

EELE 3340

Probability Theory



Chapter 2

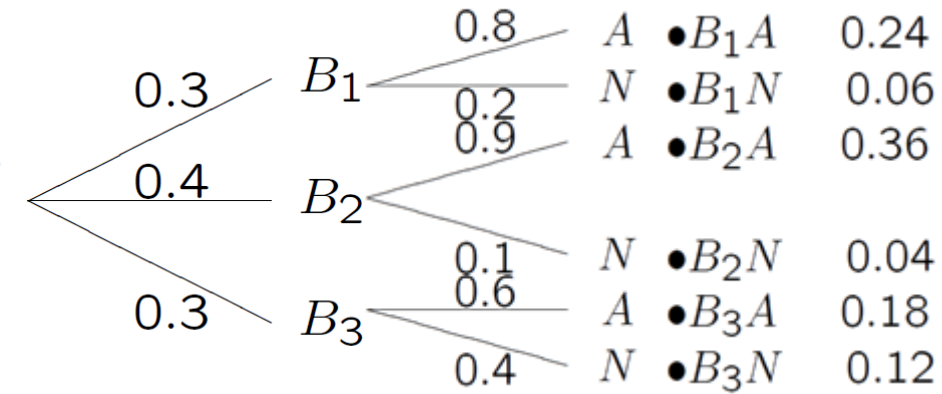
Sequential Experiments

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2.1 Tree Diagrams

Example 2.1.1

A company has three machines B_1 , B_2 , and B_3 making 1 k Ω resistors. Resistors within 50 Ω of the nominal value are considered acceptable. It has been observed that 80% of the resistors produced by B_1 and 90% of the resistors produced by B_2 are acceptable. The percentage for machine B_3 is 60%. Each hour, machine B_1 produces 3000 resistors, B_2 produces 4000 resistors, and B_3 produces 3000 resistors. All of the resistors are mixed together at random in one bin and packed for shipment. What is the probability that the company ships an acceptable resistor?



What is the probability of choosing a resistor from machine B2 that is not acceptable?

$$P[A] = P[A|B_1] P[B_1] + P[A|B_2] P[B_2] + P[A|B_3] P[B_3]$$

$$P[A|B_1] = 0.8 \quad P[A|B_2] = 0.9 \quad P[A|B_3] = 0.6$$

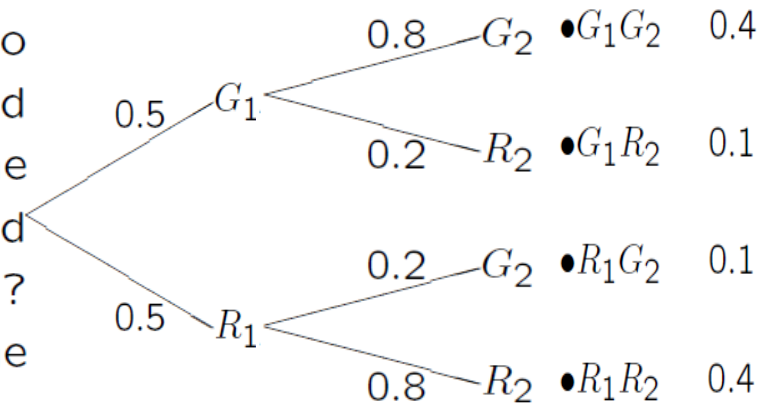
$$P[B_2 N] = P[N|B_2] P[B_2]$$

$$P[B_2 N] = (0.4)(0.1) = 0.04$$

2.1 Tree Diagrams

Example 2.1.2

Traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green, what is the probability $P[G_2]$ that the second light is green? Also, what is $P[W]$, the probability that you wait for at least one of the first two lights? Lastly, what is $P[G_1|R_2]$, the conditional probability of a green first light given a red second light?



