

3.3 Families of Discrete Random Variables

Bernoulli Random Variables

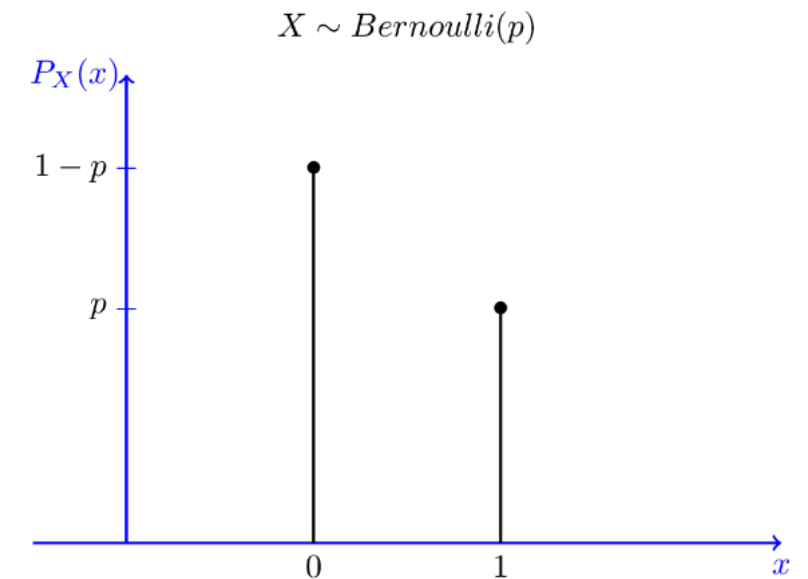
Example. Flip a coin {H,T}. Take an exam {Pass, Fail}.

$$X \sim \text{Bernoulli}(p)$$

PMF:

$$P_X(x) = \begin{cases} p & \text{for } x = 1 \\ 1 - p & \text{for } x = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Range}(X) = \{0, 1\}.$$

$$P_X(0) = 1 - p, \quad P_X(1) = p.$$



3.3 Families of Discrete Random Variables

Geometric Random Variables

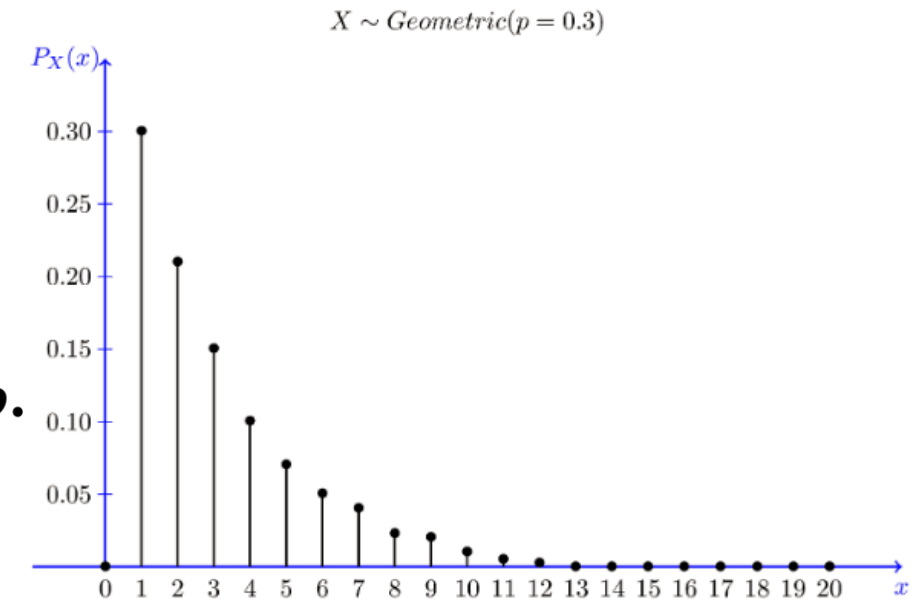
$$X \sim \text{Geometric}(p)$$

$$S_X = \text{Range}(X) = \{1, 2, 3, \dots\}$$

Random experiment: consider a coin with $P(H) = p$.
Toss the coin repeatedly until the first heads is observed.

