

Islamic University of Gaza
Electrical Engineering Department
EELE 3310 Signals & Systems
Final Exam
January, 4, 2009

OPEN BOOK & NOTE EXAM

Dr. Basil Hamed

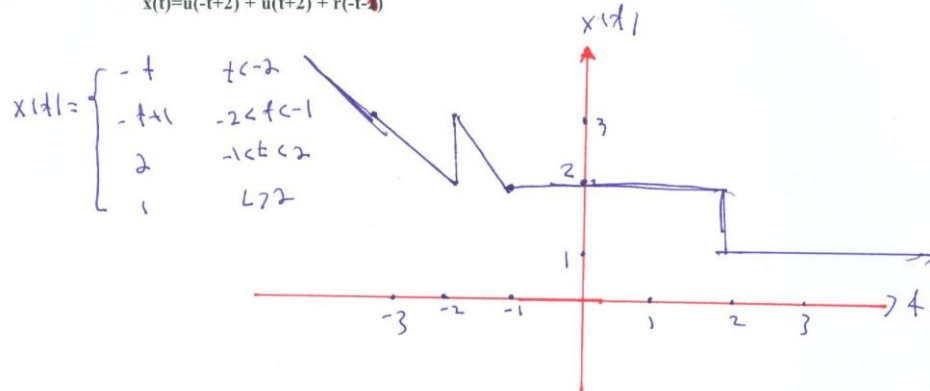
Exam Time 11:00-1:00

Name	Solution
Student ID	
Grade	

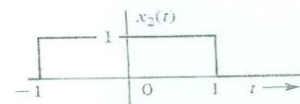
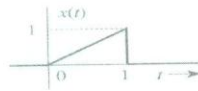
GOOD LUCK

Students who did not take the midterm exam they have to answer all five questions. But for Students who took the midterm, the fifth question will be bonus if they answer it.

1. a) Plot the graph of the signal represented in terms of unit step and unit ramp signals as $x(t) = u(-t+2) + u(t+2) + r(-t-1)$ (4 Pt.)



- b) In Figure below, express signals $x_2(t)$ in terms of signal $x(t)$ and its time-shifted, time-scaled, or time-reversed versions. (5 Points)



$$x_2(t) = x\left(\frac{t+1}{2}\right) + x\left(\frac{1-t}{2}\right)$$

OR

$$x_2(t) = x(t) + x(-t) + x(t+1) + x(-t+1)$$

2. a) A continuous-time system is given by $y(t) = \sum_{k=-\infty}^t (x(t-k))^2 \delta(k)$

Say with justification whether the system is Memoryless, Causal, Linear, Time-invariant, Stable

(5 Pt.)

Since $\delta(k) = 0$ if $k \neq 0$ $y(t) = x(t)^2 u(t)$

a) Yes memoryless. Depends only on input at that time.

b) Yes, Causal output does not depend on future.

c) Not Linear - if $ax(t)$ is input, the output is $a^2 x(t)^2 = ay(t)$

d) Not Time-invariant - if $f(t) = x(t-t_0)$ is input, then $f(t)^2 u(t) = x(t-t_0)^2 u(t)$ which is different.

e) Yes - stable, since if $|x(t)| < B$ then $|y(t)| < B^2$. BIBO

b) A continuous-time LTI system has impulse response $h(t) = e^{-t} + \delta(t)$ if $-1 \leq t \leq 2$ and $h(t) = 0$ for all other t .

Find its output if the input signal $x(t) = 2$ if $0 \leq t \leq 3$ and is 0 for all other t .

(7 Pt.)

$$y(t) = (h_1(t) + \delta(t)) * x(t) = (h_1(t) * x(t)) + (\delta(t) * x(t))$$

where $h_1(t) = e^{-t}$ for $-1 \leq t \leq 2$

stretch -1 → 5

$$y_1(t) = \int_{-\infty}^{\infty} h_1(\tau) x(t-\tau) d\tau$$

i) For $t < -1$ no overlap so $y_1(t) = 0$

ii) For $-1 < t < 2$ overlap from -1 to t

$$y_1(t) = \int_{-1}^t 2e^{-\tau} d\tau = 2(e - e^{-t})$$

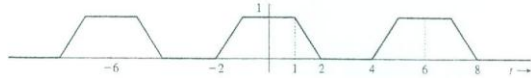
iii) For $2 < t < 5$ overlap from $t-2$ to 2

$$y_1(t) = \int_{t-2}^2 2e^{-\tau} d\tau = 2(e^{3-t} - e^{-2})$$

iv) For $t > 5$ no overlap $y_1(t) = 0$

$$\therefore y(t) = \begin{cases} 2(e - e^{-t}) + 2 & -1 < t < 2 \\ 2(e^{3-t} - e^{-2}) + 2 & 2 < t < 5 \\ 0 & \text{otherwise} \end{cases}$$

3. a) Find the Fourier-series representation of the signal shown in Figure below, Plot the magnitude and phase spectrum. (7 Pt.)