$$P[G_2] = P[G_1G_2] + P[R_1G_2]$$

$$P[G_2] = P[G_2|G_1]P[G_1] + P[G_2|R_1]P[R_1]$$

$$P[G_2] = 0.8 * 0.5 + 0.2 * 0.5 = 0.4 * 0.1 = 0.5$$

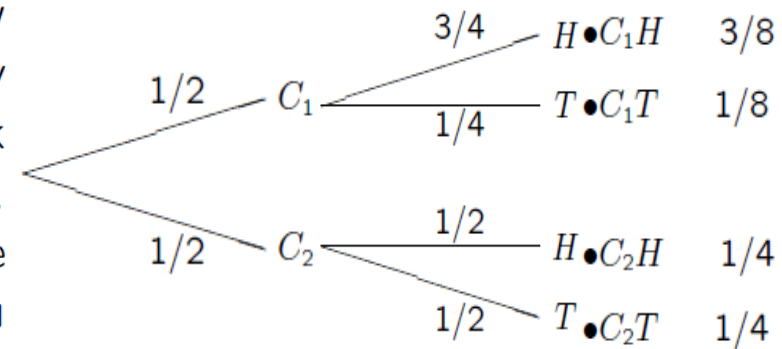
$$W = \{R_1G_2 \cup G_1R_2 \cup R_1R_2\}$$

$$P[W] = P[R_1G_2] + P[G_1R_2] + P[R_1R_2] = 0.1 + 0.1 + 0.4 = 0.6$$

2.1 Tree Diagrams

Example 2.1.3

Suppose you have two coins, one biased, one fair, but you don't know which coin is which. Coin 1 is biased. It comes up heads with probability $3/4$, while coin 2 comes up heads with probability $1/2$. Suppose you pick a coin at random and flip it. Let C_i denote the event that coin i is picked. Let H and T denote the possible outcomes of the flip. Given that the outcome of the flip is a head, what is $P[C_1|H]$, the probability that you picked the biased coin? Given that the outcome is a tail, what is the probability $P[C_1|T]$ that you picked the biased coin?



$$P[C_1|H] = \frac{P[C_1H]}{P[H]} = \frac{P[C_1H]}{P[C_1H] + P[C_2H]} = \frac{3/8}{3/8 + 1/4} = \frac{3}{5}$$

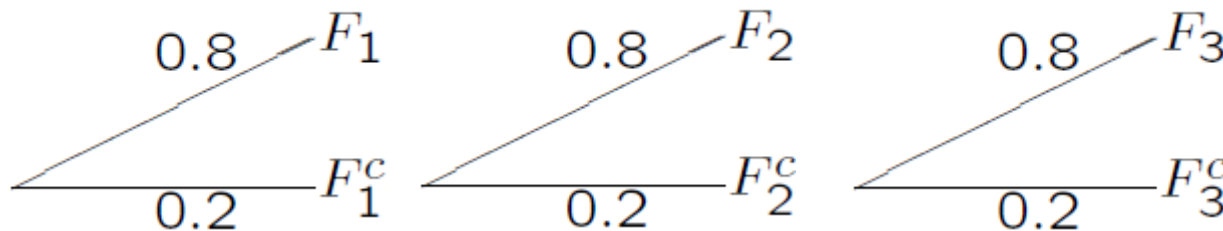
$$P[C_1|T] = \frac{P[C_1T]}{P[T]} = \frac{P[C_1T]}{P[C_1T] + P[C_2T]} = \frac{1/8}{1/8 + 1/4} = \frac{1}{3}$$

2.1 Tree Diagrams

Example 2.1.4

In a cellular phone system, a mobile phone must be paged to receive a phone call. However, paging attempts don't always succeed because the mobile phone may not receive the paging signal clearly. Consequently, the system will page a phone up to three times before giving up. If the results of all paging attempts are independent and a single paging attempt succeeds with probability 0.8, sketch a probability tree for this experiment and find the probability $P[F]$ that the phone receives the paging signal clearly.

Let F_i denote the event that the user is found on page i . The tree for the experiment is



$$P[F] = 1 - P[F_1^c F_2^c F_3^c] = 1 - (0.2)^3 = 0.992$$

$$\text{OR } P[F] = P[F_1 \cup F_1^c F_2 \cup F_1^c F_2^c F_3] = P[F_1] + P[F_1^c F_2] + P[F_1^c F_2^c F_3] = 0.8 + 0.2 \cdot 0.8 + 0.2 \cdot 0.2 \cdot 0.8 = 0.992$$