

Electrical Circuit (2)

Introduction (week1 class1)

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FIFTH EDITION

Fundamentals of Electric Circuits



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PART 2

AC Circuits 368

Chapter 9

Sinusoids and Phasors 369

- 9.1 Introduction 370
- 9.2 Sinusoids 371
- 9.3 Phasors 376
- 9.4 Phasor Relationships for
Circuit Elements 385
- 9.5 Impedance and Admittance 387
- 9.6 [†]Kirchhoff's Laws in the Frequency
Domain 389
- 9.7 Impedance Combinations 390
- 9.8 [†]Applications 396
 - 9.8.1 Phase-Shifters
 - 9.8.2 AC Bridges
- 9.9 Summary 402
 - Review Questions 403
 - Problems 403
 - Comprehensive Problems 411

9.1

Introduction

A **sinusoid** is a signal that has the form of the sine or cosine function.

A sinusoidal current is usually referred to as *alternating current (ac)*. Such a current reverses at regular time intervals and has alternately positive and negative values. Circuits driven by sinusoidal current or voltage sources are called *ac circuits*.

9.2 Sinusoids

Consider the sinusoidal voltage

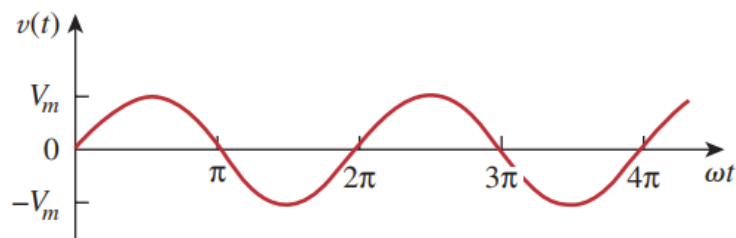
$$v(t) = V_m \sin \omega t$$

where

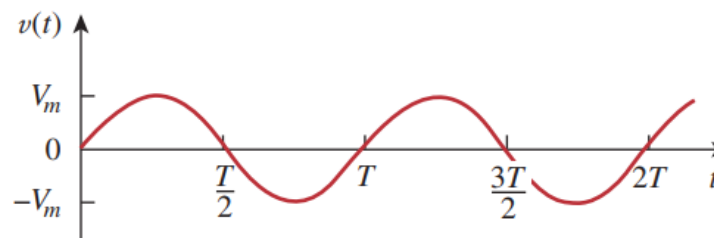
V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

ωt = the *argument* of the sinusoid



(a)



(b)

A **periodic function** is one that satisfies $f(t) = f(t + nT)$, for all t and for all integers n .

$$v(t + T) = v(t)$$

$$\begin{aligned} v(t + T) &= V_m \sin \omega(t + T) = V_m \sin \omega \left(t + \frac{2\pi}{\omega} \right) \\ &= V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t) \end{aligned}$$

$$f = \frac{1}{T} \quad \omega = 2\pi f$$

$$v(t) = V_m \sin(\omega t + \phi)$$

$$v_1(t) = V_m \sin \omega t$$

$$v_2(t) = V_m \sin(\omega t + \phi)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