Period $T=6$, $\omega_0 = \frac{\pi}{3}$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\frac{\pi}{3}t}$$

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\frac{\pi}{3}t} dt$$

$$x(t) = \begin{cases} t+2 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ -t+2 & 1 < t < 2 \end{cases}$$

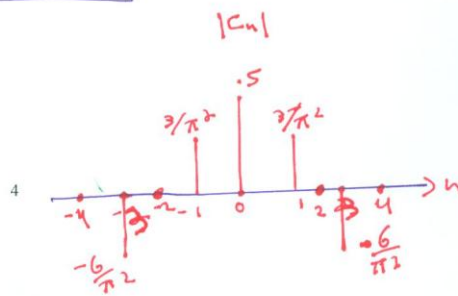
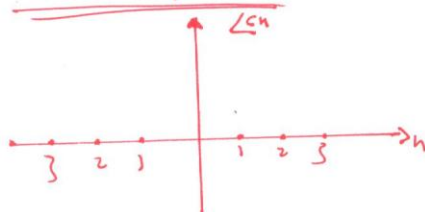
$$C_n = \frac{1}{6} \left[\int_{-2}^{-1} (t+2) e^{-jn\frac{\pi}{3}t} dt + \int_{-1}^1 e^{-jn\frac{\pi}{3}t} dt + \int_1^2 (-t+2) e^{-jn\frac{\pi}{3}t} dt \right]$$

$$C_n = \frac{3}{\pi^2 n^2} \left(\cos \frac{n\pi}{3} - \cos \frac{2n\pi}{3} \right) ; n \neq 0$$

$$C_0 = .5$$

$$\therefore x(t) = .5 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} C_n e^{jn\frac{\pi}{3}t}$$

Line Spectra:



b) Find the Fourier transform of the signal

(6 Pt.)

$$x(t) = \left(\frac{d}{dt} [te^{-|t-1|}] \right) * \cos(2t) e^{-t} u(t)$$

$$e^{-|t|} \xleftrightarrow{FT} \frac{2}{1+\omega^2}$$

$$e^{-|t-1|} \xleftrightarrow{FT} \frac{2e^{-j\omega}}{1+\omega^2}$$

$$t e^{-|t-1|} \xleftrightarrow{FT} \frac{d}{d\omega} \left\{ \frac{2e^{-j\omega}}{1+\omega^2} \right\} = \frac{-2j e^{-j\omega}}{1+\omega^2} - \frac{2e^{-j\omega} 2\omega}{(1+\omega^2)^2}$$

$$\frac{d}{dt} [t e^{-|t-1|}] \xleftrightarrow{FT} j\omega \frac{d}{d\omega} \left\{ \frac{2e^{-j\omega}}{1+\omega^2} \right\}$$

$$e^{-t} u(t) \xleftrightarrow{FT} \frac{1}{j\omega+1}$$

$$\cos 2t e^{-t} u(t) \xleftrightarrow{FT} \frac{1}{2} \left(\frac{1}{j(\omega-2)+1} \right) + \frac{1}{2} \left(\frac{1}{j(\omega+2)+1} \right)$$

$$\therefore X(j\omega) = j\omega \frac{d}{d\omega} \left(\frac{2e^{-j\omega}}{1+\omega^2} \right) \left[\frac{1}{j(\omega-2)+1} + \frac{1}{j(\omega+2)+1} \right]$$

4. a) Solve $y[n+2] - 2y[n+1] + 2y[n] = x[n]$; with $y[-1]=1$, $y[-2]=0$, and $x[n]=u[n]$.

(8 Pt.)

delay Form is

$$y[n] - 2y[n-1] + 2y[n-2] = x[n-2]$$

$$Y(z) - 2[z^{-1}Y(z) + 1] + 2[z^{-2}Y(z) + z^{-1}] = z^{-2} \frac{z}{z-1}$$

$$\left[\frac{z^2 - 2z + 2}{z^2} \right] Y(z) = \frac{2z^2 - 4z + 3}{z(z-1)}$$

$$\therefore \frac{Y(z)}{z} = \frac{2z^2 - 4z + 3}{(z-1)(z^2 - 2z + 2)} = \frac{1}{(z-1)} + \frac{z-1}{z^2 - 2z + 2}$$

$$\therefore Y(z) = \frac{z}{z-1} + \frac{z(z-1)}{z^2 - 2z + 2}$$

$$\therefore y[n] = [1 + (\sqrt{2})^n \cos(\pi/4 n)] u[n]$$

b) Solve the following differential equation:

(8 Pt.)

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 4y(t) = \frac{dx}{dt} + x(t) \quad y(0) = 2, \quad \dot{y}(0) = 1 \quad \text{and} \quad x(t) = e^{-t} u(t)$$

$$(s^2 Y(s) - 2s - 1) + 4(sY(s) - 2) + 4Y(s) = (s+1) \frac{1}{s+1}$$

$$(s^2 + 4s + 4)Y(s) = 2s + 10$$

$$\therefore Y(s) = \frac{2s + 10}{s^2 + 4s + 4} = \frac{2s' + 10}{(s' + 2)^2} = \frac{2}{s' + 2} + \frac{6}{(s' + 2)^2}$$

$$\therefore y(t) = (2 + 6t) e^{-2t} u(t)$$

5. A system has impulse response

$$h(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \delta(t-n)$$

use convolution to find the output $y(t)$ for an input $x(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$ sketch $y(t)$

Since $\delta(t-n) * x(t) = x(t-n)$

$$\therefore y(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \delta(t-n) * x(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n x(t-n)$$

$$\therefore y(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n [u(t-n+\frac{1}{2}) + u(t-n-\frac{1}{2})]$$

