

$$\rho = \frac{m}{V}$$

$$T = \frac{h}{m \lambda}$$

8542
AC

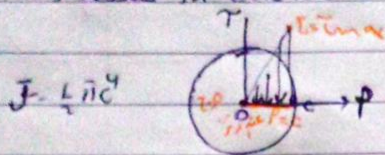


$$L\gamma = \rho\phi \text{ or } \gamma = \frac{\rho\phi}{L}$$

$$\gamma_{\text{max}} = \frac{C\phi}{L} \quad \text{and} \quad \gamma = \frac{\rho}{c} \gamma_{\text{max}}$$

$f = C \cdot u \cdot c$

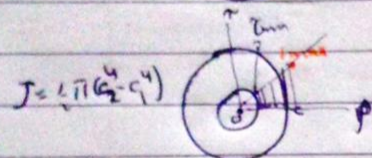
stresses in elastic Range



$$J = \frac{1}{2} \pi c^4$$

$$G \gamma = \frac{L}{c} G \gamma_{max} \quad \text{Hooke's law}$$

$$\tau_c = \frac{\rho}{c} \tau_{max}$$



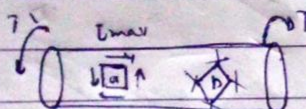
$$J = \frac{1}{2} \pi (\frac{4}{2} - 1)$$

لوعصره 'Hollow force من صنع ما

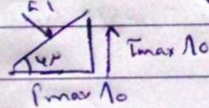
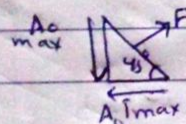
$\tau_{max} = \frac{T_c}{J}$ and $\tau = \frac{T_l}{J}$ at any point

shear stress $\rightarrow$ normal stress

الزيت



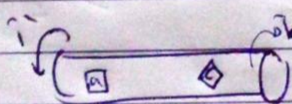
rotation ω ω



$$F = 2(C_{max} A_0) \cos 45^\circ = C_{max} A_0 \sqrt{2}$$

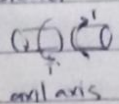
$$\underline{G_{iso.} \quad \frac{F}{A} = \frac{\tilde{\gamma}_{max} \Lambda_0 \sqrt{2}}{\Lambda_0 \sqrt{2}}}$$

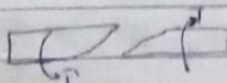
$$G_{45}^{\circ} = T_{max}$$



$$I_{\text{mass}} = \frac{r_c}{T} \left| 64.5^\circ \pm \frac{T_c}{T} \right|$$

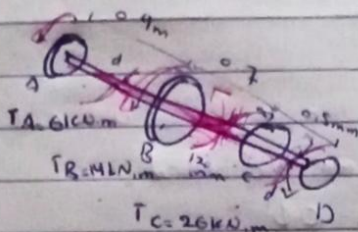
Forced Failure Modes


 elastic material
 elastic axis to shear stress

15 mm diameter

 brittle material

Sample problem

3-1



$T_D = 6 \text{ kNm}$

shaft BC is hollow with inner and outer diameter of 45 mm and 60 mm respectively.
 shaft AB and CD are solid of diameter d .

For loading shown, determine

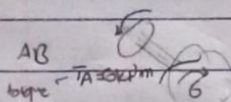
(a) the minimum and maximum shearing stress in shaft (BC)

(b) the required diameter d of shafts AB and CD if the allowable shearing stress in these shafts is 65 MPa

Check stability $i \geq 6$

$$\frac{1}{2} \left(\frac{1}{6} + \frac{1}{14} + \frac{1}{26} \right) + 6 = 0 \quad \text{OK}$$

CD, BC, AB internal force



$$\sum M = 0 = 6 \text{ kNm} - T_{AB}$$

$$T_{AB} = 6 \text{ kNm} - T_D$$

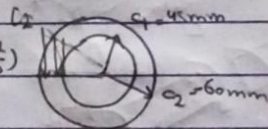
BC

internal forces

$$\sum M = 0 = 6 + 14 - T_{BC} \quad T_{BC} = 20 \text{ kNm}$$

CD

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.06^4 - 0.045^4) = 13.92 \times 10^{-6} \text{ m}^4$$



$$\tau_{max} = \tau_2 = \frac{T_{BC} c_2}{J} = \frac{(20 \text{ kNm})(0.06 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4} = 86.2 \text{ MPa}$$

$$\tau_{min} = \tau_1 = \frac{T_{BC} c_1}{J} = \frac{(20 \text{ kNm})(0.045 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4} = 64.2 \text{ MPa}$$

$$\tau_{min} = 64.2 \text{ MPa}$$

Solution: AB, CD given
allow stress τ_{max} & allow shear strain
d given
65 MPa

(W o d)

$$\tau_{max} = \frac{T C}{J \theta} = 65 \text{ MPa} = \frac{610 \text{ N.m}}{\frac{\pi C^3}{2}}$$

