

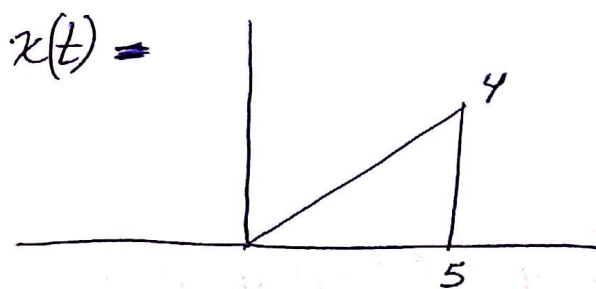
Question 01 / Part a/

even components

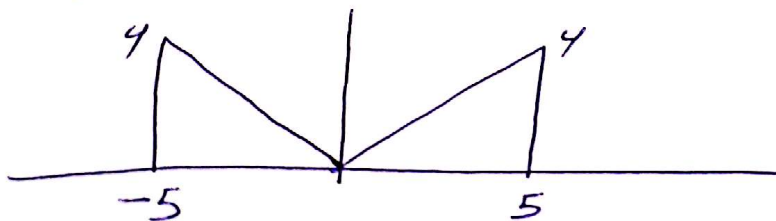
$$x(t) = \frac{1}{2} (x(t) + x(-t))$$

odd components

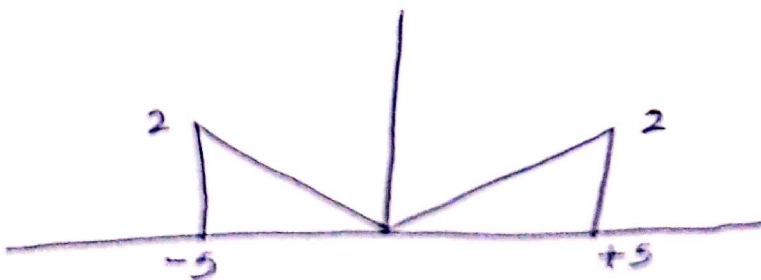
$$x(t) = \frac{1}{2} (x(t) - x(-t))$$



$x(t) + x(-t)$



$\frac{1}{2} (x(t) + x(-t))$

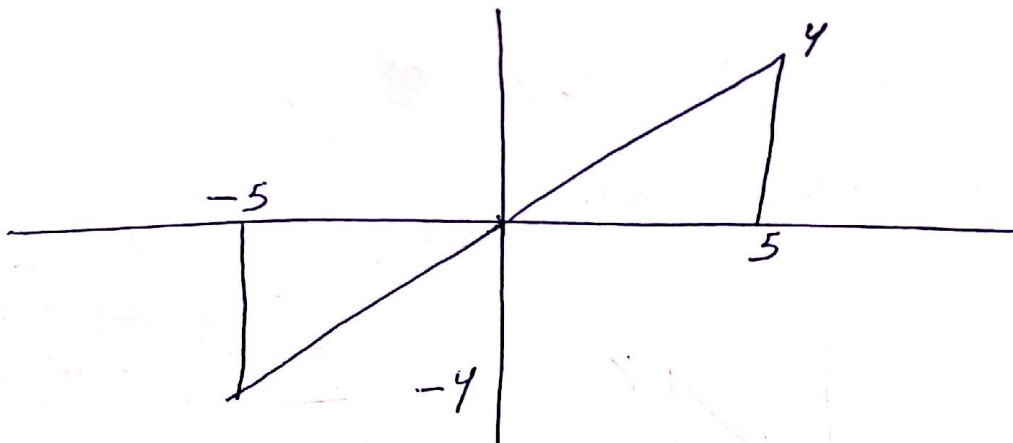


