

Electrical Circuit (2)

RMS Value (week6 class1)

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The **effective value** of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

$$P = I_{\text{eff}}^2 R$$

$$P = \frac{1}{T} \int_0^T i^2 R \, dt = \frac{R}{T} \int_0^T i^2 \, dt$$

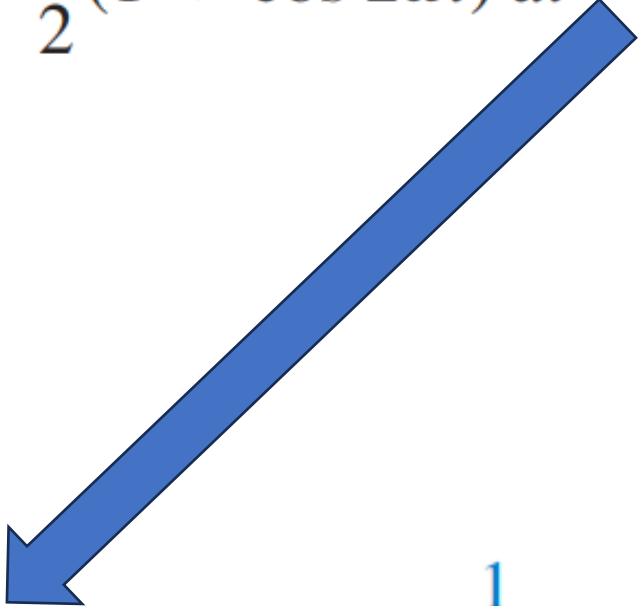
$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i^2 \, dt}$$

$$V_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T v^2 \, dt}$$

$$I_{\text{eff}} = I_{\text{rms}}, \quad V_{\text{eff}} = V_{\text{rms}}$$

$$X_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T x^2 \, dt}$$

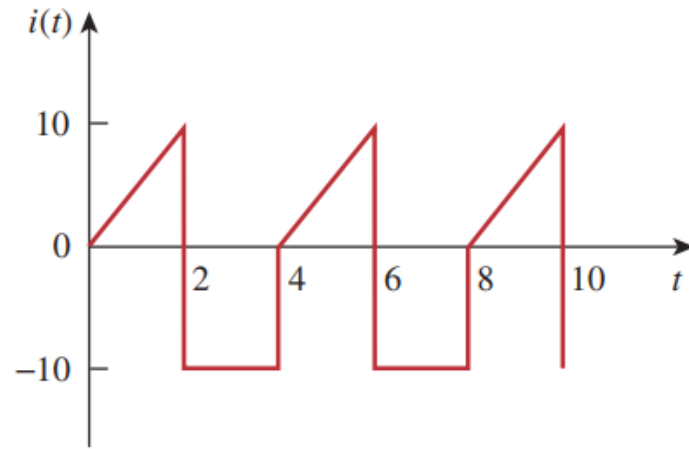
$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2 \omega t \, dt}$$
$$= \sqrt{\frac{I_m^2}{T} \int_0^T \frac{1}{2} (1 + \cos 2\omega t) \, dt} = \frac{I_m}{\sqrt{2}}$$


$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$\begin{aligned} P &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) \\ &= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \end{aligned}$$

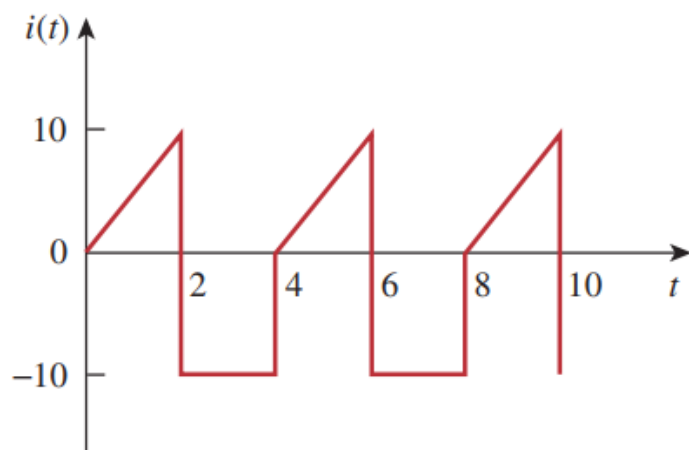
11.4 Effective or RMS Value

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a $2\text{-}\Omega$ resistor, find the average power absorbed by the resistor.



11.4 Effective or RMS Value

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a $2\text{-}\Omega$ resistor, find the average power absorbed by the resistor.



