

Islamic University of Gaza
Electrical and Computer Engineering Department
ENGE 3310 Signals & Systems
Final Exam
January, 9, 2007

OPEN BOOK & NOTE EXAM

Dr. Basil Hamed

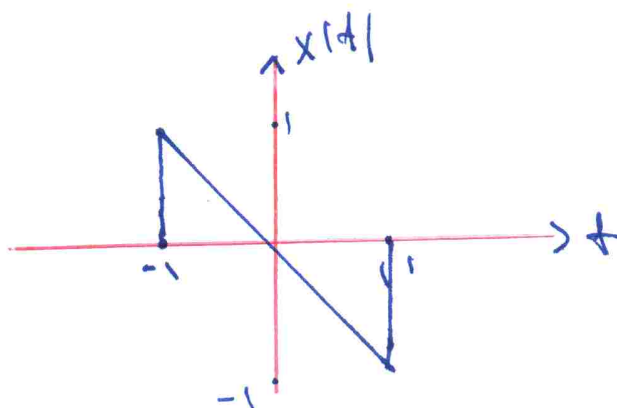
Exam Time 8:00-10:30

Name	SOLUTION
Student ID	
Grade	

GOOD LUCK

Students who did not take the midterm exam they have to answer all five questions. But for students who took the midterm, the fifth question will be bonus if they answer it.

1. a) Define $x(t) = t(u(-t-1) - u(-t+1))$, where $u(t)$ is the unit step function. Plot $x(t)$. (3 Points)



- b) In Fig. 1, express signals $x_5(t)$ in terms of signal $x(t)$ and its time-shifted, time-scaled, or time-reversed versions. (4 Points)

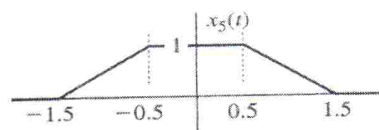
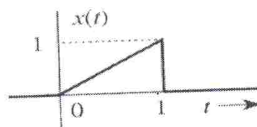


Fig. 1

$$x_5(t) = x(t+1.5) + x(1.5-t) + x(t-1.5) + x(1.5-t)$$

c) The Fourier transform of the triangular pulse $x(t)$ in Fig. 1 is expressed as

$$X(\omega) = \frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1)$$

Use this information, and Fourier properties, to find the Fourier transform of the signal $x_5(t)$ shown in Fig. 1 (3 Points)

$$X_5(\omega) = X(\omega) e^{j\omega/2} + X(-\omega) e^{-j\omega/2} + X(\omega) e^{1.5j\omega} + X(-\omega) e^{-1.5j\omega}$$

substitute

$$X(\omega) = \frac{1}{\omega^2} [e^{j\omega} - j\omega e^{j\omega} - 1]$$

2. a) Find and sketch $y(t) = x_1(t) * x_2(t)$ for the function illustrated in Fig. 2-a

(5 Points)

$$0 \leq t \leq 1$$

$$y(t) = \int_{-1+t}^0 e^{\tau} e^{-2(t-\tau)} d\tau = \frac{1}{3} [e^{-2t} - e^{t-3}]$$

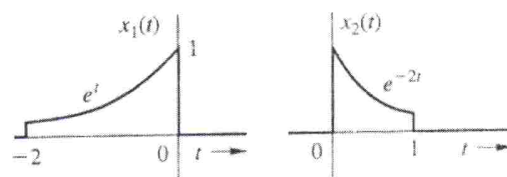


Fig. 2-a

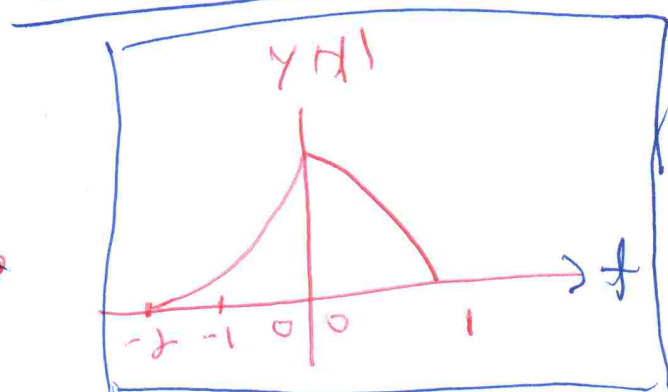
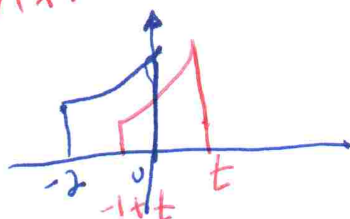
$$-1 \leq t \leq 0$$

