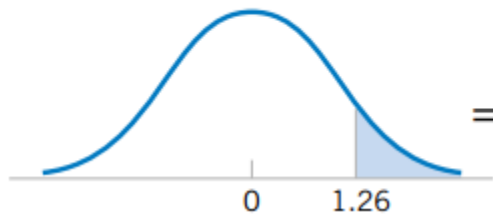


4.6 Gaussian Random Variables

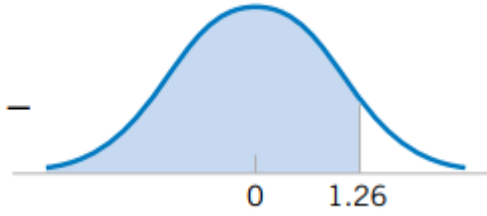
Example 4.6.3

Assume Z is a standard normal random variable.

(1) $P(Z > 1.26)$

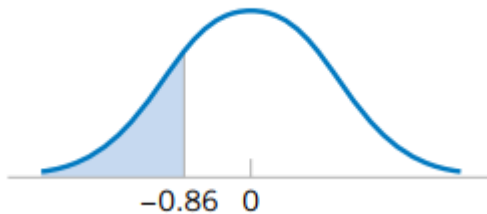


$= 1 -$

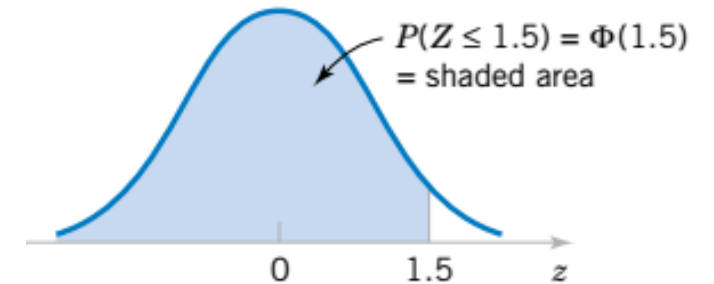


$$= 1 - P(Z \leq 1.26) = 1 - 0.89616 = 0.10384$$

(2) $P(Z < -0.86)$



$$= 0.19490.$$

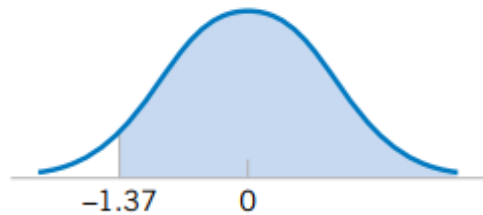


4.6 Gaussian Random Variables

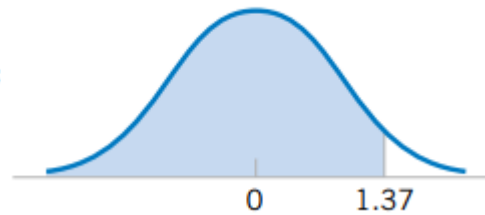
Example 4.6.3

Assume Z is a standard normal random variable.

