



Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3310 Signals & Systems
Final Exam
January, 6, 2008
OPEN BOOK & NOTE EXAM

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Exam Time 12:00-2:30

Name	Solution
Student ID	
Grade	

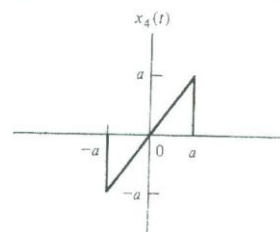
GOOD LUCK

Students who did not take the midterm exam they have to answer all five questions. But for Students who took the midterm, the fifth question will be bonus if they answer it.

1. a) For the signal shown, write an equation in terms of elementary signals.

(4 Pt.)

$$x_4(t) = (r(t) - r(-t)) (u(t+a) - u(t-a))$$



- b) Determine whether the system described by the following input/output relationships are linear or nonlinear, causal or noncausal, time variant, or invariant, memory, or memoryless. (Justify your answer)

(4 Pt.)

$$y(t) = \frac{d}{dt} x(t-1)$$

* Linearity

$$y_1(t) = \frac{d}{dt} x_1(t-1), \quad y_2(t) = \frac{d}{dt} x_2(t-1)$$

$$\therefore a x_1 + b x_2 \Rightarrow y(t) = \frac{d}{dt} (a x_1(t-1) + b x_2(t-1)) = a \frac{d}{dt} x_1(t-1)$$

$$+ b \frac{d}{dt} x_2(t-1) = a y_1 + b y_2 \quad \therefore \text{system is Linear}$$

* system is with memory. system depends on a past value of the input.

* system is causal. system does not depend on future

* system is time-invariant. no multiplication of time.

c) A signal has Fourier transform

(4 pt.)

$$X(w) = \frac{w^2 + j4w + 2}{-w^2 + j4w + 3}$$

Find the transform of $x(t) * \delta(t-1)$

$$\mathcal{F}\{x(t) * \delta(t-1)\} = e^{-jw} X(w) = e^{-jw} \frac{w^2 + j4w + 2}{-w^2 + j4w + 3}$$

2. a) for a system with transfer function

$$H(s) = \frac{2s+3}{s^2+2s+5}$$

Find the response for input $x(t) = u(t-5)$.

(6 Pt.)

$$x(t) = u(t-5) \quad ; \quad x(s) = \frac{1}{s} e^{-5s}$$

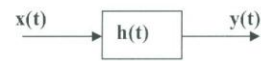
$$\therefore Y(s) = \frac{2s+3}{s(s^2+2s+5)} e^{-5s} = \left[\frac{0.6}{s} + \frac{1}{10} \left(\frac{-6s+8}{s^2+2s+5} \right) \right] e^{-5s}$$

$$\therefore y(t) = \frac{1}{10} \left(6 + 9.22 e^{-(t-5)} \cos[2(t-5) - 130.6^\circ] \right) u(t-5)$$

b) Find $h(t)$ for given input and output (6 Pt.)

$$x(t) = 2u(t);$$

$$y(t) = tu(t) - \exp(-2t)u(t)$$



$$X(s) = \frac{2}{s}$$

$$Y(s) = \frac{1}{s^2} - \frac{1}{s+2}$$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{s^2} - \frac{1}{s+2}}{\frac{2}{s}} = -\frac{1}{2} + \frac{s}{s} + \frac{1}{s+2}$$

$$\therefore h(t) = \mathcal{L}^{-1}[H(s)]$$

$$h(t) = -\frac{1}{2} \delta(t) + \frac{1}{2} u(t) + \exp(-2t) u(t)$$

3. a) Solve $4y[n+2] + 4y[n+1] + y[n] = x[n+1]$; with $y[-1]=0$, $y[-2]=1$, and $x[n]=u[n]$.

(6 Pt.)

$$4y[n+2] + 4y[n+1] + y[n] = x[n+1]$$

$$y[n+1] \leftrightarrow \frac{1}{z} Y(z)$$

$$y[n+2] \leftrightarrow \frac{1}{z^2} Y(z) + 1$$

$$x[n+1] \leftrightarrow \frac{1}{z-1}$$

∴ The z-transform of the system eq:

$$4Y(z) + \frac{4}{z} Y(z) + \frac{1}{z^2} Y(z) + 1 = \frac{1}{z-1}$$

$$\frac{4z^2 + 4z + 1}{z^2} Y(z) = \frac{z-1}{z-1}$$

$$\text{and } \frac{Y(z)}{z} = \frac{z(2-z)}{4(z-1)(z^2+z+2.5)} = \frac{z(2-z)}{4(z-1)(z+0.5)^2}$$

$$Y(z) = \frac{1}{4} \left[\frac{4/9}{z-1} - \frac{13/9}{z+0.5} + \frac{5/6}{(z+0.5)^2} \right]$$

$$y[n] = \left[\frac{1}{9} - \frac{13}{36} (-0.5)^n - \frac{5}{12} n (-0.5)^n \right] u[n]$$

b) A system with impulse response $h[n] = 2(1/3)^n u[n-1]$ produce an output $y[n] = (-2)^n u[n-1]$. Determine the corresponding input $x[n]$. (7 Pt.)