$$y(t) = \int_{-1+t}^t e^{\tau} e^{-2(t-\tau)} d\tau = \frac{1}{3} [e^t - e^{t-3}]$$

$$-2 \leq t \leq -1$$

$$y(t) = \int_{-2}^t e^{\tau} e^{-2(t-\tau)} d\tau = \frac{1}{3} [e^t - e^{-2(t+3)}]$$

$$y(t) = 0 \quad t \leq -2$$



b) For the circuit shown in Fig. 2-b, let $v_c(0) = 1$ volts, $i_L(0) = 2$ amperes, $x(t) = u(t)$. Find $y(t)$. (6 Points)

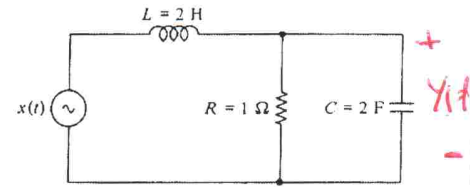
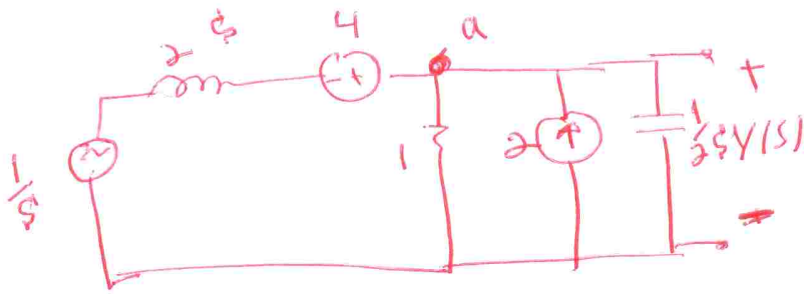


Fig. 2-b

at node a

$$\frac{1/s + 4 - Y(s)}{2s} + 2 - Y(s) - Y(s) 2s = 0$$

$$1/s + 4 - Y(s) + 4s = (1 + 2s) 2s Y(s)$$

$$Y(s) = \frac{s^2 + s + 1/4}{s(s^2 + 1/2s + 1/4)} = 1/s + \frac{1/2}{s^2 + 1/2s + 1/4}$$

$$y(t) = u(t) + \frac{2}{\sqrt{5}} e^{-1/4t} \sin \sqrt{5/4} t$$

c) For the discrete -time signal shown in Fig. 2-c, sketch $x[2-n] + x[3n-4]$ (4 Points)

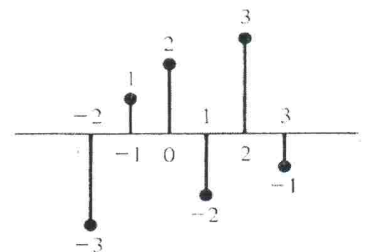
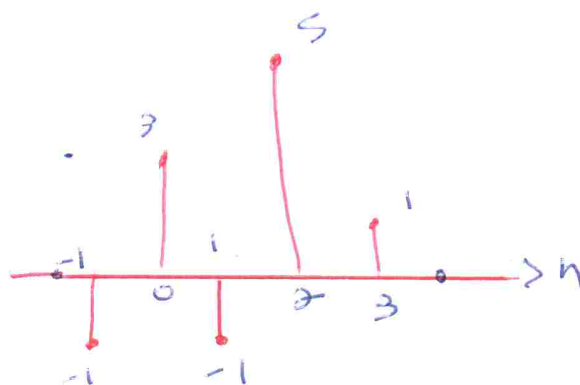


