

**Islamic University of Gaza**  
**Electrical and Computer Engineering Department**  
**EELE 3310 Signals & Systems**  
**Final Exam**  
**December, 30, 2004**  
**OPEN BOOK & NOTE EXAM**

**Dr. Basil Hamed**

**Exam Time 8:30-11:00**

|                   |                 |
|-------------------|-----------------|
| <b>Name</b>       | <u>SOLUTION</u> |
| <b>Student ID</b> |                 |
| <b>Grade</b>      |                 |

**GOOD LUCK**

1. a) Given the system

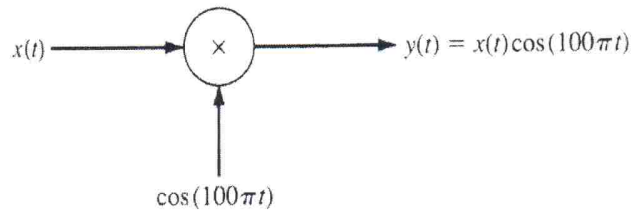


Fig. 1

Determine whether the system shown in Fig. 1 is linear or nonlinear, causal or noncausal, time variant, or invariant. (Justify your answer) (4 Pt.)

$$\begin{aligned} * y_1(t) &= x_1(t) \cos(100\pi t) \\ y_2(t) &= x_2(t) \cos(100\pi t) \end{aligned}$$

$$\begin{aligned} x(t) &= x_1(t) \cos(100\pi t) \\ &+ x_2(t) \cos(100\pi t) \end{aligned}$$

$$ay_1(t) + by_2(t) = [ax_1(t) + bx_2(t)] \cos(100\pi t)$$

$\therefore$  system is Linear

\* system is time-varying because multiplying w/  $x(t)$ .

\* It is causal because the output depends on only the input.

b) Find the step response of the system shown in Fig. 2.

(3 Pt.)

$$Y(s) = \frac{sL}{sL + R} X(s)$$

$$\begin{aligned} x(t) &= u(t) \\ \downarrow \\ X(s) &= 1/s \end{aligned}$$

$$\therefore Y(s) = \frac{sL}{sL + R} \cdot \frac{1}{s} = \frac{L}{s + R/L}$$

$$\therefore y(t) = e^{-R/L t} u(t)$$

2

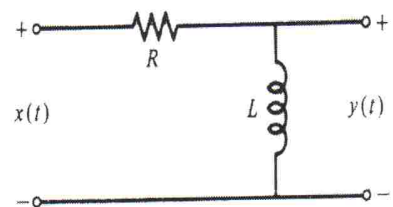
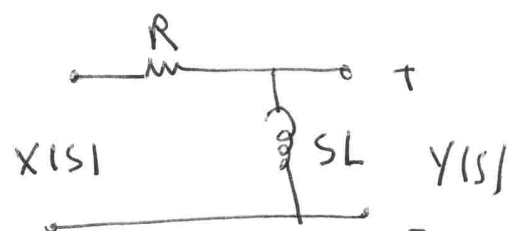


Fig. 2



2b)

$$x(t) = R i(t) + L \frac{di(t)}{dt} = R i(t) + y(t)$$

$$\therefore i(t) = \frac{1}{L} \int y(t) dt$$

$$\therefore x(t) = \frac{R}{L} \int y(t) dt + y(t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{R}{L} y(t) + \frac{dy(t)}{dt}$$

$$x(t) = s(t)$$

$$\frac{ds(t)}{dt} = \frac{dh(t)}{dt} + \frac{R}{L} u(t)$$

$$h(t) = -\frac{R}{L} e^{-R/L t} u(t) + s(t)$$

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} \left( -\frac{R}{L} e^{-R/L \tau} u(\tau) + s(\tau) \right) u(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^t -\frac{R}{L} e^{-R/L \tau} + s(\tau) d\tau$$

$$= -\frac{R}{L} \frac{e^{-R/L \tau}}{-R/L} \Big|_0^t + 1 = e^{-R/L t} - 1 + 1 = e^{-R/L t} u(t)$$

2. a) Use the Fourier transform properties to find  $X_b(\omega)$  for the signal shown in Fig. 3. (4 Pt.)

