

EELE 3340

Probability Theory



Chapter 1

Experiments, Models and Probabilities

Dr. Mohammed El-Absi

1.1 Set Theory

A set is an unordered collection of things (elements).

$$A = \{\clubsuit, \diamondsuit\} = \{\diamondsuit, \clubsuit\} \quad \diamondsuit \in A; \quad \heartsuit \notin A$$

$$B = \{1, 2, 3\};$$

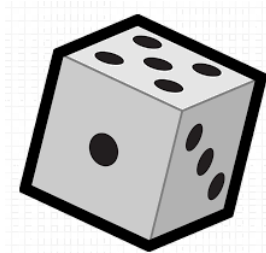
$$C = \{x : x \text{ satisfies some property}\} \quad \text{" : " are pronounced "such that."}$$

$$D = \{x^2 : x = 1, 2, 3\} = \{1, 4, 9\};$$

$$E = \{H, T\};$$



$$F = \{1, 2, 3, 4, 5, 6\};$$



1.1 Set Theory

- The set of natural numbers: $\mathbb{N} = \{1, 2, 3, \dots\}$.
- The set of integers: $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- The set of real number: $\mathbb{R}$.

1.1 Set Theory

Example:

1. $C = \{x | x \in \mathbb{Z}, -2 \leq x < 10\}$ $C = ?$

$$C = \{-2, -1, 0, \dots, 9\}$$

2. $D = \{x^2 | x \in \mathbb{N}\}$ $D = ?$

$$D = \{1, 4, 9, 16, \dots\}$$

1.1 Set Theory

Set A is a subset of set B if every element of A is also an element of B .

We write $A \subset B$, where " $\subset$ " indicates "subset".

$$A \subset B \equiv (x \in A) \Rightarrow (x \in B)$$

$B \supset A$ B is superset of A

Example:

- $E = \{1, 4\}; C = \{1, 4, 9\} \Rightarrow E \subset C$.
- $\mathbb{N} \subset \mathbb{Z}$.

1.1 Set Theory

$A = B$ if and only if $A \subset B$ and $B \subset A$.

Example:

- $\{1, 2, 3\} = \{3, 2, 1\}$
- $\{a, a, b\} = \{a, b\}$

Universal set: The set of all things that we could possibly consider in a given context.

$S = \text{Universal set} = \Omega;$

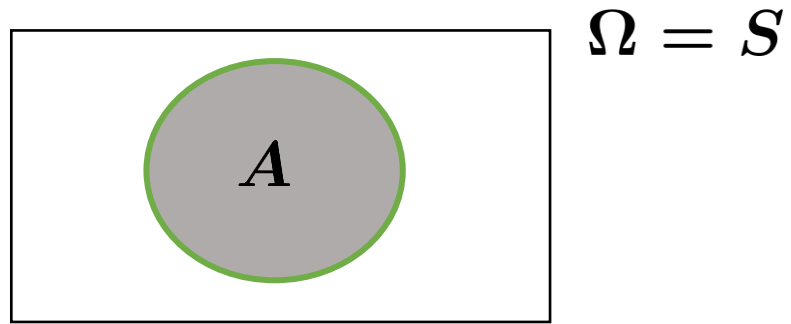
$\emptyset = \text{Null set}; \emptyset = \{\};$

For any set A ; $\emptyset \subset A$.

1.1 Set Theory

Venn Diagrams

In a Venn diagram any set is depicted by a closed surface.



1.1 Set Theory

Set Operations

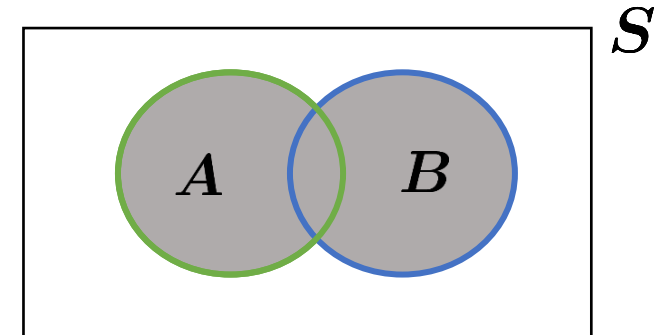
Union: The union of two sets A and B is denoted by $A \cup B$ and consist of all objects in A **or** B .

