

Electrical Circuit (2)

Average Power (week5 class1)

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11.2 Instantaneous and Average Power

$$p(t) = v(t)i(t)$$

The **instantaneous power** (in watts) is the power at any instant of time.

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

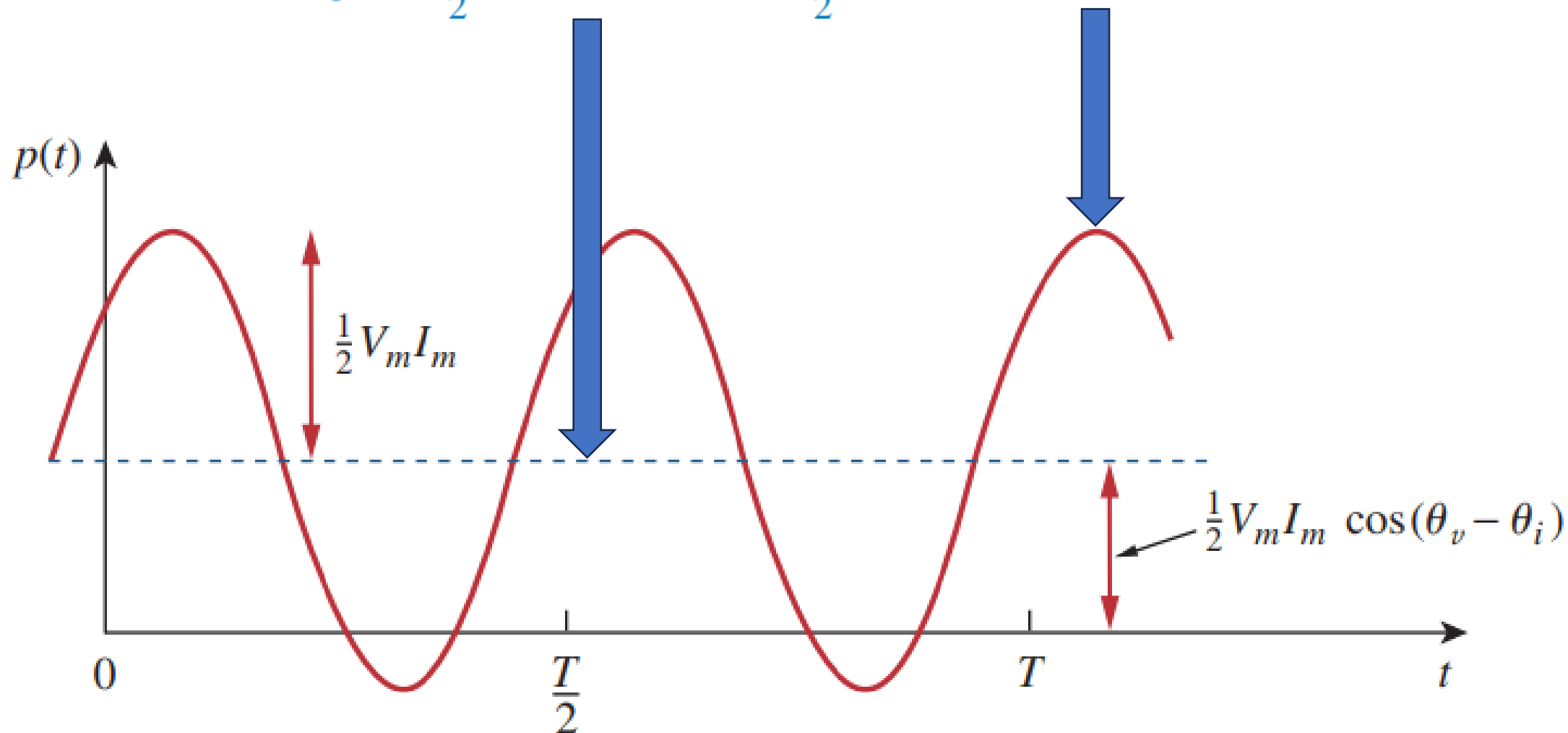
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

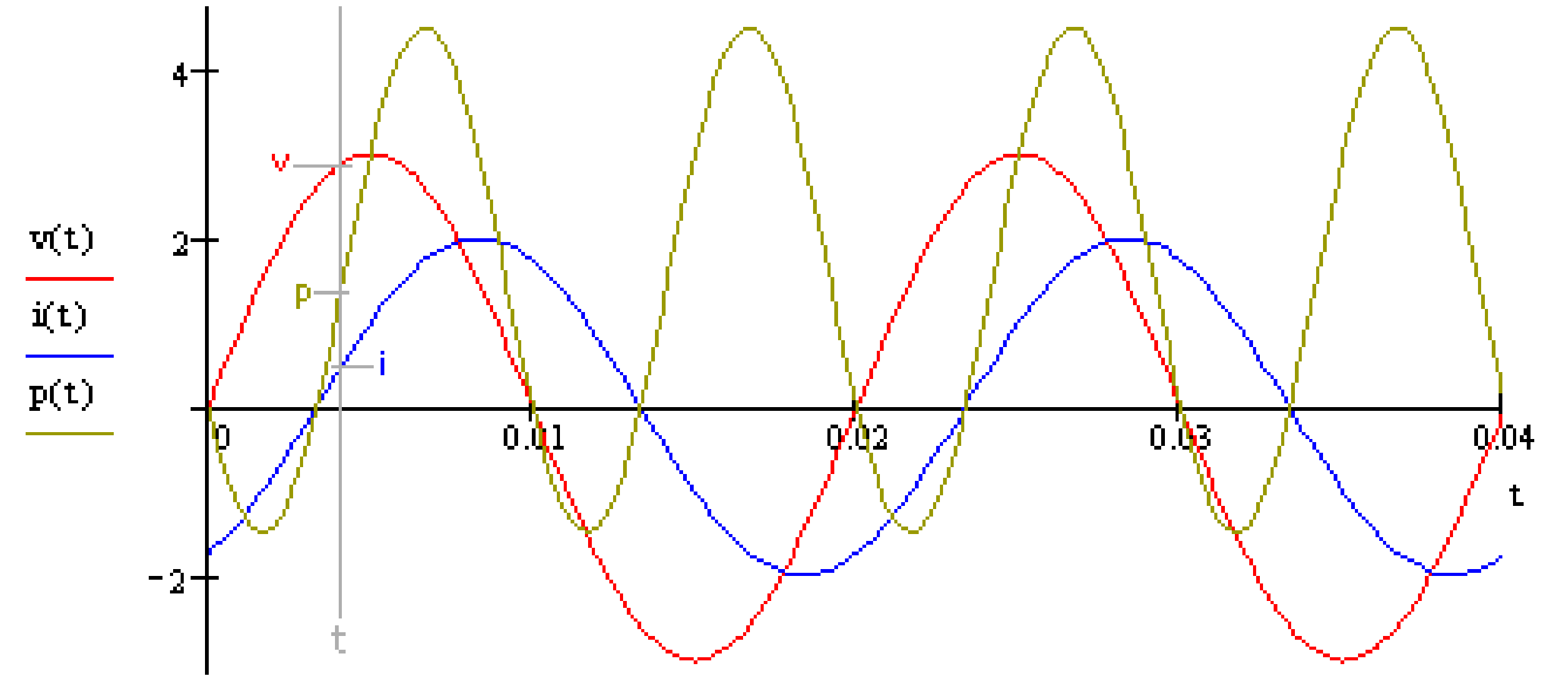
$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

11.2 Instantaneous and Average Power

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11.2 Instantaneous and Average Power



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$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\frac{1}{2} \mathbf{V} \mathbf{I}^* = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i = \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$$

$$P = \frac{1}{2} \text{Re}[\mathbf{V} \mathbf{I}^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

11.2 Instantaneous and Average Power

Given that

$$v(t) = 120 \cos(377t + 45^\circ) \text{ V} \quad \text{and} \quad i(t) = 10 \cos(377t - 10^\circ) \text{ A}$$

find the instantaneous power and the average power

Example 11.1

11.2 Instantaneous and Average Power

Given that

$$v(t) = 120 \cos(377t + 45^\circ) \text{ V} \quad \text{and} \quad i(t) = 10 \cos(377t - 10^\circ) \text{ A}$$

find the instantaneous power and the average power

Example 11.1

$$p = vi = 1200 \cos(377t + 45^\circ) \cos(377t - 10^\circ)$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$p = 600 [\cos(754t + 35^\circ) + \cos 55^\circ]$$

Instantaneous power $\rightarrow p(t) = 344.2 + 600 \cos(754t + 35^\circ) \text{ W}$

Average power

11.2 Instantaneous and Average Power

Practice Problem 11.1

Calculate the instantaneous power and average power absorbed by the passive linear network of Fig. 11.1 if

$$v(t) = 330 \cos(10t + 20^\circ) \text{ V} \quad \text{and} \quad i(t) = 33 \sin(10t + 60^\circ) \text{ A}$$

Answer: $3.5 + 5.445 \cos(20t - 10^\circ) \text{ kW}$, 3.5 kW .