Fig. 2-c



3. a) Determine $x(t)$ where

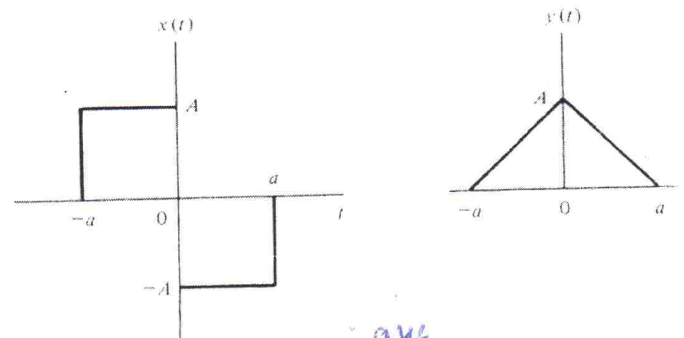
(4 Points)

$$X(s) = \frac{s^2}{e^{s+3}(s+1)(s+2)}$$

$$X(s) = e^{-3} e^{-s} \left[1 + \frac{-3s-2}{(s+1)(s+2)} \right] = e^{-3} e^{-s} \left[1 + \frac{1}{s+1} + \frac{-4}{s+2} \right]$$

$$x(t) = e^{-3} \left[\delta(t-1) + e^{-(t-1)} u(t-1) - 4 e^{-2(t-1)} u(t-1) \right]$$

b) Consider the two related signals shown in Fig. 3. Use linearity, time-shifting, and integration properties, to find $X(\omega)$ and $Y(\omega)$ (6 Points)



$$x(t) = \text{rect}\left(\frac{t+a/2}{a}\right) - \text{rect}\left(\frac{t-a/2}{a}\right)$$

$$X(\omega) = a \text{Sinc} \frac{\omega a}{2\pi} e^{j\frac{\omega a}{2}} - a \text{Sinc} \frac{\omega a}{2\pi} e^{-j\frac{\omega a}{2}}$$

Fig. 3

$$= a \text{Sinc} \frac{\omega a}{2\pi} [2j \sin \frac{\omega a}{2}]$$

$$\therefore Y(\omega) = \int_{-\infty}^{\infty} x(\tau) d\tau \quad \text{and} \quad x(0) = 0$$

$$\therefore Y(\omega) = \frac{X(\omega)}{j\omega} = \frac{2a}{\omega} \text{Sinc}\left(\frac{\omega a}{2\pi}\right) \sin\left(\frac{\omega a}{2}\right)$$

4. a) Find the unit impulse response of the system described by the following equation :

$$y[n+2] + 2y[n+1] + y[n] = 2x[n+2] - x[n+1]$$

(5 Points)

$$H(z) = \frac{2z^2 - z}{z^2 + 2z + 1} = \frac{z(2z - 1)}{(z + 1)^2}$$

$$H(z) = \frac{2}{z+1} - \frac{3}{(z+1)^2} \Rightarrow H(z) = 2 \left(\frac{z}{z+1} \right) - 3 \frac{z}{(z+1)^2}$$

$$h[n] = [2(-1)^n + 3n(-1)^n] u[n] = (2+3n)(-1)^n u[n]$$

b) Obtain the z transform of the curve shown in Fig. 4-b Assume T=1 sec.

(4 Points)

$$\begin{aligned} x(0) &= 0 \\ x(1) &= 0.25 \\ x(2) &= 0.50 \\ x(3) &= 0.75 \\ x(k) &= 1 \end{aligned}$$

$$k=4, 5, 6, \dots$$

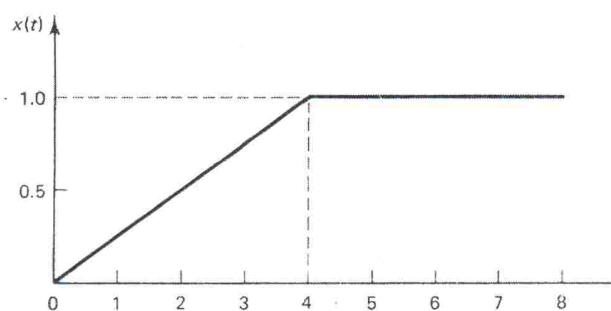


Fig. 4-b

$$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$= 0.25z^{-1} + 0.5z^{-2} + 0.75z^{-3} + z^{-4} + z^{-5} + \dots$$

$$= 0.25(z^{-1} + 2z^{-2} + 3z^{-3}) + z^{-4} \frac{1}{1-z^{-1}}$$

$$X(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4}}{4(1-z^{-1})} \cdot \frac{(1-z^{-1})}{(1-z^{-1})}$$

$$X(z) = \frac{1}{4} \frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$$

OK

$$x(t) = \frac{1}{4}t - \frac{1}{4}(t-4)u(t-4)$$

$$X(z) = \mathcal{Z}\{x(t)\} = \frac{1}{4} \frac{z^{-1}}{(1-z^{-1})^2} - \frac{1}{4} \frac{z^{-4} z^{-1}}{(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4} \frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$$

c) Find the Fourier-series representation of the signal shown in Fig. 4-c, Plot the magnitude and phase spectrum. (6 Points)

