

Electrical Circuit (2)

Balanced Delta Connection(week9 class1)

Dr. Akram Al-Mahrouk
Philadelphia University

Chapter 12 Three-Phase Circuits 503

- 12.1 Introduction 504
- 12.2 Balanced Three-Phase Voltages 505
- 12.3 Balanced Wye-Wye Connection 509
- 12.4 Balanced Wye-Delta Connection 512
- 12.5 Balanced Delta-Delta Connection 514
- 12.6 Balanced Delta-Wye Connection 516
- 12.7 Power in a Balanced System 519
- 12.8 †Unbalanced Three-Phase Systems 525
- 12.9 *PSpice* for Three-Phase Circuits 529
- 12.10 †Applications 534
 - 12.10.1 Three-Phase Power Measurement
 - 12.10.2 Residential Wiring
- 12.11 Summary 543
 - Review Questions 543
 - Problems 544
 - Comprehensive Problems 553

12.5 Balanced Delta-Delta Connection

A **balanced Δ - Δ system** is one in which both the balanced source and balanced load are Δ -connected.

$$\mathbf{V}_{ab} = V_p \angle 0^\circ$$

$$\mathbf{V}_{bc} = V_p \angle -120^\circ, \quad \mathbf{V}_{ca} = V_p \angle +120^\circ$$

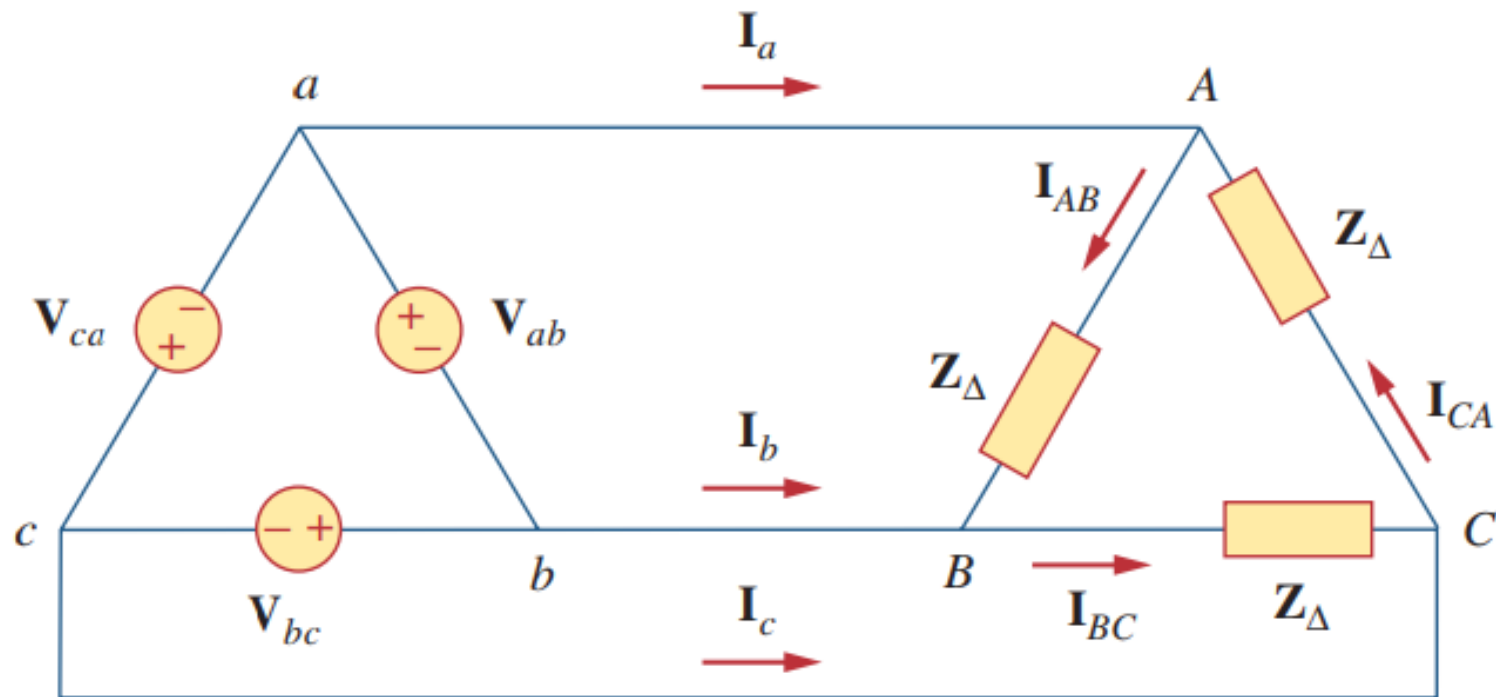
$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \quad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}, \quad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta}$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \quad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \quad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$I_L = \sqrt{3}I_p$$



12.5 Balanced Delta-Delta Connection

A balanced Δ -connected load having an impedance $20 - j15 \, \Omega$ is connected to a Δ -connected, positive-sequence generator having $\underline{V}_{ab} = 330 \angle 0^\circ \text{ V}$. Calculate the phase currents of the load and the line currents.

12.5 Balanced Delta-Delta Connection

A balanced Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected, positive-sequence generator having $V_{ab} = 330 \angle 0^\circ \text{ V}$. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$Z_{\Delta} = 20 - j15 = 25 \angle -36.87^\circ \Omega$$

Since $V_{AB} = V_{ab}$, the phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{330 \angle 0^\circ}{25 \angle -36.87^\circ} = 13.2 \angle 36.87^\circ \text{ A}$$

$$I_{BC} = I_{AB} \angle -120^\circ = 13.2 \angle -83.13^\circ \text{ A}$$

$$I_{CA} = I_{AB} \angle +120^\circ = 13.2 \angle 156.87^\circ \text{ A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\begin{aligned} I_a &= I_{AB} \sqrt{3} \angle -30^\circ = (13.2 \angle 36.87^\circ)(\sqrt{3} \angle -30^\circ) \\ &= 22.86 \angle 6.87^\circ \text{ A} \end{aligned}$$

$$I_b = I_a \angle -120^\circ = 22.86 \angle -113.13^\circ \text{ A}$$

$$I_c = I_a \angle +120^\circ = 22.86 \angle 126.87^\circ \text{ A}$$

12.5 Balanced Delta-Delta Connection

A positive-sequence, balanced Δ -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18 + j12 \Omega$ and $\mathbf{I}_a = 9.609 \angle 35^\circ \text{ A}$, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $5.548 \angle 65^\circ \text{ A}$, $120 \angle 98.69^\circ \text{ V}$.

12.5

Balanced Delta-Delta Connection

A positive-sequence, balanced Δ -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18 + j12 \Omega$ and $\mathbf{I}_a = 9.609 \angle 35^\circ \text{ A}$, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $5.548 \angle 65^\circ \text{ A}$, $120 \angle 98.69^\circ \text{ V}$.

$$\mathbf{I}_a = 9.609 \angle 35^\circ \quad \mathbf{I}_b = 9.609 \angle -85^\circ \quad \mathbf{I}_c = 9.609 \angle -205^\circ$$

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_a}{\sqrt{3}} \angle 30^\circ = \frac{9.609 \angle (35^\circ - 30^\circ)}{\sqrt{3}} = 5.548 \angle 65^\circ \quad \mathbf{I}_{BC} = 5.548 \angle -55^\circ \quad \mathbf{I}_{CA} = 5.548 \angle -175^\circ$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB} \mathbf{Z}_\Delta = 5.548 \angle 65^\circ * (18 + j12) = -18.134 + j118.6 = 120 \angle 98.7^\circ$$

$$\mathbf{V}_{BC} = 120 \angle -21.3^\circ \quad \mathbf{V}_{CA} = 120 \angle -141.3^\circ$$

12.6 Balanced Delta-Wye Connection

A **balanced Δ -Y system** consists of a balanced Δ -connected source feeding a balanced Y-connected load.

$$\begin{aligned} \mathbf{V}_{ab} &= V_p \angle 0^\circ, & \mathbf{V}_{bc} &= V_p \angle -120^\circ \\ \mathbf{V}_{ca} &= V_p \angle +120^\circ \end{aligned}$$

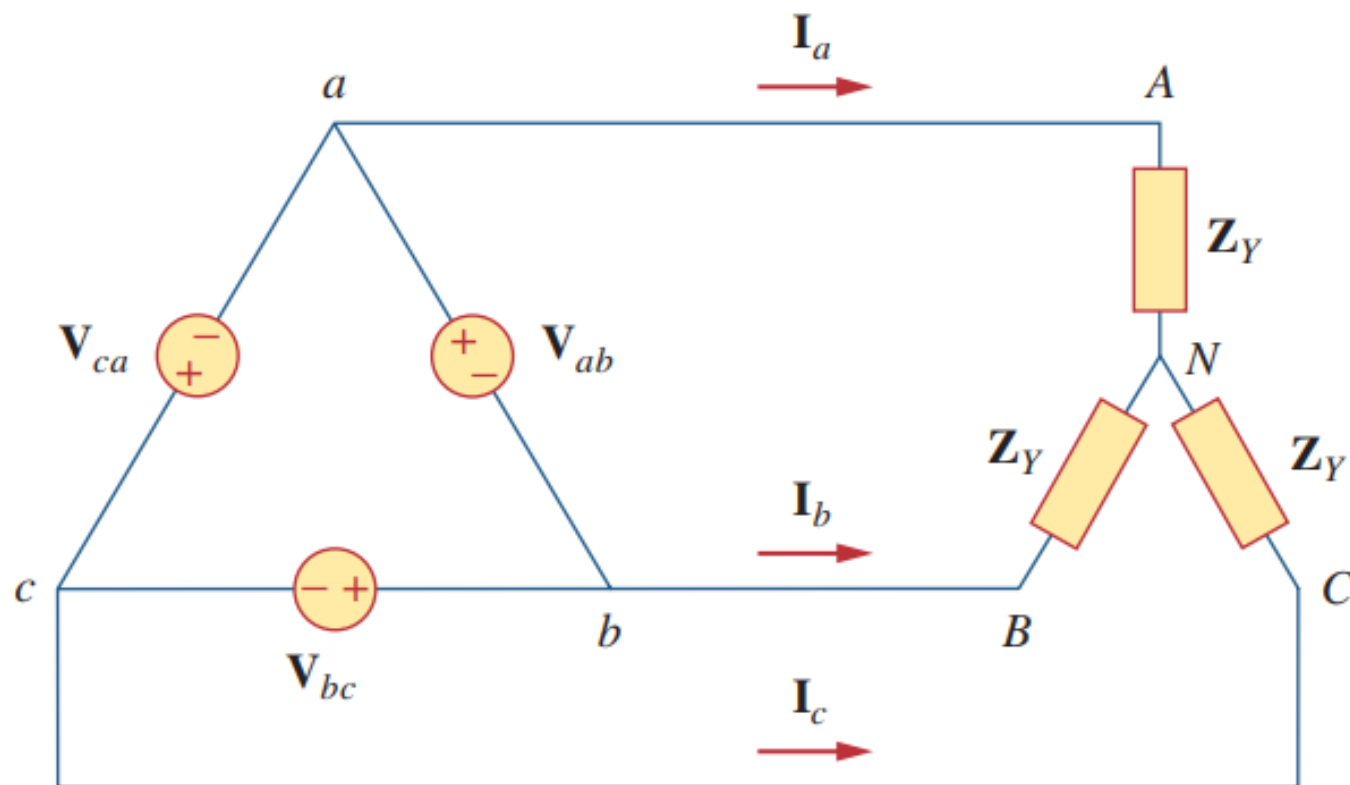
$$-\mathbf{V}_{ab} + \mathbf{Z}_Y \mathbf{I}_a - \mathbf{Z}_Y \mathbf{I}_b = 0$$

$$\mathbf{Z}_Y (\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p \angle 0^\circ$$

