

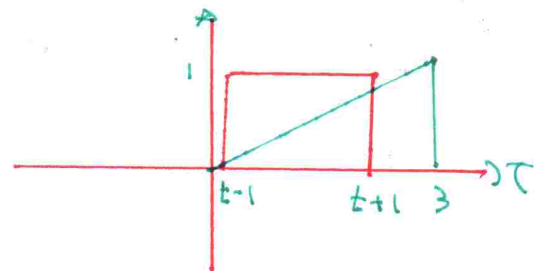
$$-1 \leq t \leq 1$$



$$y(t) = \int_{t-1}^{t+1} (1) \left(\frac{1}{3}\tau\right) d\tau$$

$$y(t) = \frac{2}{3}t$$

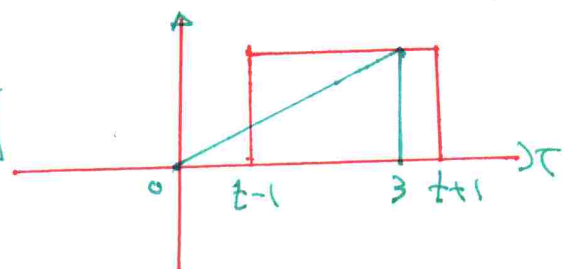
$$1 \leq t \leq 2$$



$$y(t) = \int_{t-1}^3 (1) \left(\frac{1}{3}\tau\right) d\tau$$

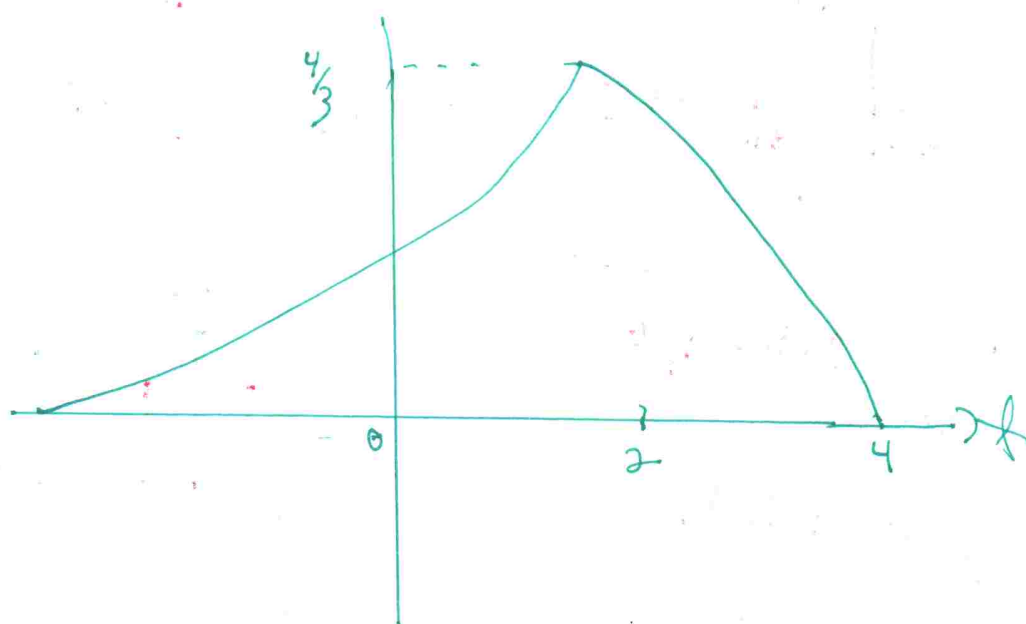
$$y(t) = -\frac{1}{6}(t^2 - 2t - 8)$$

$$2 \leq t \leq 4$$



$$y(t) = 0$$

$$t \geq 4$$



3. a- Find the exponential Fourier series representation of the signal shown in Fig 3. (4 Points)

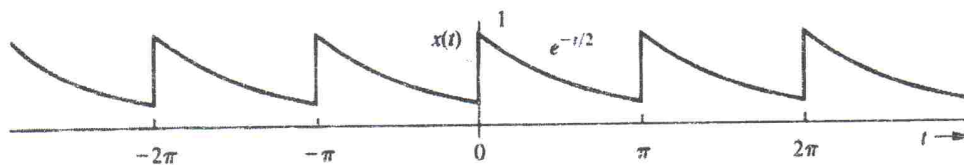


Fig. 3

$$T = \pi, \quad \omega_0 = 2\pi/T = 2$$

over one period
 $x(t) = e^{-t/2}$ for $0 \leq t < \pi$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 n t}$$

$$C_n = \frac{1}{T} \int_0^{\pi} x(t) e^{-j\omega_0 n t} dt = \frac{1}{\pi} \int_0^{\pi} e^{-t/2} e^{-j2n t} dt$$

$$= \frac{1}{\pi} \int_0^{\pi} e^{-(1/2 + j2n)t} dt = \frac{-1}{\pi (1/2 + j2n)} e^{-(1/2 + j2n)t} \Big|_0^{\pi}$$

$$C_n = \frac{0.504}{1 + j4n}$$

$$\therefore x(t) = 0.504 \sum_{n=-\infty}^{\infty} \frac{1}{1 + j4n} e^{j2n t}$$

b- Plot the amplitude and phase spectrum for $n = \pm 1, \pm 2, \pm 3, \pm 4$.

