

Electrical Circuit (2)

Impedance and Admittance (week2 class1)

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PART 2

AC Circuits 368

Chapter 9

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9.5

Impedance and Admittance

$$V = RI, \quad V = j\omega LI, \quad V = \frac{I}{j\omega C}$$

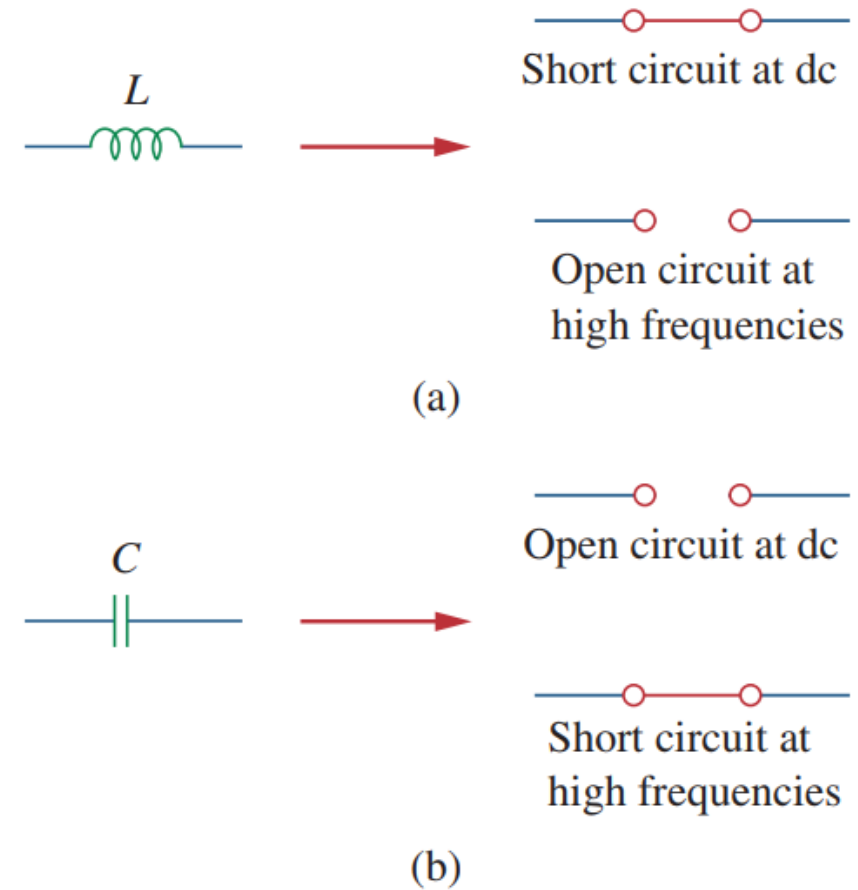
$$\frac{V}{I} = R, \quad \frac{V}{I} = j\omega L, \quad \frac{V}{I} = \frac{1}{j\omega C}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{ZI}$$

$$\mathbf{Z} = R + jX$$

$$\mathbf{Z} = |\mathbf{Z}| \angle \theta$$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1} \frac{X}{R}$$



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Impedance and Admittance

The **admittance** $\mathbf{Y}$ is the reciprocal of impedance, measured in siemens (S).

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$

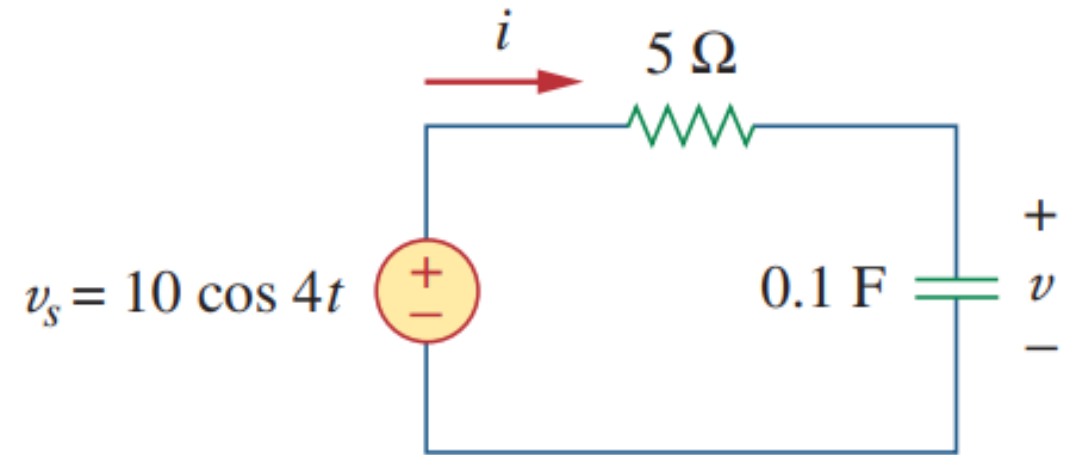
$$\mathbf{Y} = G + jB$$

where $G = \text{Re } \mathbf{Y}$ is called the *conductance* and $B = \text{Im } \mathbf{Y}$ is called the *susceptance*.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