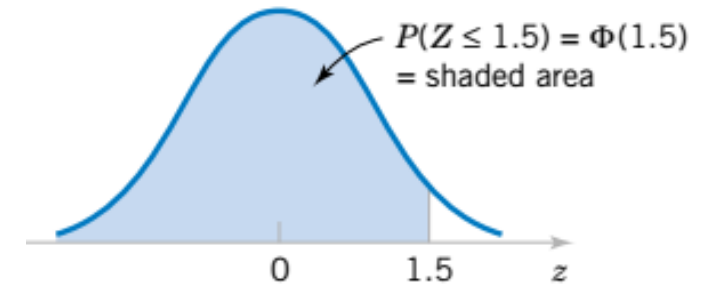
$$(3) \quad P(Z > -1.37)$$



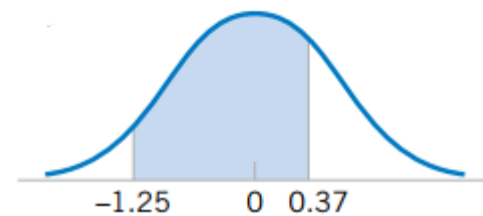
=



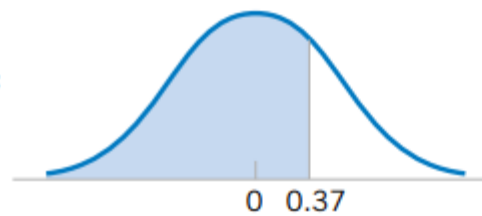
$$= P(Z < 1.37) = 0.91465$$



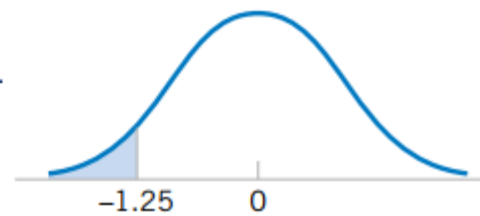
$$(4) \quad P(-1.25 < Z < 0.37)$$



=



-



$$= 0.64431 - 0.10565 = 0.53866$$

4.6 Gaussian Random Variables

Key Point

If the behaviour of a continuous random variable X is described by the distribution $N(\mu, \sigma^2)$ then the behaviour of the random variable $Z = \frac{X - \mu}{\sigma}$ is described by the standard normal distribution $N(0, 1)$.

We call Z the **standardised normal variable** and we write

$$Z \sim N(0, 1)$$

4.6 Gaussian Random Variables

Example 4.6.4

The random variable X follows a normal distribution with mean 1000 and variance 100. When X takes values between 1005 and 1010, between which values does the standardised normal variable Z lie?

The transformation is $Z = \frac{X - 1000}{10}$

When $X = 1005$, $Z = \frac{5}{10} = 0.5$

When $X = 1010$, $Z = \frac{10}{10} = 1$.

Hence Z lies between 0.5 and 1.

4.6 Gaussian Random Variables

Example 4.6.5

Piston rings are mass-produced. The target internal diameter is 45 mm but records show that the diameters are normally distributed with mean 45 mm and standard deviation 0.05 mm. An acceptable diameter is one within the range 44.95 mm to 45.05 mm. What proportion of the output is unacceptable?

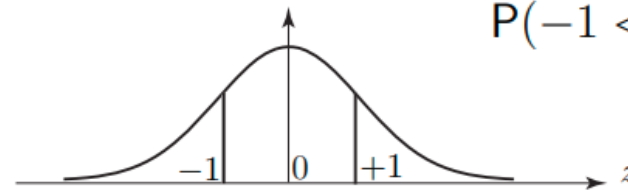
If X is the diameter of a piston ring then $X \sim N(45, (0.05)^2)$

The transformation is $Z = \frac{X - \mu}{\sigma} = \frac{X - 45}{0.05}$.

The upper limit of acceptability is $x_2 = 45.05$ so that $z_2 = (45.05 - 45)/0.05 = 1$.

The lower limit of acceptability is $x_1 = 44.95$ so that $z_1 = (44.95 - 45)/0.05 = -1$.

The range of 'acceptable' Z values is therefore -1 to 1 .



$$P(-1 < Z < 1) = 2 \times P(0 < Z < 1) = 2 \times 0.3413 = 0.6826.$$

Thus the proportion of unacceptable items is $1 - 0.6826 = 0.3174$, or 31.74%.

4.6 Gaussian Random Variables

Example 4.6.5

If the standard deviation is halved by improved production practices what is now the proportion of unacceptable items?

$$\sigma = 0.025$$

$$z_2 = \frac{45.05 - 45}{0.025} = 2 \qquad z_1 = -2$$

$$P(-2 < Z < 2) = 2 \times P(0 < Z < 2) = 2 \times 0.4772 = 0.9544.$$

unacceptable items is reduced to $1 - 0.9544 = 0.0456$ or 4.56%

4.6 Gaussian Random Variables

Example 4.6.6

Assume that in the detection of a digital signal the background noise follows a normal distribution with a mean of 0 volt and standard deviation of 0.45 volt. The system assumes a digital 1 has been transmitted when the voltage exceeds 0.9. What is the probability of detecting a digital 1 when none was sent?

Let the random variable N denote the voltage of noise. The requested probability is

$$P(N > 0.9) = P\left(\frac{N}{0.45} > \frac{0.9}{0.45}\right) = P(Z > 2) = 1 - 0.97725 = 0.02275$$

4.6 Gaussian Random Variables

Example 4.6.7

The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of 4 kilobits per second.

A) What is the probability that the file will transfer at a speed of 70 kilobits per second or more?

$$P(X > 70) = P\left(Z > \frac{70 - 60}{4}\right) = 1 - P(Z < 2.5) = 1 - 0.99379 = 0.00621$$

B) What is the probability that the file will transfer at a speed of less than 58 kilobits per second?

$$P(X < 58) = P\left(Z < \frac{58 - 60}{4}\right) = P(Z < -0.5) = 0.308538$$