

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3310 Signals & Systems
Med-Term Exam
November 4, 2008

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Exam Time 10:00-11:30

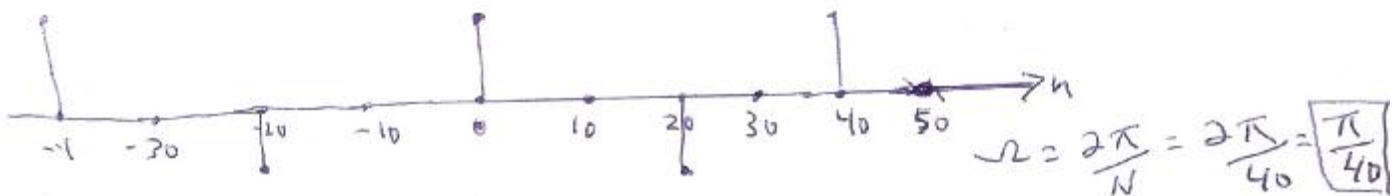
1. a) Is the following discrete-time signal periodic? If so find its period N and fundamental frequency. If not explain why.

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-10k] \cos\left(\frac{\pi}{2}k\right)$$

$$\delta[n-10k] = 1 \text{ when } \boxed{n-10k=0} \quad (4 \text{ Pt.})$$

$$\therefore \boxed{N=40}$$

Yes it is periodic.



- b) Determine whether the system described by the following input/output relationships are linear or nonlinear, causal or noncausal, time variant, or invariant, memory, or memoryless. (Justify your answer) (4 Pt.)

$$y(t) = \frac{d}{dt} x(t-1)$$

* Linearity

$$y_1(t) = \frac{d}{dt} x_1(t-1), \quad y_2(t) = \frac{d}{dt} x_2(t-1)$$

$$\therefore ax_1 + bx_2 \Rightarrow y(t) = \frac{d}{dt} [ax_1(t-1) + bx_2(t-1)]$$

$$= a \frac{d}{dt} x_1(t-1) + b \frac{d}{dt} x_2(t-1) = ay_1 + by_2 \quad \therefore \boxed{\text{system is Linear}}$$

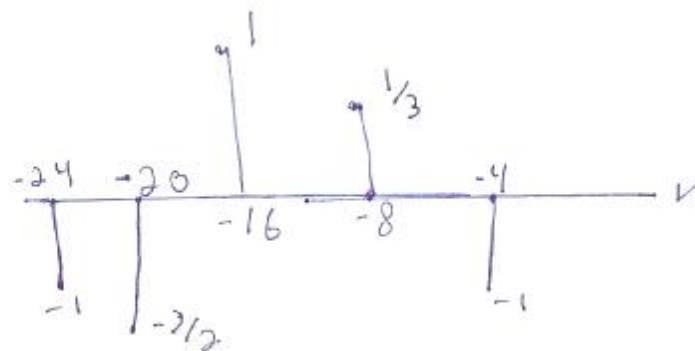
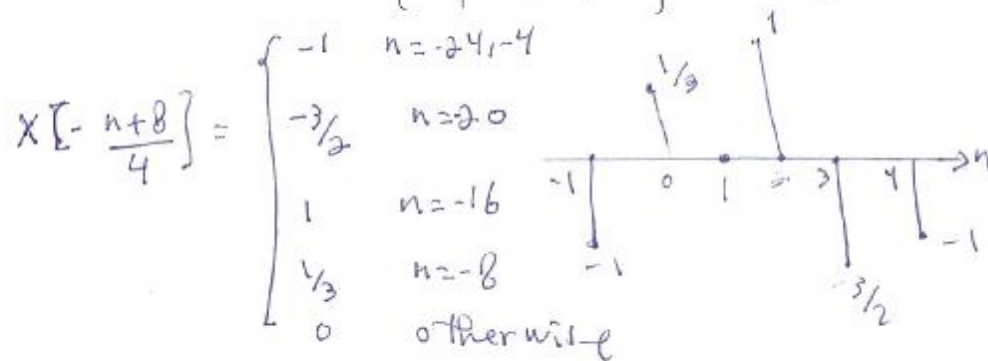
* System is with memory. System depends on past values.