X = The total number of coin tosses

$$X \sim \text{Geometric}(p)$$



3.3 Families of Discrete Random Variables

Geometric Random Variables

Definition. A random variable X is said to be a *geometric* random variable with parameter p , shown as $X \sim \text{Geometric}(p)$, if

$$S_X = \{1, 2, 3, \dots\} = \mathbb{N},$$

$$P_X(k) = p(1 - p)^{k-1}, \quad k = 1, 2, 3, \dots$$

3.3 Families of Discrete Random Variables

Binomial Random Variables

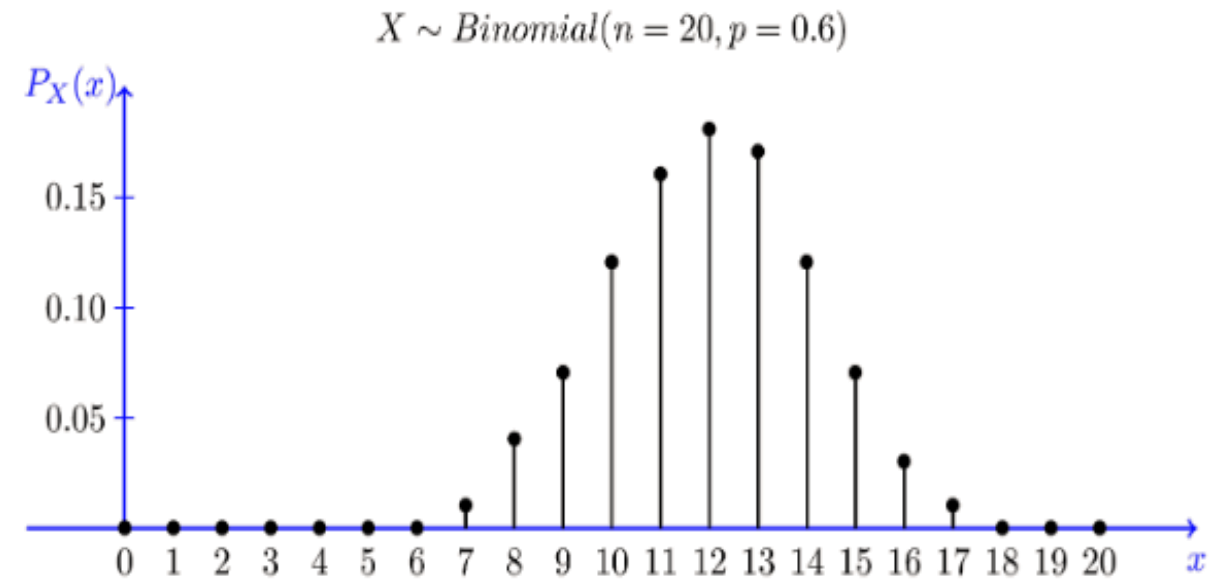
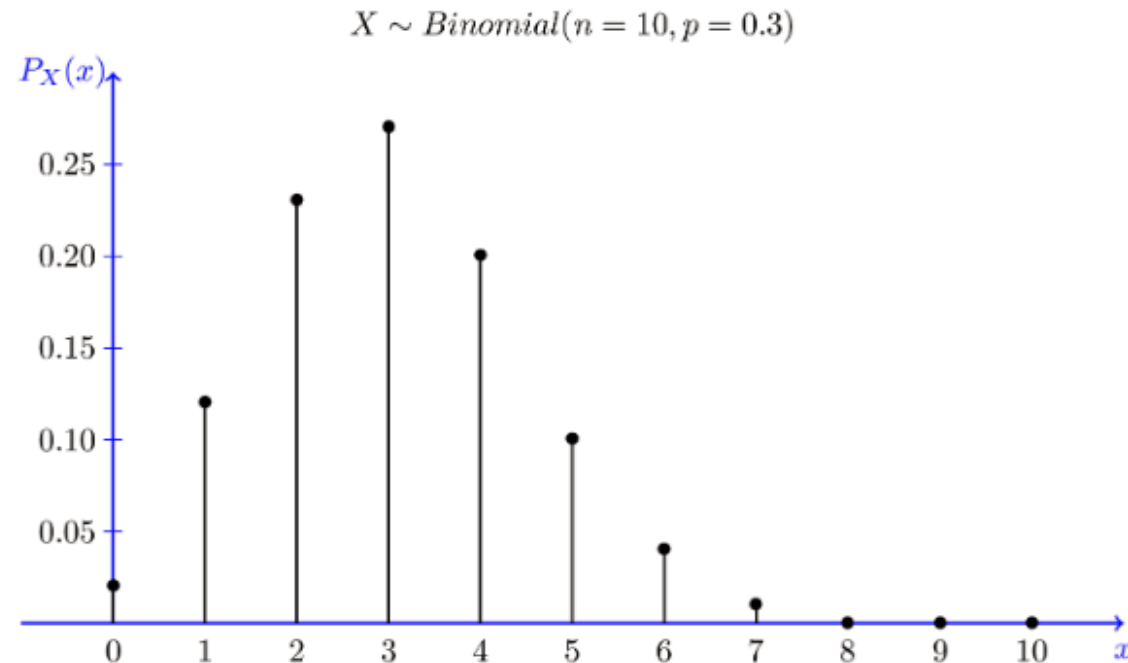
Definition. A random variable X is said to be a **binomial random variable** with parameters n and p ($P(H) = p$), shown as $X \sim \text{Binomial}(n, p)$, if

$$S_X = \{0, 1, 2, \dots, n\}$$

$$P_X(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad k = 0, 1, 2, \dots, n$$

3.3 Families of Discrete Random Variables

Binomial Random Variables



3.3 Families of Discrete Random Variables

Pascal Distribution:

Example. You flip a coin until you observe m heads.