$$H(z) = z^{-1} \frac{2z/3}{z-1/3}, \quad Y(z) = z^{-1} \frac{-2z}{z+2}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{z^{-1} \frac{-2z}{z+2}}{z^{-1} \frac{2z/3}{z-1/3}} = -3 \frac{z-1/3}{z+2}$$

$$\therefore X[n] = -3 \left((-2)^n u[n] - \frac{1}{3} (-2)^{n-1} u[n-1] \right)$$

OR

$$X[n] = -3 \delta[n] + 7 (-2)^{n-1} u[n-1]$$

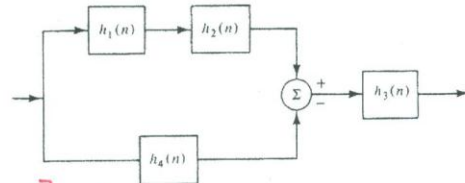
4. a) Find the impulse response of the system shown in the Figure below. Assume that

(8 Pt.)

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$h_2[n] = \delta[n]$$

$$h_3[n] = h_4[n] = \left(\frac{1}{3}\right)^n u[n].$$



$$h[n] = \left[\left(\frac{1}{2}\right)^n u[n] * \delta[n] - \left(\frac{1}{3}\right)^n u[n] \right] * \left(\frac{1}{3}\right)^n u[n]$$

$$= \left[\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n] \right] - \left[\left(\frac{1}{3}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n] \right]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k] \left(\frac{1}{3}\right)^{n-k} u[n-k] - \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u[k] \left(\frac{1}{3}\right)^{n-k} u[n-k]$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^n \left(\frac{3}{2}\right)^k - \left(\frac{1}{3}\right)^n \sum_{k=0}^n 1^k =$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{\left(\frac{3}{2}\right)^{n+1} - 1}{\frac{3}{2} - 1} \right] - \left(\frac{1}{3}\right)^n \left(1 + \sum_{k=1}^n 1^k \right)$$

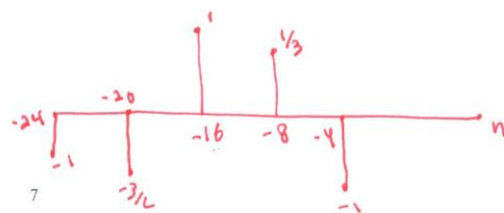
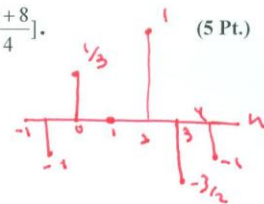
$$= \left[3 \left(\frac{1}{2}\right)^n - 2 \left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^n (n+1) \right] u[n]$$

$$= \left[3 \left(\frac{1}{2}\right)^n - 3 \left(\frac{1}{3}\right)^n - n \left(\frac{1}{3}\right)^n \right] u[n]$$

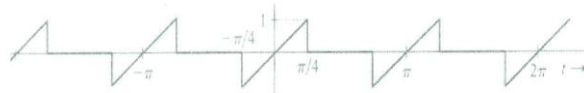
b) For the signal described by $x[n] = \left\{ -1, \frac{1}{3}, 0, 1, -\frac{3}{2}, -1 \right\}$, find $x\left[-\frac{n+8}{4}\right]$.

(5 Pt.)

$$x\left[-\frac{n+8}{4}\right] = \begin{cases} -1 & n = -24, -4 \\ -3/2 & n = -20 \\ 1 & n = -16 \\ 1/3 & n = -8 \\ 0 & \text{otherwise} \end{cases}$$



5. For the periodic signal shown in the Figure below find exponential Fourier series and sketch the line spectra.



Period $T = \pi$

$\omega_0 = 2$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j n \omega_0 t}$$

$x(t)$ over one period = $\frac{4}{\pi} t$

$$\therefore C_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j 2 n t} dt$$

$$= -\frac{j}{\pi n} \left(\frac{2}{\pi n} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right)$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} -\frac{j}{\pi n} \left(\frac{2}{\pi n} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right) e^{j 2 n t}$$

Line Spectra:

