

Electrical Circuit (2)

Power in Balanced System (week9 class2)

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Chapter 12 Three-Phase Circuits 503

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12.7 Power in a Balanced System

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \quad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ) \\ v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ)$$

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \quad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ) \\ i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)$$

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ = 2V_p I_p [\cos \omega t \cos(\omega t - \theta) \\ + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$p = V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ + \cos(2\omega t - \theta + 240^\circ)] \\ = V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\ + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\ \text{where } \alpha = 2\omega t - \theta \\ = V_p I_p \left[3 \cos \theta + \cos \alpha + 2 \left(-\frac{1}{2} \right) \cos \alpha \right] = 3V_p I_p \cos \theta$$

12.7 Power in a Balanced System

$$P_p = V_p I_p \cos \theta \quad Q_p = V_p I_p \sin \theta \quad S_p = V_p I_p$$

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3} V_L I_L \cos \theta$$

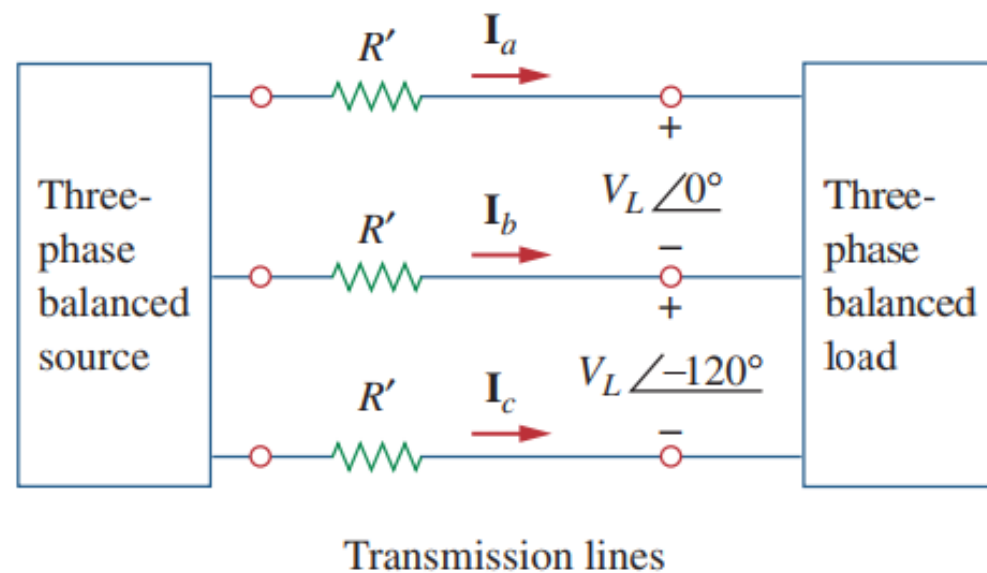
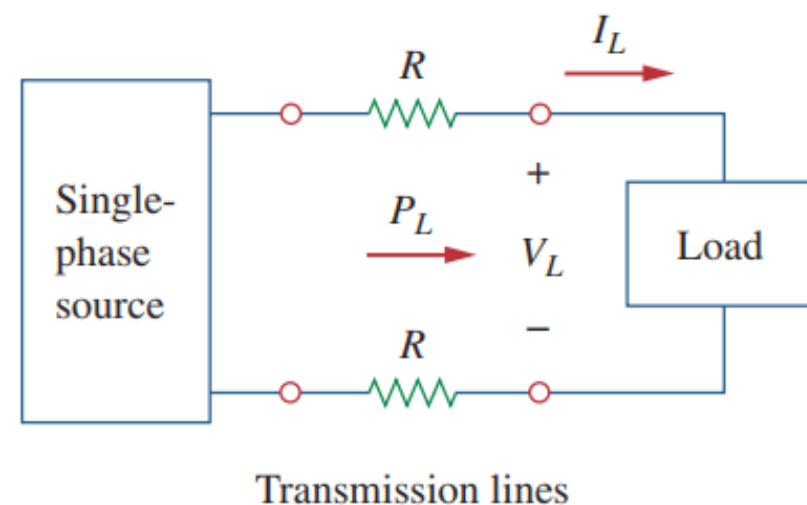
$$Q = 3V_p I_p \sin \theta = 3Q_p = \sqrt{3} V_L I_L \sin \theta$$

