

Islamic University of Gaza
Electrical Engineering Department
EELE 3310 Signals & Systems
Med-Term Exam
November 4, 2012

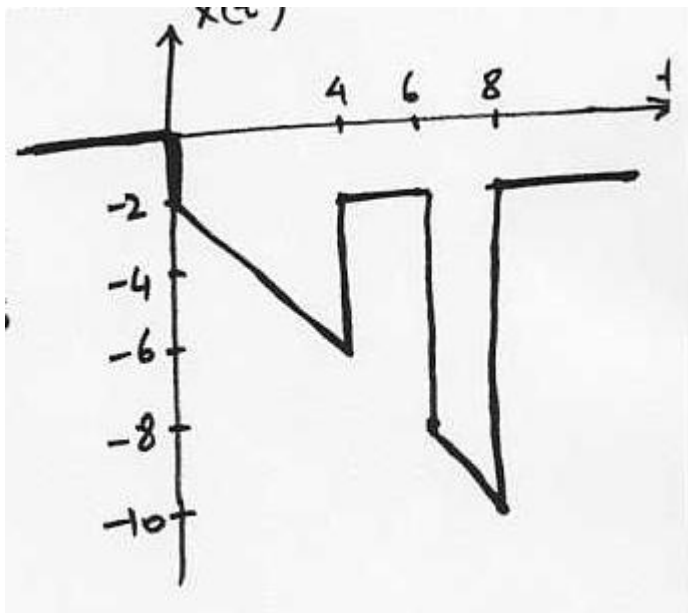


Dr. Basil Hamed
Name:

Exam Time 11:00-12:30
ID #

1. a) Plot the signal; $x(t) = -(t+2)u(t) + t(u(t-4) - u(t-6) + u(t-8))$

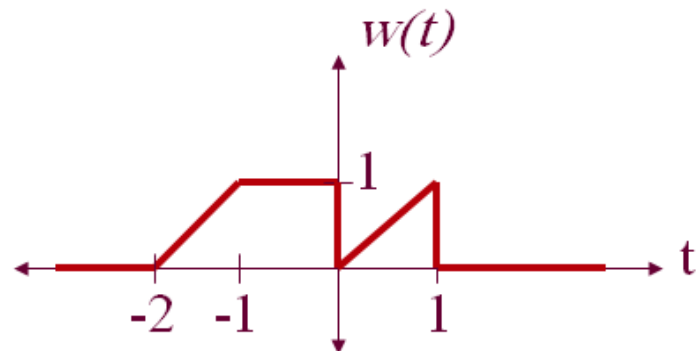
(4 PT.)



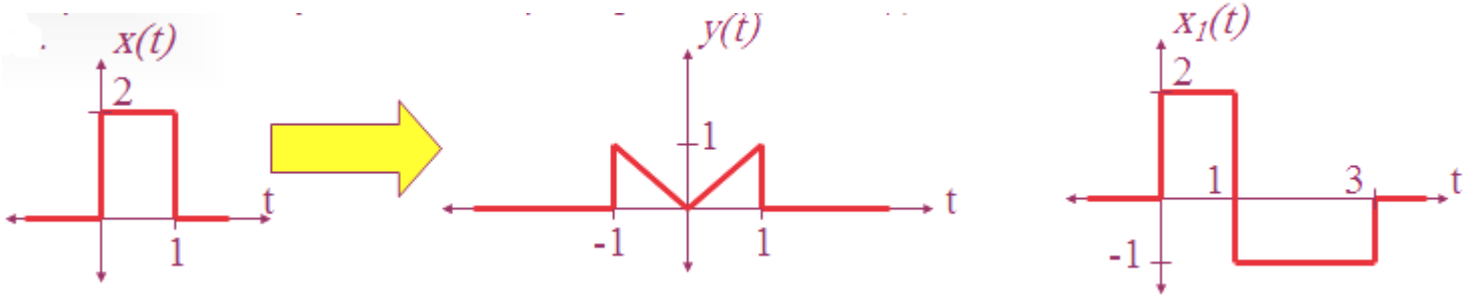
b) Express the signal shown in the Fig by single expression (4 Pt.)

