

[1]  $A - B = A \cap B^c$

[2]  $A$  and  $B$  are mutually exclusive if  $A \cap B = \emptyset$  (Disjoint)

[3] collection of sets  $A_1, A_2, A_3, \dots, A_n$  is a partition of  $S$  if:

$\Rightarrow$  They are disjoint

$\Rightarrow A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n = S$

[4]  $(A \cup B)^c = (\overline{A \cup B}) = A^c \cap B^c$  "De Morgan's Law"

[5]  $\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$  "Distributive Law"

$\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

[6] For any event  $A$ ,  $P(A) \geq 0$

[7] Probability of the Sample Space  $S$ ,  $P(S) = 1$

[8] For any countable collection  $A_1, A_2, A_3, \dots$  of disjoint events:

$\Rightarrow P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

[9] For mutually exclusive events  $A_1$  and  $A_2$ :  $\Rightarrow$

$\Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2)$

[10] If  $A = (A_1 \cup A_2 \cup \dots \cup A_m)$  and  $A_i \cap A_j = \emptyset$  for  $i \neq j$ :  $P(A) = \sum_{i=1}^m P[A_i]$

[11] The probability of an event  $B = \{s_1, s_2, \dots, s_m\}$  is:  $P(B) = \sum_{i=1}^m P[\{s_i\}]$

[12] For an experiment with sample space  $S = \{s_1, s_2, s_3, \dots, s_n\}$  in which each outcome  $s_i$  is equally likely:

$P(s_i) = \frac{1}{n}$ ,  $1 \leq i \leq n$

[13]  $P(A^c) = 1 - P(A)$

[14]  $P(\emptyset) = P(S^c) = 0$

[15]  $0 \leq P(A) \leq 1$

[16]  $P(A - B) = P(A) - P(A \cap B)$

[17]  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[18]  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$$[19] A \subset B \Rightarrow P(A) \leq P(B)$$

$$[20] P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ when } P(B) > 0$$

$$[21] P(A|B) \geq 0$$

$$[22] P(B|B) = 1$$

$$[23] \text{ If } A_1, A_2, A_3, \dots \text{ are disjoint events, then: } \Rightarrow P(A_1 \cup A_2 \cup A_3 \cup \dots | B) = P(A_1|B) + P(A_2|B) + P(A_3|B) + \dots$$

$$\Rightarrow P(A_1 \cup A_2 \cup A_3 \cup \dots | B) = P(A_1|B) + P(A_2|B) + P(A_3|B) + \dots$$

$$[24] \text{ If } A \text{ and } B \text{ are disjoint } \Rightarrow P(A|B) = 0 = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)}$$

$$[25] \text{ If } A \subset B \Rightarrow P(A|B) = \frac{P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

$$[26] \text{ If } B \subset A \Rightarrow P(A|B) = 1 = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)}$$

$$[27] P(A) = \sum_{i=1}^m P(A_i) = \sum_{i=1}^m P(A \cap B_i) \quad \text{--- "Law of Total Probability"}$$

$$\Rightarrow P(A \cap B_i) = P(A_i)$$

$$[28] P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)} \Rightarrow P(A \cap B_i) = P(A|B_i) P(B_i) = P(A_i)$$

$$\Rightarrow \text{In general, } P(A) = \sum_{i=1}^m P(A_i) = \sum_{i=1}^m P(A \cap B_i) = \sum_{i=1}^m P(A|B_i) P(B_i) \quad \left( \text{Law of Total Probability} \right)$$

$$[29] P(B|A) = \frac{P(A|B) P(B)}{P(A)} \quad \text{--- "Bayes' Rule"}$$

$$[30] \text{ Two events } A \text{ and } B \text{ are independent if } A \text{ occurs without affecting the probability of } B \text{ occurring:}$$

$$P(A|B) = P(A), \text{ equivalently } P(A \cap B) = P(A) P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) P(B)}{P(B)} = P(A)$$

[31] In Independency:  $P(A|B) = P(A) \Rightarrow P(B|A) = P(B)$ , ( $P(A), P(B) \neq 0$ )

[32] If A and B are independent, then:

$\Rightarrow$  A and  $B^c$  are independent

$\Rightarrow A^c$  and B are independent

$\Rightarrow A^c$  and  $B^c$  are independent

[32] In Independency:  $P(A, \cup A_2) = 1 - (1 - P(A_1))(1 - P(A_2))$

[33] Three events A, B and C are <sup>mutually</sup> independent if all these conditions hold:

$$\left. \begin{aligned} \Rightarrow P(A \cap B) &= P(A)P(B) \\ \Rightarrow P(A \cap C) &= P(A)P(C) \\ \Rightarrow P(B \cap C) &= P(B)P(C) \\ \Rightarrow P(A \cap B \cap C) &= P(A)P(B)P(C) \end{aligned} \right\} \longrightarrow \begin{array}{l} \text{إذا حدث كل واحد منهم} \\ \text{بشكل مستقل} \\ \text{pair-wise independent} \end{array}$$

mutually independent: إذا حدث كل واحد منهم بشكل مستقل

1) For a finite sample space  $S$  with equally likely outcomes, the probability of an event  $A$  is given by:

$$P(A) = \frac{|A|}{|S|} = \frac{M}{N}$$

بعض الاحتمالات  
مساوية  
الاحتمال

in order (الترتيب) Cardinality

2) If we have  $r$  experiments, each experiment has  $n_i$  possible outcomes, then the total outcomes of the sequence of the  $r$  experiments =  $n_1 \times n_2 \times n_3 \times \dots \times n_r$

3) In the Ordered Sampling with Replacement (repetition allowed):

⇒ Number of outcomes =  $n^k$  → عدد مرات التجربة Exp.  
عدد النتائج في Sample Space

4) In the Ordered Sampling without Replacement (repetition not allowed):

⇒ Number of outcomes =  $n \times (n-1) \times (n-2) \times \dots \times (n-k+1)$

(الترتيب)  $nPr$

Exp.

⇒ Number of  $k$ -permutations of  $n$  distinguishable objects:

$$P_k^n = \frac{n!}{(n-k)!}, \text{ for } 0 \leq k \leq n$$

الترتيب

$$P_k^n = n \cdot nPr \cdot k$$

استخدام

Permutations

(التباديل)

OR:

$$P_k^n = k! \cdot \binom{n}{k}$$

الترتيب

استخدام

Combinations

(التوافيق)

$$P_k^n = k! \times (n \cdot nCr \cdot k)$$

\*  $\{ \}$  ⇒ الترتيب غير مهم

\*  $( )$  ⇒ الترتيب مهم

5 In the Unordered Sampling without Replacement (Combination):

⇒ the number of  $k$ -combinations of  $n$  objects is :

$n$  choose  $k$  →  $\binom{n}{k} = \frac{P_k^n}{k!} = \frac{n!}{k!(n-k)!}$

⇒ Number of ways to choose  $k$  objects out of  $n$  distinguishable objects =  $\binom{n}{k}$

→  $\binom{n}{k} = n \text{ Cr } k$

6 If we have  $n = n_1 + n_2 + \dots + n_r$ , then the number of possible divisions of  $n$  distinct objects into  $r$  distinct groups of respective sizes  $n_1, n_2, n_3, \dots, n_r$  is :

$\binom{n}{n_1, n_2, n_3, \dots, n_r} = \frac{n!}{n_1! n_2! n_3! \dots n_r!}$  "Multinomial Coefficients"

مثال 140 ← 140 شخص في فريق 4 أشخاص  
 30 شخص في فريق 50 شخص في فريق 20 شخص في فريق 40 شخص في فريق



7 For  $n$  independent Bernoulli (Binomial) trials where each trial has success probability  $p$ , the probability of  $k$  success is given by :

$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$  ← outcome probability

\* Note:

$\binom{N}{M} = \binom{8}{3} = \binom{8}{5} = 56$

3+5=8  
 3 و 5  
 3 و 5

3

8 In the Unordered Sampling with Replacement (repetition allowed) :

⇒ the total number of distinct  $k$  samples from an  $n$ -element set such that repetition is allowed and ordering does not matter is :

$$\binom{n+k-1}{k}$$

9 The probability of  $n_0$  failures and  $n_1$  successes in  $n = n_0 + n_1$  independent trials

is :

$$P[E_{\substack{n_0 \\ \text{failure}}, \substack{n_1 \\ \text{success}}}] = \binom{n}{n_1} (1-p)^{n-n_1} p^{n_1} = \binom{n}{n_0} (1-p)^{n_0} p^{n-n_0}$$

$\Downarrow$   
using success

$\Downarrow$   
using failure