9.5

Impedance and Admittance



Solution:

From the voltage source $10 \cos 4t$, $\omega = 4$,

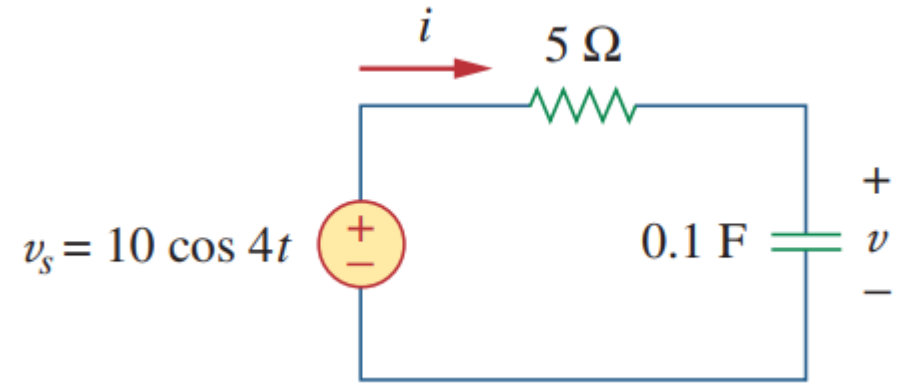
$$\mathbf{V}_s = 10 \angle 0^\circ \text{ V}$$

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5 \, \Omega$$

$$\begin{aligned} \mathbf{I} &= \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10 \angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \\ &= 1.6 + j0.8 = 1.789 \angle 26.57^\circ \text{ A} \end{aligned} \quad (9.9.1)$$

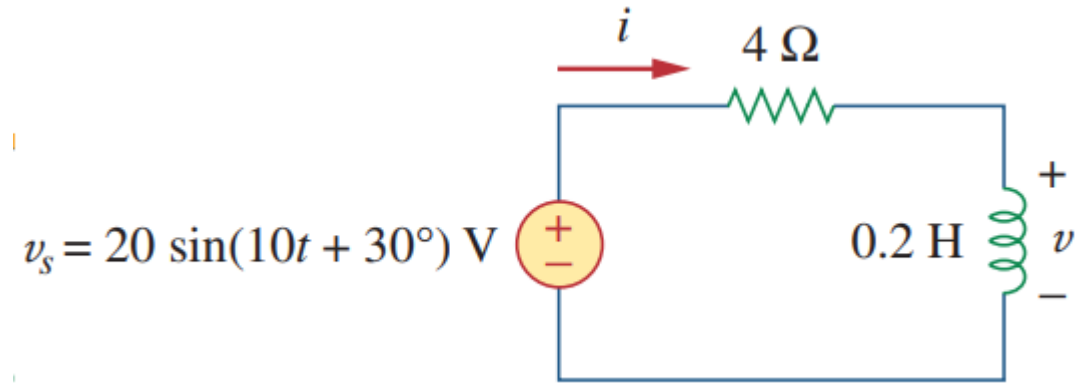
The voltage across the capacitor is

$$\begin{aligned} \mathbf{V} &= \mathbf{I} \mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789 \angle 26.57^\circ}{j4 \times 0.1} \\ &= \frac{1.789 \angle 26.57^\circ}{0.4 \angle 90^\circ} = 4.47 \angle -63.43^\circ \text{ V} \end{aligned} \quad (9.9.2)$$



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Impedance and Admittance

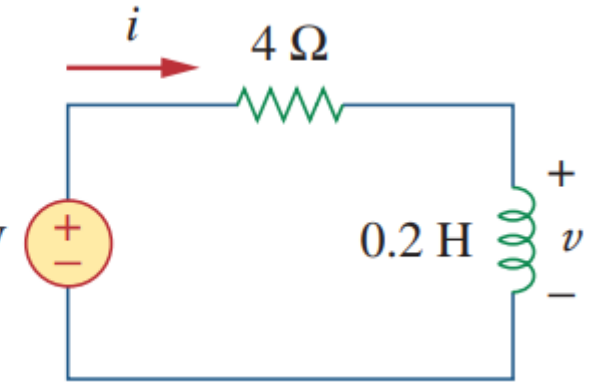


9.5

Impedance and Admittance

Answer: $8.944 \sin(10t + 93.43^\circ)$ V, $4.472 \sin(10t + 3.43^\circ)$ A.

