

Electrical Circuit (2)

Apparent Power and Power Factor (week6 class2)

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11.5 Apparent Power and Power Factor

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

The **apparent power** (in VA) is the product of the rms values of voltage and current.

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

The **power factor** is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

$$\begin{aligned}\text{Complex Power} = \mathbf{S} &= P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^* \\ &= |\mathbf{V}_{\text{rms}}| |\mathbf{I}_{\text{rms}}| \angle \theta_v - \theta_i\end{aligned}$$

$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}| |\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S \cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S \sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

11.5 Apparent Power and Power Factor

A series-connected load draws a current $i(t) = 4 \cos(100\pi t + 10^\circ)$ A when the applied voltage is $v(t) = 120 \cos(100\pi t - 20^\circ)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

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Solution:

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \quad (\text{leading})$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120 \angle -20^\circ}{4 \angle 10^\circ} = 30 \angle -30^\circ = 25.98 - j15 \, \Omega$$
$$\text{pf} = \cos(-30^\circ) = 0.866 \quad (\text{leading})$$

The load impedance $\mathbf{Z}$ can be modeled by a $25.98\text{-}\Omega$ resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

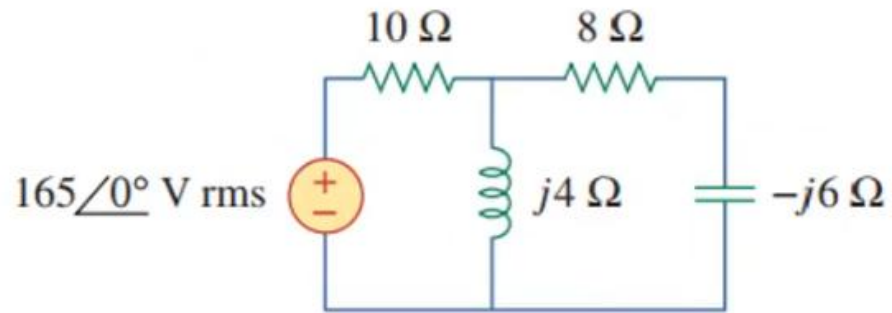
or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2 \, \mu\text{F}$$

11.5 Apparent Power and Power Factor

Calculate the power factor of the entire circuit of Fig. 11.19 as seen by the source. What is the average power supplied by the source?

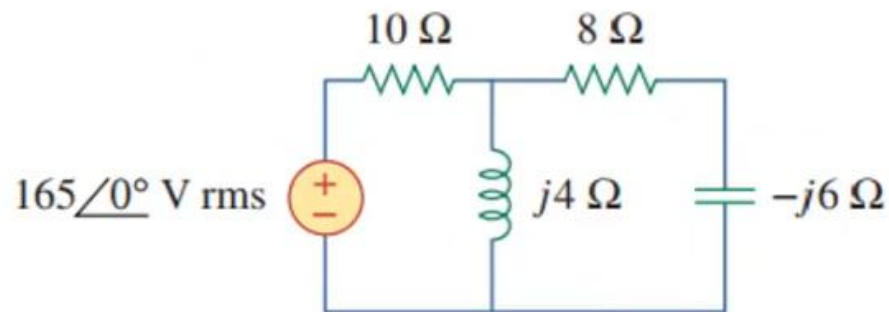
Answer: 0.936 lagging, 2.008 kW.



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Answer: 0.936 lagging, 2.008 kW.



$$z = 10 + \frac{(8 - j6) * j4}{8 - j2} = 11.88 + j4.47$$

$$I = \frac{165}{11.88 + j4.47} = 12.16 - j4.577 = 13\angle -20.62$$

$$P = (165 * 13) \cos(20.62) = 2008 \text{ Watt}$$

$$Pf = \cos(20.62) = 0.936$$

11.6 Complex Power

For a load, $\mathbf{V}_{\text{rms}} = 110\angle 85^\circ \text{ V}$, $\mathbf{I}_{\text{rms}} = 0.4\angle 15^\circ \text{ A}$. Determine: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Answer: (a) $44\angle 70^\circ \text{ VA}$, 44 VA , (b) 15.05 W , 41.35 VAR , (c) 0.342 lagging, $94.06 + j258.4 \Omega$.

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Answer: (a) $44\angle 70^\circ \text{ VA}$, 44 VA , (b) 15.05 W , 41.35 VAR , (c) 0.342 lagging, $94.06 + j258.4 \Omega$.

$$S = V * I^* = 110\angle 85 * 0.4\angle -15 = 15.05 + j41.35 = 44\angle 70$$

$$P = 15.05 \text{ W}$$

$$Q = 41.35 \text{ VAR}$$

$$Pf = \cos(70) = 0.342$$

$$Z = \frac{V}{I} = \frac{110\angle 85}{0.4\angle 15} = 94 + j258.4$$

11.6 Complex Power

A sinusoidal source supplies 100 kVAR reactive power to load $\mathbf{Z} = 250 \angle -75^\circ \Omega$. Determine: (a) the power factor, (b) the apparent power delivered to the load, and (c) the rms voltage.

Answer: (a) 0.2588 leading, (b) 103.53 kVA, (c) 5.087 kV.

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$$Pf = \cos(-75) = 0.2588$$

$$S = \frac{Q}{pf} = \frac{100 * 10^3}{\sin(75)} = 103.53 \text{ KVA}$$

$$P = S * pf = 103.53 * 0.2588 = 26792 \text{ W}$$

$$V = \sqrt{S * Z} = \sqrt{103530(250)} = 5.087 \text{ V}$$