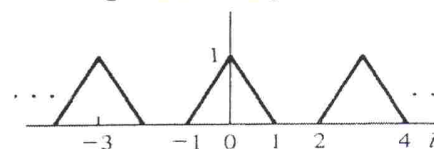


Fig. 4-c

$x(t)$ is even function

$$C_n = \frac{1}{T} \int_T x(t) e^{-jn\omega_0 t} dt = \frac{2}{T} \int_0^{T/2} x(t) \cos n\omega_0 t dt$$

$$T=3 \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}, \quad C_0 = 0$$

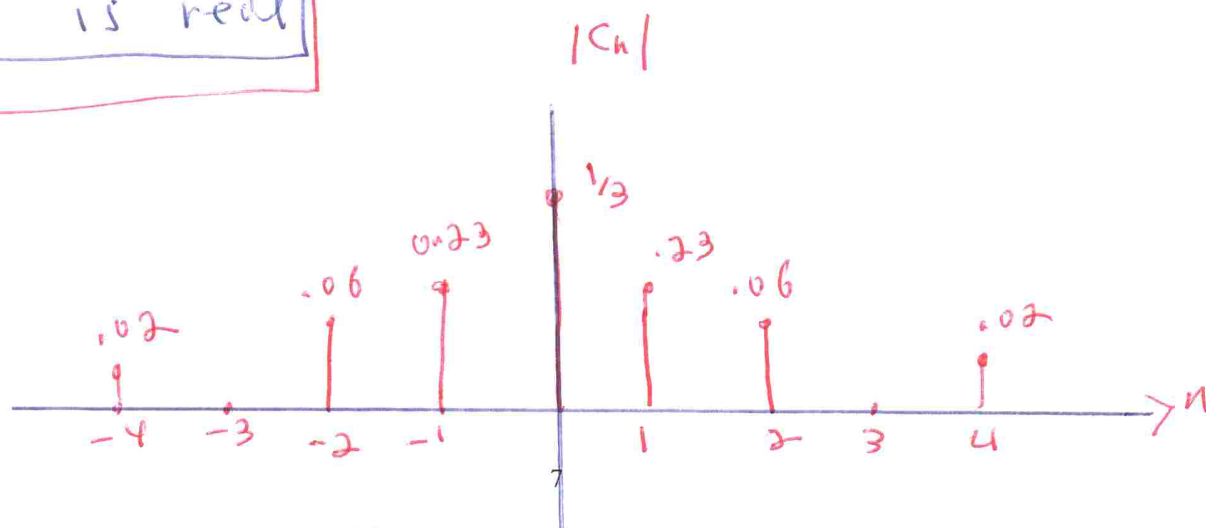
$$C_n = \frac{2}{3} \int_0^1 (1-t) \cos \frac{2n\pi}{3} t dt$$

$$= \frac{2}{3} \left[\frac{3}{2n\pi} \sin \left(\frac{2n\pi}{3} \right) - \frac{3}{2n\pi} \left[\sin \frac{2n\pi}{3} + \frac{3}{2n\pi} (\cos \frac{2n\pi}{3} - 1) \right] \right]$$

$$= -\frac{2}{3} \left[\left(\frac{3}{2n\pi} \right)^2 (\cos \frac{2n\pi}{3} - 1) \right]$$

$$= \frac{-3 [\cos(\frac{2n\pi}{3}) - 1]}{(2\pi^2)n^2}$$

C_n is real



$$\angle C_n = 0 \text{ for all } n$$

$$X(z) = \frac{1}{(z-1)^2 z^2} = \frac{1}{(z-1)^2} - \frac{2}{z-1} + \frac{1}{z^2} + \frac{2}{z}$$

$$X(z) = \frac{z^{-2}}{(1-z^{-1})^2} - \frac{2z^{-1}}{1-z^{-1}} + z^{-2} + 2z^{-1}$$

$$\mathcal{Z}^{-1}\left[\frac{z^{-2}}{(1-z^{-1})^2}\right] = \begin{cases} k-1 & k=1, 2, 3, \dots \\ 0 & k \leq 0 \end{cases}$$

$$\mathcal{Z}^{-1}\left[\frac{z^{-1}}{1-z^{-1}}\right] = \begin{cases} 1 & k=1, 2, 3, \dots \\ 0 & k \leq 0 \end{cases}$$

