

Electrical Circuit (2)

Introduction (week1 class2)

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PART 2

AC Circuits 368

Chapter 9

Sinusoids and Phasors 369

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9.3 Phasors

A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.

A complex number z can be written in rectangular form as

$$z = x + jy \quad (9.14a)$$

The complex number z can also be written in polar or exponential form as

$$z = r \angle \phi = re^{j\phi} \quad (9.14b)$$

$$z = x + jy \quad \text{Rectangular form}$$

$$z = r \angle \phi \quad \text{Polar form}$$

$$z = re^{j\phi} \quad \text{Exponential form}$$

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1} \frac{y}{x}$$

$$x = r \cos \phi, \quad y = r \sin \phi$$

$$z = x + jy = r \angle \phi = r(\cos \phi + j \sin \phi)$$

$$z = x + jy = r \angle \phi, \quad z_1 = x_1 + jy_1 = r_1 \angle \phi_1 \quad z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \quad (9.18a)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \quad (9.18b)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2 \quad (9.18c)$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2 \quad (9.18d)$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \angle -\phi \quad (9.18e)$$

Square Root:

$$\sqrt{z} = \sqrt{r} \angle \phi/2 \quad (9.18f)$$

Complex Conjugate:

$$z^* = x - jy = r \angle -\phi = re^{-j\phi} \quad (9.18g)$$

$$\frac{1}{j} = -j$$

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Example 9.3

Evaluate these complex numbers:

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

(b) $\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Evaluate these complex numbers:

Example 9.3

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

(b) $\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Solution:

(a) Using polar to rectangular transformation,

$$40\angle 50^\circ = 40(\cos 50^\circ + j \sin 50^\circ) = 25.71 + j30.64$$

$$20\angle -30^\circ = 20[\cos(-30^\circ) + j \sin(-30^\circ)] = 17.32 - j10$$

Adding them up gives

$$40\angle 50^\circ + 20\angle -30^\circ = 43.03 + j20.64 = 47.72\angle 25.63^\circ$$

Taking the square root of this,

$$(40\angle 50^\circ + 20\angle -30^\circ)^{1/2} = 6.91\angle 12.81^\circ$$

(b) Using polar-rectangular transformation, addition, multiplication, and division,

$$\begin{aligned} \frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} &= \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 + j5)} \\ &= \frac{11.66 - j9}{-14 + j22} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ} \\ &= 0.565\angle -160.13^\circ \end{aligned}$$

Practice Problem 9.3

Evaluate the following complex numbers:

(a) $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^*$

(b) $\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ + j5$

Practice Problem 9.3

Evaluate the following complex numbers:

(a) $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^*$

(b) $\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ + j5$

Answer: (a) $-15.5 - j13.67$, (b) $8.293 + j7.2$.

9.4

Phasor Relationships for Circuit Elements

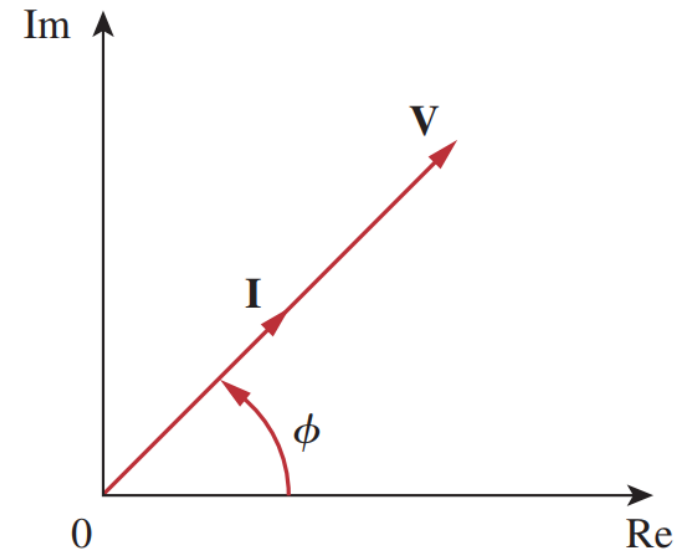
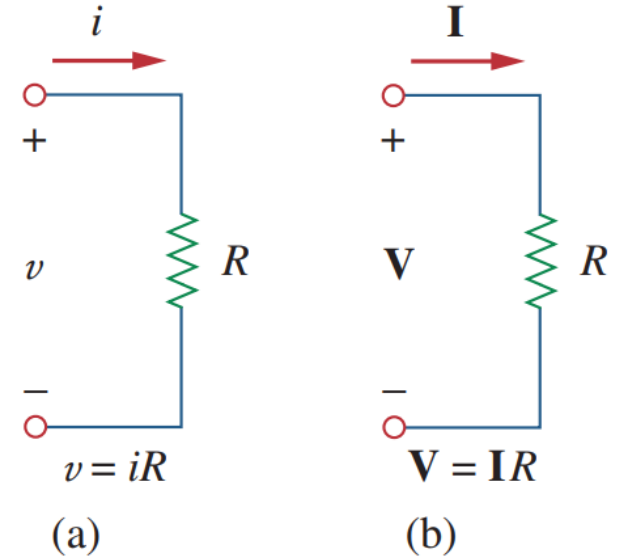
9.4

