

Islamic University of Gaza
Electrical Engineering Department
EELE 3310 Signals & Systems
Med-Term Exam
December 2, 2014

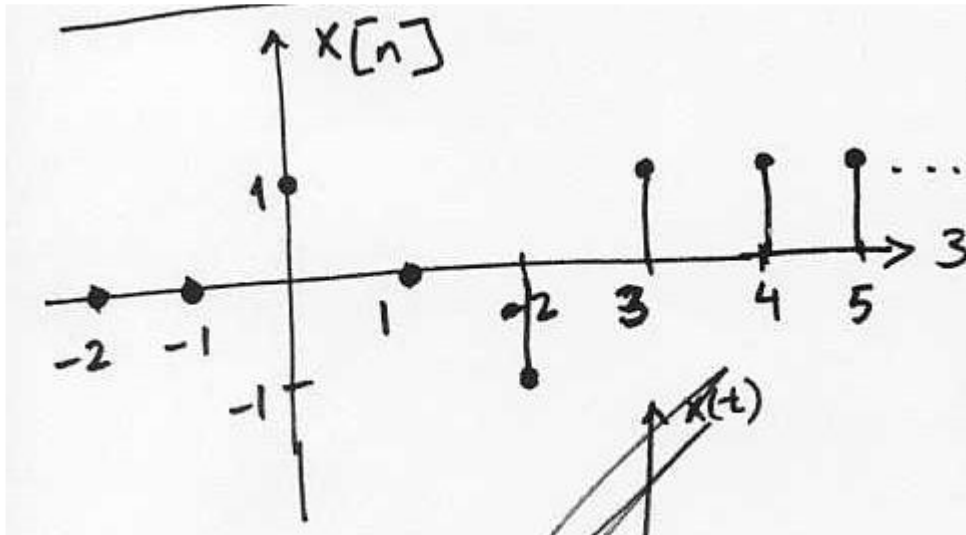


Dr. Basil Hamed
Name:

Exam Time 11:00-12:00
ID #

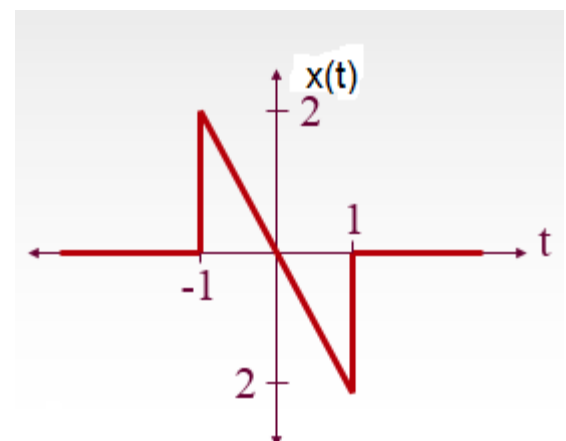
1. a) sketch the following signal $x[n] = (1-n)u[n] + n u[n-3]$

(4 PT)



b) Express the signal shown in the Fig by single expression

(5 PT)



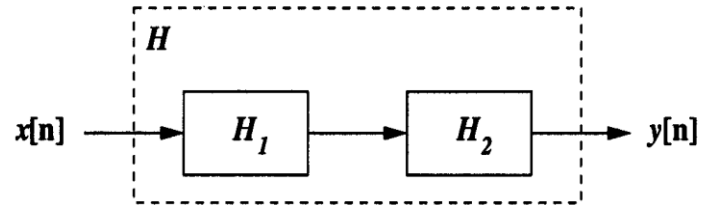
$$x(t) = -2t(u(t+1) - u(t-1))$$

2. a) A new system H is formed by two discrete-time LTI systems H_1 , and H_2 as shown in the figure. (6 Pt)

The impulse response of H_1 is $h_1[n] = 2^n u[-n]$

The impulse response of H_2 is

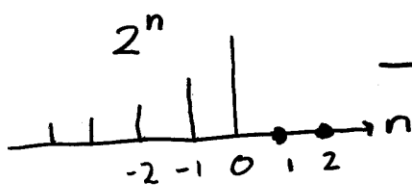
$$h_2[n] = \delta[n] - \frac{1}{2}\delta[n+1]$$



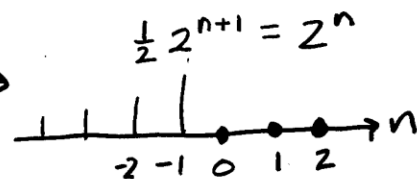
Find the impulse response $h[n]$ of the overall system H

