

## 2.3 Independent Trials

### Example 2.3.1

What is the probability  $P[E_{2,3}]$  of two failures and three successes in five independent trials with success probability  $p$ .

each outcome with three successes has probability  $(1 - p)^2 p^3$ .

$$P[E_{2,3}] = \binom{5}{3} (1 - p)^2 p^3.$$

## 2.3 Independent Trials

### Theorem

The probability of  $n_0$  failures and  $n_1$  successes in  $n = n_0 + n_1$  independent trials is

$$\mathbb{P}[E_{n_0, n_1}] = \binom{n}{n_1} (1-p)^{n-n_1} p^{n_1} = \binom{n}{n_0} (1-p)^{n_0} p^{n-n_0}.$$

## 2.3 Independent Trials

### Example 2.3.2

To communicate one bit of information reliably, cellular phones transmit the same binary symbol five times. Thus the information “zero” is transmitted as 00000 and “one” is 11111. The receiver detects the correct information if three or more binary symbols are received correctly. What is the information error probability  $P[E]$ , if the binary symbol error probability is  $q = 0.1$ ?

The probability of a success is  $p = 1 - q = 0.9$ .

$$\begin{aligned} P[E] &= P[E_{0,5}] + P[E_{1,4}] + P[E_{2,3}] \\ &= \binom{5}{0}q^5 + \binom{5}{1}pq^4 + \binom{5}{2}p^2q^3 = \underline{0.00856}. \end{aligned}$$

By increasing the number of binary symbols per information bit from 1 to 5, the cellular phone reduces the probability of error by more than one order of magnitude, from 0.1 to 0.0081.

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### Example 2.3.3

Data packets containing 100 bits are transmitted over a communication link. A transmitted bit is received in error (either a 0 sent is mistaken for a 1, or a 1 sent is mistaken for a 0) with probability  $\epsilon = 0.01$ , independent of the correctness of any other bit. The packet has been coded in such a way that if three or fewer bits are received in error, then those bits can be corrected. If more than three bits are received in error, then the packet is decoded with errors.

- (a) Let  $E_{k,100-k}$  denote the event that a received packet has  $k$  bits in error and  $100 - k$  correctly decoded bits. What is  $P[E_{k,100-k}]$  for  $k = 0, 1, 2, 3$ ?
- (b) Let  $C$  denote the event that a packet is decoded correctly. What is  $P[C]$ ?

## 2.3 Independent Trials

### Example 2.3.3

(a) In this problem,  $k$  bits received in error is the same as  $k$  failures in 100 trials. The failure probability is  $\epsilon = 1 - p$  and the success probability is  $1 - \epsilon = p$ . That is, the probability of  $k$  bits in error and  $100 - k$  correctly received bits is

For  $\epsilon = 0.01$ ,

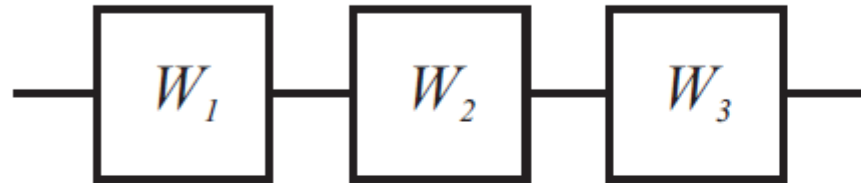
$$\begin{aligned} P[E_{k,100-k}] &= \binom{100}{k} \epsilon^k (1 - \epsilon)^{100-k} \\ P[E_{0,100}] &= (1 - \epsilon)^{100} = (0.99)^{100} = 0.3660. \\ P[E_{1,99}] &= 100(0.01)(0.99)^{99} = 0.3700. \\ P[E_{2,98}] &= 4950(0.01)^2(0.99)^{98} = 0.1849. \\ P[E_{3,97}] &= 161,700(0.01)^3(0.99)^{97} = 0.0610. \end{aligned}$$

