

Electrical Circuit (2)

Maximum Average Power Transfer (week5 class2)

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11.3

Maximum Average Power Transfer

$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L$$

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\text{Th}}}{(R_{\text{Th}} + jX_{\text{Th}}) + (R_L + jX_L)}$$

$$P = \frac{1}{2} |\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{\text{Th}}|^2 R_L / 2}{(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2}$$

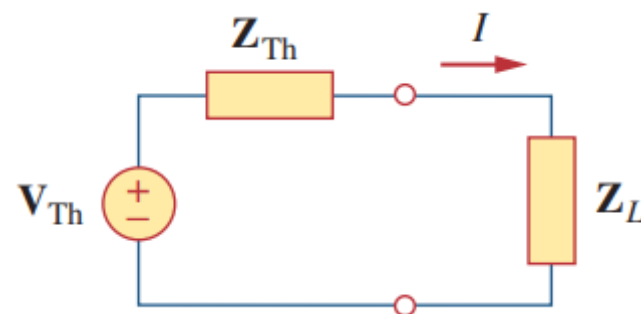
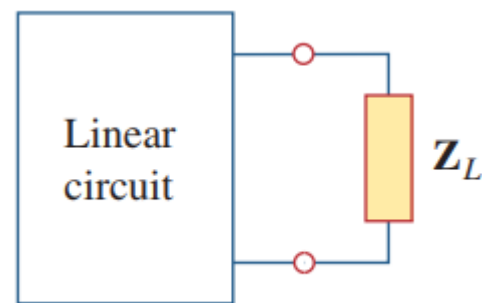
$$X_L = -X_{\text{Th}} \quad R_L = \sqrt{R_{\text{Th}}^2 + (X_{\text{Th}} + X_L)^2}$$

$$\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*$$

For **maximum average power transfer**, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.

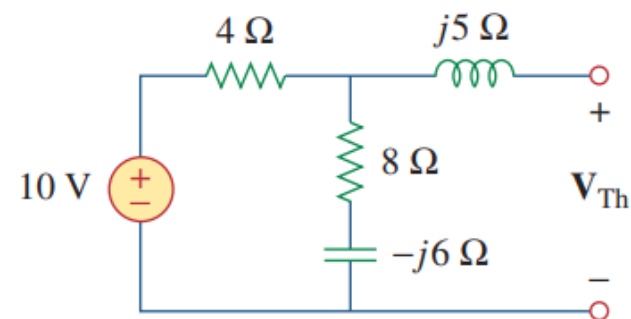
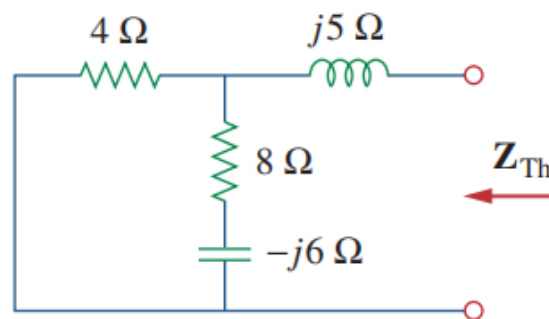
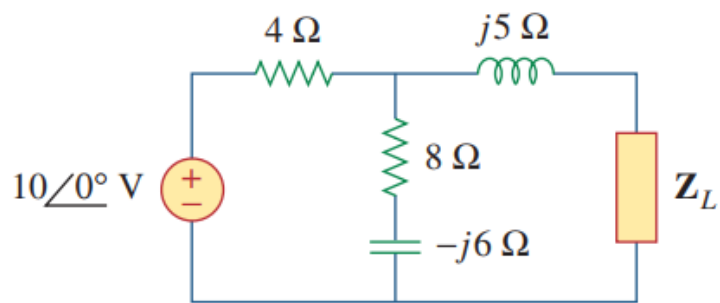
$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}$$

