

مراجعة الاحتمالات
ch 1

1.1 Set Theory

* $A \subset B$ and $B \supset A$

iff $(x \in A) \Rightarrow (x \in B)$

* $A = B$ iff $A \subset B$ and $B \subset A$

* For every set A : $\emptyset \subset A$

Set operations

[1] Union $A \cup B$

where $x \in (A \cup B)$ iff $x \in A$
or $x \in B$

[2] Intersection $A \cap B$

$x \in (A \cap B)$

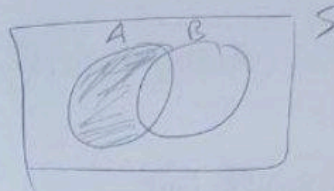
iff $x \in A$ and $x \in B$

[3] Complement

A^c is the set of all elements in S
 that are Not in A

[4] Difference (subtraction)

$(A - B) \rightarrow$ مجموعة كل العناصر الموجودين في A وليس في B

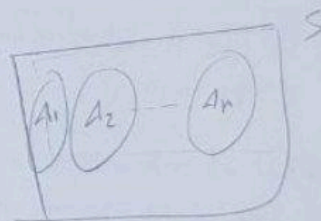


$$A - B = A \cap B^c$$

where $x \in A$ and $x \notin B$

[5] Mutually exclusive set (disjoint)

- A and B are disjoint iff $A \cap B = \emptyset$

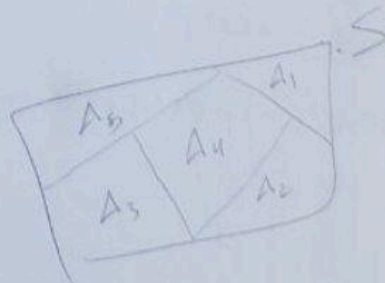


[6] Partition

- $A_1, A_2, \dots, A_n$ is a partition iff

1) They are disjoint

2) $A_1 \cup A_2 \cup \dots \cup A_n = S$



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[7] De Morgan's Law

$$-(A \cup B)^c = \overline{(A \cup B)} = A^c \cap B^c$$

[8] Distributive Law

$$- A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$- A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

1.2 Applying Set Theory to Probability

- Outcome: is the result of a random experiment

- Event: is a set of outcomes of an experiment (one or more outcomes)

1.3 Axioms of Probability

① For any event A , $0 \leq P(A) \leq 1$

② $P(S) = 1$

③ For any countable collection $A_1, A_2, A_3, \dots$ of disjoint events

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$

$$\textcircled{4} \text{ If } B = \{s_1, s_2, \dots, s_m\}$$

$$\therefore P(B) = \sum_{i=1}^m P[\{s_i\}]$$

$$\textcircled{5} S = \{s_1, s_2, \dots, s_n\}$$

حسب احتمال حدوث كل outcome صاري الآخر

$$\therefore P(s_i) = \frac{1}{n} \quad 1 \leq i \leq n$$

$$\textcircled{6} A \cup A^c = S$$

$$\therefore P(A) + P(A^c) = 1$$

$$\textcircled{7} P(\emptyset) = 0$$

$$\textcircled{8} S^c = \emptyset$$

$$\textcircled{9} P(A - B) = P(A) - P(A \cap B)$$

$$\textcircled{10} P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\textcircled{11} P(A \cup B \cup C) =$$

$$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$\textcircled{12} \text{ If } A \subset B$$

$$\therefore P(A) \leq P(B)$$

13)

1.4 Conditional Probability

* The conditional probability is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$$

* Special cases

1) if A and B are disjoint

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

2) if $A \subset B$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

3) if $B \subset A$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

* For any event A, $P(A|B) \geq 0$

* $P(B|B) = 1$

* If $A_1, A_2, A_3, \dots$ are disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \dots | B) = P(A_1|B) + P(A_2|B) + P(A_3|B) + \dots$$

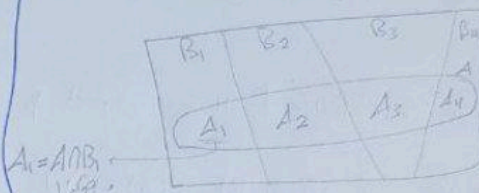
1.5 Law of Total Probability

* Definition

Let $B_1, B_2, B_3, \dots$ be partition of the sample space S with $P(B_i) > 0$.

For any event A we have

$$P(A) = \sum_{i=1}^m P(A \cap B_i) = \sum_{i=1}^m P(A|B_i)P(B_i)$$



$$* P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)P(B)$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

* For any two events A and B where $P(A) \neq 0$ we have

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Bayes' Rule

1.6 Independence

* $P(A|B) = P(A)$ for A and B are independent (B is independent of A and vice versa)

* For two events A and B are independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

[4]

Independence في الاحتمال

$$\Rightarrow P(A \cap B) = P(A)P(B)$$

$$* P(A|B) = P(A), \text{ equivalents}$$

$$P(A \cap B) = P(A)P(B)$$

$$\Rightarrow P(A|B)P(B) = P(A)P(B)$$

$$\Rightarrow P(A \cap B) = P(A)P(B)$$

الفرق بين Independent و disjoint

$$\text{Disjoint: } A \cap B = \emptyset, P(A \cup B) = P(A) + P(B)$$

في حال disjoint A و B لا يوجد بينهما تقاطع
لو كانت A و B disjoint
فإن $P(A \cap B) = 0$

$$\text{Independent: } P(A \cap B) = P(A)P(B),$$

$$P(A|B) = P(A), P(B|A) = P(B)$$

* Suppose A and B are disjoint:

$$\textcircled{1} \text{ If } P(B) \neq 0, P(A|B) = 0$$

$$\textcircled{2} \text{ If } P(B) \neq 0, P(A) \neq 0, \& \text{ disjoint} \\ \Rightarrow \text{not Independent.}$$

* If A & B are Independent, then

$$\textcircled{1} A \& B^c \text{ are Independent}$$

$$\textcircled{2} A^c \& B \text{ are Independent}$$

$$\textcircled{3} A^c \& B^c \text{ are Independent}$$

$$\textcircled{4} P(A_1 \cup A_2) = 1 - [(1 - P(A_1))(1 - P(A_2))]$$

* Three events A, B and C are independent if all of the following conditions hold

$$\textcircled{1} P(A \cap B) = P(A)P(B)$$

$$\textcircled{2} P(A \cap C) = P(A)P(C)$$

$$\textcircled{3} P(B \cap C) = P(B)P(C)$$

$$\textcircled{4} P(A \cap B \cap C) = P(A)P(B)P(C)$$

لو كانت A و B و C مستقلة
فإن $P(A \cap B \cap C) = P(A)P(B)P(C)$

A_1 and A_2 and A_3 are Mutually independent

لو كانت A_1 و A_2 و A_3 مستقلة
فإن $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$

A_1 and A_2 and A_3 are Pairwise independent

لو كانت A_1 و A_2 و A_3 مستقلة
فإن $P(A_1 \cap A_2) = P(A_1)P(A_2)$
 $P(A_1 \cap A_3) = P(A_1)P(A_3)$
 $P(A_2 \cap A_3) = P(A_2)P(A_3)$