even
Component

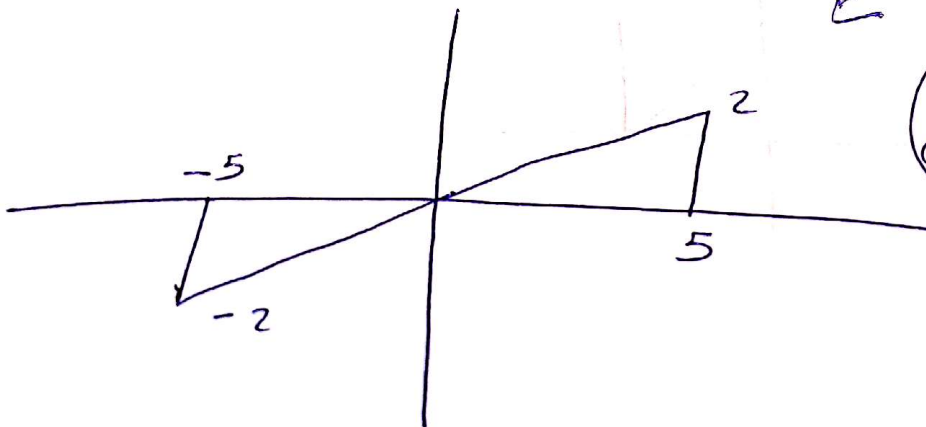


Question 01 / Part a / continue

$$x(t) - x(-t)$$



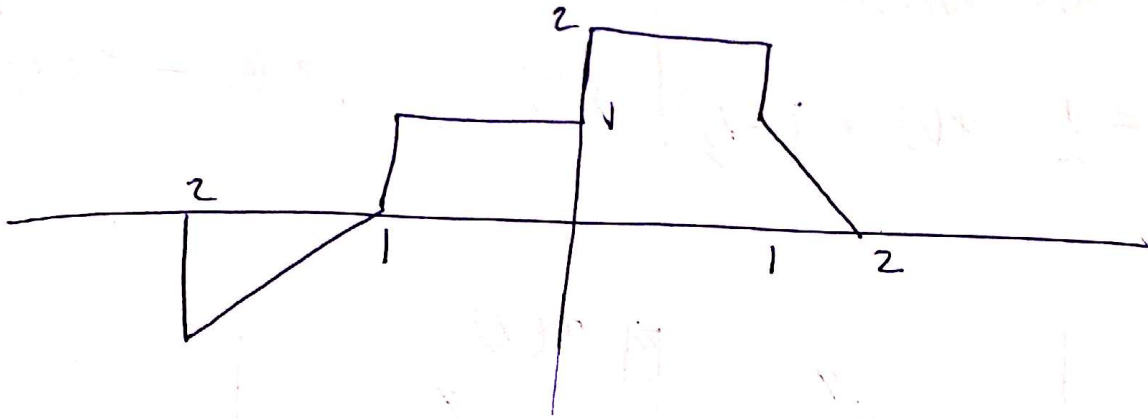
$$x_{odd} = \frac{1}{2} (x(t) - x(-t))$$



odd
Component

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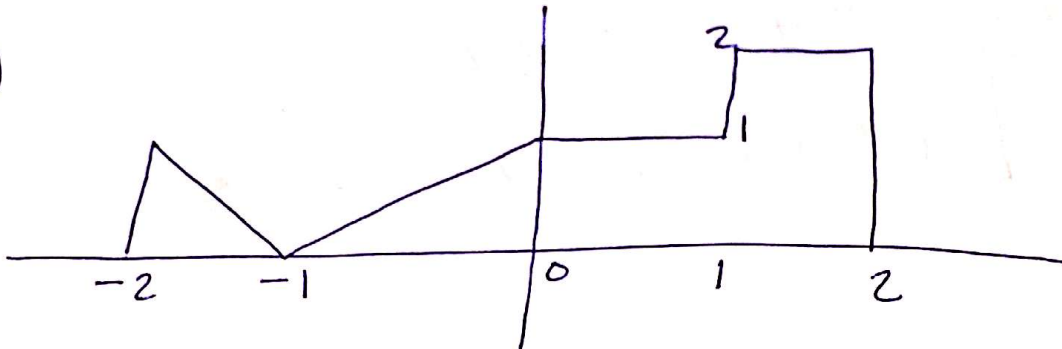
Question 04/61