$$v_s = 20 \sin(10t + 30^\circ) \text{ V}$$



9.6

† Kirchhoff's Laws in the Frequency Domain

$$v_1 + v_2 + \cdots + v_n = 0$$

$$V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2) \\ + \cdots + V_{mn} \cos(\omega t + \theta_n) = 0$$

$$\text{Re}(V_{m1}e^{j\theta_1}e^{j\omega t}) + \text{Re}(V_{m2}e^{j\theta_2}e^{j\omega t}) + \cdots + \text{Re}(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

$$\text{Re}[(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n})e^{j\omega t}] = 0$$

$$\text{Re}[(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}] = 0$$

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

$$i_1 + i_2 + \cdots + i_n = 0$$

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0$$

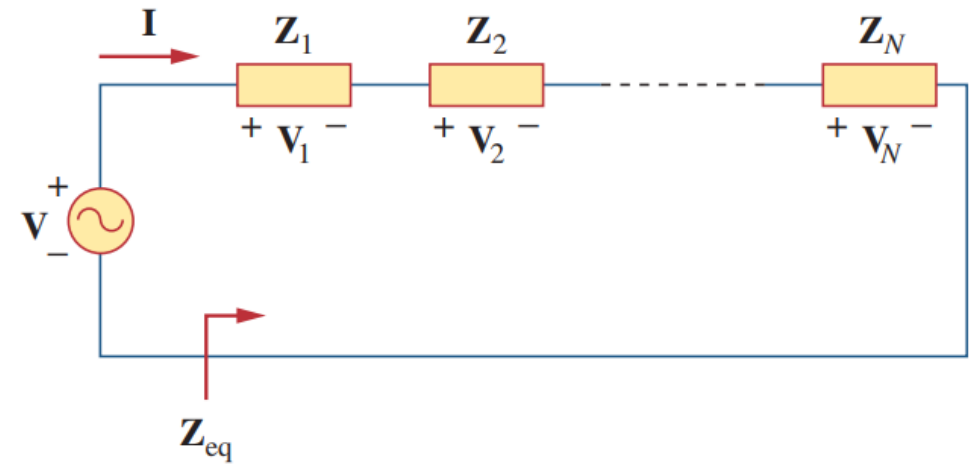
9.7 Impedance Combinations

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Since $\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I}$ and $\mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}$, then

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}, \quad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N = \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N)$$



$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

9.7 Impedance Combinations

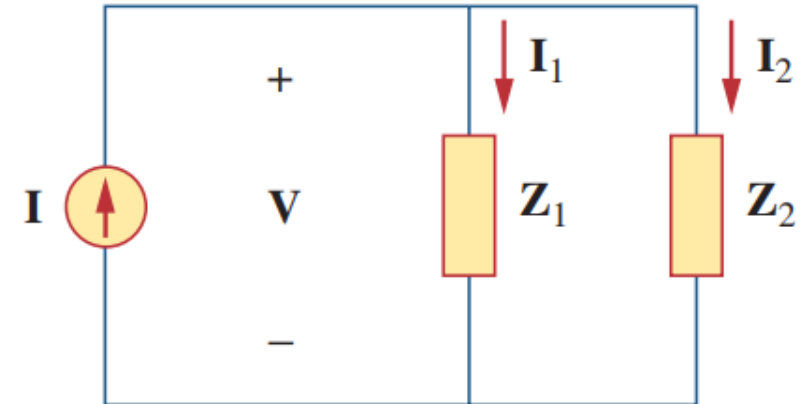
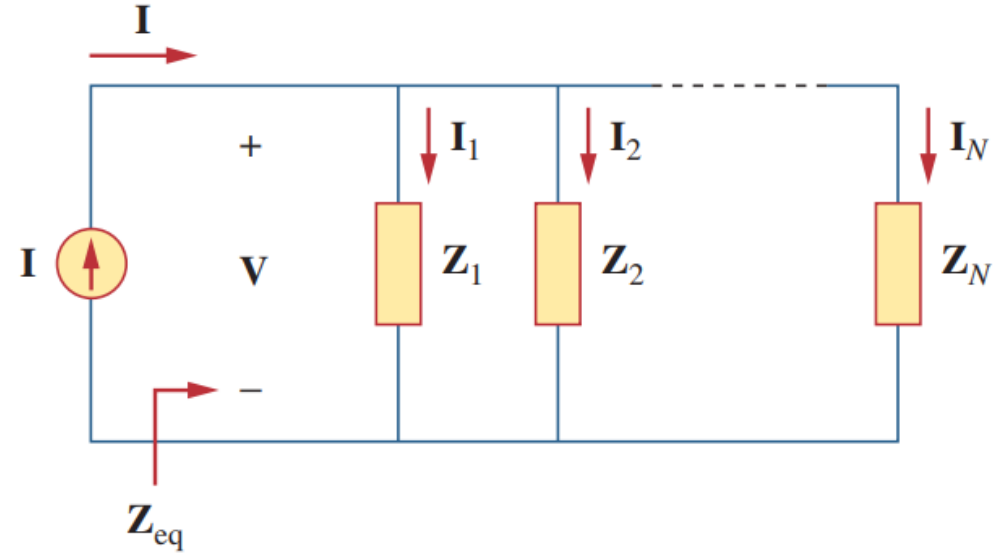
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = \mathbf{V} \left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N} \right)$$