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}$$

$$\mathbf{S} = P + jQ = \sqrt{3} V_L I_L \angle \theta$$

$$P_{\text{loss}} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2}$$

$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2}$$



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Power in a Balanced System

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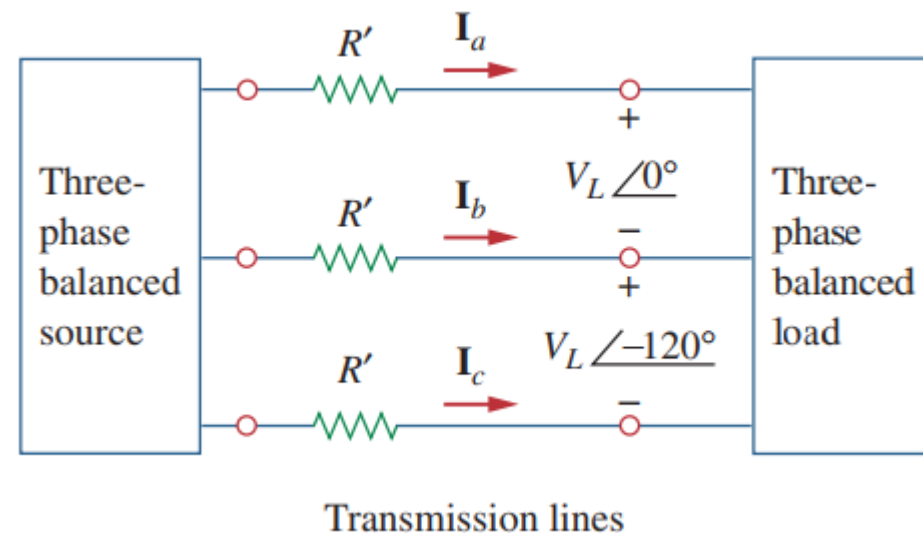
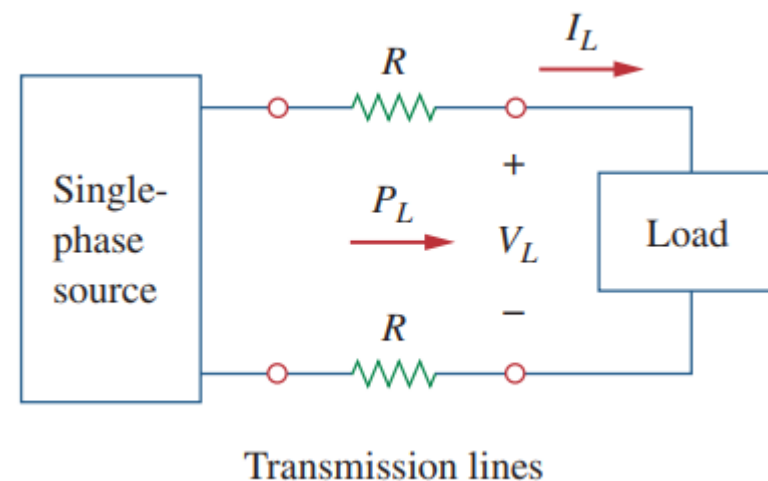
$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'}$$

But from Chapter 2, $R = \rho \ell / \pi r^2$ and $R' = \rho \ell / \pi r'^2$, where r and r' are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \quad (12.57)$$