$$X_b(\omega) = \text{Rect}(t-1.5) - \text{Rect}(t+1.5)$$

$$X_b(\omega) = e^{-j\omega t_0} X(\omega)$$

$$\therefore X_b(\omega) = e^{-j1.5\omega} \text{Sinc} \frac{\omega}{2\pi} - e^{j1.5\omega} \text{Sinc} \frac{\omega}{2\pi}$$

$$\therefore X_b(\omega) = \text{Sinc} \frac{\omega}{2\pi} [e^{-j1.5\omega} - e^{j1.5\omega}]$$

$$= \text{Sinc} \frac{\omega}{2\pi} (2j \sin 1.5\omega) = \boxed{-2j \text{Sinc} \frac{\omega}{2\pi} (\sin 1.5\omega)}$$

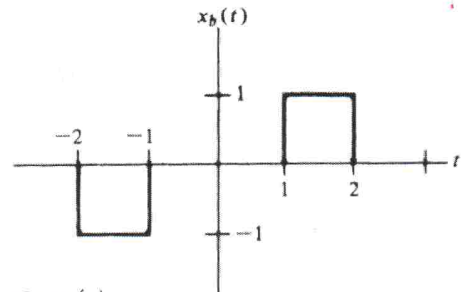


Fig. 3

- b) A modulation system is defined by  $y(t) = x(t) f(t)$ , where  $X(\omega)$  given in Fig. 4. (7 Pt.)

i. Sketch  $Y(\omega)$  when  $f(t) = \cos 10t$ .

ii. Sketch  $Y(\omega)$  when

$$f(t) = \sum_{n=-\infty}^{\infty} \delta(t - n(\pi/15))$$

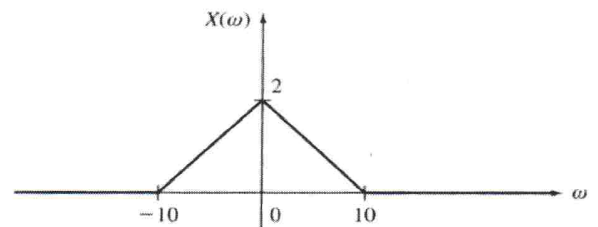


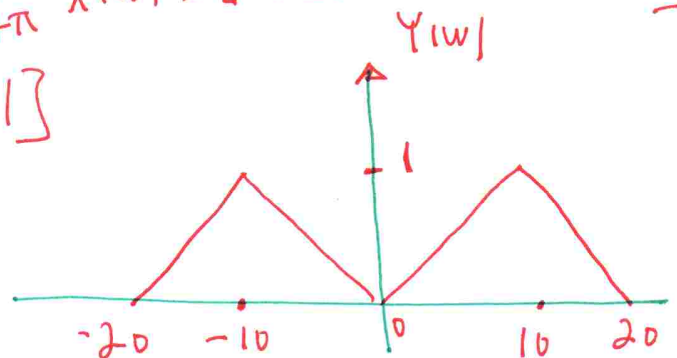
Fig. 4

$$i) y(t) = x(t) f(t)$$

$$Y(\omega) = \frac{1}{2\pi} X(\omega) * F(\omega) = \frac{1}{2\pi} X(\omega) * [\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]]$$

$$\therefore Y(\omega) = \frac{1}{2} [X(\omega - \omega_0) + X(\omega + \omega_0)]$$