* System is Causal. System does not depend on future.

* System is time-invariant. (No multiplication of time.)

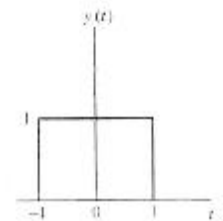
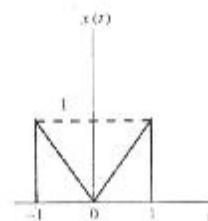
2. a) For the signal described by $x[n] = \left\{ -1, \frac{1}{3}, 0, 1, -\frac{3}{2}, -1 \right\}$, find $x\left[-\frac{n+8}{4}\right]$.

(4 Pt.)

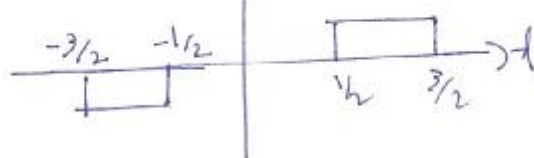
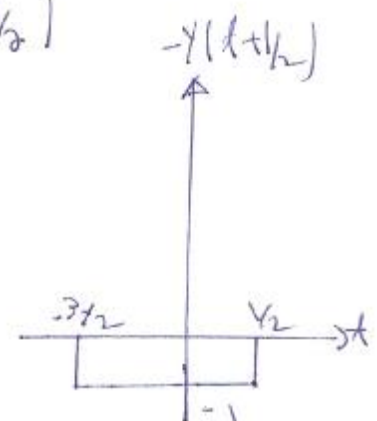
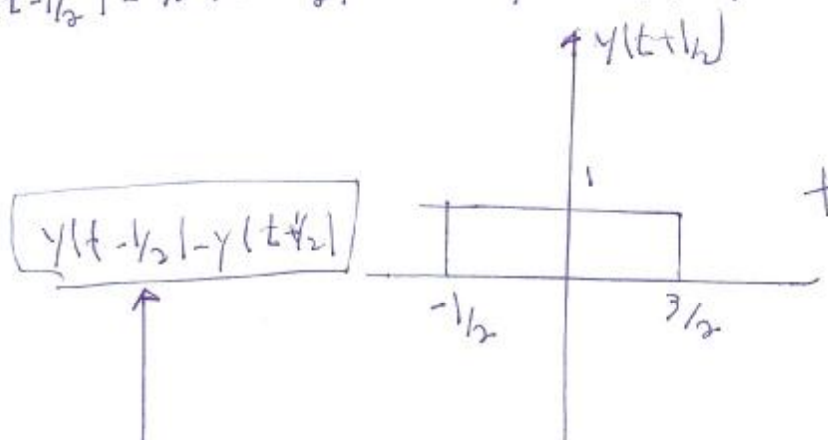


b) The input $x(t)$ and the output $y(t)$ of LTI system are shown in Figure. Sketch the response to $x(t-1/2) - x(t+1/2)$ (4 Points)

LTI System



$$\therefore x(t-1/2) - x(t+1/2) \longrightarrow y(t-1/2) - y(t+1/2)$$



3. a) In the following LTI system, find the impulse response $h(t)$ of the equivalent system.

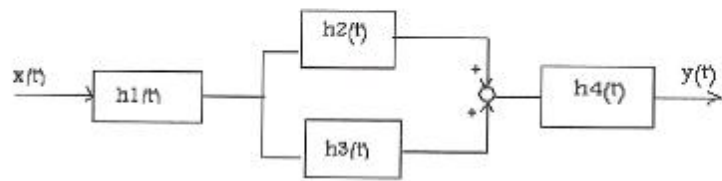
(8 Points)

$$h_1(t) = e^{-2t} u(t+1)$$

$$h_2(t) = u(t-3)$$

$$h_3(t) = 2\delta(t)$$

$$h_4(t) = 3\delta(t-1)$$



$$h(t) = h_1(t) * [h_2(t) + h_3(t)] * h_4(t)$$

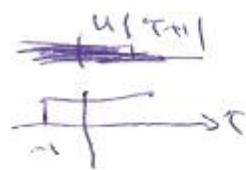
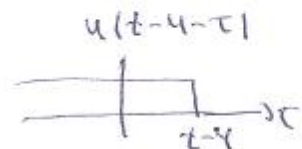
$$= e^{-2t} u(t+1) * [u(t-3) + 2\delta(t)] * 3\delta(t-1)$$