$$-u(t+2) + r(t+2) - r(t+1) + u(t+1) + u(t) - \\ u(t-1) - r(t-1) + r(t-2)$$

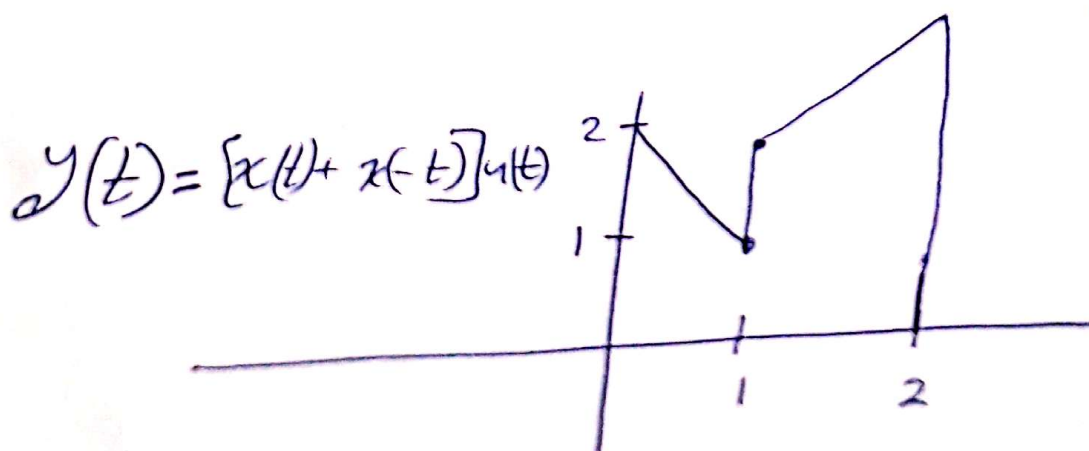
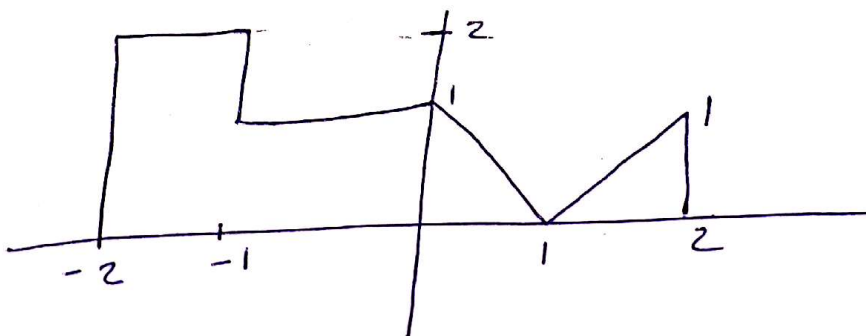
Question 2/A/

$x(t)$



$$y(t) = [x(t) + x(-t)] u(t) \rightarrow \text{From } 0 \text{ to } \infty$$

$x(-t)$



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Continue Question 2/A/

$$y(t) = \begin{cases} -t + 2 & 0 \leq t < 1 \\ +t + 1 & 1 \leq t < 2 \end{cases}$$

Question 2/b1

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$$T = 2\pi$$

$$\omega = \frac{2\pi}{2\pi} = 1$$

$$x(t) = \begin{cases} -t & , -\pi \leq t < 0 \\ -\pi & , 0 \leq t < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^0 -t \, dt + \frac{1}{2\pi} \int_0^{\pi} \pi \, dt$$

$$\frac{-1}{2\pi \cdot 2} \cdot \frac{t^2}{2} \Big|_{-\pi}^0 + \frac{1}{2\pi} \cdot \pi t \Big|_0^{\pi} = \boxed{\frac{3\pi}{4}}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 -t \cos(n\omega_0 t) \, dt + \frac{1}{\pi} \int_0^{\pi} \pi \cos(n\omega_0 t) \, dt$$

$$= \frac{-1}{\pi} \left(\frac{1}{(n\omega_0)^2} \cos(n\omega_0 t) + \frac{t}{n\omega_0} \cdot \sin(n\omega_0 t) \right) \Big|_{-\pi}^0$$

$$+ \frac{1}{n\omega_0} \cdot \sin(n\omega_0 t) \Big|_0^{\pi} = \boxed{\frac{1}{\pi n^2} (\cos(n\pi) - 1)}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^0 -t \sin(n\omega_0 t) \, dt + \frac{1}{\pi} \int_0^{\pi} \pi \sin(n\omega_0 t) \, dt$$