$$Y(\omega) = \frac{1}{2} [X(\omega - 10) + X(\omega + 10)]$$



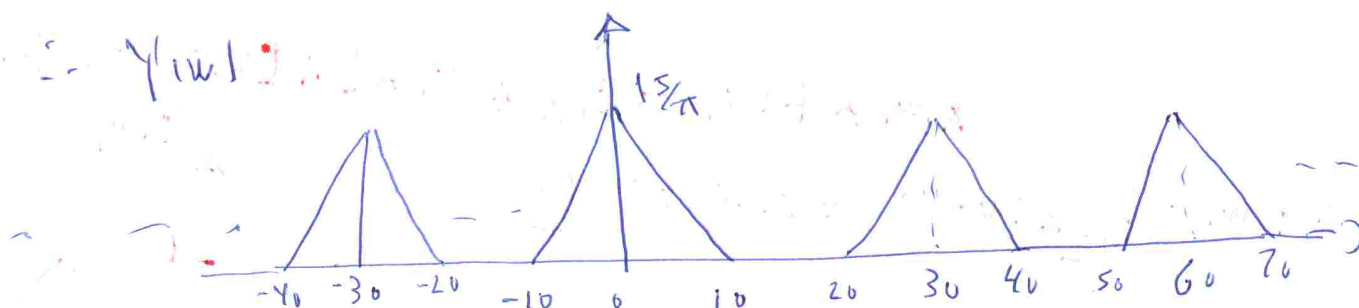
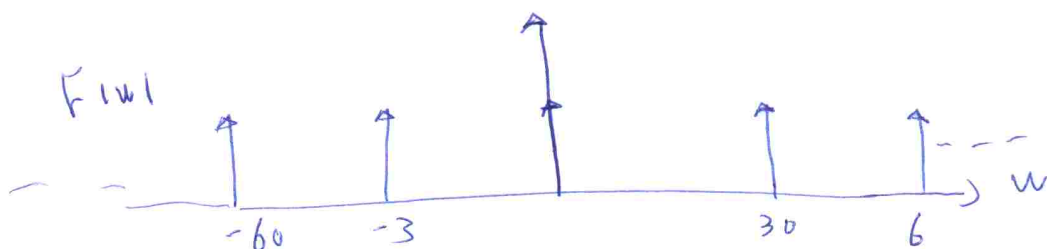
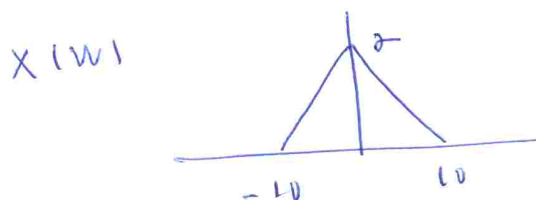
(i)

$$F(\omega) = \frac{2\pi}{\pi/15} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{\pi/15})$$

$$Y(\omega) = \frac{1}{2}\pi X(\omega) * F(\omega)$$

$$= \frac{1}{2}\pi X(\omega) * 30 \sum_{n=-\infty}^{\infty} \delta(\omega - 30n)$$

$$Y(\omega) = \frac{30}{2\pi} [X(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - 30n)]$$



3. a) In the circuit shown in Fig. 5, obtain the current,  $i(t)$ , for  $t \geq 0$  using the Laplace transform. (7 Pt.)

$$\begin{aligned} X(t) &= t \exp(-2t) u(t) \\ L &= 1 \text{ H}, \\ C &= 0.625 \text{ F}, \\ R &= 1 \Omega \\ v_C(0) &= i_L(0) = 0 \end{aligned}$$

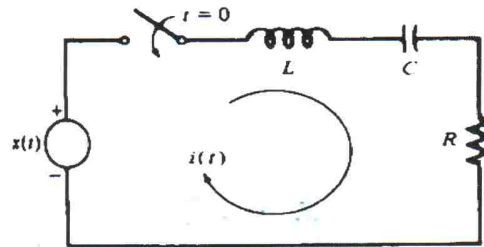
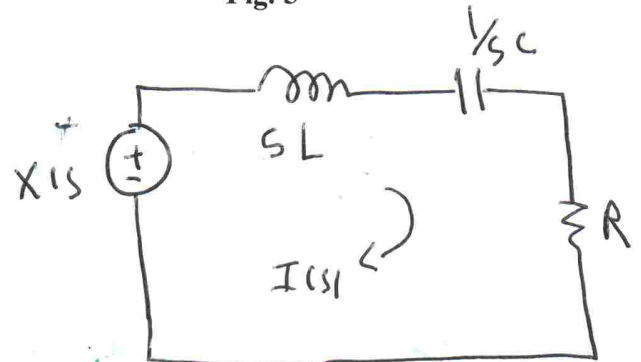


