

Electrical Circuit (2)

Balanced Wye Connection (week8 class2)

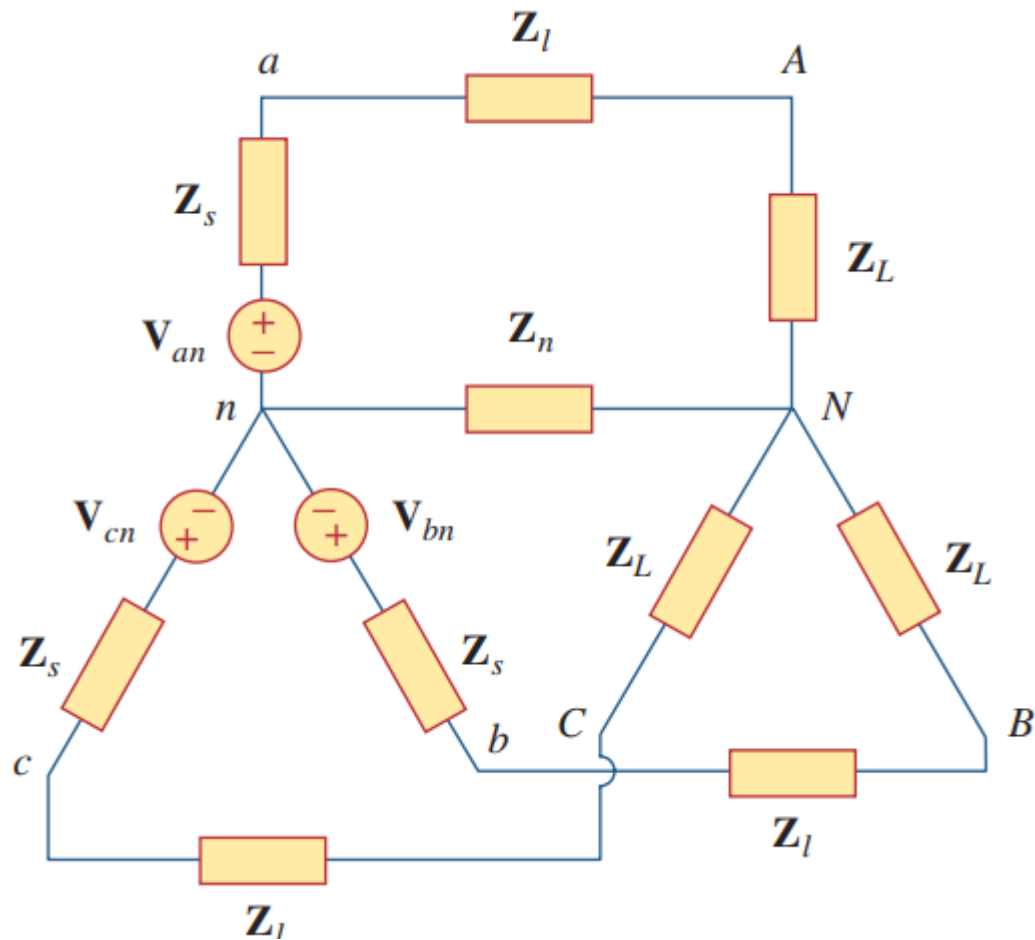
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12.3 Balanced Wye-Wye Connection

A **balanced Y-Y system** is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



12.3 Balanced Wye-Wye Connection

$$\mathbf{V}_{an} = V_p \angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p \angle -120^\circ, \quad \mathbf{V}_{cn} = V_p \angle +120^\circ$$

$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p \angle 0^\circ - V_p \angle -120^\circ \\ &= V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = \sqrt{3} V_p \angle 30^\circ\end{aligned}$$