Phasor Relationships for Circuit Elements

$$v = iR = RI_m \cos(\omega t + \phi)$$

$$\mathbf{V} = RI_m \angle \phi$$

$$\mathbf{V} = R\mathbf{I}$$

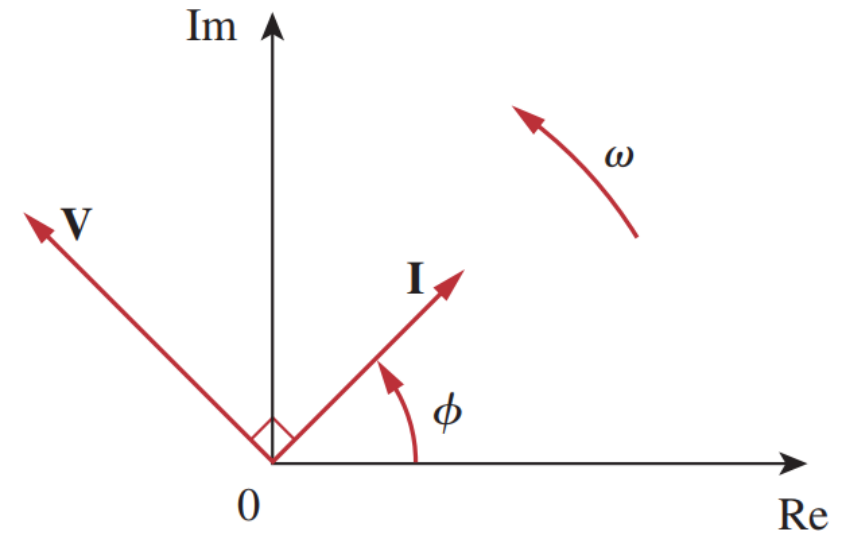
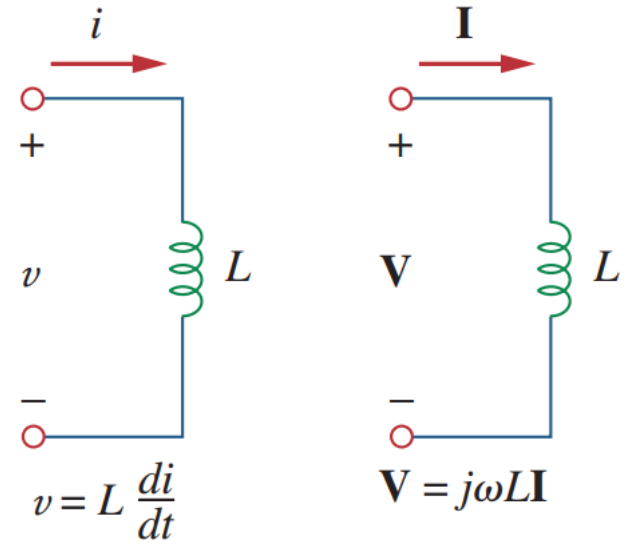


9.4

Phasor Relationships for Circuit Elements

$$v = L \frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

$$\mathbf{V} = j\omega L \mathbf{I}$$

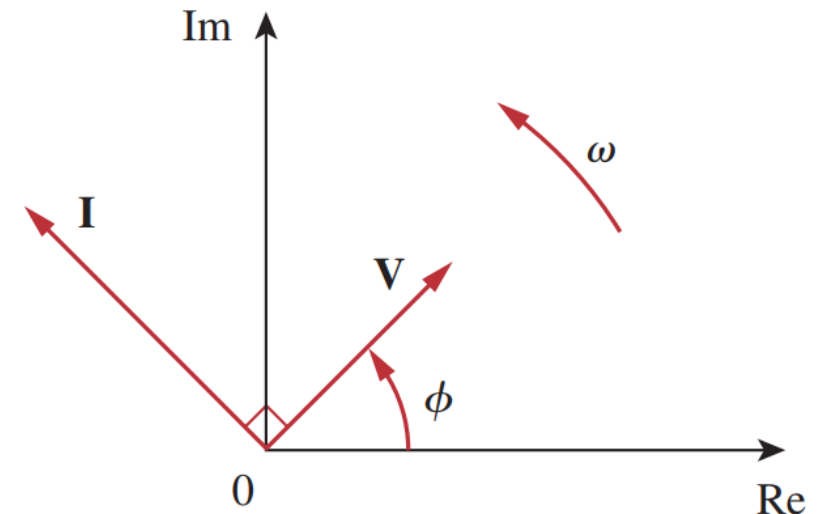
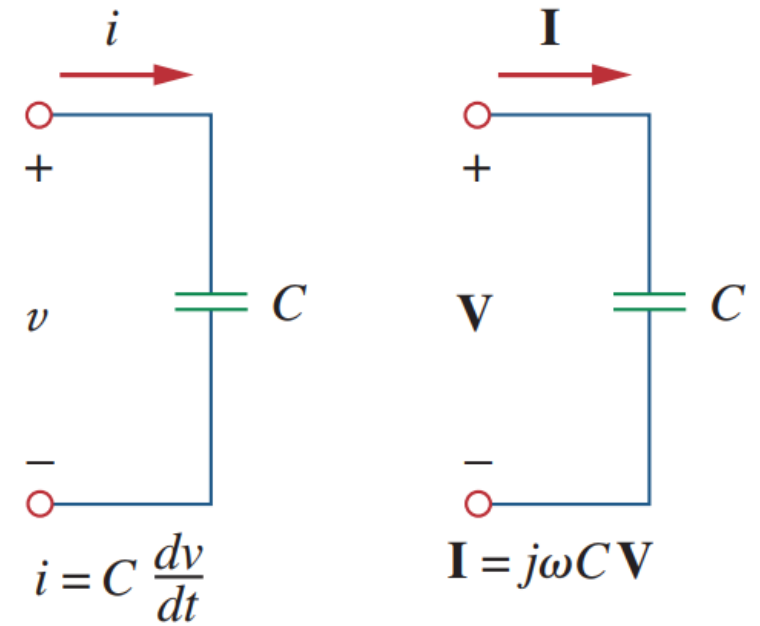


9.4

Phasor Relationships for Circuit Elements

$$i = C \frac{dv}{dt}$$

$$\mathbf{I} = j\omega C \mathbf{V} \quad \Rightarrow \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$



9.4**Phasor Relationships
for Circuit Elements****TABLE 9.2**

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L \frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C \frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

9.4

Phasor Relationships for Circuit Elements

Example 9.8

The voltage $v = 12 \cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\angle 45^\circ$ V. Hence,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\angle 45^\circ}{j60 \times 0.1} = \frac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ \text{ A}$$

Converting this to the time domain,

$$i(t) = 2 \cos(60t - 45^\circ) \text{ A}$$

9.4

Phasor Relationships for Circuit Elements

Practice Problem 9.8

If voltage $v = 10 \cos(100t + 30^\circ)$ is applied to a $50 \mu\text{F}$ capacitor, calculate the current through the capacitor.

Answer: $50 \cos(100t + 120^\circ) \text{ mA}$.