Fig. 5



$$I(s) = \frac{X(s)}{sL + \frac{1}{sC} + R}$$

$$I(s) = \frac{sX(s)}{s^2L + \frac{1}{C} + Rs}$$

$$\therefore I(s) = \frac{sX(s)}{.1s^2 + 1.6 + s} = \frac{10sX(s)}{s^2 + 10s + 16}$$

$$X(s) = \frac{1}{(s+2)^2}$$

$$\therefore I(s) = \frac{10s}{(s+2)^2(s+2)(s+8)} = \frac{10s}{(s+2)^3(s+8)}$$

$$= \frac{A}{(s+2)^3} + \frac{B}{(s+2)^2} + \frac{C}{s+2} + \frac{D}{s+8}$$

4

$$I(s) = \frac{-10/3}{(s+2)^3} + \frac{20/9}{(s+2)^2} + \frac{-10/27}{s+2} + \frac{10/27}{s+8}$$

$$\therefore i(t) = \frac{10}{27} e^{-8t} - \frac{10}{27} e^{-2t} - \frac{5}{3}$$

$$i(t) = \left[ -\frac{10}{3} \frac{t^2 e^{-2t}}{2!} + \frac{20}{9} t e^{-2t} - \frac{10}{27} e^{-2t} + \frac{10}{27} e^{-8t} \right] u(t)$$

$$i(t) = \left[ -\frac{5}{3} t^2 e^{-2t} + \frac{20}{9} t e^{-2t} - \frac{10}{27} e^{-2t} + \frac{10}{27} e^{-8t} \right] u(t)$$

$$= \left[ \left( -\frac{5}{3} t^2 + \frac{20}{9} t - \frac{10}{27} \right) e^{-2t} + \frac{10}{27} e^{-8t} \right] u(t)$$

b) A signal has Laplace transform

$$X(s) = \frac{s+2}{s^2+4s+5}$$

Find the Laplace transform,  $Y(s)$ , of the following signals

(4 Pt.)

i)  $y_1(t) = dx/dt$

ii)  $y_2(t) = e^{-3t} x(t)$

i)  $\mathcal{L}\left\{\frac{dx}{dt}\right\} = sX(s) - X(0)$

$X(0) = \lim_{s \rightarrow \infty} \frac{s(s+2)}{s^2+4s+5} = 1$

$Y(s) = s \frac{s+2}{s^2+4s+5} - 1 = -\frac{2s+5}{s^2+4s+5}$

4. a) Convolve the two functions shown in Fig. 6 and plot the result

(5 Pt.)

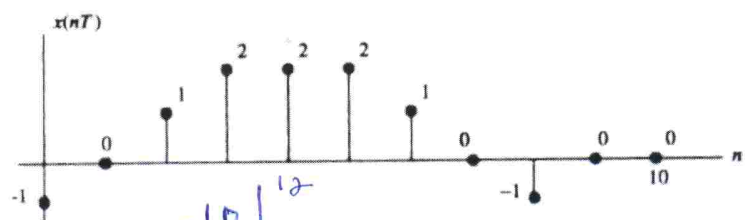
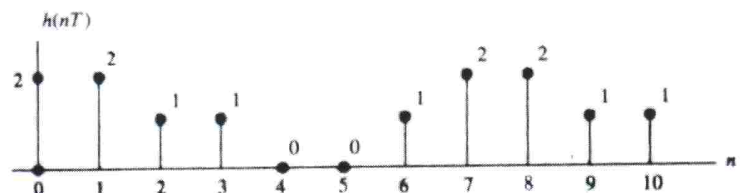
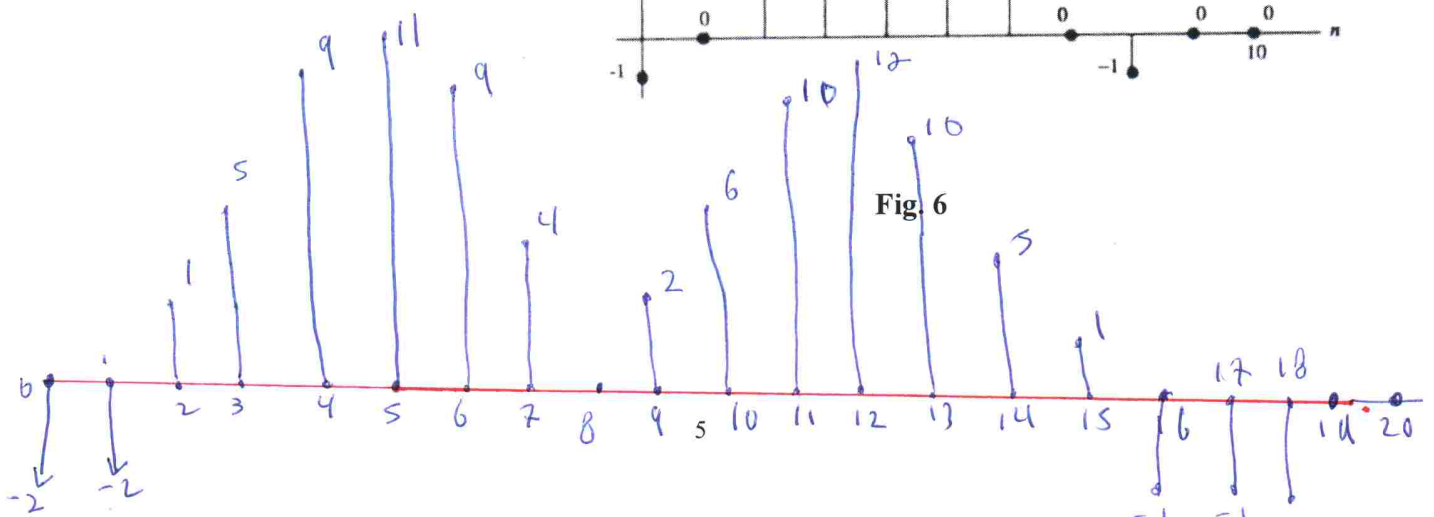