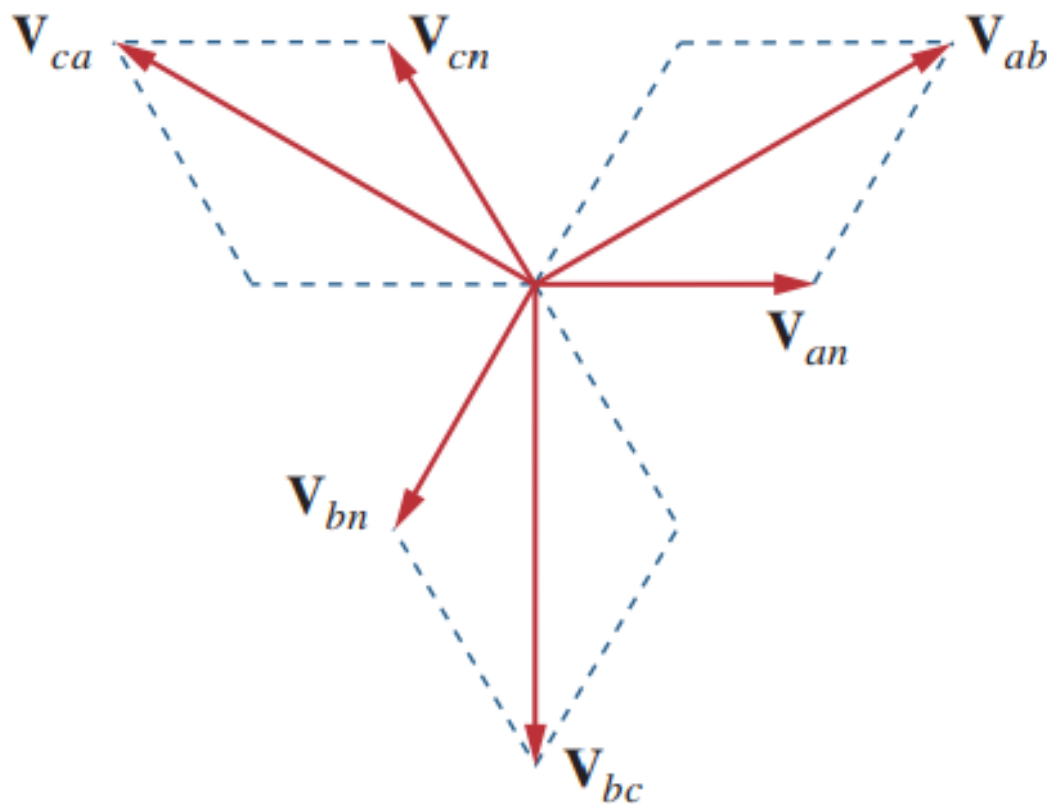
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3} V_p \angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3} V_p \angle -210^\circ$$

$$V_L = \sqrt{3} V_p$$

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$



12.3 Balanced Wye-Wye Connection

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \quad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an} \angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a \angle -120^\circ$$

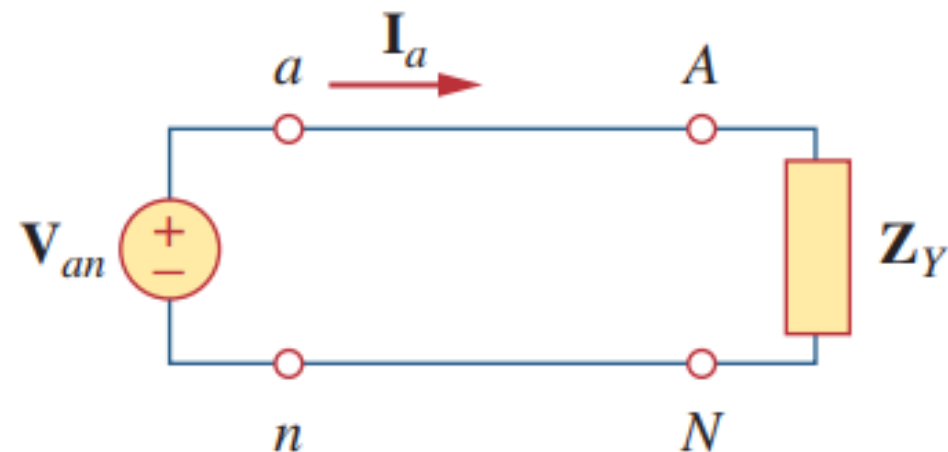
$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an} \angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a \angle -240^\circ$$