Solution

$$w(t) = (t+2)u(t+2) - (t+1)u(t+1) + (t-1)u(t) - (t-2)u(t-1)$$



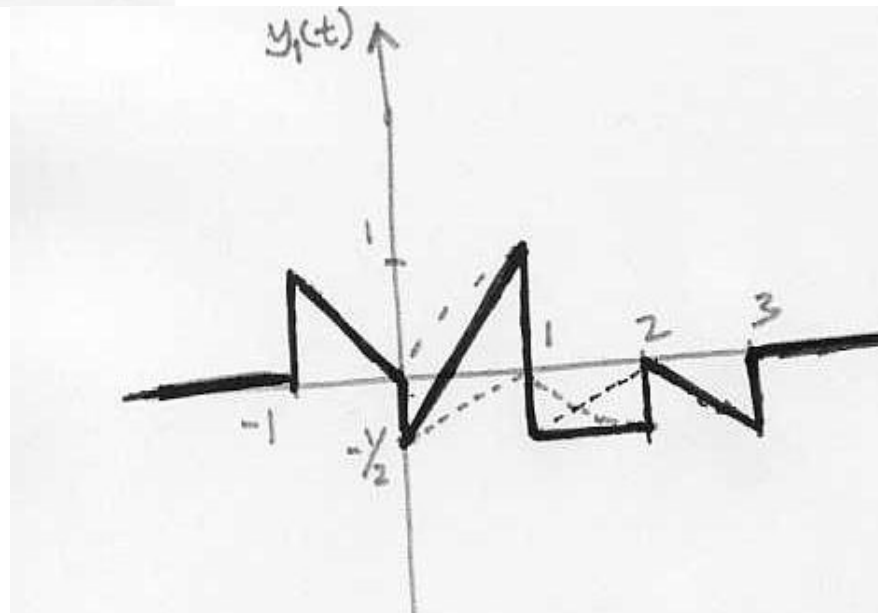
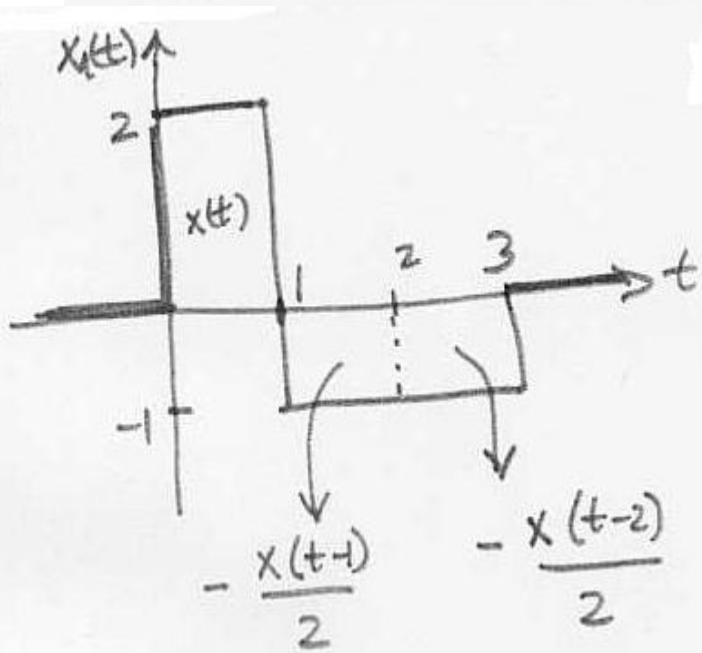
2.a) Given response $y(t)$ of an LTI system to the input signal $x(t)$ below, find response of that system to the input signals $x_1(t)$ shown below. (5 Pt.)



Solution

$$x_1(t) = x(t) - \frac{1}{2}x(t-1) - \frac{1}{2}x(t-2)$$

$$\Rightarrow y_1(t) = y(t) - \frac{y(t-1)}{2} - \frac{y(t-2)}{2}$$



b) Determine if the following systems are time-invariant, linear, causal, and/or memoryless? (4 PT.)

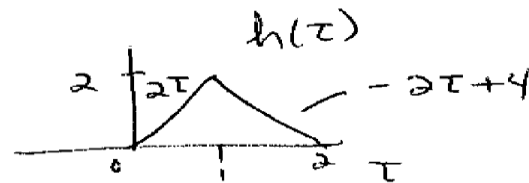
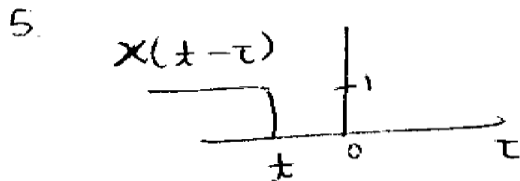
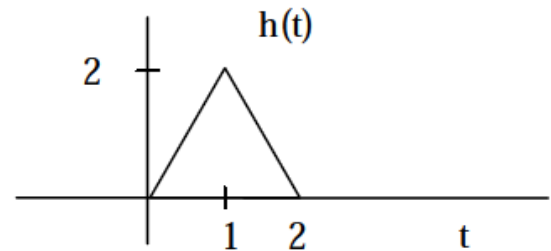
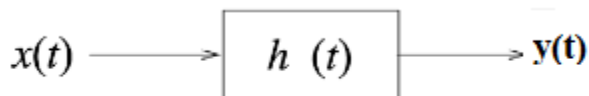
$$y[n + 1] + 4y[n] = 3x[n + 1] - x[n]$$

Solution

Rewrite the equation as $y[n] + 4y[n - 1] = 3x[n] - x[n - 1]$ by decreasing the index.