$$= e^{-2t} u(t+1) * [3u(t-4) + 6\delta(t-1)]$$

$$h(t) = 3e^{-2t} u(t+1) * u(t-4) + 6e^{-2(t-1)} u(t)$$

$$3e^{-2t} u(t+1) * u(t-4) = 3 \int_{-\infty}^{\infty} e^{-2\tau} u(\tau+1) u(t-4-\tau) d\tau$$

$$u(\tau+1) u(t-4-\tau) = 1 \quad -1 < \tau < t-4$$



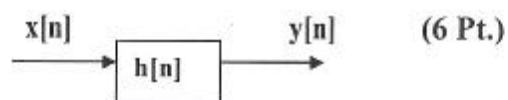
$$3 \int_{-\infty}^{\infty} e^{-2\tau} u(\tau+1) u(t-4-\tau) d\tau = 3 \int_{-1}^{t-4} e^{-2\tau} d\tau$$

$$= 3 \left[\frac{e^{-2\tau}}{-2} \right]_{-1}^{t-4} = -\frac{3}{2} e^{-2(t-4)} + \frac{3}{2} e^2 = -\frac{3}{2} [e^{-2(t-4)} - e^2] u(t-3)$$

$$\therefore h(t) = 0 \quad t < 3$$

$$h(t) = -\frac{3}{2} [e^{-2(t-4)} - e^2] u(t-3) + 6e^{-2(t-1)} u(t)$$

b) Find $y[n]$ for given $x[n]$, and $h[n]$.

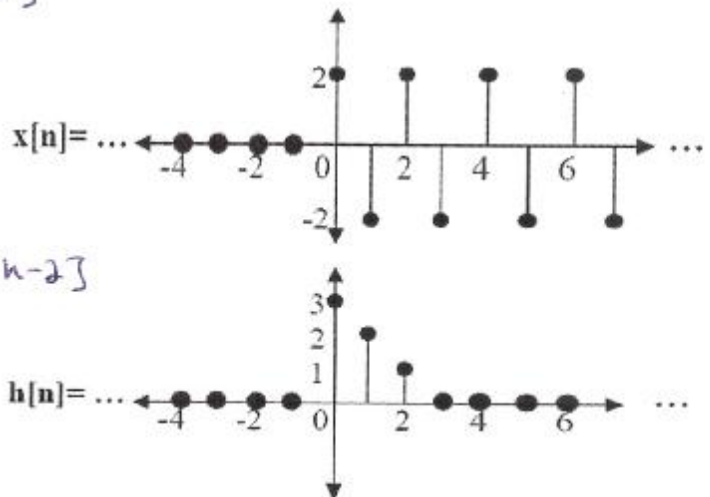


$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$y[n] = x[n] * h[n]$$

$$\therefore y[n] = 3x[n] + 2x[n-1] + x[n-2]$$

$y[n]$ starts from 0 $\rightarrow 9$



$$\therefore y[0] = 3x[0] + 2x[-1] + x[-2] = 6$$

$$y[1] = 3x[1] + 2x[0] + x[-1] = -6 + 4 = -2$$

$$y[2] = 3x[2] + 2x[1] + x[0] = 6 - 4 + 2 = 4$$

$$y[3] = 3x[3] + 2x[2] + x[1] = -6 + 4 - 2 = -4$$

$$y[4] = 3x[4] + 2x[3] + x[2] = 6 - 4 + 2 = 4$$

$$y[5] = 3x[5] + 2x[4] + x[3] = -6 + 4 - 2 = -4$$

$$y[6] = 3x[6] + 2x[5] + x[4] = 6 - 4 + 2 = 4$$

$$y[7] = 3x[7] + 2x[6] + x[5] = -6 + 4 - 2 = -4$$

$$y[8] = 3x[8] + 2x[7] + x[6] = 0 - 4 + 2 = -2$$

$$y[9] = 3x[9] + 2x[8] + x[7] = 0 + 0 - 2 = -2$$