X : total number of coin toss

$$S_X = \{m, m + 1, m + 2, \dots\}.$$

Find PMF.

- $A = \{X = k\}$, or $A = B \cap C$,
- C is the event that we observe $m - 1$ heads in the first $k - 1$ trials.
- B is the event that we observe a heads in the k th (the last) trial.

3.3 Families of Discrete Random Variables

Pascal Distribution:

Definition. A random variable X is said to be a **Pascal random variable** with parameters m and p ($P(H) = p$), shown as $X \sim \text{Pascal}(m, p)$, if

$$P_X(k) = \binom{k-1}{m-1} p^m (1-p)^{k-m} \quad k = m, m+1, m+2, \dots$$

Where $0 < p < 1$.

3.3 Families of Discrete Random Variables

Uniform Random Variables

Definition. A random variable X is said to be a **Uniform random variable**, shown as $X \sim \text{Uniform}(S_X)$, if

$$S_X = \{x_1, x_2, x_3, \dots\},$$

$$P_X(x_i) = \frac{1}{|S_X|}.$$

3.3 Families of Discrete Random Variables

Poisson Random Variable:

Poisson RVs are used to model

- Arrival of customers at a service facility
- Arrival of information request at a server

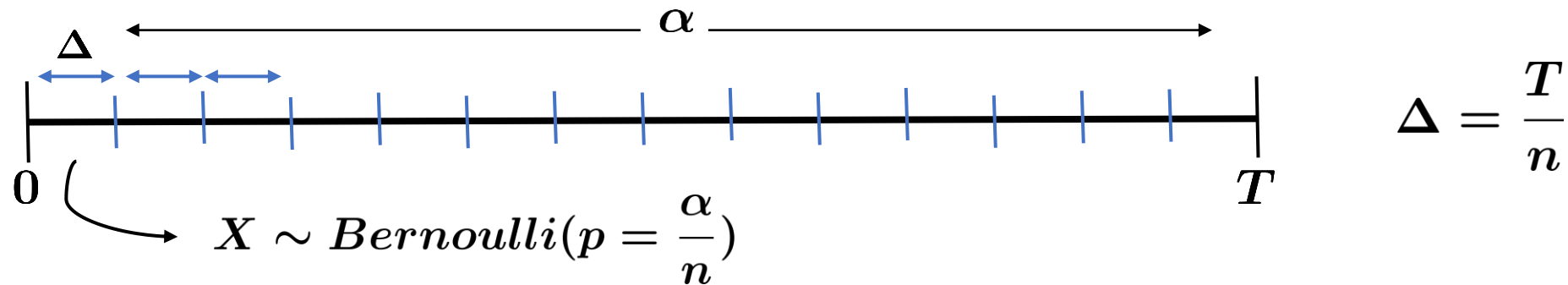
Counting the occurrence of certain events in an interval of time or space.

3.3 Families of Discrete Random Variables

Poisson Random Variable:

Arrival of customers in an interval:

α : the average number of arrivals in that interval



3.3 Families of Discrete Random Variables

Poisson Random Variable:

X : the total number of customers = $X_1, X_2, \dots, X_n$

$$X \sim \text{Binomial}(n, p = \frac{\alpha}{n})$$

$$P(X = k) = P_X(k) = \binom{n}{k} p^k (1 - p)^{n-k} = \binom{n}{k} \left(\frac{\alpha}{n}\right)^k \left(1 - \frac{\alpha}{n}\right)^{n-k}.$$

Thus,

$$\lim_{n \rightarrow \infty} P_X(k) = \frac{e^{-\alpha} \alpha^k}{k!}.$$

3.3 Families of Discrete Random Variables

Poisson Random Variable:

Definition. A random variable X is said to be a **Poisson random variable** with parameter α , shown as $X \sim \text{Poisson}(\alpha)$, if

$$S_X = \{0, 1, 2, 3, \dots\},$$

$$P_X(k) = \frac{e^{-\alpha} \alpha^k}{k!}, \quad k = 0, 1, 2, 3, \dots$$