(b) The probability a packet is decoded correctly is just

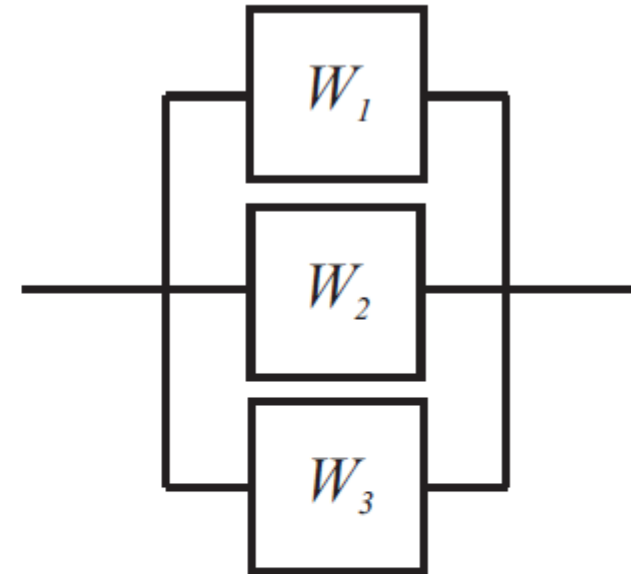
$$P[C] = P[E_{0,100}] + P[E_{1,99}] + P[E_{2,98}] + P[E_{3,97}] = 0.9819.$$

## 2.4 Reliability Analysis

### Introduction



*Components in Series*



*Components in Parallel*

Serial and parallel devices.

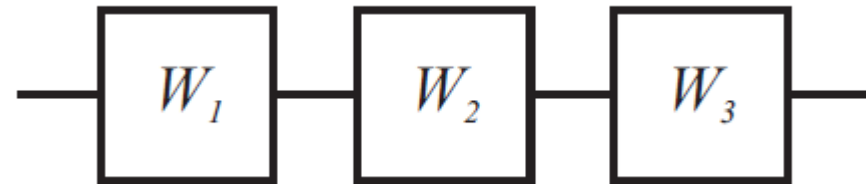
## 2.4 Reliability Analysis

### Example 2.4.1

A memory module consists of nine chips. The device is designed with redundancy so that it works even if one of its chips is defective. Each chip contains  $n$  transistors and functions properly only if all of its transistors work. A transistor works with probability  $p$  independent of any other transistor.

(a) What is the probability  $P[C]$  that a chip works?

$$P[C] = p^n$$



## 2.4 Reliability Analysis

### Example 2.4.1

(b) What is the probability  $P[M]$  that the memory module works?

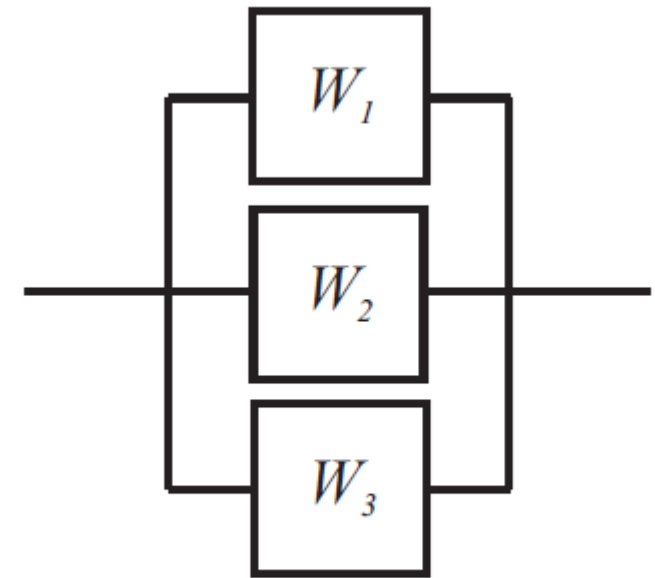
Let  $C_k$  denote the event that exactly  $k$  chips work.

