



Islamic University of Gaza
Electrical Engineering Department
EELE 3310 Signals & Systems
Final Exam
December, 30, 2013
OPEN BOOK & NOTE EXAM

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Exam Time 11:30-1:30

Name	SOLUTION
Student ID	
Grade	

GOOD LUCK

1. a) Consider a discrete-time signal $x[n]$, fed as input into a system. The system produces the discrete-time output $y[n]$ such that

$$y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

- (i) Is the system described above memoryless? Explain. (1 PT)
(ii) Is the system described above causal? Explain. (1 PT)
(iii) Is the system described above linear? Explain. (2 PT)
(iv)) Is the system described above time-invariant? Explain. (2 PT)

Solution

- (i) It is memoryless since the output at time instant n depends on the input only at time instant n and not past or future time instants.
(ii) It is causal since the output at time instant n depends on the input only at time instant n and not future time instants.

$$(iii) \quad y[n] = \frac{x[n] + (-1)^n x[n]}{2}$$

From this we see that if the input signal $x_1[n]$ produces an output signal $y_1[n]$ and the input signal $x_2[n]$ produces an output signal $y_2[n]$ then the input signal $a_1 x_1[n] + a_2 x_2[n]$ produces the output

$$y_3[n] = \frac{(a_1 x_1[n] + a_2 x_2[n]) + (-1)^n (a_1 x_1[n] + a_2 x_2[n])}{2} = a_1 y_1[n] + a_2 y_2[n].$$

Therefore, the system is linear.

- (iv) However, if the input signal $x[n]$ produces an output signal $y[n]$ then the input signal $x[n - n_o]$ produces the output $y_1[n] = \frac{x[n - n_o] + (-1)^n x[n - n_o]}{2}$.

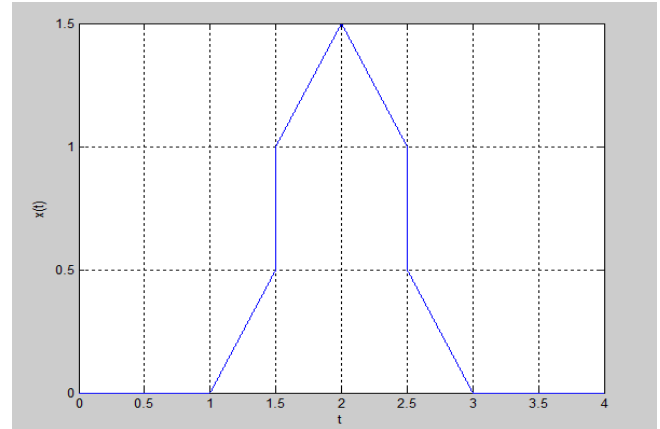
$$\text{We see that } y[n - n_o] = \frac{x[n - n_o] + (-1)^{n - n_o} x[n - n_o]}{2} \neq y_1[n]$$

Therefore, the system is time varying.

b) Write the signal in the following Fig. in terms of unit step and unit ramp basic function

(3 PT)

$$x(t) = r(t-1) + 0.5u(t-1.5) - 2r(t-2) - 0.5u(t-2.5) + r(t-3)$$



c) Given, $x(t) = e^{2t} \leftrightarrow X(\omega)$

What is the Fourier transform of $y(t) = e^{-t-1} + te^{2t} + 2e^{2t}$ in terms of $X(\omega)$? (4 PT)

$$y(t) = x\left(-\frac{t}{2} - \frac{1}{2}\right) + t x(t) + \frac{d x(t)}{dt}$$

$$\text{Let } x_1(t) = x\left(t - \frac{1}{2}\right) \leftrightarrow X(\omega) \cdot e^{-j\omega/2}$$

$$\text{Let } x_2(t) = x_1(-t) = x\left(-t - \frac{1}{2}\right) \leftrightarrow X(-\omega) e^{j\omega/2}$$

$$\text{Let } y_1(t) = x_2\left(\frac{t}{2}\right) = x\left(-\frac{t}{2} - \frac{1}{2}\right) \leftrightarrow 2 \cdot X(-2\omega) e^{j\omega}$$

$$\text{Let } y_2(t) = \frac{t x(t)}{\text{Diff. in Frequency}} \leftrightarrow j \frac{d}{d\omega} X(\omega)$$

$$\text{Let } y_3(t) = \frac{d x(t)}{dt} \leftrightarrow j\omega X(\omega)$$

Combine $y_1(t)$, $y_2(t)$, $y_3(t)$:

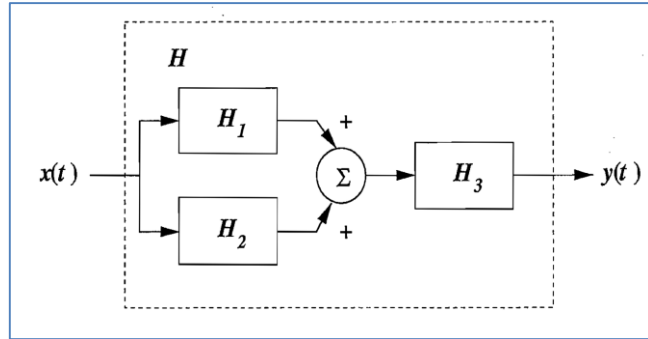
$$Y(\omega) = 2 e^{j\omega} X(-2\omega) + j \frac{d}{d\omega} X(\omega) + j\omega X(\omega)$$

