

**Islamic University of Gaza**  
**Electrical and Computer Engineering Department**  
**EELE 3310 Signals & Systems**  
**Final Exam**  
**December, 31, 2005**  
**OPEN BOOK & NOTE EXAM**

**Dr. Basil Hamed**

**Exam Time 8:30-11:00**

|                   |          |
|-------------------|----------|
| <b>Name</b>       | Solution |
| <b>Student ID</b> |          |
| <b>Grade</b>      |          |

**GOOD LUCK**

Students who did not take the midterm exam they have to answer all four questions. But for students who took the midterm, the fourth question will be bonus if they answer it.

1. a) In Fig. 1, express signals  $x_1(t)$  in terms of signal  $x(t)$  and its time-shifted, time-scaled, or time-reversed versions. (4 PT.)

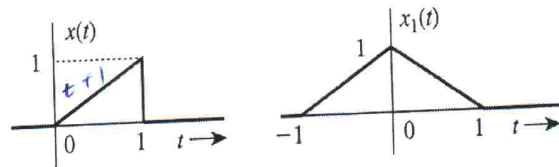


Fig 1

$$x_1(t) = x(t+1) + x(1-t)$$

- b) The Fourier transform of the triangular pulse  $x(t)$  in Fig. 1 is expressed as (5 PT.)

$$X(\omega) = \frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1)$$

Use this information, and Fourier properties, to find the Fourier transform of the signal  $x_1(t)$  shown in Fig. 1.

$$x(t+1) \longleftrightarrow X(\omega) e^{j\omega}$$

$$x(1-t) \longleftrightarrow X(-\omega) e^{-j\omega}$$

$$\therefore x_1(t) = x(t+1) + x(1-t)$$

$$x_1(\omega) = X(\omega) e^{j\omega} + X(-\omega) e^{-j\omega}$$

$$\therefore x_1(\omega) = \frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1) e^{j\omega} + \frac{1}{\omega^2} (e^{-j\omega} + j\omega e^{-j\omega} - 1) e^{-j\omega}$$

c) A system is given by

(6 PT.)

$$y(t) = \begin{cases} x(t), & x(t) > 0 \\ 0, & x(t) \leq 0 \end{cases}$$

- i. Is the system stable?
  - ii. Is the system linear?
  - iii. Is the system causal?
  - iv. Is the system time invariant?
- Justify your answer.

i) Yes it is stable if  $|x(t)| < \infty$  then  $|y(t)| < \infty$

ii) No the system is not linear if  $|x(t)| \leq 0$  then  $|y(t)| = 0$

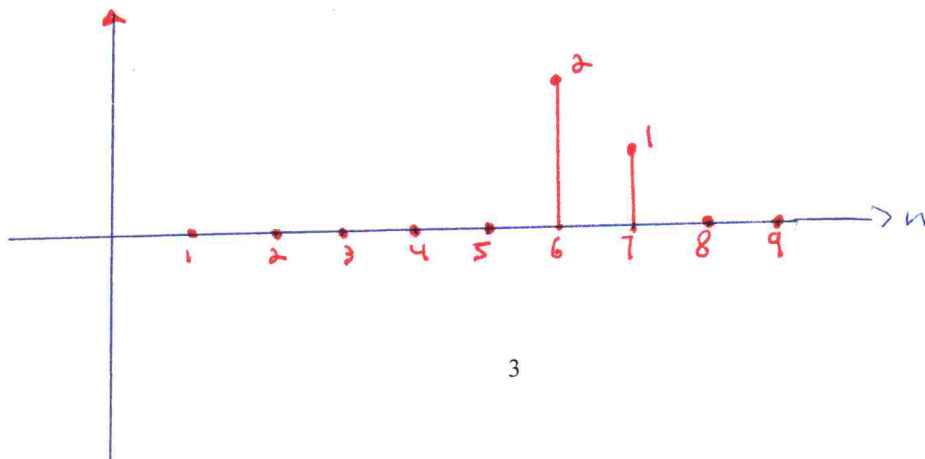
iii) Yes the system is causal because it does not depend on future

iv) Yes the system is time invariant, no multiplication of  $t$ .

2. a) Sketch the following signal.

(4 PT.)

$$(-n+8)[u(n-6)-u(n-9)]$$



b) Describe the signal in Fig. 2 an expression valid for all n.