$x \in (A \cup B)$ if and only if $(x \in A)$ **or** $(x \in B)$.

$$\{1, 2\} \cup \{3\} = \{1, 2, 3\}$$

$A_1, A_2, A_3, \dots, A_n$ n sets their union:

$$A_1 \cup A_2 \cup A_3 \cdots \cup A_n = \bigcup_{i=1}^n A_i.$$



1.1 Set Theory

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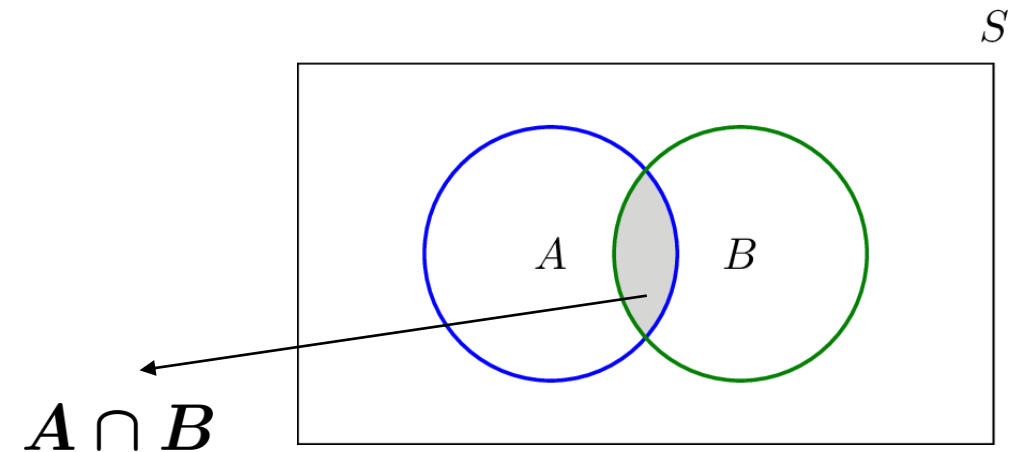
Intersection:

The intersection of two sets A and B is denoted by $A \cap B$ and consist of all objects in both A **and** B .

$$\{1, 2, 3\} \cap \{3\} = \{3\}$$

$A_1, A_2, A_3, \dots, A_n$ n sets their intersestion:

$$A_1 \cap A_2 \cap A_3 \cdots \cap A_n = \bigcap_{i=1}^n A_i$$



1.1 Set Theory

Set Operations

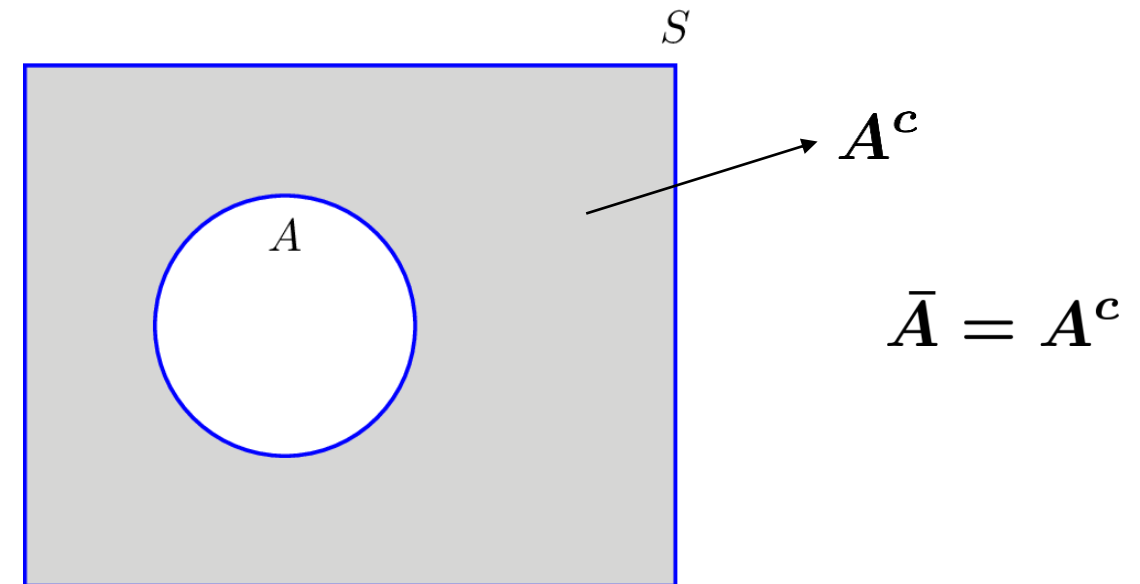
Complement:

The complement of a set A , denoted by A^c , is the set of all elements in S (Ω) that are **Not** in A .

$$S = \{2, 4, 6, 8\};$$

$$A = \{2, 8\}$$

$$A^c = \{4, 6\}$$



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Difference (subtraction):

The subtraction of set B from A ($A - B$) is all elements in A that are not in B .