11.2 Instantaneous and Average Power

Practice Problem 11.1

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Answer: $3.5 + 5.445 \cos(20t - 10^\circ) \text{ kW}$, 3.5 kW .

$$i(t) = 33 \sin(10t + 60 + 90 - 90) = 33 \cos(10t - 30)$$

$$p(t) = 330 * 33 * (\cos 10t + 20) \cos(10t - 30)$$

Instantaneous power $\rightarrow$
$$p(t) = \frac{330 * 33}{2} * (\cos 50) + \frac{330 * 33}{2} \cos(20t - 10)$$

$\uparrow$
Average power

11.2 Instantaneous and Average Power

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70 \, \Omega$ when a voltage $\mathbf{V} = 120 \angle 0^\circ$ is applied across it.

Example 11.2

11.2 Instantaneous and Average Power

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Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70 \Omega$ when a voltage $\mathbf{V} = 120\angle 0^\circ$ is applied across it.

Solution:

The current through the impedance is

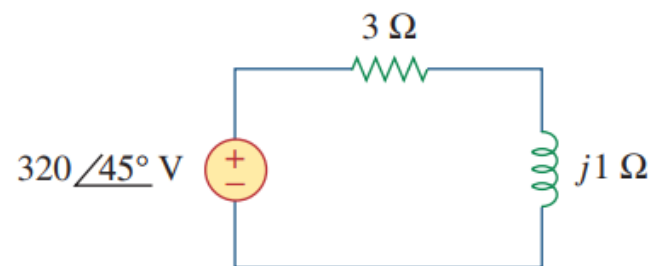
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\angle 0^\circ}{30 - j70} = \frac{120\angle 0^\circ}{76.16\angle -66.8^\circ} = 1.576\angle 66.8^\circ \text{ A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576) \cos(0 - 66.8^\circ) = 37.24 \text{ W}$$

11.2 Instantaneous and Average Power

Practice Problem 11.3

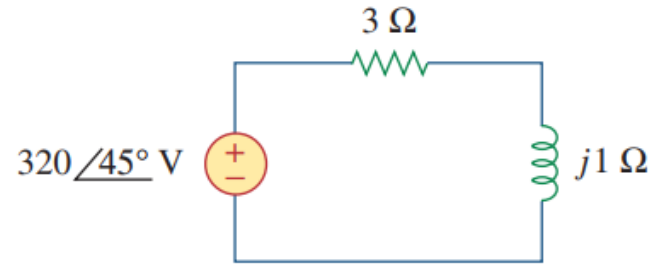


In the circuit of Fig. 11.4, calculate the average power absorbed by the resistor and inductor. Find the average power supplied by the voltage source.

Answer: 15.361 kW, 0 W, 15.361 kW.

11.2 Instantaneous and Average Power

Practice Problem 11.3



In the circuit of Fig. 11.4, calculate the average power absorbed by the resistor and inductor. Find the average power supplied by the voltage source.

Answer: 15.361 kW, 0 W, 15.361 kW.

$$i_X = \frac{320\angle 45}{3 + j} = 90.51 + j45.25 = 101.19\angle 26.56^\circ \text{ A}$$

$$V_{@3\Omega} = i_X * R = (90.51 + j45.25) * 3 = 271.53 + j135.76 = 303.58\angle 26.56^\circ \text{ V}$$