(4 PT.)

$$x[n] = -3n [u(n+3) - u(n-4)]$$

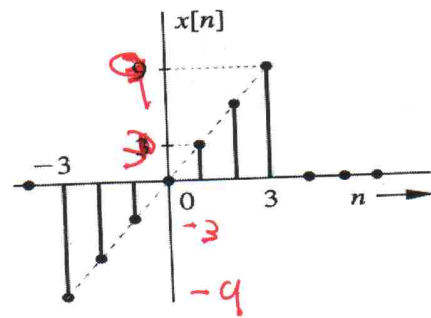


Fig 2

c) A system with impulse response  $h(n) = 2(\frac{1}{3})^n u(n-1)$  produces an output  $y(n) = (-2)^n u(n-1)$ . Determine the corresponding input  $x(n)$ . (12 PT.)

$$H(z) = z^{-1} \frac{2z/3}{z - 1/3}$$

$$Y(z) = z^{-1} \frac{-2z}{z+2}$$

$$Y(z) = H(z) X(z) \Rightarrow X(z) = \frac{Y(z)}{H(z)} = \frac{z^{-1} \frac{-2z}{z+2}}{z^{-1} \frac{2z/3}{z-1/3}}$$

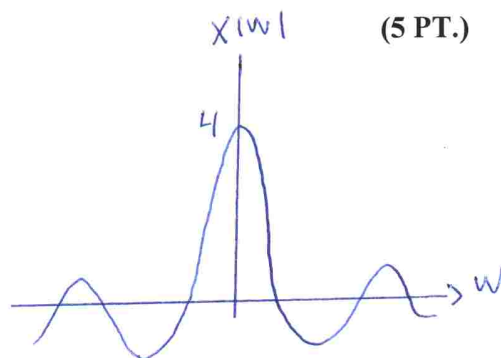
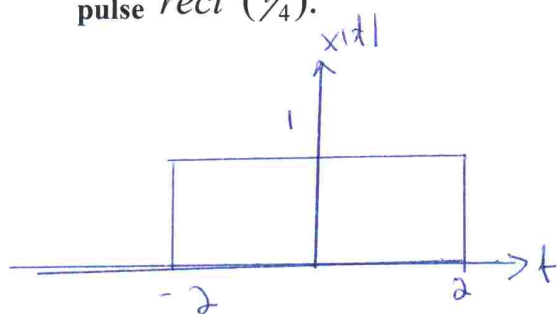
$$\therefore X(z) = -3 \frac{z - 1/3}{z+2} = -3 \left[ \frac{z}{z+2} - \frac{1/3}{z+2} \right]$$

$$\therefore x[n] = -3 \left[ (-2)^n u[n] - \frac{1}{3} (-2)^{n-1} u[n-1] \right]$$

OR

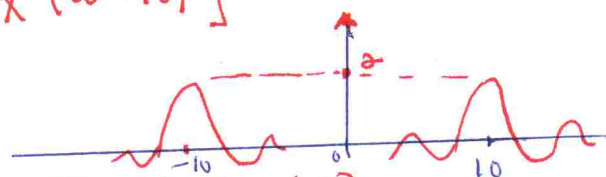
$$x[n] = -3\delta[n] + 7(-2)^{n-1} u[n-1]$$

3. a) Find and sketch the Fourier transform of the modulated signal  $x(t) \cos 10t$ , in which  $x(t)$  is pulse  $\text{rect}(t/4)$ . (5 PT.)



$$x(t) \cos 10t \longleftrightarrow \frac{1}{2} [X(w+10) + X(w-10)]$$

$$X(w) = 4 \text{sinc}(2w)$$



$$\therefore x(t) \cos 10t \longleftrightarrow 2 \text{sinc}[2(w+10)] + 2 \text{sinc}[2(w-10)]$$

b)  $X(s) = \frac{2s-5}{(s-2)(s-3)}$

$2 < \text{Re}\{s\} < 3$ , Find  $x(t)$

(5 PT.)

$$X(s) = \frac{1}{s-2} + \frac{1}{s-3}$$

$$2 < \text{Re}\{s\} < 3$$

$$\therefore x(t) = e^{2t} u(t) - e^{3t} u(1-t)$$

The pole at  $-2$  lies to the left and that lies to the right of the origin.

c) Find the loop current  $y_1(t)$  and  $y_2(t)$  for  $t \geq 0$  in the circuit of Fig. 3-a for the input  $x(t)$  in Fig 3-b, where  $y_1(0) = 2$  and  $y_2(0) = 1$  (10 PT.)

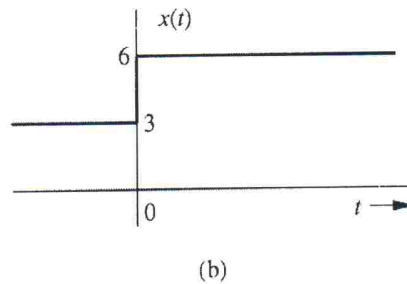
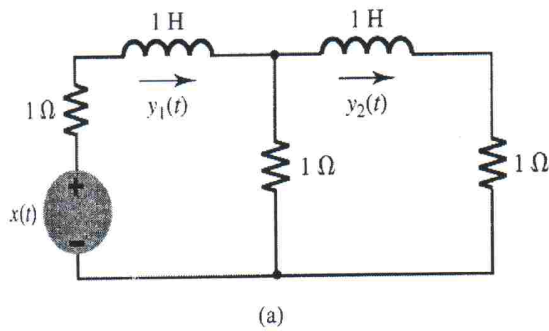
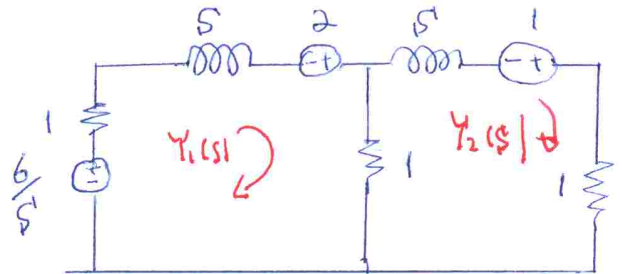


Fig. 3



$$\begin{aligned} (\delta + 2)Y_1(s) - Y_2(s) &= 2 + \frac{6}{s} \\ -Y_1(s) + (\delta + 2)Y_2(s) &= 1 \end{aligned}$$

Using Cramer's rule:

$$Y_1(s) = \frac{2s^2 + 11s + 12}{s(s+1)(s+3)} = \frac{4}{s} - \frac{3/2}{s+1} - \frac{1/2}{s+3}$$

$$Y_2(s) = \frac{s^2 + 4s + 6}{s(s+1)(s+3)} = \frac{2}{s} - \frac{3/2}{s+1} + \frac{1/2}{s+3}$$

$$y_1(t) = (4 - \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t})u(t)$$

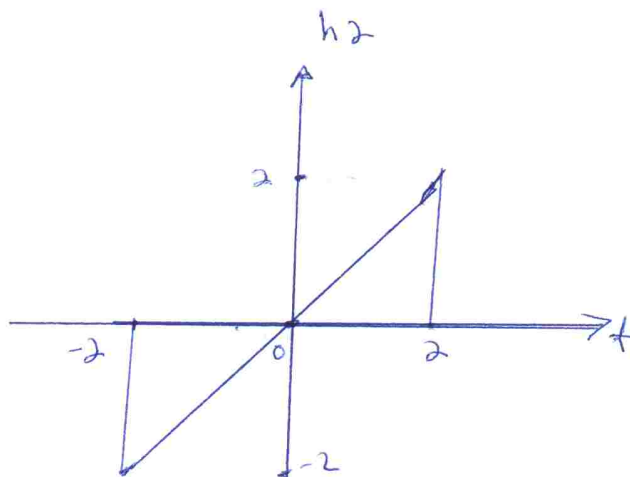
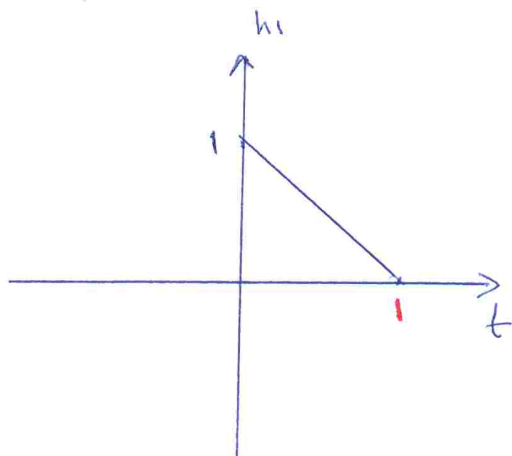
$$y_2(t) = (2 - \frac{3}{2}e^{-t} + \frac{1}{2}e^{-3t})u(t)$$

4. Two systems have impulse response functions given by

$$h_1 = (1-t)[u(t) - u(t-1)],$$

$$h_2 = t[u(t+2) - u(t-2)].$$

a) Sketch the functions  $h_1(t)$  and  $h_2(t)$ .



b) Assume that the two systems are connected in cascade as shown in Fig. 4. Find the equivalent impulse response function,  $h_s(t)$ .  
(Work only on time domain)