This is a difference equation with constant coefficients, so it is linear and time-invariant. The output does not depend on future values of the input, so it is causal. It has memory.

3) Determine $y(t)$ where $x(t) = u(t)$ and (7 Pt)



i) $t < 0, y(t) = 0$

ii) $0 \leq t < 1, y(t) = \int_0^t 2\tau d\tau = t^2$

iii) $1 \leq t < 2, y(t) = \int_0^1 2\tau d\tau + \int_1^t (-2\tau + 4) d\tau$
 $= 1 + (-\tau^2 + 4\tau) \Big|_1^t = 1 - t^2 + 4t + 1 - 4$
 $= -2 - t^2 + 4t$

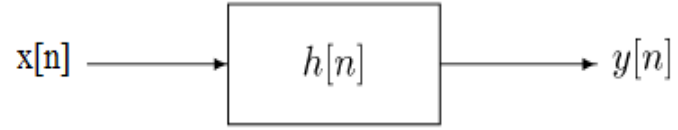
iv) $2 \leq t, y(t) = \int_0^1 2\tau d\tau + \int_1^2 (-2\tau + 4) d\tau$
 $= 2$

$$y(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t < 1 \\ -t^2 + 4t - 2 & 1 \leq t < 2 \\ 2 & 2 \leq t \end{cases}$$

4. Consider the discrete-time system

(6 Pt.)

$$y[n] = \frac{x[n] + x[n-1]}{2}$$



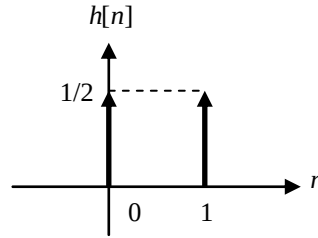
- i. Find the impulse response of the system $h[n]$, expressing it compactly as a function. Sketch the impulse response.
- ii. Find the output $y[n]$ when

$$x[n] = \begin{cases} n, & n = 0, 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

Solution:

The impulse response of the system $h[n]$ is defined as the output of the system when the input is the impulse function $\delta[n]$.

Therefore, $h[n] = \frac{\delta[n] + \delta[n-1]}{2}$. This function is shown below:



- (iii) This is defined as $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$. In that case $x[k]$ is non-zero if $0 \leq k \leq 4$ and $h[n-k]$ is non-zero if $0 \leq n-k \leq 1 \Rightarrow -1 \leq -n+k \leq 0 \Rightarrow n-1 \leq k \leq n$. We may find three separate cases for which the two intervals overlap, and therefore the convolution is non-zero.

1. The lower bound of the function $x[k]$ lies within the bounds of the function $h[n-k]$, i.e., $n-1 \leq 0 \leq n \Rightarrow 0 \leq n \leq 1$.

In that case $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=0}^n \frac{1}{2}k = \frac{n(n+1)}{4}$. (The well known result $\sum_{k=0}^n k = \frac{n(n+1)}{2}$ is used).

2. The bounds of the function $h[n-k]$ are included within the bounds of the function $x[k]$, $n-1 > 0 \Rightarrow n > 1$ and $n \leq 4$, i.e., $1 < n \leq 4$

In that case $y[n] = \sum_{k=n-1}^n \frac{1}{2}k = \frac{1}{2}[(n-1) + n] = \frac{1}{2}(2n-1) = n - \frac{1}{2}$.

3. The upper bound of the function $x[k]$ lies within the bounds of the function $h[n-k]$, i.e., $n-1 \leq 4 < n \Rightarrow 4 < n \leq 5 \Rightarrow n = 5$.

In that case $y[n] = \sum_{k=n-1}^4 \frac{1}{2}k = \sum_{k=5-1}^4 \frac{1}{2}k = 2$.

Thus,

$$y[n] = \begin{cases} 0 & n \leq 0, n \geq 6 \\ 1/2 & n = 1 \\ 3/2 & n = 2 \\ 5/2 & n = 3 \\ 7/2 & n = 4 \\ 2 & n = 5 \end{cases}$$