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

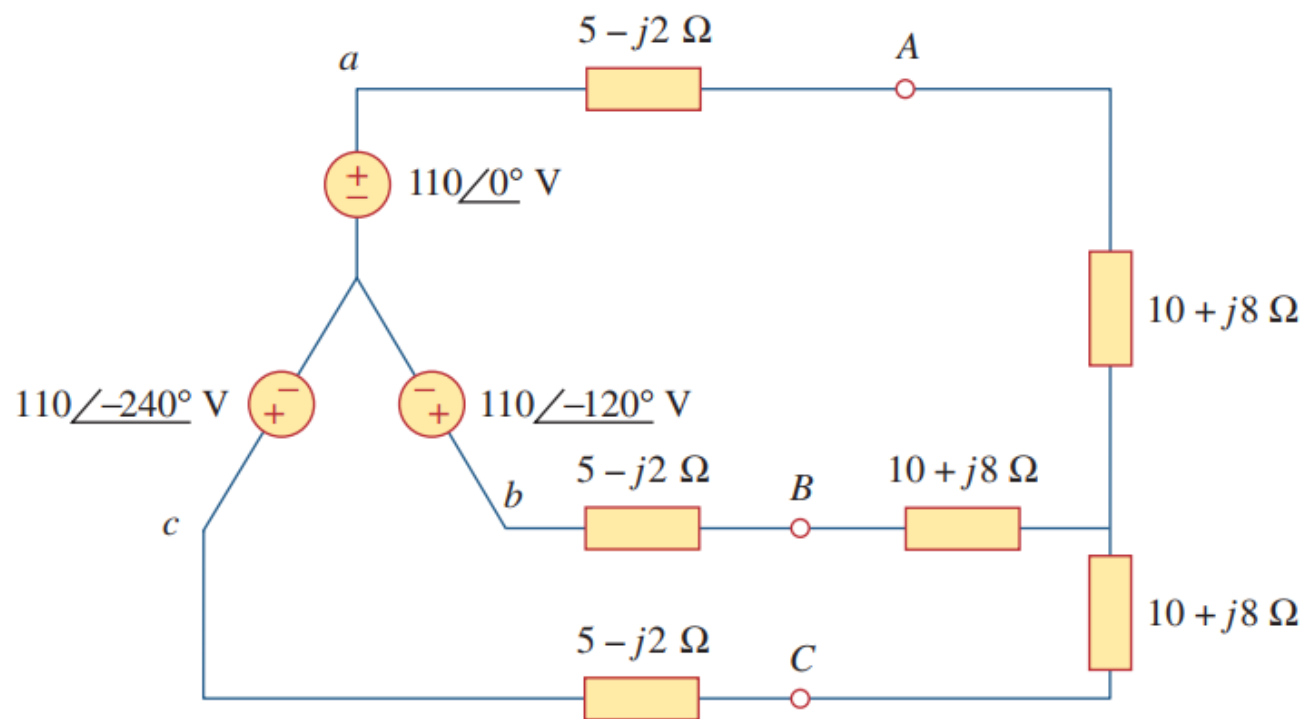
$$\mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0$$

$$\boxed{\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}}$$

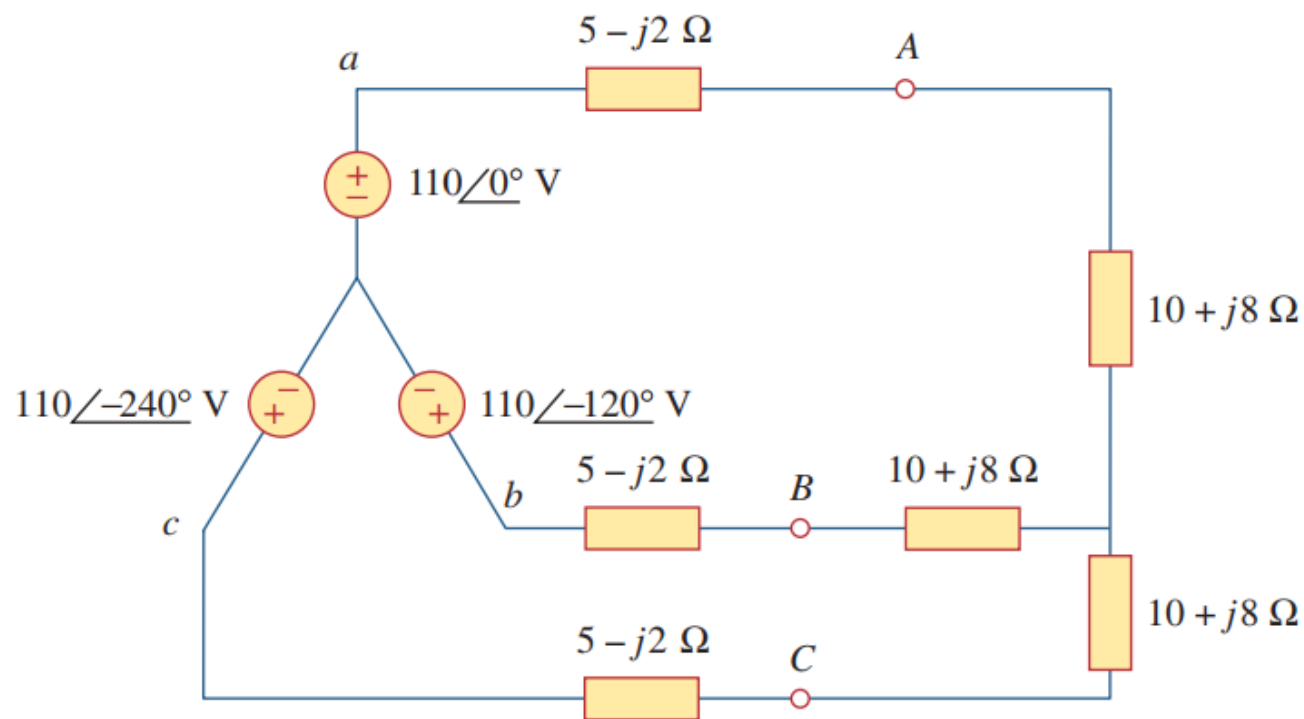


12.3

Balanced Wye-Wye Connection



12.3 Balanced Wye-Wye Connection



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

12.3 Balanced Wye-Wye Connection

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3 \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19 \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7 \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\mathbf{V}_{an} = 120 \angle 30^\circ \text{ V}$, find: (a) the line voltages, (b) the line currents.

Answer: (a) $207.8 \angle 60^\circ \text{ V}$, $207.8 \angle -60^\circ \text{ V}$, $207.8 \angle -180^\circ \text{ V}$,
(b) $3.75 \angle -8.66^\circ \text{ A}$, $3.75 \angle -128.66^\circ \text{ A}$, $3.75 \angle 111.34^\circ \text{ A}$.

12.3 Balanced Wye-Wye Connection

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Answer: (a) $207.8\angle 60^\circ$ V, $207.8\angle -60^\circ$ V, $207.8\angle -180^\circ$ V,
(b) $3.75\angle -8.66^\circ$ A, $3.75\angle -128.66^\circ$ A, $3.75\angle 111.34^\circ$ A.

$$V_{an} = 120\angle 30^\circ \rightarrow V_{bn} = 120\angle -90^\circ \rightarrow V_{cn} = 120\angle 150^\circ$$

$$V_{ab} = V_{an} - V_{bn} = 120\angle 30^\circ - 120\angle -90^\circ = 207.8\angle 60^\circ$$

$$V_{bc} = V_{ab}\angle \theta_{ab} - 120 = 207.8\angle -60^\circ$$

$$V_{ca} = V_{ab}\angle \theta_{ab} + 120 = 207.8\angle 180^\circ$$

$$I_a = \frac{V_{an}}{Z_G + Z_T + Z_L} = \frac{120\angle 30^\circ}{0.4 + j0.3 + 24 + j19 + 0.6 + j0.7} = 3.75\angle -8.66^\circ$$

$$I_b = \frac{V_{bn}}{Z_G + Z_T + Z_L} = \frac{120\angle -90^\circ}{0.4 + j0.3 + 24 + j19 + 0.6 + j0.7} = 3.75\angle -128.66^\circ$$

$$I_c = \frac{V_{cn}}{Z_G + Z_T + Z_L} = \frac{120\angle 150^\circ}{0.4 + j0.3 + 24 + j19 + 0.6 + j0.7} = 3.75\angle 111.34^\circ$$

12.4

Balanced Wye-Delta Connection

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ \end{aligned}$$

$$\begin{aligned} V_{ab} &= \sqrt{3}V_p \angle 30^\circ = V_{AB}, \quad V_{bc} = \sqrt{3}V_p \angle -90^\circ = V_{BC} \\ V_{ca} &= \sqrt{3}V_p \angle -150^\circ = V_{CA} \end{aligned}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}, \quad I_{BC} = \frac{V_{BC}}{Z_\Delta}, \quad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

$$-V_{an} + Z_\Delta I_{AB} + V_{bn} = 0$$

$$I_{AB} = \frac{V_{an} - V_{bn}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} = \frac{V_{AB}}{Z_\Delta}$$

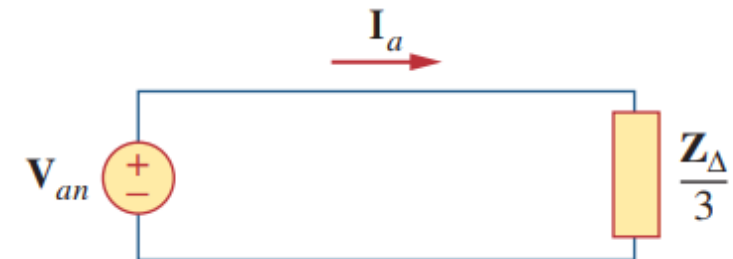
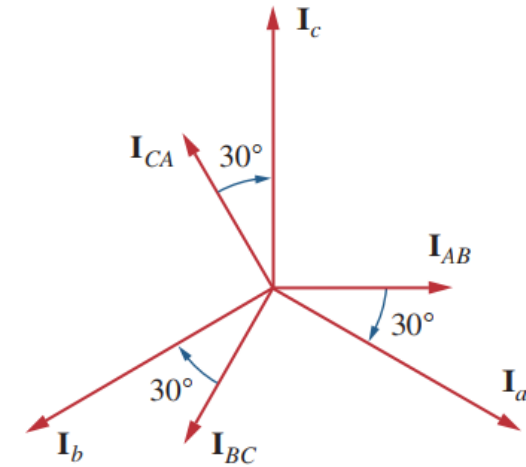
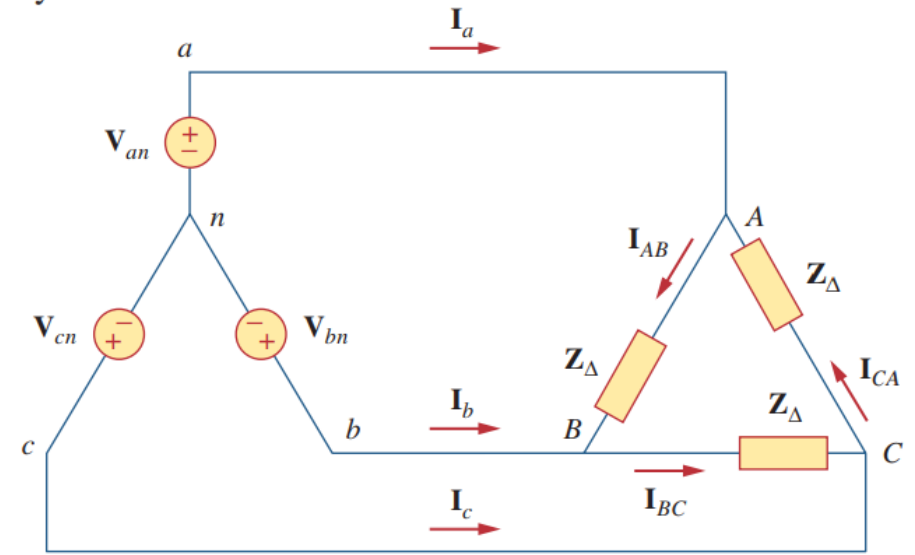
$$I_a = I_{AB} - I_{CA}, \quad I_b = I_{BC} - I_{AB}, \quad I_c = I_{CA} - I_{BC}$$