Fig. 6



$y(n) = \{-2, -2, 1, 5, 9, 11, 9, 4, 0, 2, 6, 10, 12, 10, 5, 1, 0, -1, -1, 0, 0\}$

$$\exp(s_0 t) x(t) \longleftrightarrow X(s-s_0)$$

$$\boxed{\text{ii}} \quad Y(s) = X(s+3) = \frac{(s+3)+2}{(s+3)^2 + 4(s+3) + 5}$$

$$\boxed{Y(s) = \frac{s+5}{s^2 + 10s + 26}}$$

b) Determine  $y(n)$  as the convolution of  $h(n)$  and  $x(n)$ , where

(8 Pt.)

$$x(n) = \left(\frac{1}{4}\right)^n u(n-3)$$

$$h(n) = \left(\frac{1}{3}\right)^n u(n-5)$$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u(k-3) \left(\frac{1}{3}\right)^{n-k} u(n-k-5)$$

$$u(k-3) u(n-k-5) = \begin{cases} 1 & 3 \leq k \leq n-5 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore n-5 \geq 3 \quad \therefore n \geq 8$$

$$y(n) = 0 \quad \text{for } n < 8$$

$y(n)$  for  $n \geq 8$ :

$$y(n) = \sum_{k=3}^{n-5} \left(\frac{1}{4}\right)^k \left(\frac{1}{3}\right)^{n-k} = \left(\frac{1}{3}\right)^n \sum_{k=3}^{n-5} \left(\frac{3}{4}\right)^k$$

Let  $m = k+3$   $\therefore y(n) = \left(\frac{1}{3}\right)^n \sum_{m=3}^{n-5-3} \left(\frac{3}{4}\right)^{m+3}$

$$\therefore y(n) = \left(\frac{1}{3}\right)^n \sum_{m=0}^{n-8} \left(\frac{3}{4}\right)^3 \left(\frac{3}{4}\right)^m = \left(\frac{1}{3}\right)^n \left(\frac{27}{64}\right) \sum_{m=0}^{n-8} \left(\frac{3}{4}\right)^m$$

$$\sum_{m=0}^{n-8} \left(\frac{3}{4}\right)^m = \frac{\left(\frac{3}{4}\right)^{n-7} - 1}{\frac{3}{4} - 1}$$

$$= \frac{\left(\frac{3}{4}\right)^{n-7} - 1}{-1/4}$$

$$y(n) = \left(\frac{1}{3}\right)^n \left(27/16\right) (-1) \left[\left(\frac{3}{4}\right)^{n-7} - 1\right]$$

$$\therefore y(n) = \left(27/16\right) \left[\frac{1}{3}\right]^n \left[\left(\frac{3}{4}\right)^{n-7} - 1\right] \quad \text{for } n \geq 8$$

$$\therefore y(n) = \begin{cases} 0 & n < 8 \\ \left(27/16\right) \left(\frac{1}{3}\right)^n \left[1 - \left(\frac{3}{4}\right)^{n-7}\right] & n \geq 8 \end{cases}$$

[or]

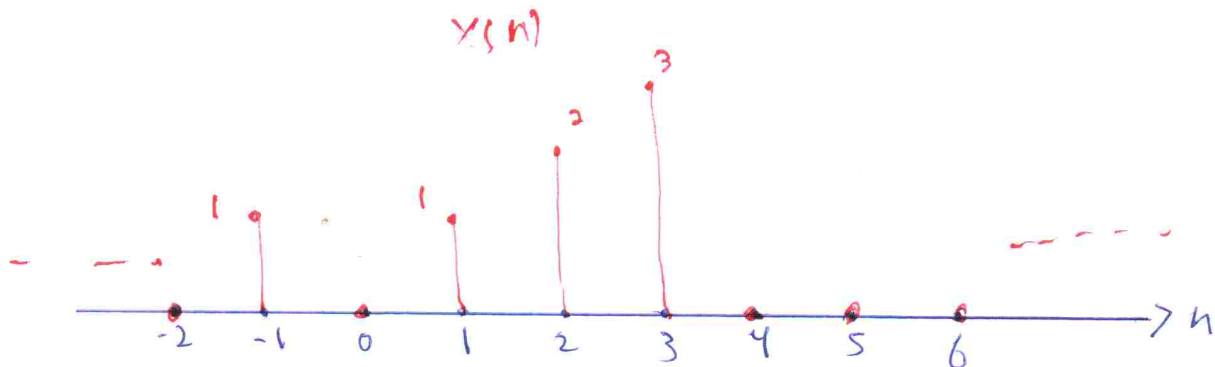
$$y(n) = \left(27/16\right) \left(\frac{1}{3}\right)^n \left(1 - \left(\frac{3}{4}\right)^{n-7}\right) u(n-8)$$



C) Sketch the following signal.

(2 Pt.)

$$X(n) = (n+2) u(n+2) - 2 u(n) - n u(n-4)$$



5. Choose the right answer;

(6 Pt.)

I. Final value theorem is valid for a transfer function  $F(s)$  only if

- a) Poles and zeros of  $F(s)$  lie in the LHP.
- b) Poles and zeros of  $F(s)$  lie in the LHP.
- c) Zeros of  $F(s)$  lie in the LHP.
- ☒ d) Poles of  $F(s)$  lie in the LHP.

II. If  $f(t) = (t-2) e^{-(2t-4)} u(t-2)$  then  $F(s)$  will be

- a)  $1/(s+2)^2$
- ☒ b)  $e^{-2s}/(s+2)^2$
- c)  $e^{2s}/(s+2)^2$
- d)  $1/(s-2)^2$

III. The non-linear input output relation is given by an expression

- a)  $y(t) = k x(t)$
- b)  $y(t) = x(t/3)$
- ☒ c)  $y(t) = x(t/3) + 1$
- d)  $y(t) = x(t) + x(t-2)$

IV. The output of a system with transfer function  $5/(s+2)$  for an input  $e^{-t}$  in time domain is

- a)  $4(e^{-t} - e^{-2t})$
- ☒ b)  $5(e^{-t} - e^{-2t})$
- c)  $5(e^{-2t} - e^{-t})$
- d)  $5e^{-2t} - 4e^{-t}$

$$\textcircled{\text{IV}} \quad Y(s) = \left( \frac{1}{s+1} \right) \left( \frac{s}{s+2} \right) = \frac{s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$Y(s) = \frac{s}{s+1} + \frac{-s}{s+2}$$

$$\therefore y(t) = 5e^{-t} - 5e^{-2t} = \underline{\underline{5(e^{-t} - e^{-2t})}} \quad u(t)$$

$$\text{II} \quad f(t) = (t-2)e^{-2(t-2)} u(t-2)$$

$$f(t) = t e^{-2t} u(t) \Big|_{t=t-2}$$

$$\therefore F(s) = e^{-2s} \frac{1}{(s+2)^2} = \frac{e^{-2s}}{(s+2)^2}$$