$$R_L = \sqrt{R_{\text{Th}}^2 + X_{\text{Th}}^2} = |\mathbf{Z}_{\text{Th}}|$$

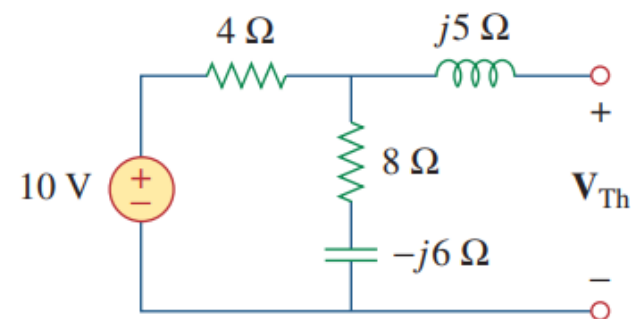
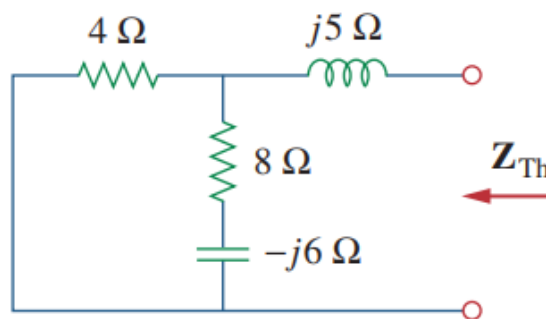
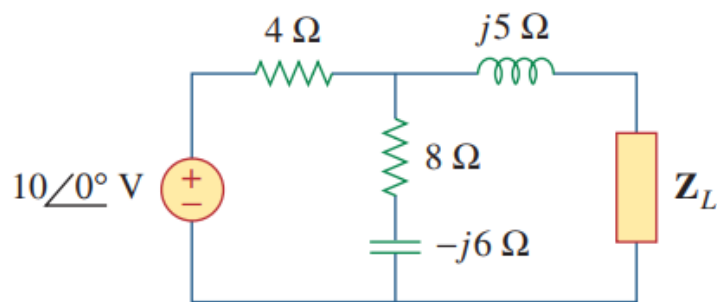


11.3 Maximum Average Power Transfer

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?



11.3 Maximum Average Power Transfer



$$\mathbf{Z}_{Th} = j5 + 4 \parallel (8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467 \, \Omega$$

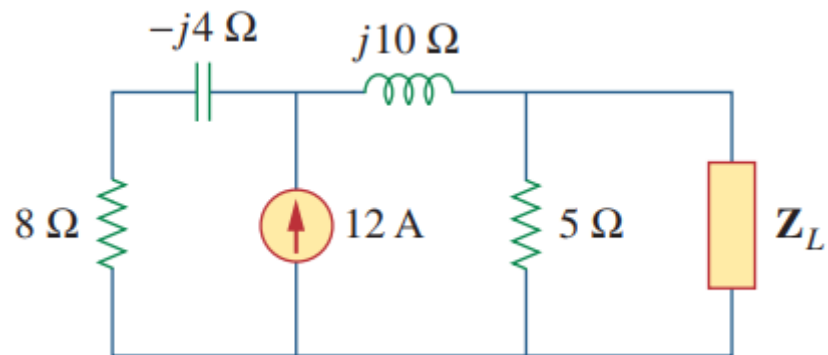
$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454 \angle -10.3^\circ \, \text{V}$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 2.933 - j4.467 \, \Omega$$

$$P_{\max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368 \, \text{W}$$

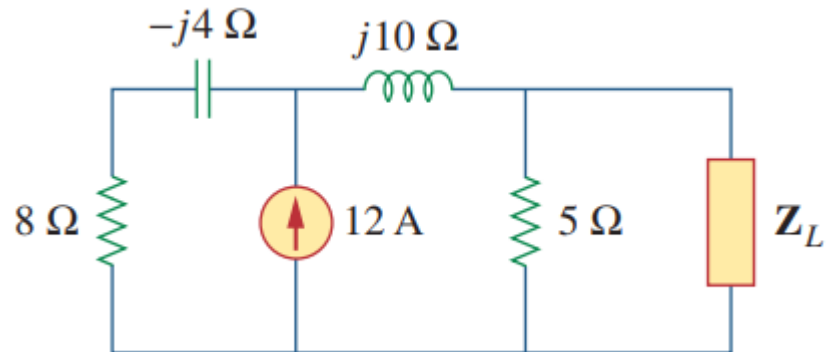
11.3**Maximum Average Power Transfer**

For the circuit shown in Fig. 11.10, find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate that maximum average power.