$$= \frac{-1}{\pi} \left(-t \frac{\cos(n\omega_0 t)}{n\omega_0} + \frac{\sin(n\omega_0 t)}{(n\omega_0)^2} \right) \Big|_{-\pi}^0$$

$$- \frac{1}{n\omega_0} \cos(n\omega_0 t) \Big|_0^{\pi}$$

⑥

Continue Question 2/b/

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$$b_n = \frac{1}{n}$$

$$\frac{3\pi}{4} + \sum_{n=0}^{\infty} \frac{\cos(n\pi - 1)}{\pi n^2} \cdot (\cos(n\omega_0 t) + \frac{1}{n} \sin(n\omega_0 t))$$

Question 3/a/

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$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{other wise} \end{cases}$$

$$x(t) \cos(t) \xrightarrow{F.t} \frac{1}{2} [x(j(\omega+1)) + x(j(\omega-1))]$$

$$\frac{1}{2} (x(j(\omega-1)) + x(j(\omega+1))) = \begin{cases} 1, & \omega \leq 2 \\ 0, & \text{other wise} \end{cases}$$

Band width of $G(j\omega) = 4$, from $(\omega = -2 \text{ to } \omega = 2)$

" $x(j\omega) \neq 0$ for $|\omega| \leq 1$

$$x(j\omega) = \begin{cases} 2, & |\omega| \leq 1 \\ 0, & \text{other wise} \end{cases}$$

$$\underline{\underline{\frac{1}{2} [x(j(\omega-1)) + x(j(\omega+1))] = 1}}$$

(8)

Continuec Question 3/a/

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$$x(t) = \mathcal{F}^{-1} \{x(j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) e^{j\omega t} d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-1}^1 x e^{j\omega t} d\omega = \frac{1}{\pi} \left[\frac{e^{j\omega t}}{jt} \right]_{-1}^1$$

$$x(t) = \frac{1}{\pi jt} (e^{jt} - e^{-jt})$$

$$x(t) = \frac{2 \sin t}{\pi t} \rightarrow \text{sinc}(t)$$

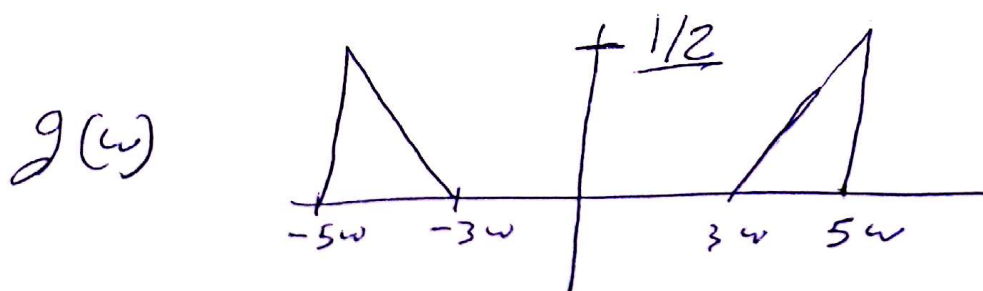
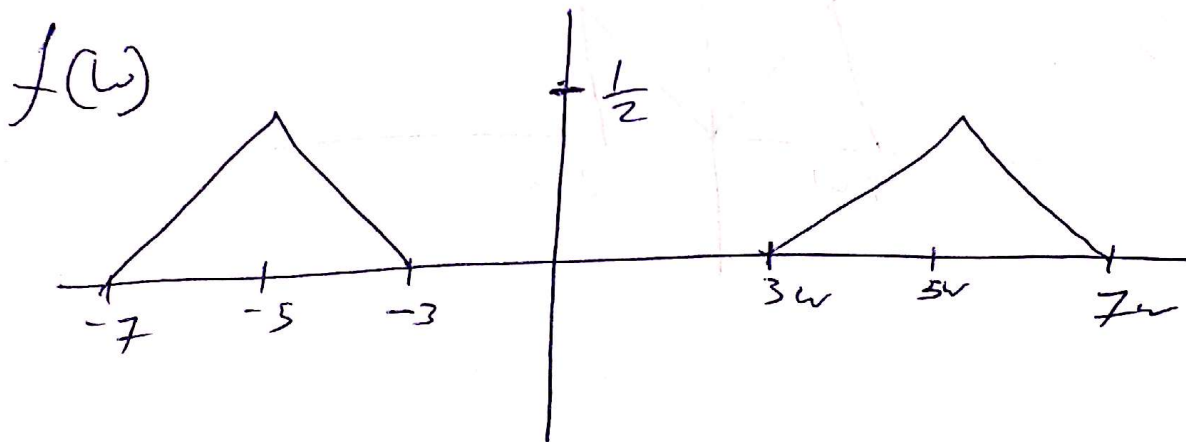
$$x(t) = \frac{2}{\pi} \cdot \text{sinc}(t)$$