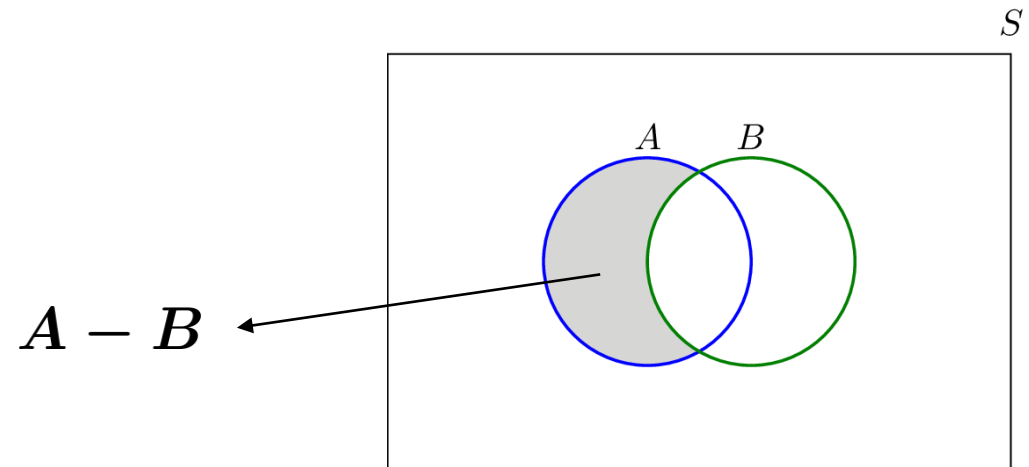
$$A - B = A \cap B^c; (x \in A) \text{ and } (x \notin B).$$

$$A = \{1, 2\}$$

$$B = \{2, 3, 4\}$$

$$A - B = \{1\}$$

$$B - A = \{3, 4\}$$



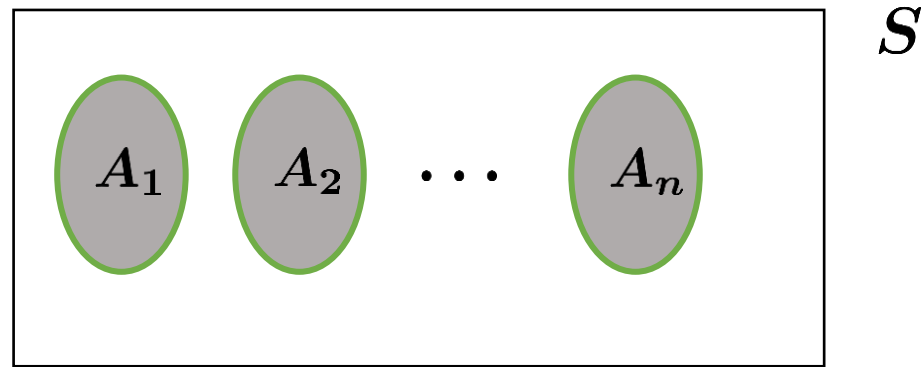
1.1 Set Theory

Set Operations

Mutually exclusive set (disjoint):

Two sets A and B are mutually exclusive (or disjoint) if $A \cap B = \emptyset$.

❖ $A_1, A_2, \dots, A_n$ are m.e. if $A_i \cap A_j = \emptyset, i \neq j$.



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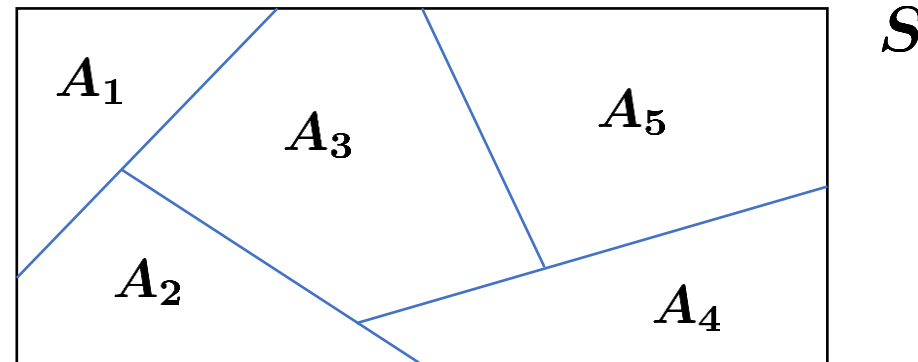
Set Operations

Partition:

A collection of sets $A_1, A_2, \dots, A_n$ is a Partition of S if

a) They are disjoint .

b) $A_1 \cup A_2 \cup A_3 \cup \dots A_n = S$



1.1 Set Theory

Set Operations

Theorem : De Morgan's law

$$(A \cup B)^c = \overline{A \cup B} = A^c \cap B^c$$

Theorem : Distributive law

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

1.1 Set Theory

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Example:

Let $S = \{1, 2, 3, 4, 5, 6\}$, and $A = \{1, 2\}$, $B = \{2, 4, 6\}$.