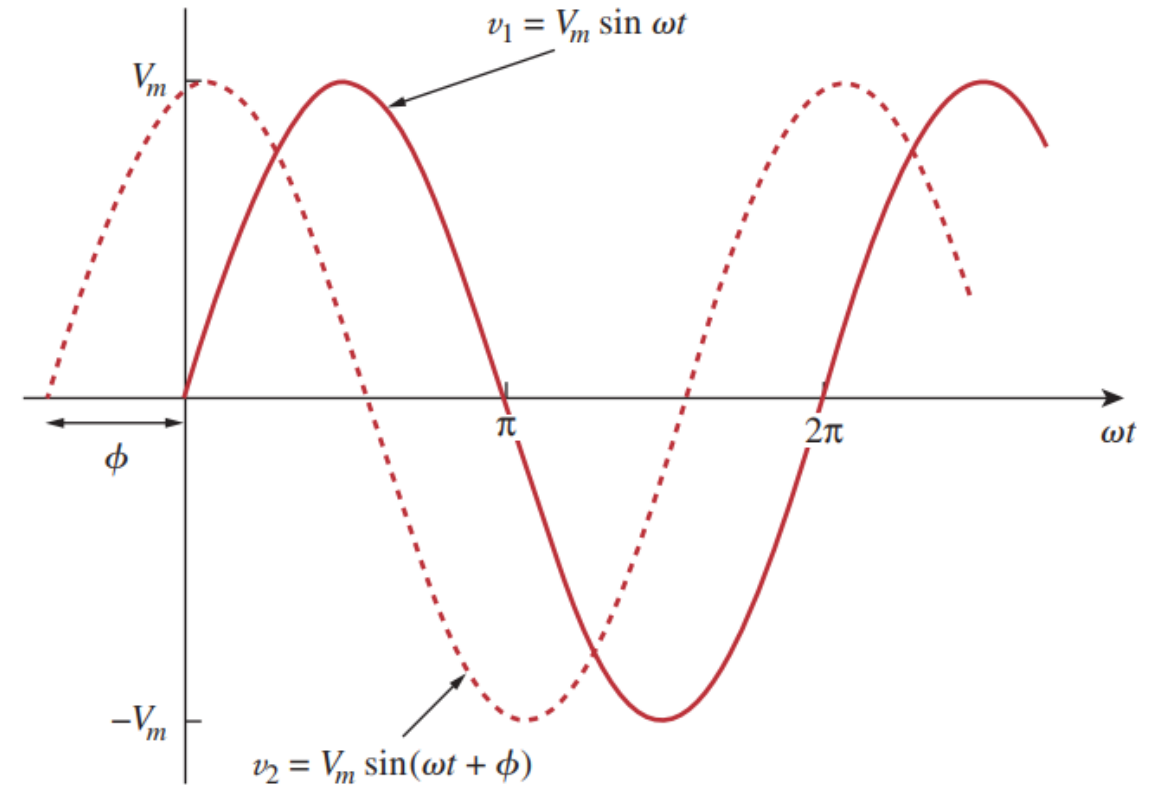


Figure 9.2

Two sinusoids with different phases.

Example 9.1

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12 \cos(50t + 10^\circ)$$

Example 9.1

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12 \cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \frac{1}{T} = 7.958$ Hz.

Given the sinusoid $30 \sin(4\pi t - 75^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Practice Problem 9.1

Practice Problem 9.1

Given the sinusoid $30 \sin(4\pi t - 75^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Answer: 30, -75° , 12.57 rad/s, 0.5 s, 2 Hz.

Calculate the phase angle between $v_1 = -10 \cos(\omega t + 50^\circ)$ and $v_2 = 12 \sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution:

Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

■ **METHOD 1** In order to compare v_1 and v_2 , we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$\begin{aligned} v_1 &= -10 \cos(\omega t + 50^\circ) = 10 \cos(\omega t + 50^\circ - 180^\circ) \\ v_1 &= 10 \cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10 \cos(\omega t + 230^\circ) \end{aligned} \quad (9.2.1)$$