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] \\ &= h_1[n] * \left(\delta[n] - \frac{1}{2}\delta[n+1] \right) \\ &= h_1[n] * \delta[n] - \frac{1}{2} h_1[n] * \delta[n+1] \\ &= h_1[n] - \frac{1}{2} h_1[n+1] \end{aligned}$$

Graph of $h_1[n]$:



Graph of $\frac{1}{2}h_1[n+1]$



$$\begin{aligned} \text{So } h[n] &= 2^n u[-n] - \frac{1}{2} 2^{n+1} u[-(n+1)] \\ &= 2^n u[-n] - 2^n u[-n-1] \end{aligned}$$

$$= \begin{cases} 2^n - 2^n, & n < 0 \\ 2^0 - 0, & n = 0 \\ 0 - 0, & n > 0 \end{cases} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} = \delta[n]$$

$$h[n] = \delta[n]$$

(H_2 is the inverse of system H_1).

b) A discrete time system H has input $x[n]$ and output $y[n]$ related by $y(t) = \frac{1}{2}x(-t)$ (5 PT)

i. Is the system H memoryless? Justify your answer

ii. Is the system H causal? Justify your answer

iii. Is the system H linear? Justify your answer

iv. Is the system H time invariant? Justify your answer

v. Is the system H BIBO stable? Justify your answer

(a) 3 pts. Is the system H memoryless? Justify your answer.

Consider the point $t=1$. Here, the value of the output is $y(1) = \frac{1}{2}x(-1)$, which depends on the input from the prior time $t=-1$. Therefore H is NOT memoryless.

(b) 3 pts. Is the system H causal? Justify your answer.

As shown in part (a), at $t=1$ the output depends on a previous value $x(-1)$ at the input. Therefore H is NOT causal.

(c) 5 pts. Is the system H linear? Justify your answer.

Let $y_1(t) = H\{x_1(t)\} = \frac{1}{2}x_1(-t)$ and
let $y_2(t) = H\{x_2(t)\} = \frac{1}{2}x_2(-t)$. Let $a, b \in \mathbb{C}$
be constants and let $x_3(t) = ax_1(t) + bx_2(t)$.

$$\begin{aligned}\text{Then } y_3(t) &= H\{x_3(t)\} = \frac{1}{2}x_3(-t) \\ &= \frac{1}{2}[ax_1(-t) + bx_2(-t)] \\ &= a\frac{1}{2}x_1(-t) + b\frac{1}{2}x_2(-t) \\ &= ay_1(t) + by_2(t) \checkmark\end{aligned}$$

Therefore, H IS Linear.

$$\text{Now, } y(t) = H\{x(t)\} = \frac{1}{2}x(-t),$$

$$\text{so } |y(t)| = \frac{1}{2}|x(-t)|. \text{ But } |x(t)| \text{ is always } \leq B,$$

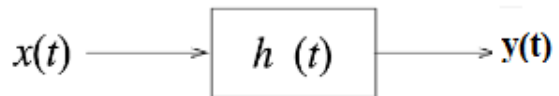
$$\text{so } |y(t)| \leq \frac{1}{2}B. \text{ Therefore } y(t) \text{ is bounded.}$$

Therefore, H
is NOT Time
Invariant.

3) Determine $y(t)$ where $x(t)$ and $h(t)$ is as shown in the figure below.

(10 Pt)

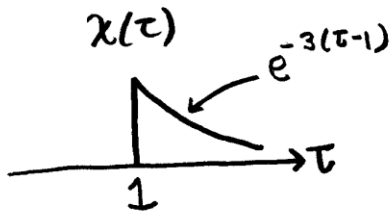
$$x(t) = e^{-3(t-1)} u(t-1)$$



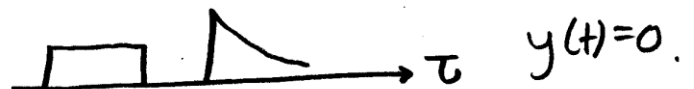
$$h(t) = u(t+2) - u(t-2)$$

Find the system output $y(t)$.