$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

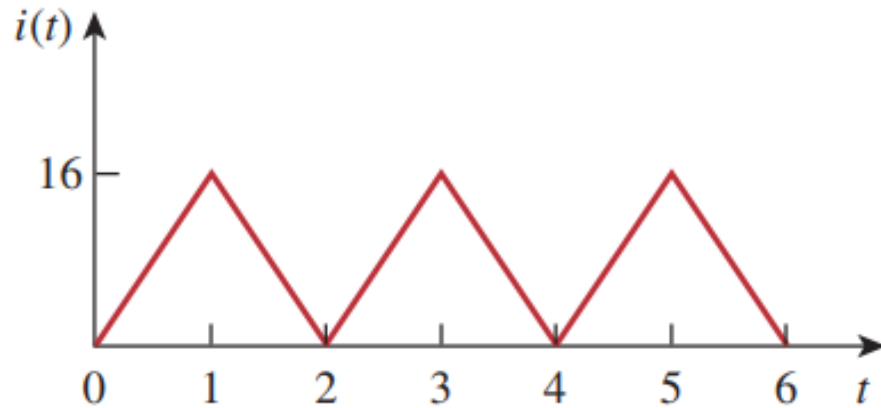
$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T i^2 dt} = \sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt \right]} \\ &= \sqrt{\frac{1}{4} \left[25 \frac{t^3}{3} \Big|_0^2 + 100t \Big|_2^4 \right]} = \sqrt{\frac{1}{4} \left(\frac{200}{3} + 200 \right)} = 8.165 \text{ A} \end{aligned}$$

The power absorbed by a $2\text{-}\Omega$ resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2 (2) = 133.3 \text{ W}$$

11.4 Effective or RMS Value

Find the rms value of the current waveform of Fig. 11.15. If the current flows through a $9\text{-}\Omega$ resistor, calculate the average power absorbed by the resistor.



$$i(t) = \begin{cases} 16t & 0 < t < 1 \\ -16t + 32 & 1 < t < 2 \end{cases}$$

$$I_{rms}^2 = \frac{1}{T} \int_0^T i(t)^2 dt = \frac{1}{2} \left[\int_0^1 (16t)^2 + \int_1^2 (-16t + 32)^2 \right] dt$$

$$I_{rms}^2 = \frac{1}{2} \left[\int_0^1 (16t)^2 + \int_1^2 256t^2 - 1024t + 1024 \right] dt$$

$$I_{rms}^2 = \frac{1}{2} \left\{ \left| \frac{256 * t^3}{3} \right|_0^1 + \left| \frac{256 * t^3}{3} \right|_1^2 - \left| \frac{1024 * t^2}{2} \right|_1^2 + |1024 * t|_0^1 \right\}$$

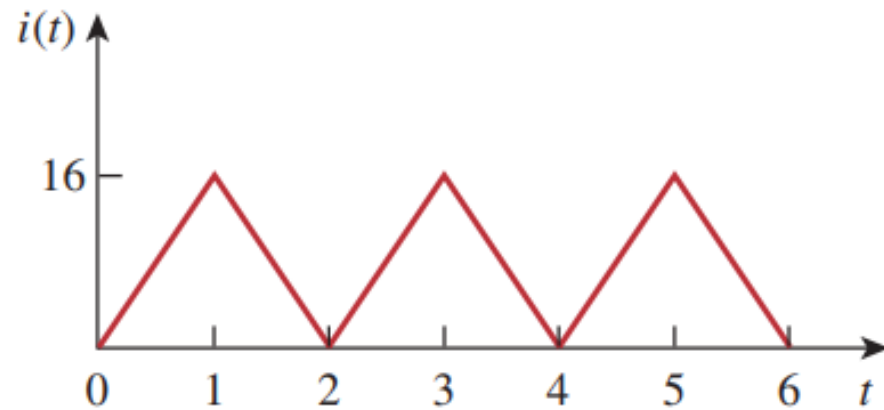
$$= \frac{1}{2} \left[\left(\frac{256}{3} \right) + \left[\left(\frac{256 * 2^3}{3} - \frac{1024 * 2^2}{2} \right) + (1024 * 2) \right] - \left[\frac{256}{3} - \frac{1024}{2} + 1024 \right] \right]$$