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$$Y(\omega) = ??$$

$$f(\omega) = \frac{1}{2\pi} \left[x(\omega) \cdot \pi \left[(\delta + 5) + (\delta - 5) \right] \right]$$

$$= \frac{1}{2} \left[x(\omega + 5) + x(\omega - 5) \right]$$



$$h(t) = g(t) * \cos(3\omega t)$$

$$h(t) = \frac{1}{2\pi} g(\omega) * \left[\pi \left[\delta(\omega - 3\omega) + \delta(\omega + 3\omega) \right] \right]$$

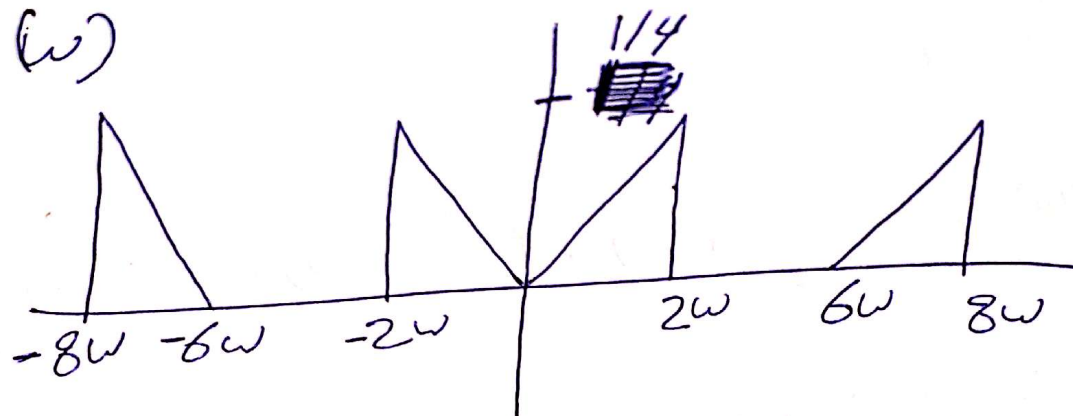
$$h(\omega) = \frac{1}{2} \left[g(\omega - 3\omega) + g(\omega + 3\omega) \right]$$

Continue Question 13/4

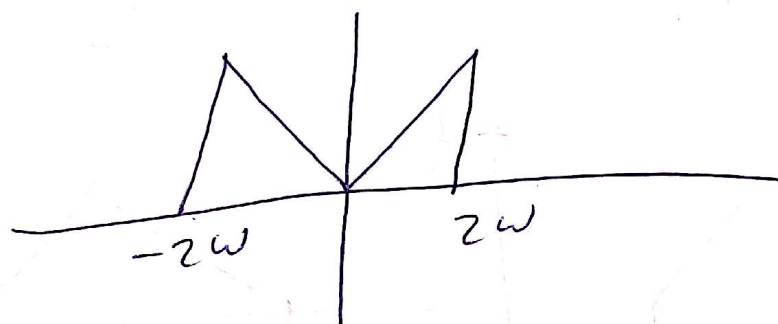
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$h(\omega)$