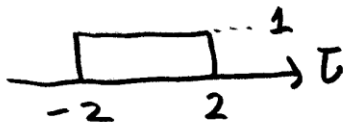
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$



$$\text{I) } t+2 < 1 : t < -1 :$$

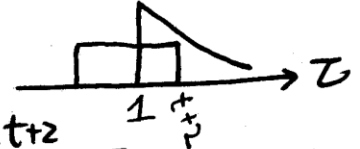


$$h(\tau)$$



$$\text{II) } t+2 \geq 1 \text{ and } t-2 < 1 : t \geq -1 \text{ and } t < 3 : -1 \leq t < 3 :$$

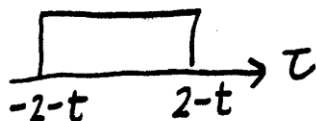
$$y(t) = \int_1^{t+2} e^{-3(\tau-1)} d\tau$$



$$h(\tau-t) = h(t+\tau)$$

$$= \int_1^{t+2} e^{-3\tau} e^3 d\tau = e^3 \int_1^{t+2} e^{-3\tau} d\tau$$

$$= e^3 \left(-\frac{1}{3}\right) \left[e^{-3\tau}\right]_{\tau=1}^{t+2} = -\frac{e^3}{3} \left[e^{-3(t+2)} - e^{-3}\right]$$

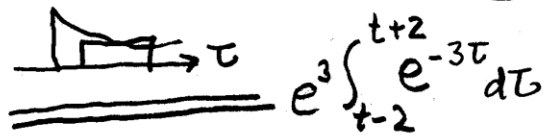


$$= -\frac{1}{3} \left[e^3 e^{-3t} e^{-6} - 1\right] = \frac{1}{3} \left[1 - e^{-3(t+1)}\right]$$

$$h(t-\tau)$$

$$\text{III) } t-2 \geq 1 : t \geq 3 :$$

$$y(t) = \int_{t-2}^{t+2} e^{-3(\tau-1)} d\tau$$



$$= -\frac{e^3}{3} \left[e^{-3\tau}\right]_{\tau=t-2}^{t+2} = -\frac{e^3}{3} \left[e^{-3(t+2)} - e^{-3(t-2)}\right]$$

$$= -\frac{e^3}{3} \left[e^{-3t} e^{-6} - e^{-3t} e^6\right] = \frac{1}{3} \left[e^{-3t} e^9 - e^{-3t} e^{-3}\right]$$

$$= \frac{1}{3} e^{-3t} \left[e^9 - e^{-3}\right]$$

ALL TOGETHER:
$$y(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{3} \left[1 - e^{-3(t+1)}\right] & -1 \leq t < 3 \\ \frac{1}{3} e^{-3t} \left[e^9 - e^{-3}\right] & t \geq 3 \end{cases}$$

More Workspace for Problem 3...

OTHER WAY:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

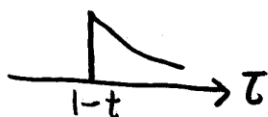
$h(\tau)$



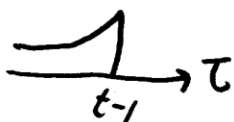
$x(\tau)$



$x(\tau+t)$



$x(t-\tau)$



I) $t-1 < -2$; $t < -1$: $y(t) = 0$

II) $-2 \leq t-1 < 2$; $-1 \leq t < 3$:

$$y(t) = \int_{-2}^{t-1} e^{-3(t-\tau-1)} d\tau = \int_{-2}^{t-1} e^{-3t} e^{3\tau} e^3 d\tau$$

$$= e^{-3t} e^3 \int_{-2}^{t-1} e^{3\tau} d\tau = e^{-3(t-1)} \left(\frac{1}{3} \right) [e^{3\tau}]_{\tau=-2}^{t-1}$$

$$= \frac{1}{3} e^{-3(t-1)} [e^{3(t-1)} - e^{-6}]$$

$$= \frac{1}{3} e^{-3t} e^3 [e^{3t-3} - e^{-6}] = \frac{1}{3} [1 - e^{-3t} e^{-3}]$$

$$= \frac{1}{3} [1 - e^{-3(t+1)}]$$

III) $t-1 \geq 2$; $t \geq 3$:

$$y(t) = \int_{-2}^2 e^{-3(t-\tau-1)} d\tau = e^{-3(t-1)} \int_{-2}^2 e^{3\tau} d\tau$$

$$= \frac{1}{3} e^{-3(t-1)} [e^{3\tau}]_{\tau=-2}^2 = \frac{1}{3} e^{-3(t-1)} [e^6 - e^{-6}]$$

$$= \frac{1}{3} e^{-3t} e^3 [e^6 - e^{-6}] = \frac{1}{3} e^{-3t} [e^9 - e^{-3}]$$

ALL TOGETHER:

$$y(t) = \begin{cases} 0 & , t < -1 \\ \frac{1}{3} [1 - e^{-3(t+1)}] & , -1 \leq t < 3 \\ \frac{1}{3} e^{-3t} [e^9 - e^{-3}] & , t \geq 3 \end{cases}$$