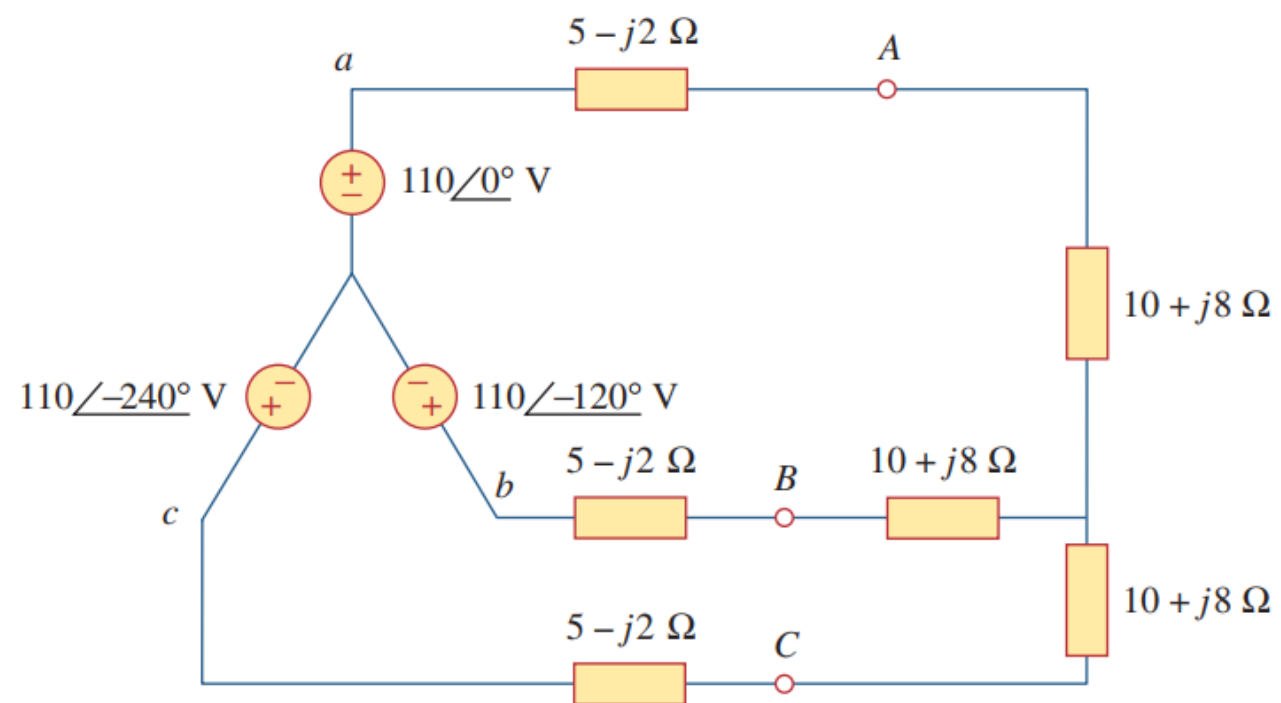
$$\begin{aligned} \frac{\text{Material for single-phase}}{\text{Material for three-phase}} &= \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2} \\ &= \frac{2}{3}(2) = 1.333 \end{aligned}$$

since $r^2 = 2r'^2$. Equation (12.58) shows that the single-phase system uses 33 percent more material than the three-phase system or that the three-phase system uses only 75 percent of the material used in the equivalent single-phase system. In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.



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Power in a Balanced System



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Power in a Balanced System

$$\mathbf{V}_p = 110\angle 0^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_p = 6.81\angle -21.8^\circ \text{ A}$$

