



Final Exam

QUESTIONS	MARKS
Ques. 1 (10)	
Ques. 2 (8)	
Ques. 3 (6)	
Ques. 4 (6)	
Ques. 5 (10)	
Ques. 6 (10)	
Total (50)	

Time Allowed: 120 minutes

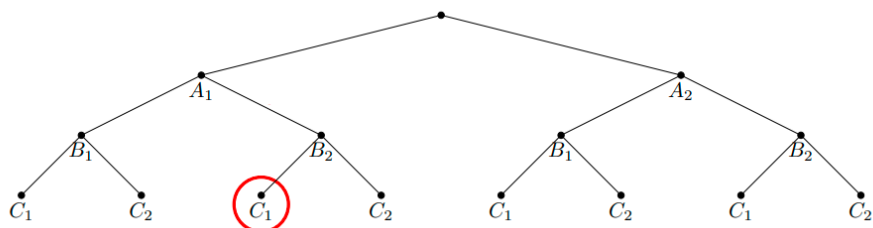
Student Name:.....

Student ID:.....

Question (1): Answer the following questions.

- a) What is the probability of 10 people in a room that no two of them have a birthday in the same month?
- b) In a die game, each person rolls a fair die which has the numbers 1 to 6 on it. The Player wins if the die shows up as a ‘doubles’ (such as 3-3 or 6-6) or if the sum of the die is 7. What is the probability that the player wins?
- c) Suppose now that events A, B and C are mutually independent with $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.5$. Compute $P(A^c \cap B \cap C)$.
- d) In a binary communication channel, it is equally likely to send either a 1 or a 0. The probability of error given that a 1 was sent is $1/9$, while the probability of an error given that a 0 was sent is $2/9$. What is the probability that a 1 is received?
- e) For the following tree, which does represent the probability of the circled node?

- A. C_1 B. $B_2 \cap C_1$ C. $A_1 \cap B_2 \cap C_1$ D. $C_1|B_2 \cap A_1$.



QUESTION (2):

A) The number of hits at a website per second is a Poisson random variable with mean 10 hits/second. Using the central limit theorem, estimate the probability that more than 650 hits occurred in one minute. (Hint: For a Poisson random variable X , $E[X]=\text{Var}(X)=\alpha$ (*Poisson Parameter*))

B) Let $\{X_n, n \in \mathbb{Z}\}$ be a discrete-time random process, defined as

$$X_n = 2 \cos\left(\frac{\pi n}{8} + \Phi\right),$$

where $\Phi \sim \text{Uniform}(0, 2\pi)$

- 1) Find the mean function $\mu_X(n)$.
- 2) Find the correlation function $R_X(m, n)$.

QUESTION (3):

A) Any student needs 0.55 to pass a test graded on the scale 0 to 1. Student scores are modeled by the following probability density function:

$$f(x) = \begin{cases} Ax & 0 \leq x \leq 0.5 \\ 4 - 4x & 0.5 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Where A is a constant. Find:

- 1) A .
- 2) The probability that a random student passes the exam.

B) Toss a fair coin 3 times. Let X = the number of heads on the first toss, Y = the total number of heads on the last two tosses.

- 1) Give the joint probability mass function table for X and Y .
- 2) Compute $\text{Cov}(X, Y)$. Are X and Y independent?

QUESTION (4):

Consider two random variables X and Y with joint PMF given in the following table.

- [A] Find $P(X \leq 2, Y > 1)$.
- [B] Find the marginal PMFs of X and Y .
- [C] Find $E[X]$.

	$Y = 1$	$Y = 2$
$X = 1$	$\frac{1}{3}$	$\frac{1}{12}$
$X = 2$	$\frac{1}{6}$	0
$X = 4$	$\frac{1}{12}$	$\frac{1}{3}$

QUESTION (5):

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{4}x^2 + \frac{1}{6}y & -1 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
- (b) Find $P(X > 0, Y < 1)$.
- (c) Find $P(X > 0 \text{ or } Y < 1)$.

QUESTION (6):

The average life of an electronic device follows a Gaussian distribution with average of 10 years with a standard deviation of 3 years. If the device fails while under guarantee, the manufacturer replaces it free.

- a) What fraction of the devices will be functional for more than 10 years?
- b) What fraction of the devices will be functional for less than 3 year?
- c) If the manufacturer plans to replace only 3% of the devices that could fail, how long the manufacturer should offer the guarantee to be?