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p \angle 0^\circ}{\mathbf{Z}_Y}$$

$$\begin{aligned} \mathbf{I}_a - \mathbf{I}_b &= \mathbf{I}_a (1 - 1 \angle -120^\circ) \\ &= \mathbf{I}_a \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = \mathbf{I}_a \sqrt{3} \angle 30^\circ \end{aligned}$$

$$\mathbf{I}_a = \frac{V_p / \sqrt{3} \angle -30^\circ}{\mathbf{Z}_Y}$$



12.6 Balanced Delta-Wye Connectio

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$

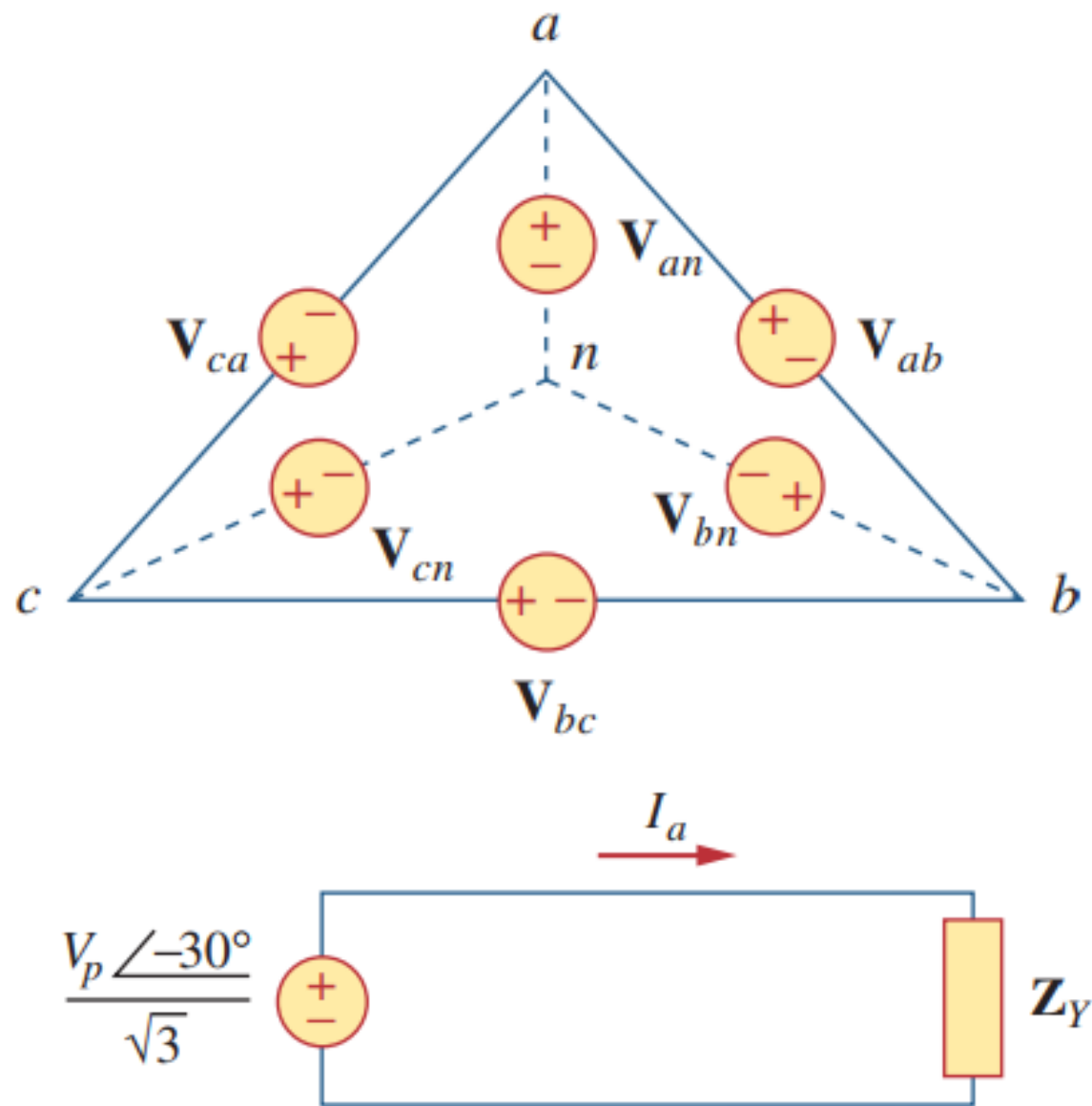
$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}} \angle -150^\circ, \quad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}} \angle +90^\circ$$