2) Consider, the continuous LTI system H shown below;

$$H_1 \text{ is given : } h_1(t) = -\frac{1}{2}e^{4t}u(-t),$$

$$H_2 \text{ is given : } 4y_2(t) - 2\frac{dy_2}{dt} = x_2(t)$$

$$H_3 \text{ is given : } h_3(t) = \delta(t-2)$$



a) Find the transfer function $H(s)$ of overall system (4 PT)

b) Is $H(s)$ system stable (2 PT)

c) Find the system output $y(t)$ when the input $x(t)$ is given by $x(t) = \delta'(t) - 2\delta(t)$ (3 PT)

$$\boxed{a} \quad h_1(t) = -\frac{1}{2}e^{4t}u(-t) \Rightarrow H_1(s) = \frac{1}{2} \cdot \frac{1}{s-4}$$

$$\uparrow \quad 4Y_2(s) - 2sY_2(s) = X_2(s) \Rightarrow H_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{1}{-2s+4}$$

$$\bullet \quad H_3(s) = e^{2s}$$

$$\therefore H(s) = [H_1(s) + H_2(s)] H_3(s) = \boxed{\frac{e^{-2s}}{(s-4)(s-2)}} \quad 2 < \operatorname{Re}\{s\} < 4$$

$\boxed{b}$ system is not stable both poles RHP

$$\boxed{c} \quad X(s) = s - 2 \quad Y(s) = X(s) H(s) = \frac{(s-2)e^{2s}}{(s-4)(s-2)}$$

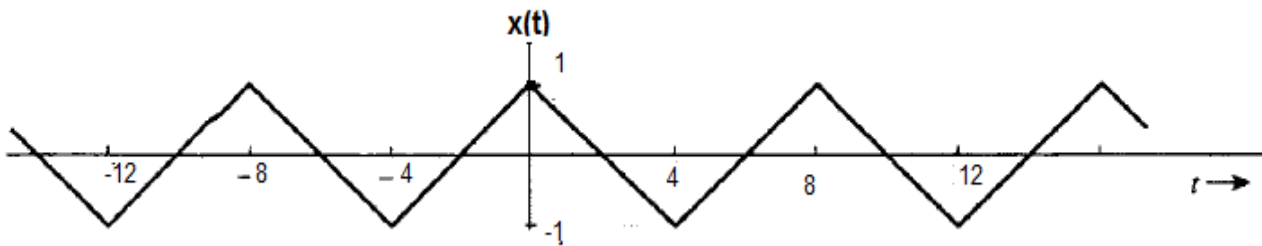
$$\boxed{Y(s) = \frac{e^{2s}}{s-4}} \quad \operatorname{Re}\{s\} < 4 \quad \text{Table : } -e^{4t}u(-t) \xrightarrow{\mathcal{L}} \frac{1}{s-4}$$

Shift Property

$$u(t-2) \xrightarrow{-e^{-2s}} u(-(t-2)) \xrightarrow{\mathcal{L}} \frac{e^{-2s}}{s-4}$$

$$\therefore y(t) = -e^{4(t-2)}u(2-t)$$

3. a) Find the exponential Fourier-series representation of the signal shown in Figure below, (6 PT)



$$T = 8 \quad \omega_0 = \pi/4 \quad C_0 = 0 \text{ (by Inspection)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\pi/4 t}$$

$x(t)$ over one Period:

$$x(t) = \begin{cases} (t/2 + 1) & -4 < t < 0 \\ (-t/2 + 1) & 0 < t < 4 \end{cases}$$

$$C_n = 1/8 \left[\int_{-4}^0 (t/2 + 1) e^{-j2\pi(\pi/4)t} dt + \int_0^4 (-t/2 + 1) e^{-j2\pi(\pi/4)t} dt \right]$$

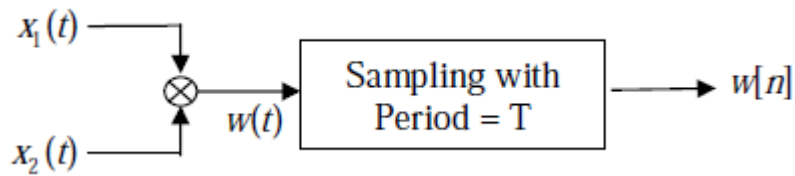
$$\therefore C_n = \begin{cases} \frac{4}{\pi^2 n^2} \\ 0 \end{cases}$$

$$n = \pm 1, \pm 3, \pm 5, \dots$$

0 otherwise

$$\therefore x(t) = \sum_{\substack{n=1 \\ n: \text{odd}}}^{\infty} \frac{4}{\pi^2 n^2} e^{jn\pi/4 t}$$

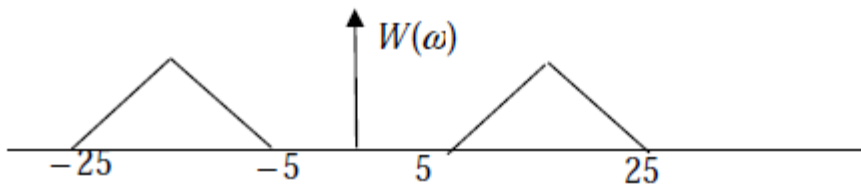
b) In the following system, two continuous-time functions $x_1(t)$ and $x_2(t)$ are MULTIPLIED and the product $w(t)$ is sampled with sampling period T . (3PT)



If the spectrums of $x_1(t)$ and $x_2(t)$ are given as follows, sketch the continuous time spectrum $W(\omega)$ of $w(t)$.