$$\frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{\text{eq}} = \mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2$$

$$\mathbf{Z}_{\text{eq}} = \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{1/\mathbf{Z}_1 + 1/\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}, \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}$$

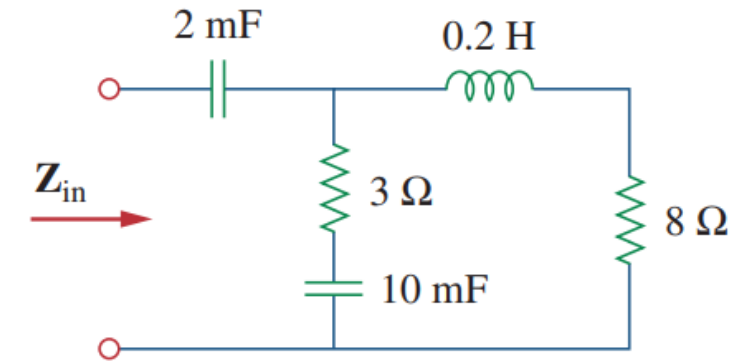


Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

Z_1 = Impedance of the 2-mF capacitor

Z_2 = Impedance of the 3- Ω resistor in series with the 10-mF capacitor

Z_3 = Impedance of the 0.2-H inductor in series with the 8- Ω resistor



Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

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$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8- Ω resistor

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10 \Omega$$

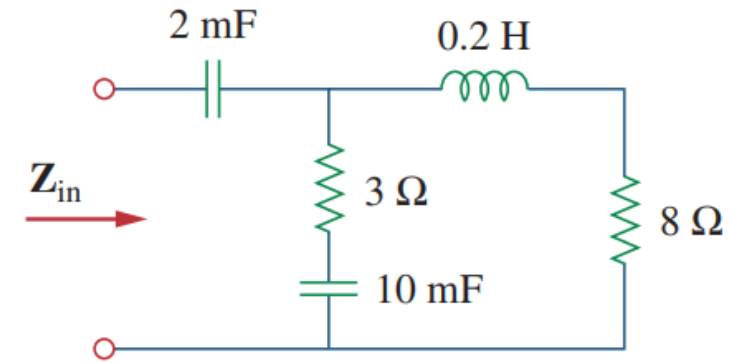
$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2) \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10) \Omega$$

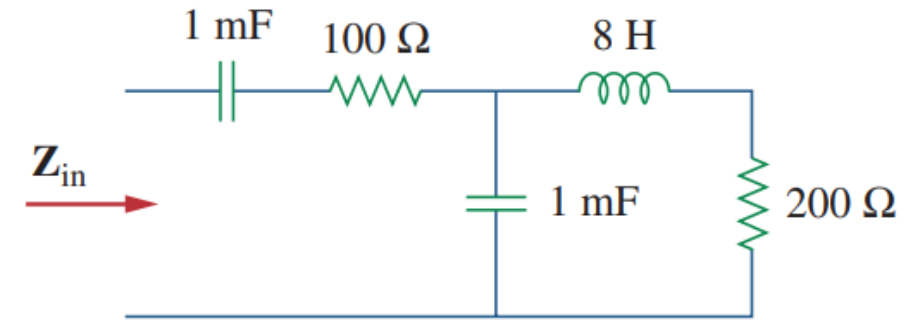
$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07 \Omega$$

$$\mathbf{Z}_{\text{in}} = 3.22 - j11.07 \Omega$$



Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 10$ rad/s.



Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 10$ rad/s.

Answer: $(149.52 - j195)$