(3 Points)

$$C_n = \frac{0.504}{1+j4n}$$

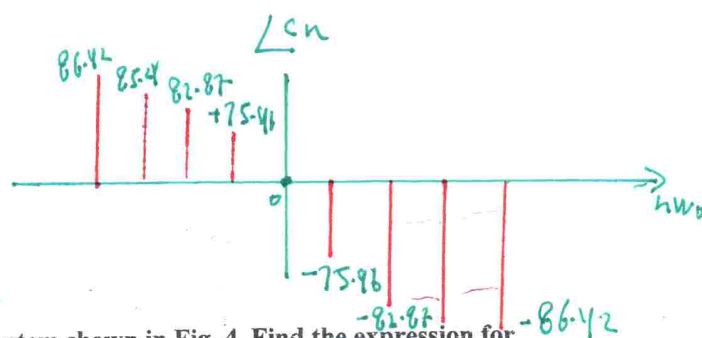
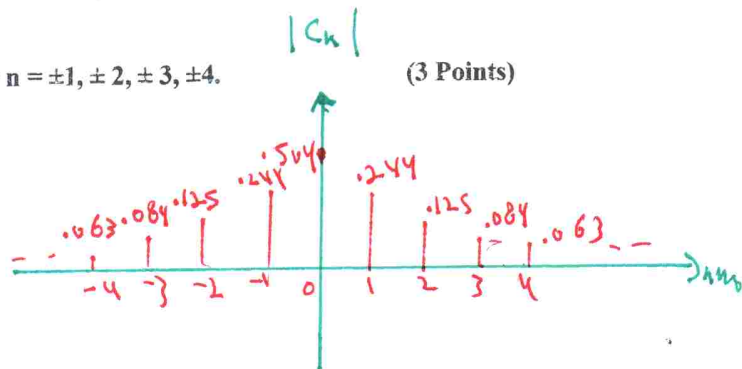
$$C_0 = 0.504 \angle 0$$

$$C_1 = \frac{0.504}{1+j4} = 0.244 \angle -75.96$$

$$C_2 = \frac{0.504}{1+j8} = 0.125 \angle -82.87$$

$$C_3 = \frac{0.504}{1+j12} = 0.084 \angle -85.24$$

$$C_4 = \frac{0.504}{1+j16} = 0.063 \angle -86.42$$



c- The signal $x(t)$ is applied at the input of the system shown in Fig. 4. Find the expression for the output $y(t)$. (3 Points)

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$Y(\omega) = \frac{1}{1 + \frac{1}{j\omega}} X(\omega)$$

$$\omega_0 = 2$$

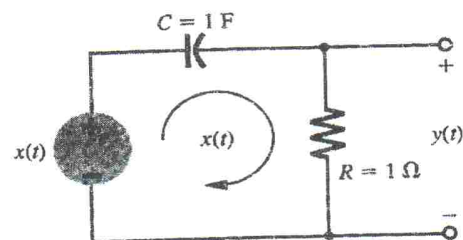
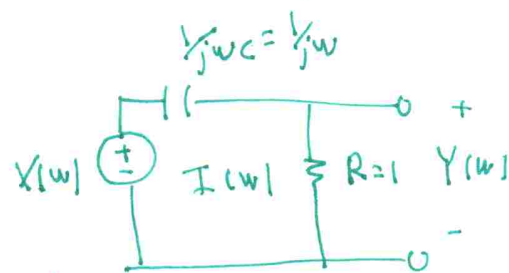


Fig. 4

$$\therefore H(j\omega_0) = \frac{1}{1 + \frac{1}{j\omega_0}} = \frac{j2n}{j2n+1}$$

$$X(t) = \sum_{n=-\infty}^{\infty} \frac{0.504}{1+j4n} e^{j2nt}$$

from Part a



$$\therefore Y(t) = \sum_{n=-\infty}^{\infty} \left(\frac{0.504}{1+j4n} \right) \left(\frac{j2n}{j2n+1} \right) e^{j2nt}$$