$Y(\omega)$



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Question 4/a/

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$$y' + 3y(t) = t^2 e^{-3t} + t e^{-2t} + t, \quad y(0) = 1$$

$$s y(s) - y(0) + 3 y(s) = \frac{2}{(s+3)^3} + \frac{1}{(s+2)^2} + \frac{1}{s^2}$$

$$y(s)(s+3) - 1 = \frac{2}{(s+3)^3} + \frac{1}{(s+2)^2} + \frac{1}{s^2}$$

$$y(s) = \frac{2}{(s+3)^4} + \frac{1}{(s+2)^2(s+3)} + \frac{1}{s^2(s+3)} + \frac{1}{(s+3)}$$

$$\mathcal{L}^{-1} \left\{ \frac{2}{(s+3)^4} \right\} = \frac{2t^3 e^{-3t}}{3!} = \frac{t^3 e^{-3t}}{3}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s+3)} \right\} = e^{-3t}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s+3)(s+2)^2} \right\} = \frac{K_1}{s+3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{(s+2)}$$

Question 4/a) (continued) أحمد م. الشيخ خليل
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$$K_1 = \frac{1}{(s+2)^2} \Big|_{s=-3} = \boxed{1}$$

$$K_2 = \frac{1}{s+3} \Big|_{s=-2} = \boxed{1}$$

$$K_3 = \frac{1}{1!} \frac{d}{ds} \left[\frac{1}{s'+3} \right] \Big|_{s=-2} = \frac{-1}{(s+3)^2} = \boxed{-1}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s+3)(s'+2)^2} \right\} = \frac{1}{s'+3} + \frac{1}{(s'+2)^2} - \frac{1}{(s'+2)}$$

$$= e^{-3t} + t e^{-2t} - e^{-2t}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s'^2(s'+3)} \right\} = \frac{K_1}{(s+3)} + \frac{K_2}{s} + \frac{K_3}{s^2}$$

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Question 4/a/ (continued)

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$$K_1 = \frac{(s+3) \cdot 1}{s^2 (s^2+3)} \Big|_{s=-3} = \boxed{\frac{1}{9}}$$

$$K_2 = \frac{1}{s^2+3} \Big|_{s=0} = \boxed{\frac{1}{3}}$$

$$K_3 = \frac{1}{1!} \frac{d}{ds} \frac{1}{s^2+3} \Big|_{s=0} = \frac{-1}{(s^2+3)^2} \Big|_{s=0} = \boxed{\frac{-1}{9}}$$

$$\mathcal{L}^{-1} \left[\frac{1}{s^2 (s^2+3)} \right] = \frac{1}{9(s^2+3)} + \frac{1}{3s} + \frac{-1}{9s^2}$$

$$\frac{1}{9} e^{-3t} + \frac{1}{3} - \frac{1}{9} t$$

$$y(t) = e^{-3t} + \frac{t^3}{3} e^{-3t} + e^{-3t} + t e^{-2t} - e^{-2t} +$$

$$\frac{1}{9} e^{-3t} - \frac{1}{9} t + \frac{1}{3}$$

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Question 4/6/

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$$x(s) = \frac{s+2}{s-2}, \quad x(s) = 0 \text{ for } x > 0$$

$$y(t) = -\frac{2}{3} e^{2t} u(t) + \frac{1}{3} e^{-t} u(t)$$

$$y(s) = x(s) H(s)$$

$$y(s) = \frac{2}{3} \cdot \frac{1}{s-2} + \frac{1}{3} \cdot \frac{1}{s+1}$$

$[Re] > -1$

$$\{Re\} < +2$$

$$y(s) = \frac{2}{3(s-2)} + \frac{1}{3(s+1)} = \frac{6(s+1) + 3(s-2)}{9(s-2)(s+1)}$$

$$H(s) = \frac{y(s)}{x(s)} = \frac{6(s+1) + 3(s-2)}{9(s-2)(s+1)} \cdot \frac{(s+2)}{(s-2)}$$

$$H(s) = \frac{s}{(s+2)(s+1)}$$

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Continue Question 4/b/

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$$\textcircled{1} \quad H(s) = \frac{s}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = \left. \frac{s}{s+1} \right|_{s=-2} = \underline{2}$$

$$B = \left. \frac{s}{s+2} \right|_{s=-1} = \underline{-1}$$

$$H(s) = \frac{2}{s+2} + \frac{-1}{s+1}$$

$$\textcircled{2} \quad H(t) = 2e^{-2t} u(t) - e^{-t} u(t)$$

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