$$\mathbf{I}_a = \frac{V_p / \sqrt{3} \angle -30^\circ}{\mathbf{Z}_Y}$$

$$\mathbf{I}_a = \frac{V_p / \sqrt{3} \angle -30^\circ}{\mathbf{Z}_Y}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a \mathbf{Z}_Y = \frac{V_p}{\sqrt{3}} \angle -30^\circ$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN} \angle -120^\circ, \quad \mathbf{V}_{CN} = \mathbf{V}_{AN} \angle +120^\circ$$



Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p \angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p \angle 30^\circ$
	$\mathbf{V}_{bn} = V_p \angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab} \angle -120^\circ$
	$\mathbf{V}_{cn} = V_p \angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab} \angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an} / \mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a \angle +120^\circ$
Y- Δ	$\mathbf{V}_{an} = V_p \angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p \angle 30^\circ$
	$\mathbf{V}_{bn} = V_p \angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab} \angle -120^\circ$
	$\mathbf{V}_{cn} = V_p \angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab} \angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB} / \mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC} / \mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA} / \mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB} \sqrt{3} \angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a \angle +120^\circ$
Δ - Δ	$\mathbf{V}_{ab} = V_p \angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p \angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p \angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab} / \mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc} / \mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca} / \mathbf{Z}_\Delta$	
Δ -Y	$\mathbf{V}_{ab} = V_p \angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p \angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p \angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \frac{V_p \angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a \angle +120^\circ$

12.6 Balanced Delta-Wye Connection

A balanced Y-connected load with a phase impedance of $40 + j25 \, \Omega$ is supplied by a balanced, positive sequence Δ -connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as a reference.

12.6 Balanced Delta-Wye Connection

A balanced Y-connected load with a phase impedance of $40 + j25 \Omega$ is supplied by a balanced, positive sequence Δ -connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as a reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17 \angle 32^\circ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210 \angle 0^\circ \text{ V}$$

When the Δ -connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30^\circ = 121.2 \angle -30^\circ \text{ V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2 \angle -30^\circ}{47.12 \angle 32^\circ} = 2.57 \angle -62^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 2.57 \angle -178^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = 2.57 \angle 58^\circ \text{ A}$$

12.6 Balanced Delta-Wye Connection

In a balanced Δ -Y circuit, $\mathbf{V}_{ab} = 240\angle 15^\circ$ and $\mathbf{Z}_Y = (12 + j15) \Omega$. Calculate the line currents.

Answer: $7.21\angle -66.34^\circ$ A, $7.21\angle +173.66^\circ$ A, $7.21\angle 53.66^\circ$ A.

12.6 Balanced Delta-Wye Connection

In a balanced Δ -Y circuit, $V_{ab} = 240\angle 15^\circ$ and $Z_Y = (12 + j15) \Omega$. Calculate the line currents.

Answer: $7.21\angle -66.34^\circ$ A, $7.21\angle +173.66^\circ$ A, $7.21\angle 53.66^\circ$ A.

$$Z_{\Delta} = 3 * (12 + j15) = 36 + j45 = 57.63\angle 51.34$$

$$I_{AB} = \frac{V_{ab}}{Z_{\Delta}} = \frac{240\angle 15}{57.63\angle 51.34} = 3.355 - j2.47 = 4.165\angle -36.34$$

$$I_{BC} = 4.165\angle -156.34 \quad I_{CA} = 4.165\angle -276.34$$

$$I_a = I_{AB} - I_{BC} = 4.165\angle -36.34 - 4.165\angle -156.34 = 7.21\angle -6.34$$

$$I_b = 7.21\angle -126.34 \quad I_c = 4.165\angle -246.34$$