Since multiplication corresponds to convolution, the output spectrum looks like



C) Continue from part b, determine the maximum sampling period T such that $w(t)$ is recoverable from $w[n]$. (3 PT)

Since the bandwidth is 25 rad/s, the Nyquist frequency is $\omega_s \geq 2\omega_B$, $\omega_s = 50$ rad/sec
 The sampling period is $2\pi/50 = 0.126$ second

4. Consider a discrete-time system

$$y[n] + y[n-1] - 2y[n-2] = 2x[n] - x[n-1]$$

Find an input $x[n]$ with $x[n] = 0$ for $n < 0$ that gives the output response $y[n] = 2(u[n] - u[n-3])$, with initial condition $y[-2] = 2$, $y[-1] = 0$ (8 PT)

Solution

$$Y(z) + z^{-1}Y(z) + \cancel{y[-1]} - 2z^{-2}Y(z) - 2\cancel{y[-1]} - 2z^{-1}\cancel{y[-1]} = 2X(z) - z^{-1}X(z)$$

$$Y(z)[1 + z^{-1} - 2z^{-2}] = 4 + (2 - z^{-1})X(z)$$

$$Y(z) \left[\frac{z^2 + z - 2}{z^2} \right] = 4 + \frac{2z - 1}{z} X(z)$$

$$\therefore Y(z) = \frac{4z^2}{z^2 + z - 2} + \frac{2z^2 - z}{z^2 + z - 2} X(z)$$

Given: $y[n] = 2(u[n] - u[n-3])$

$$\Rightarrow Y(z) = 2 \left[\frac{z}{z-1} - z^{-3} \frac{z}{z-1} \right] = 2 \left[\frac{z^2 + z + 1}{z^2} \right]$$

$$\therefore X(z) = \left[Y(z) - \frac{4z^2}{z^2 + z - 2} \right] \frac{z^2 + z - 2}{2z^2 - z}$$

$$\frac{X(z)}{z} = \frac{-z^4 + 2z^3 - z - 2}{(z - 0.5)z^4} = \frac{k_1}{z^4} + \frac{k_2}{z^3} + \frac{k_3}{z^2} + \frac{k_4}{z} + \frac{k_5}{(z - 0.5)}$$

$$\therefore \frac{X(z)}{z} = \frac{4}{z^4} + \frac{10}{z^3} + \frac{20}{z^2} + \frac{36}{z} - \frac{37}{z - 0.5}$$

$$\therefore X[n] = 36\delta[n] + 20\delta[n-1] + 10\delta[n-2] + 4\delta[n-3] - 37(0.5)^n u[n]$$

5. Consider the LTI system; $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 2\dot{x}(t) + 2x(t)$
 $y(0) = 1, \quad \dot{y}(0) = 2$ and for $t > 0, x(t) = e^{-t}u(t)$

Determine the zero-input and zero state response of above system.

(8 PT)

$$s^2 Y(s) - s y(0) - \dot{y}(0) + 5[s Y(s) - y(0)] + 6 Y(s) = 2[s X(s) - x(0)] + 2 X(s)$$

$$Y(s)[s^2 + 5s + 6] - s - 7 = [2s + 2] X(s) \quad ; \quad \boxed{X(s) = \frac{1}{s+1}}$$

$$Y(s)[s^2 + 5s + 6] = \overset{\text{Input}}{\frac{2s+2}{s+1}} + \overset{\text{I.C}}{(s+7)}$$

$$Y(s) = \overset{\text{Input}}{\frac{2}{s^2+5s+6}} + \overset{\text{I.C}}{\frac{(s+7)}{s^2+5s+6}} = \left(\overset{\text{Input}}{\frac{k_1}{s+2}} + \overset{\text{Input}}{\frac{k_2}{s+3}} \right) + \left(\overset{\text{I.C}}{\frac{k_3}{s+2}} + \overset{\text{I.C}}{\frac{k_4}{s+3}} \right)$$

$$k_1 = 2, k_2 = -2, k_3 = 5, k_4 = -4$$

$$\therefore y(t) = \underbrace{(2e^{-2t} - 2e^{-3t})}_{\text{Input}} u(t) + \underbrace{(5e^{-2t} - 4e^{-3t})}_{\text{I.C}} u(t)$$

$$\therefore y(t) = (7e^{-2t} - 6e^{-3t}) u(t)$$

$$\boxed{\text{Total response}}$$