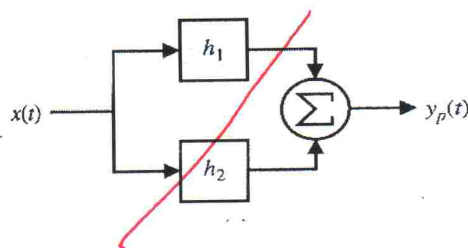


Fig. 4

$$h_s(t) = h_1 * h_2$$

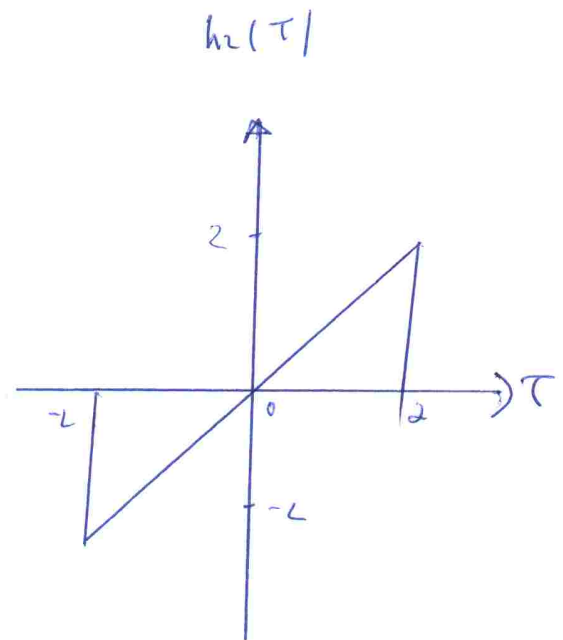
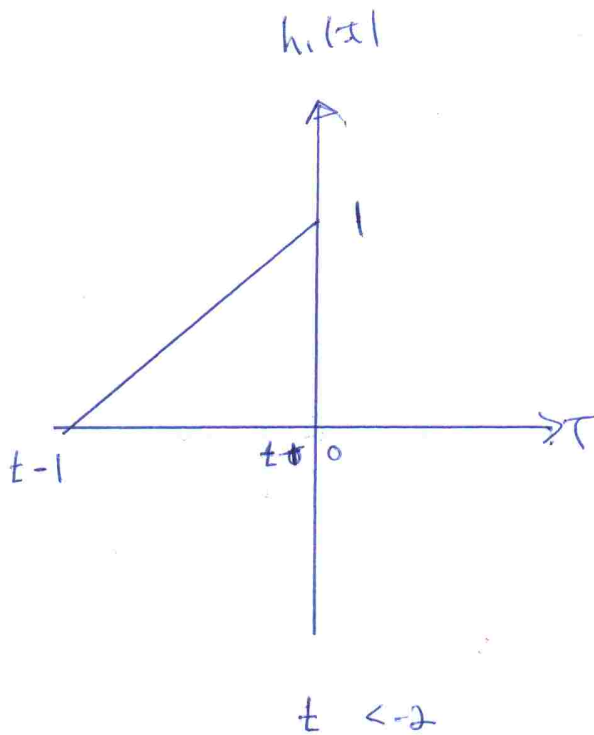


$$h_s(t) = \begin{cases} +\frac{t^3}{3} + \frac{t^2}{2} + \frac{2}{3} & -2 \leq t < -1 \\ \frac{t^3}{3} - \frac{1}{6} & -1 \leq t < 2 \\ -\frac{t^3}{3} + \frac{t^2}{2} + \frac{9}{2} & 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$2 < t < 3$$

$$h_s(t) = \int_{t-1}^2 (\tau+1) \tau d\tau = \left. \frac{\tau^3}{3} + \frac{\tau^2}{2} \right|_{t-1}^2 = \left[ \frac{8}{3} + \frac{4}{2} \right] - \left[ \frac{(t-1)^3}{3} + \frac{(t-1)^2}{2} \right]$$

$$h_s(t) = -\frac{t^3}{3} + \frac{t^2}{2} + \frac{9}{2}$$



$$h_s(t) = 0$$

$$-2 < t < -1$$

$$h_s(t) = \int_{-2}^t (\tau+1) \tau d\tau = \left. \frac{\tau^3}{3} + \frac{\tau^2}{2} \right|_{-2}^t = \left( \frac{t^3}{3} + \frac{t^2}{2} \right) - \left( -\frac{8}{3} + \frac{4}{2} \right)$$

$$h_s(t) = \frac{t^3}{3} + \frac{t^2}{2} + \frac{2}{3}$$

$$-2 < t < -1$$

$$-1 < t < 2$$

$$h_s(t) = \int_{t-1}^t (\tau+1) \tau d\tau = \left. \frac{\tau^3}{3} + \frac{\tau^2}{2} \right|_{t-1}^t = \left[ \frac{t^3}{3} + \frac{t^2}{2} \right] - \left[ \frac{(t-1)^3}{3} + \frac{(t-1)^2}{2} \right]$$

$$h_s(t) = t^2 - \frac{1}{6}$$

$$-1 < t < 2$$