Answer: $3.415 - j0.7317\ \Omega$, 51.47 W .

11.3 Maximum Average Power Transfer



Answer: $3.415 - j0.7317\ \Omega$, 51.47 W .

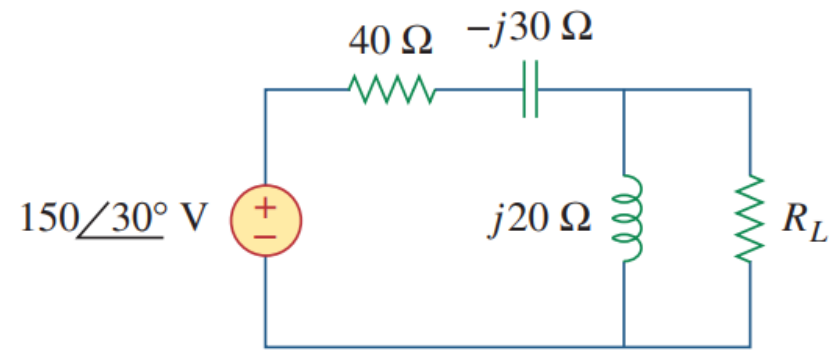
$$Z_{th} = \frac{(8 + 10j - 4j) * 5}{8 + 10j - 4j + 5} = 3.415 + j0.7317 = 3.492\angle 12.09$$

$$V_{th} \rightarrow I_{@5\Omega} = 12 * \frac{(8 - 4j)}{8 + 10j - 4j + 5} = 4.683 - j5.854\text{ A}$$

$$V_{th} = I_{@5\Omega} * 5 = 23.414 - j29.67\text{ V} = 37.48\angle - 51.34$$

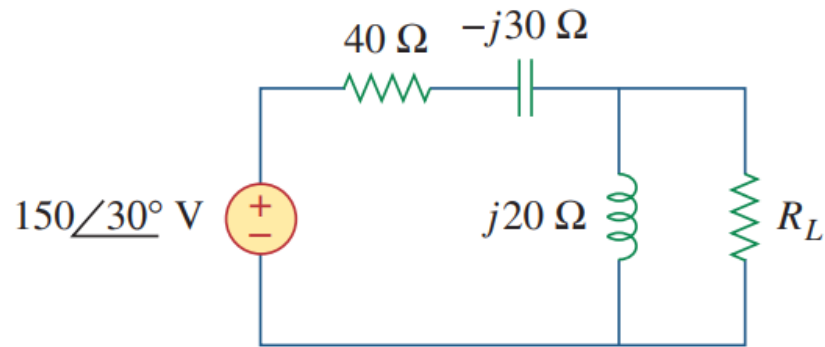
$$P_{Max} = \frac{|V_{th}|^2}{8 * R_{th}} = \frac{37.48^2}{8 * 3.415} = 51.4\text{ Watt}$$