$$\begin{aligned} I_a &= I_{AB} - I_{CA} = I_{AB}(1 - 1 \angle -240^\circ) \\ &= I_{AB}(1 + 0.5 - j0.866) = I_{AB}\sqrt{3} \angle -30^\circ \end{aligned}$$

$$I_L = \sqrt{3}I_p$$

$$I_L = |I_a| = |I_b| = |I_c|$$

$$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$$



12.4**Balanced Wye-Delta Connection**

A balanced abc -sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ \text{ V}$ is connected to a Δ -connected balanced load $(8 + j4) \Omega$ per phase. Calculate the phase and line currents.

12.4

Balanced Wye-Delta Connection

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ -connected balanced load $(8 + j4) \Omega$ per phase. Calculate the phase and line currents.

■ **METHOD 1** The load impedance is

$$\mathbf{Z}_{\Delta} = 8 + j4 = 8.944\angle 26.57^\circ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30^\circ = 100\sqrt{3}\angle 10^\circ + 30^\circ = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\angle 40^\circ \text{ V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

The line currents are

$$\begin{aligned}\mathbf{I}_a &= \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ \\ &= 33.53\angle -16.57^\circ \text{ A}\end{aligned}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

■ **METHOD 2** Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{\Delta}/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ \text{ A}$$

12.4

Balanced Wye-Delta Connection

One line voltage of a balanced Y-connected source is $\mathbf{V}_{AB} = 120\angle -20^\circ \text{ V}$. If the source is connected to a Δ -connected load of $20\angle 40^\circ \Omega$, find the phase and line currents. Assume the abc sequence.

Answer: $6\angle -60^\circ \text{ A}$, $6\angle -180^\circ \text{ A}$, $6\angle 60^\circ \text{ A}$, $10.392\angle -90^\circ \text{ A}$,
 $10.392\angle 150^\circ \text{ A}$, $10.392\angle 30^\circ \text{ A}$.

12.4

Balanced Wye-Delta Connection

One line voltage of a balanced Y-connected source is $V_{AB} = 120\angle -20^\circ$ V. If the source is connected to a Δ -connected load of $20\angle 40^\circ \Omega$, find the phase and line currents. Assume the abc sequence.

Answer: $6\angle -60^\circ$ A, $6\angle -180^\circ$ A, $6\angle 60^\circ$ A, $10.392\angle -90^\circ$ A, $10.392\angle 150^\circ$ A, $10.392\angle 30^\circ$ A.

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle (\theta_{ab} - 30) = \frac{120\angle -20 - 30}{\sqrt{3}} = 69.28\angle -50 \rightarrow V_{bn} = 69.28\angle -170 \rightarrow V_{cn} = 69.28\angle -290$$

$$I_a = \frac{V_{an}}{Z_{\Delta} \div 3} = \frac{69.28\angle -50}{20\angle 40 \div 3} = -j10.392 \rightarrow I_{bn} = 10.392\angle -210 \rightarrow I_{cn} = 10.392\angle 330$$

$$I_{ab} = \frac{V_{an} - V_{bn}}{Z_{\Delta}} = \frac{69.28\angle -50 - 69.28\angle -170}{20\angle 40} = 6\angle -60$$

$$I_{bc} = \frac{V_{bn} - V_{cn}}{Z_{\Delta}} = \frac{69.28\angle -170 - 69.28\angle -290}{20\angle 40} = 6\angle -180$$

$$I_{ca} = \frac{V_{cn} - V_{an}}{Z_{\Delta}} = \frac{69.28\angle -290 - 69.28\angle -50}{20\angle 40} = 6\angle -300$$