$$P[M] = P[C_8] + P[C_9]$$

$$P[C_8] = \binom{9}{8} (P[C])^8 (1 - P[C])^{9-8} = 9p^{8n}(1 - p^n)$$

$$P[C_9] = (P[C])^9 = p^{9n}$$

$$P[M] = P[C_8] + P[C_9] = p^{8n}(9 - 8p^n).$$



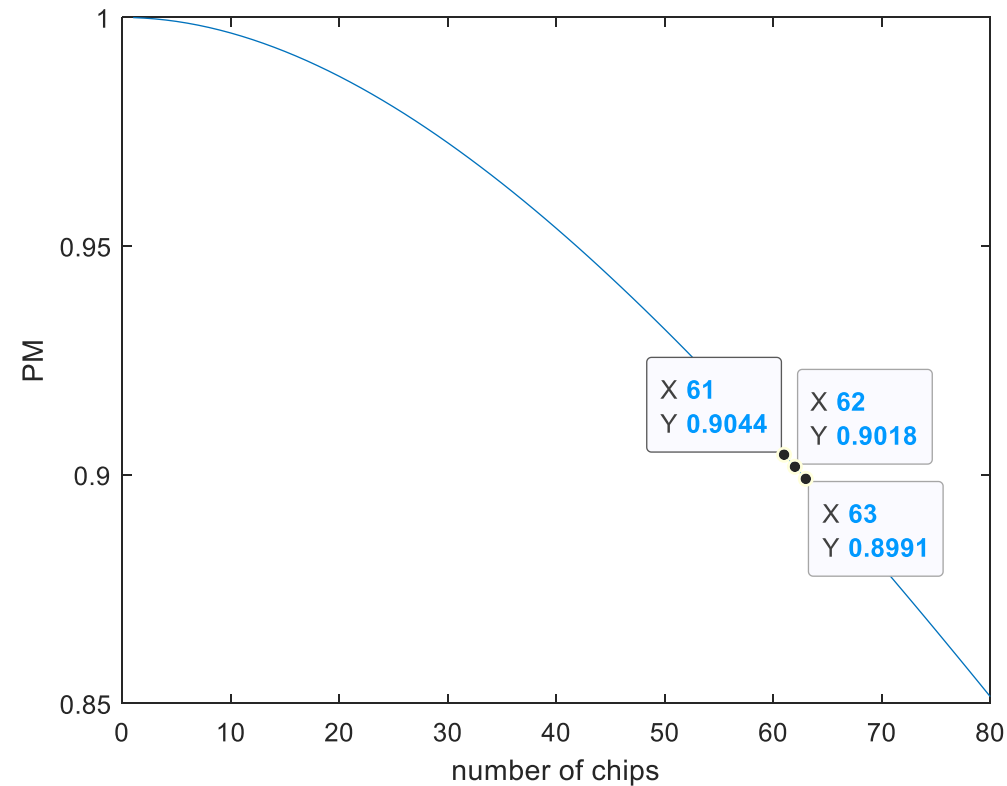
*Components in Parallel*

## 2.4 Reliability Analysis

### Example 2.4.1

(c) If  $p = 0.999$ , what is the maximum number of transistors per chip  $n$  that produces  $P[M] \geq 0.9$  (a 90% success probability for the memory module)?

**nmax = 62**



## 2.4 Reliability Analysis

### Example 2.4.1

- (d) If the memory module can tolerate two defective chips, what is the maximum number of transistors per chip  $n$  that produces  $P[M] \geq 0.9$ ?

$$P[M] = P[C_7] + P[C_8] + P[C_9]$$

$$\begin{aligned} P[C_7] &= \binom{9}{7} (P[C])^7 (1 - P[C])^2 = 36p^{7n}(1 - p^n)^2 \\ &= 36p^{7n} - 72p^{8n} + 36p^{9n}. \end{aligned}$$

$$\begin{aligned} P[M] &= P[C_7] + P[C_8] + P[C_9] \\ &= 36p^{7n} - 72p^{8n} + 36p^{9n} + p^{8n}(9 - 8p^n) \\ &= 36p^{7n} - 63p^{8n} + 28p^{9n}. \end{aligned}$$

$$n_{\max} = 138$$

The additional redundancy at the chip level to enable one more defective chip allows us to more than double the number of transistors per chip.

# Selected Problems

2.1.1, 2.1.4, 2.1.5, 2.1.6, 2.1.7, 2.1.8, 2.2.2, 2.2.7, 2.2.8, 2.3.1, 2.3.3, 2.4.1,  
2.4.2, 2.4.3