11.3 Maximum Average Power Transfer



11.3

Maximum Average Power Transfer



$$\mathbf{Z}_{\text{Th}} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

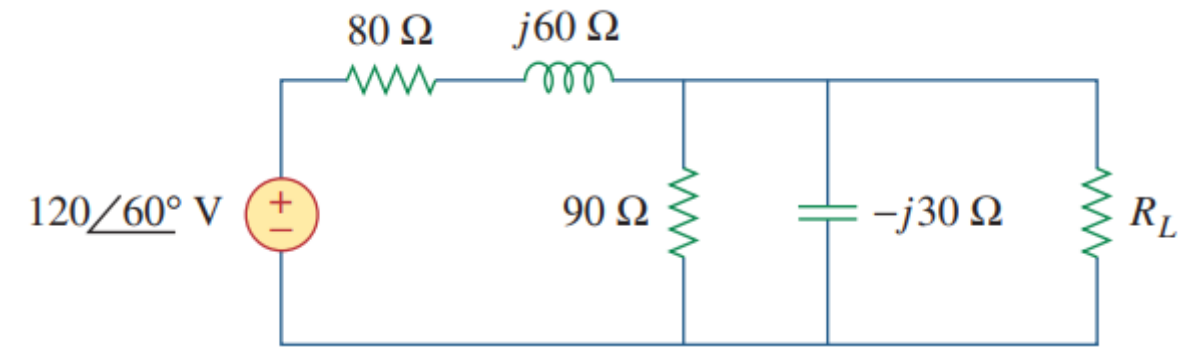
$$R_L = |\mathbf{Z}_{\text{Th}}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{j20}{j20 + 40 - j30} (150\angle 30^\circ) = 72.76\angle 134^\circ \text{ V}$$

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + R_L} = \frac{72.76\angle 134^\circ}{33.66 + j22.35} = 1.8\angle 100.42^\circ \text{ A}$$

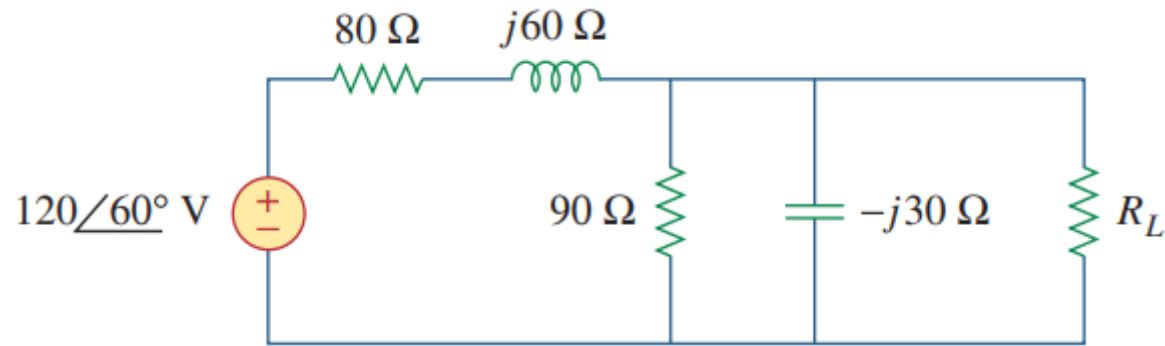
$$P_{\text{max}} = \frac{1}{2} |\mathbf{I}|^2 R_L = \frac{1}{2} (1.8)^2 (24.25) = 39.29 \text{ W}$$

11.3 Maximum Average Power Transfer



11.3

Maximum Average Power Transfer



Answer: $30\ \Omega$, $6.863\ \text{W}$.

$$Z_{th} = \frac{(80 + 60j) * 90}{80 + 60j + 90} \parallel -j30 = (47.63 + j14.95) \parallel -j30 = \frac{(47.63 + j14.95) * -j30}{(47.63 + j14.95) - j30} = 17.18 - j24.57 = 30\angle -55$$

$$Z_{Vth} = \frac{90 * -j30}{90 - j30} = 9 - j27 = 28.46\angle -71.57$$

$$R_L = 17.18 - j24.57 = 30\angle -55 = 30$$

$$V_{th} = 120\angle 60 * \frac{9 - j27}{80 + j60 + 9 - j27} = 30.543 - j19.02 = 36\angle -32$$

$$I = \frac{36\angle -32}{17.18 - j24.57 + 30} = 0.675 - j0.053 = 0.677\angle -4.49$$

$$P_{Max} = \frac{1}{2} * I^2 R_L = \frac{1}{2} * 0.677^2 * 30 = 6.874 \text{ Watt}$$