$$\mathcal{Z}^{-1}[z^{-2}] = \begin{cases} 1 & k=2 \\ 0 & k \neq 2 \end{cases}$$

$$\mathcal{Z}^{-1}[z^{-1}] = \begin{cases} 1 & k=1 \\ 0 & k \neq 1 \end{cases}$$

$$\therefore x(k) = \begin{cases} 0-0+0+0=0 & k=0 \\ 0-2+0+2=0 & k=1 \\ 1-2+1+0=0 & k=2 \\ k-1-2+0+0=k-3 & k=3, 4, 5, \dots \end{cases}$$

$$\boxed{\therefore x(k) = \begin{cases} 0 & k=0, 1, 2 \\ k-3 & k=4, 5, 6, \dots \end{cases}}$$

5. a) Obtain the inverse z transform of

$$X(z) = \frac{1}{(z-1)^2 z^2}$$

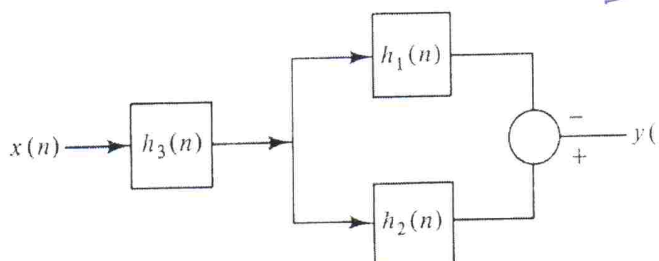
$$X(z) z^{k-1} = \frac{z^{k-1}}{(z-1)^2 z^2} = \frac{1}{(z-1)^2 z^{3-k}}$$

$X(z) z^{k-1}$ has double pole at $z=1$ and $(3-k)$ multiple pole at $z=0$

For $k=0$ double pole at $z=1$ and triple pole at $z=0$
 $k=1$ " " " double " "
 $k=2$ " " " simple " "
 $k=3, 4, 5, \dots$ " " " no " "

b) Find the transfer function of the system

if $h_1[n] = [n-1]u[n]$
 $h_2[n] = \delta[n] + nu[n-1] + \delta[n-2]$
 $h_3[n] = (1/2)^n u[n]$



$$H_1(z) = \frac{z}{(z-1)^2} - \frac{z}{z-1} = \frac{2z - z^2}{(z-1)^2}$$

$$H_2(z) = 1 + \frac{z}{(z-1)^2} + z^{-2} = \frac{z^4 - z^3 + 2z^2 - 2z + 1}{z^2(z-1)^2}$$

$$H_3(z) = \frac{z}{z-1/2}$$

$$H(z) = H_3(z) [H_2(z) - H_1(z)]$$

$$= \frac{z}{z-1/2} \left[\frac{2z^4 - 3z^3 + 2z^2 - 2z + 1}{z^2(z-1)^2} \right]$$

$$H(z) = \frac{2z^3 - z^2 + z - 1}{z(z-1/2)(z-1)}$$

For $k=0$

$$x(0) = \left[\text{residue of } \frac{1}{(z-1)^2 z^3} \text{ at } z=1 \right] + \left[\text{residue of } \frac{1}{(z-1)^2 z^3} \text{ at triple } z=0 \right]$$

$$= \frac{1}{(2-1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{1}{(z-1)^2 z^3} \right] + \frac{1}{(3-1)!} \lim_{z \rightarrow 0}$$

$$\frac{d^2}{dz^2} \left[z^3 \frac{1}{(z-1)^2 z^3} \right] = -3 + 3 = 0$$

For $k=1$

$$x(1) = \frac{1}{(2-1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{1}{(z-1)^2 z^2} \right] +$$

$$\frac{1}{(2-1)!} \lim_{z \rightarrow 0} \frac{d}{dz} \left[z^2 \frac{1}{(z-1)^2 z^2} \right] = -2 + 2 = 0$$

For $k=2$

$$x(2) = -1 + 1 = 0$$

For $k=3, 4, 5, \dots$

$$x(k) = \frac{1}{(2-1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{z^{k-3}}{(z-1)^2} \right] = k-3$$

$$\therefore x(k) = \begin{cases} 0 & k=0, 1, 2, 3 \\ k-3 & k=4, 5, 6, \dots \end{cases}$$