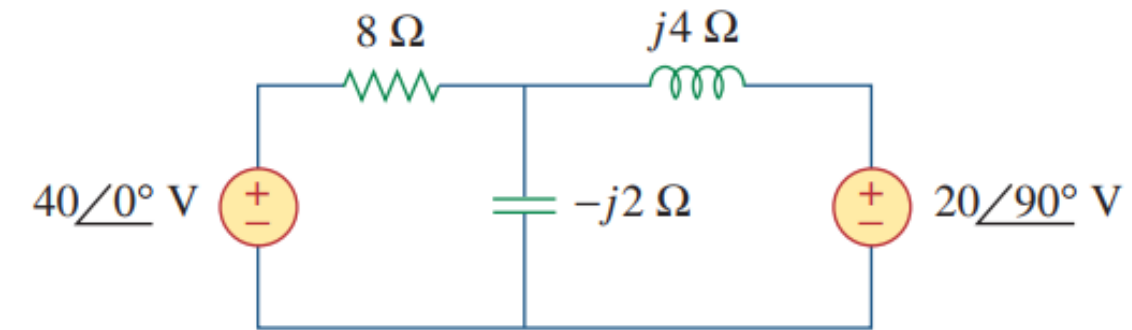
$$V_{@j\Omega} = i_X * R = (90.51 + j45.25) * j = -45.25 + j90.51 = 101.19\angle 116.56^\circ \text{ V}$$

$$P_{@3\Omega} = V_{@3\Omega} * i_X = \frac{1}{2} * (303.58) * (101.19) * \cos(26.56^\circ - 26.56^\circ) = 15359 \text{ W}$$

$$P_{@j\Omega} = V_{@j\Omega} * i_X = \frac{1}{2} * (101.19) * (101.19) * \cos(116.56^\circ - 26.56^\circ) = 0 \text{ W}$$

11.2 Instantaneous and Average Power

Calculate the average power absorbed by each of the five elements in the circuit of Fig. 11.6.



11.2 Instantaneous and Average Power

Calculate the average power absorbed by each of the five elements in the circuit of Fig. 11.6.

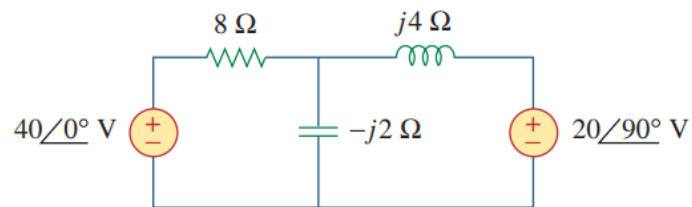


Figure 11.6
For Practice Prob. 11.4.

Answer: 40-V Voltage source: -60 W; $j20$ -V Voltage source: -40 W; resistor: 100 W; others: 0 W.

$$\frac{V_x - 40\angle 0}{8} + \frac{V_x}{-j2} + \frac{V_x - 20\angle 90}{j4} = 0$$

$$\frac{V_x}{8} + \frac{V_x}{-j2} + \frac{V_x}{j4} = \frac{40\angle 0}{8} + \frac{20\angle 90}{j4} \rightarrow (0.125 + j0.25)V_x = 10$$

$$V_x = \frac{10}{0.125 + j0.25} = 16 - 32j = 35.78 \angle -63.435$$

$$I_{@8\Omega} = \frac{16 - 32j - 40}{8} = \frac{40\angle -126.9}{8} = 5\angle -126.9 \text{ V}$$

$$I_{@-2j\Omega} = \frac{16 - 32j}{-2j} = \frac{35.78\angle -63.435}{-2j} = 17.89\angle 26.57 \text{ V}$$

$$I_{@4j\Omega} = \frac{16 - 32j - 20\angle 90}{4j} = -\frac{54.4\angle -72.9}{4j} = 13.6\angle -162.9 \text{ V}$$

$$P_{@8\Omega} = \frac{1}{2} 40 * 5 * \cos(-126.9 + 126.9) = 100 \text{ watt}$$

$$P_{@-2j\Omega} = \frac{1}{2} 35.78 * 17.89 * \cos(-63.44 - 26.56) = 0 \text{ watt}$$

$$P_{@4j\Omega} = \frac{1}{2} 54.4 * 13.6 * \cos(-72.9 + 162.9) = 0 \text{ watt}$$

$$P_{@40V} = \frac{1}{2} 40 * 5 * \cos(0 + 126.9) = -60 \text{ watt}$$

$$P_{@20jV} = \frac{1}{2} 20 * 13.6 * \cos(90 + 162.9) = -40 \text{ watt}$$