a) $A \cup B = \{1, 2, 4, 6\}$

b) $A \cap B = \{2\}$

c) $A^c = \{3, 4, 5, 6\}$

d) $B^c = \{1, 3, 5\}$

e) $(A \cup B)^c = \{3, 5\}$

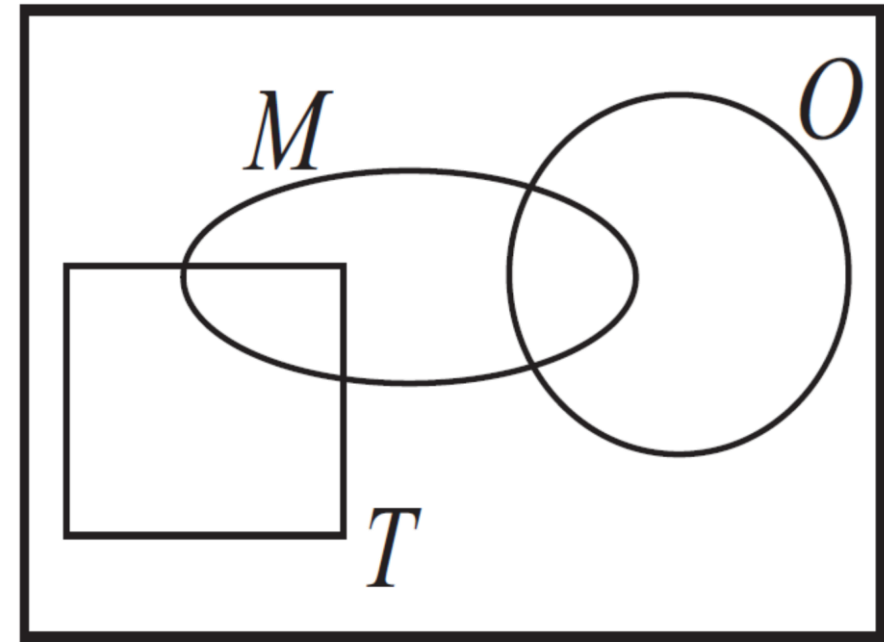
f) $A^c \cap B^c = \{3, 5\}$

1.1 Set Theory

Set Operations

Example: Gerlandas offers customers two kinds of pizza crust, Tuscan (T) and Neapolitan (N). In addition, each pizza may have mushrooms (M) or onions (O) as described by the Venn diagram at right. For the sets specified below, shade the corresponding region of the Venn diagram.

- a) N
- b) $N \cup M$
- c) $N \cap M$
- d) $T^c \cap M^c$

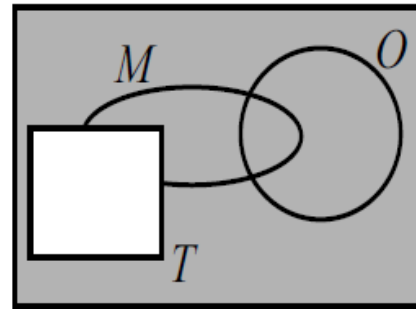


1.1 Set Theory

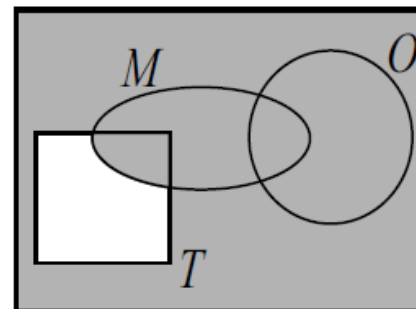
Set Operations

Solution:

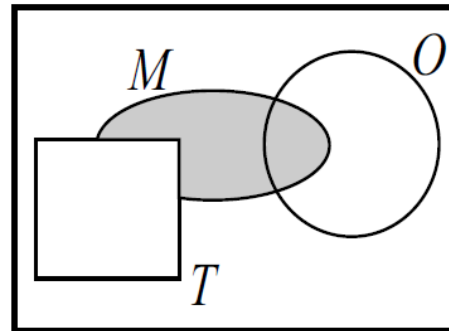
- a) N
- b) $N \cup M$
- c) $N \cap M$
- d) $T^c \cap M^c$



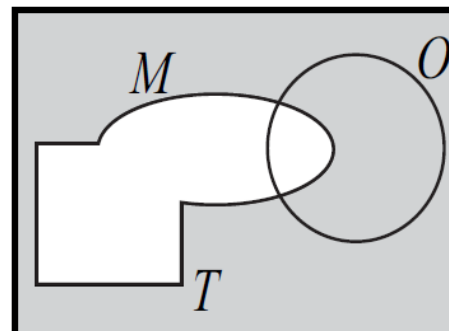
(a) $N = T^c$



(b) $N \cup M$



(c) $N \cap M$



(d) $T^c \cap M^c$