$$I_{rms}^2 = \frac{1}{2} (170) \Rightarrow I_{rms} = 9.237 \text{ A}$$

$$P = I_{rms}^2 R = 9.237^2 * 9 = 768 \text{ Watt}$$

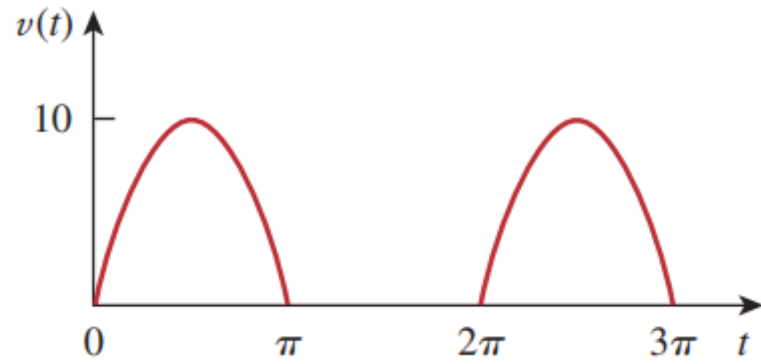
11.4 Effective or RMS Value

Find the rms value of the current waveform of Fig. 11.15. If the current flows through a $9\text{-}\Omega$ resistor, calculate the average power absorbed by the resistor.



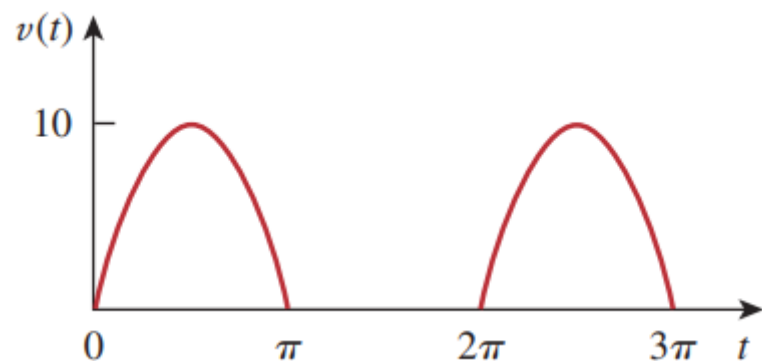
11.4 Effective or RMS Value

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a $10\text{-}\Omega$ resistor.



11.4 Effective or RMS Value

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a $10\text{-}\Omega$ resistor.



Solution:

The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T} \int_0^T v^2(t) dt = \frac{1}{2\pi} \left[\int_0^{\pi} (10 \sin t)^2 dt + \int_{\pi}^{2\pi} 0^2 dt \right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$\begin{aligned} V_{\text{rms}}^2 &= \frac{1}{2\pi} \int_0^{\pi} \frac{100}{2} (1 - \cos 2t) dt = \frac{50}{2\pi} \left(t - \frac{\sin 2t}{2} \right) \Big|_0^{\pi} \\ &= \frac{50}{2\pi} \left(\pi - \frac{1}{2} \sin 2\pi - 0 \right) = 25, \quad V_{\text{rms}} = 5 \text{ V} \end{aligned}$$

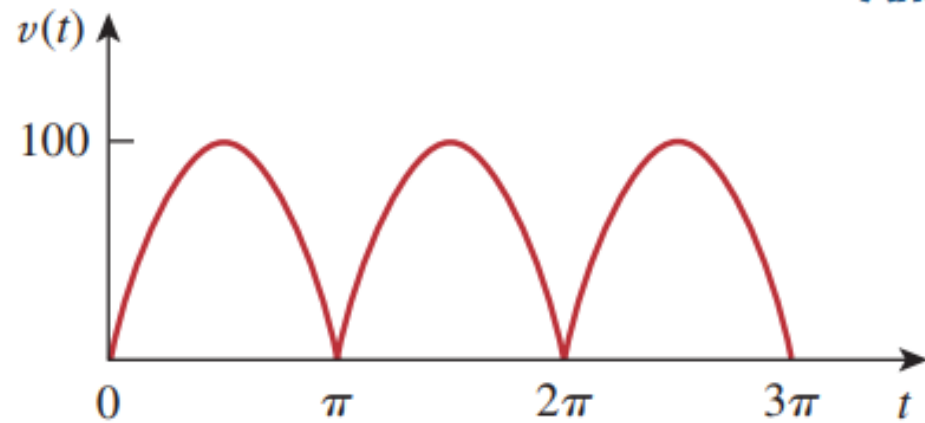
The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5 \text{ W}$$

11.4 Effective or RMS Value

Find the rms value of the full-wave rectified sine wave in Fig. 11.17. Calculate the average power dissipated in a $6\text{-}\Omega$ resistor.

Answer: 70.71 V, 833.3 W.



11.4 Effective or RMS Value

Find the rms value of the full-wave rectified sine wave in Fig. 11.17. Calculate the average power dissipated in a $6\text{-}\Omega$ resistor.

Answer: 70.71 V, 833.3 W.