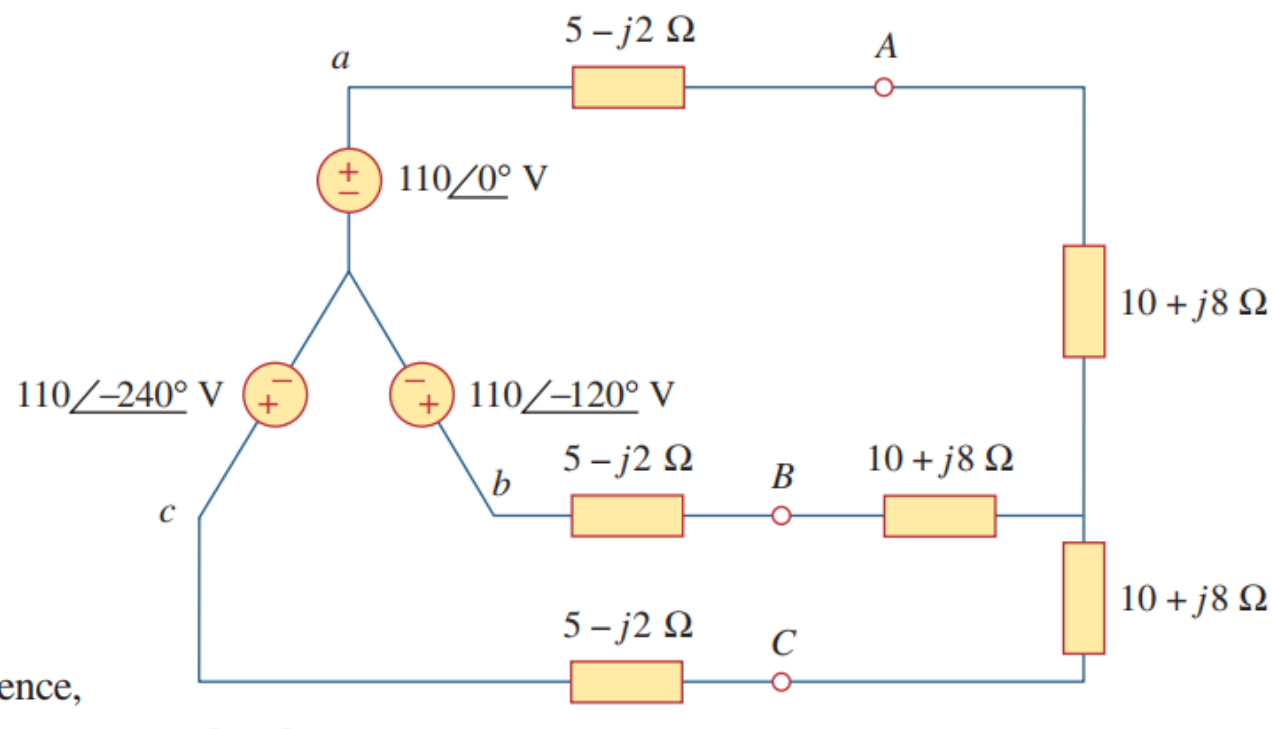
$$\begin{aligned} \mathbf{S}_s &= -3\mathbf{V}_p \mathbf{I}_p^* = -3(110\angle 0^\circ)(6.81\angle 21.8^\circ) \\ &= -2247\angle 21.8^\circ = -(2087 + j834.6) \text{ VA} \end{aligned}$$

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2 \mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81\angle 38.66^\circ$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\angle -21.8^\circ$. Hence,

$$\begin{aligned} \mathbf{S}_L &= 3(6.81)^2 12.81\angle 38.66^\circ = 1782\angle 38.66^\circ \\ &= (1392 + j1113) \text{ VA} \end{aligned}$$

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2 \mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3 \text{ VA}$$



12.7

Power in a Balanced System

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3 \, \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19 \, \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7 \, \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\mathbf{V}_{an} = 120 \angle 30^\circ \text{ V}$, find: (a) the line voltages, (b) the line currents.

For the Y-Y circuit in Practice Prob. 12.2, calculate the complex power at the source and at the load.

Answer: $-(1054.2 + j843.3) \text{ VA}$, $(1012 + j801.6) \text{ VA}$.

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Answer: $-(1054.2 + j843.3) \text{ VA}$, $(1012 + j801.6) \text{ VA}$.

$$I_a = \frac{V_{an}}{Z_G + Z_T + Z_L} = \frac{120\angle 30}{0.4 + j0.3 + 0.6 + j0.7 + 24 + j19} = 3.75\angle -8.66$$

$$S_{total} = 3 * V_{an} * I_a = 3 * 120\angle 30 * 3.75\angle -8.66 = -(1054.2 + j843.3) = 1350\angle 37.66 \text{ VA}$$

$$V_L = I_a * Z_L = 3.75\angle -8.66 * (24 + j19) = 99.7 + j56.89 \text{ V}$$

$$S_L = 3 * V_L * I_a = 3 * 99.7 + j56.89 * 3.75\angle 8.66 = 1012.5 + j801.6 \text{ VA}$$

12.7 Power in a Balanced System

Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

Answer: 46.31 A.

12.7

Power in a Balanced System

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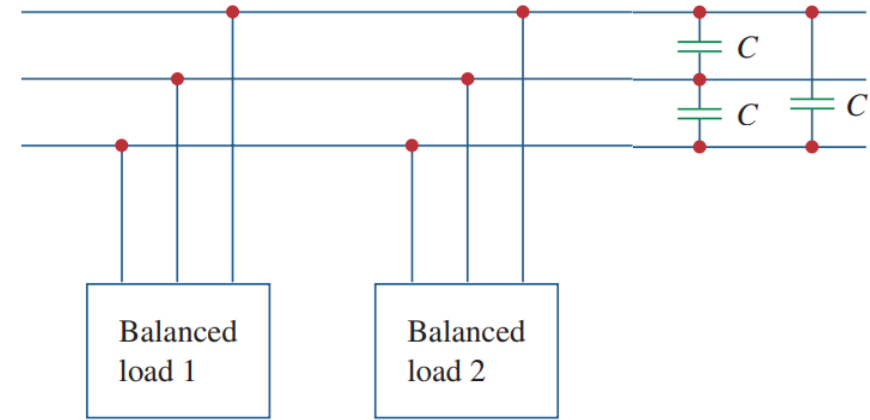
Answer: 46.31 A.

$$I_a = \frac{P_1}{3 * \frac{V_L}{\sqrt{3}} * pf} = \frac{30000}{03 * \frac{440}{\sqrt{3}} * 0.85} = 46.31$$

12.7 Power in a Balanced System

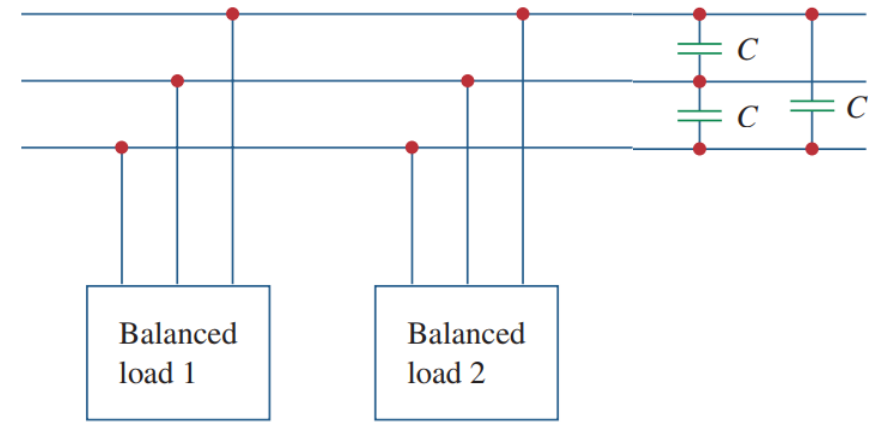
Assume that the two balanced loads in Fig. 12.22(a) are supplied by an 840-V rms 60-Hz line. Load 1 is Y-connected with $30 + j40 \, \Omega$ per phase, while load 2 is a balanced three-phase motor drawing 48 kW at a power factor of 0.8 lagging. Assuming the *abc* sequence, calculate: (a) the complex power absorbed by the combined load, (b) the kVAR rating of each of the three capacitors Δ -connected in parallel with the load to raise the power factor to unity, and (c) the current drawn from the supply at unity power factor condition.

Answer: (a) $56.47 + j47.29$ kVA, (b) 15.76 kVAR, (c) 38.81 A.



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Answer: (a) $56.47 + j47.29$ kVA, (b) 15.76 kVAR, (c) 38.81 A.

$$\text{Load}_1 \quad I_1 = \frac{\frac{V_L}{\sqrt{3}}}{30 + j40} = 9.7 \angle -53.13 \quad S_1 = 3 * \frac{V_L}{\sqrt{3}} * I_1^* = 3 * \frac{840}{\sqrt{3}} * 9.7 \angle 53.13 = 8467 + j11289$$

$$\text{Load}_2 \quad Q_2 = P_2 \tan(\cos^{-1} 0.8) = 36 \text{ KVAR} \quad S_2 = P_2 + jQ_2 = 48 + j36 \text{ KVA}$$

$$S_{total} \quad S_{total} = S_1 + S_2 = 8467 + j11289 + 48000 + j36000 = 56.47 + j47.29 \text{ KVA}$$

$$Q_{\Delta} \quad Q_{\Delta} = \frac{Q_Y}{3} = \frac{47.29 \text{ KVA}}{3} = j15.76 \text{ KVAR}$$

$$I_{@pf=1} \quad I_{@pf=1} = \frac{S_{total} - j47.29}{3 * \frac{V_L}{\sqrt{3}}} = \frac{56470}{3 * \frac{840}{\sqrt{3}}} = 38.81 \text{ A}$$