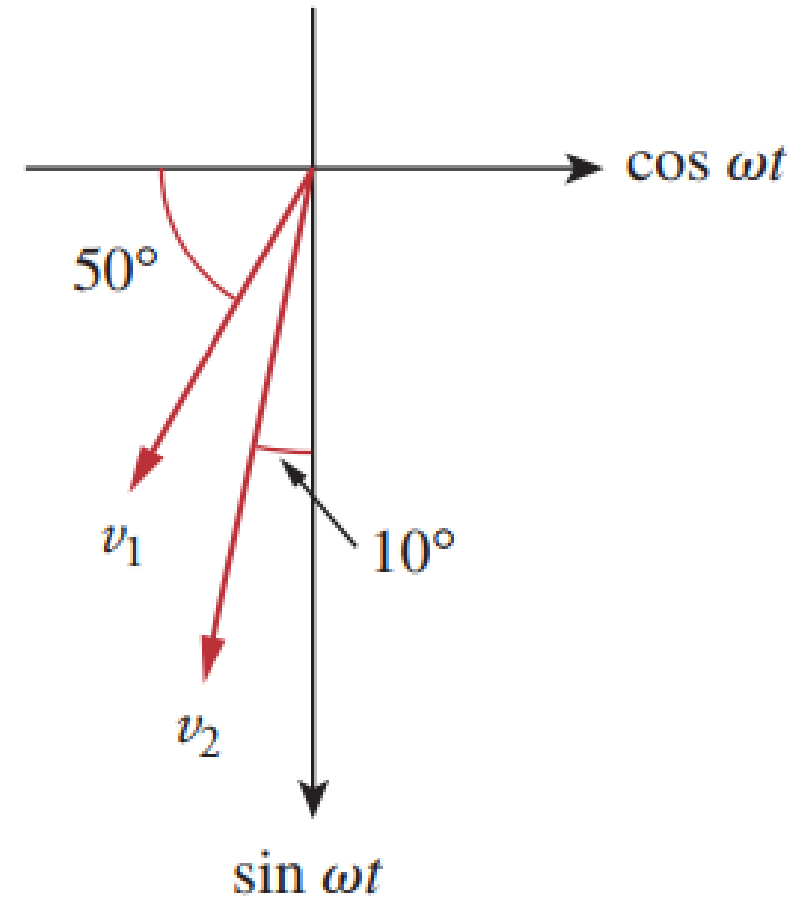
and

$$\begin{aligned} v_2 &= 12 \sin(\omega t - 10^\circ) = 12 \cos(\omega t - 10^\circ - 90^\circ) \\ v_2 &= 12 \cos(\omega t - 100^\circ) \end{aligned} \quad (9.2.2)$$

It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30° . We can write v_2 as

$$v_2 = 12 \cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12 \cos(\omega t + 260^\circ) \quad (9.2.3)$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that v_2 leads v_1 by 30° .

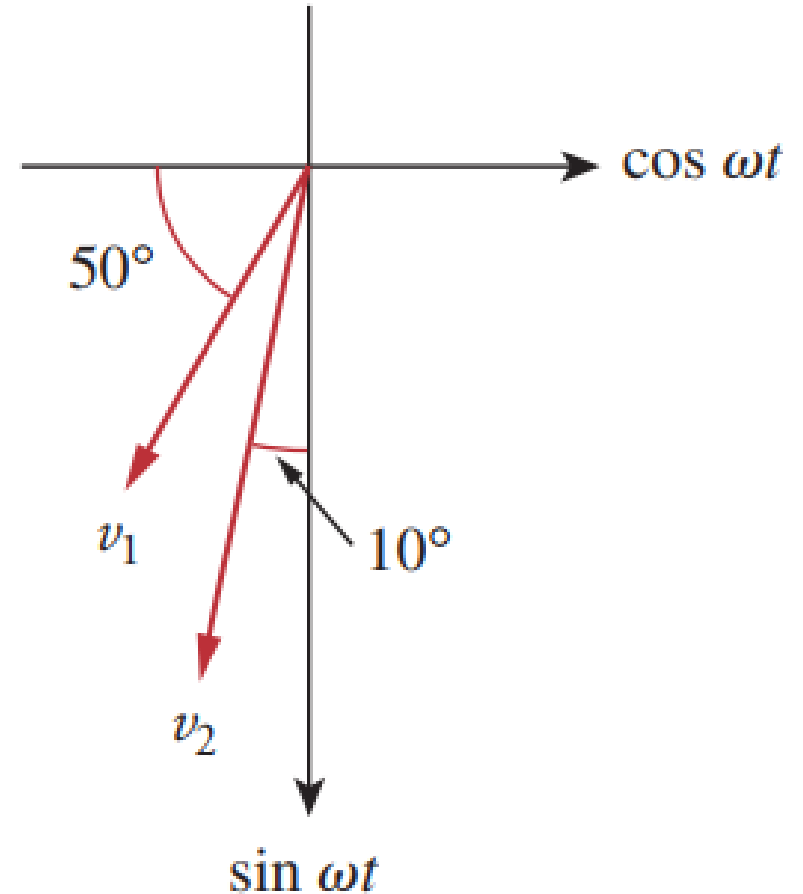


Calculate the phase angle between $v_1 = -10 \cos(\omega t + 50^\circ)$ and $v_2 = 12 \sin(\omega t - 10^\circ)$. State which sinusoid is leading.

■ **METHOD 2** Alternatively, we may express v_1 in sine form:

$$\begin{aligned} v_1 &= -10 \cos(\omega t + 50^\circ) = 10 \sin(\omega t + 50^\circ - 90^\circ) \\ &= 10 \sin(\omega t - 40^\circ) = 10 \sin(\omega t - 10^\circ - 30^\circ) \end{aligned}$$

But $v_2 = 12 \sin(\omega t - 10^\circ)$. Comparing the two shows that v_1 lags v_2 by 30° . This is the same as saying that v_2 leads v_1 by 30° .



Practice Problem 9.2

Find the phase angle between

$$i_1 = -4 \sin(377t + 55^\circ) \quad \text{and} \quad i_2 = 5 \cos(377t - 65^\circ)$$

Does i_1 lead or lag i_2 ?

Practice Problem 9.2

Find the phase angle between

$$i_1 = -4 \sin(377t + 55^\circ) \quad \text{and} \quad i_2 = 5 \cos(377t - 65^\circ)$$

Does i_1 lead or lag i_2 ?

Answer: 210° , i_1 leads i_2 .