Ratio $C = 38.9 \times 10^{-3}$
- Example

$$d = 2C = 77.8 \text{ mm} \approx 78 \text{ mm}$$

Angle of twist in elastic range

part 2

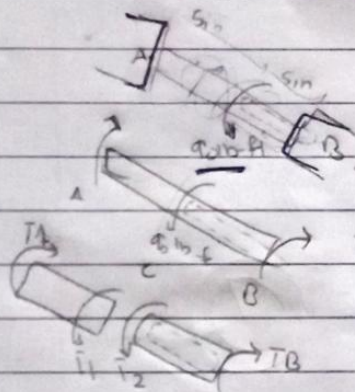
Recall that the angle of twist and maximum shearing strain are related

$$\gamma_{max} = \frac{C \phi}{L}$$

$$\gamma_{max} \tau_{max} = \frac{T C}{J G}$$

shear deformation $\phi = \int \frac{T L}{J G} dL$
modulus of shear rigidity

statically indeterminate shaft



$$\tau_A + \tau_B = 90$$

$$\phi = \phi_1 + \phi_2 = 0$$

$$\left(\begin{matrix} \phi_1, \phi_2 \end{matrix} \right)$$

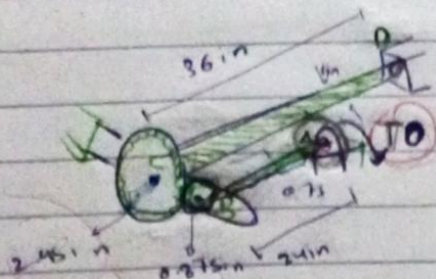
$$\phi_1 + \phi_2 = 0$$

$$\phi = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0 \quad T_B = \frac{L_1 T_A}{L_2 J_1}$$

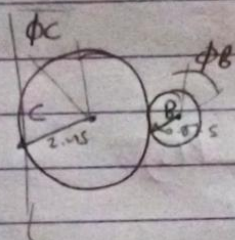
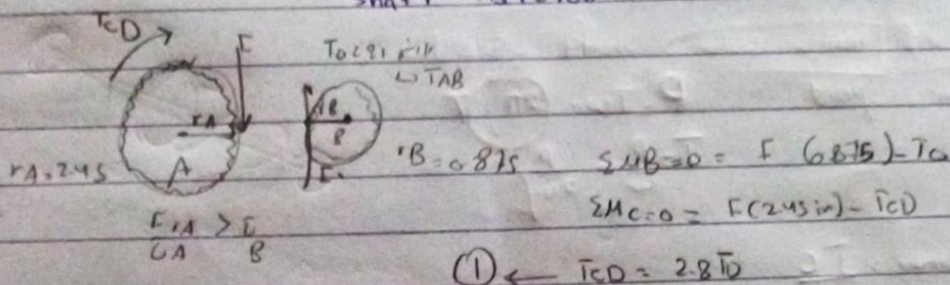
$$T_A + L_1 \frac{T_A}{L_2 J_1} = 90 \text{ lb.ft}$$

Sample Problem

3.4



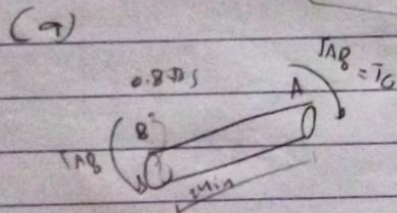
Two solid steel shafts are connected by gears, knowing that for each shaft $G = 11.2 \times 10^6$ psi and that the allowable shearing stress is 8 ksi, determine (a) the largest torque T_0 that may be applied to the end of shaft AB, (b) the corresponding angle through which end A of shaft AB rotates.



$$r_B \phi_B = r_A \phi_A$$

$$\phi_B = \frac{r_A}{r_B} \phi_A = \frac{2.45 \sin}{0.875} \phi_A$$

$$\phi_B = 2.8 \phi_A$$



$\tau_{max} = \tau_{ABC} = 8000 \text{ psi} = \frac{T_0(0.875)}{\frac{\pi}{32}(0.875)^4}$
 $T_0 = 56310 \text{ in}\cdot\text{lb}$

$\phi_{max} = \frac{T_0 L}{J G} = \frac{56310(24)}{\frac{\pi}{32}(0.875)^4 (11.2 \times 10^6)}$

$T_0 = 56110 \text{ in}\cdot\text{lb}$
 $\phi_B = 0.014 \text{ rad}$
 $\phi_A = 0.005 \text{ rad}$

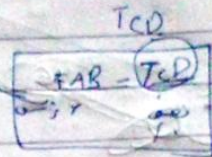
$$\phi_{AB} = \tau_{AB} L = \frac{(561) \text{ bin } (24 \text{ in})}{J_{ABG}} = 0.387 \text{ rad} = 2.22^\circ$$

$$J_{ABG} = \frac{\pi}{32} (0.875^4) (11.2 \times 10^6 \text{ psi})$$

$$\phi_{CD} = \frac{\tau_{CD} L}{J_{CDG}} = \frac{2.9 (561) (36 \text{ in})}{\frac{\pi}{32} (0.875^4) (11.2 \times 10^6 \text{ psi})} = 0.514 \text{ rad} = 2.95^\circ$$

$$\phi_B = 2.8 \phi_C = 2.8 (2.95^\circ) = 8.26^\circ$$

$$\phi_A = \phi_B + \phi_{AB} = 8.26 + 2.22 = 10.48^\circ$$



design of Transmission shaft conditions

Power, speed

$$P = T\omega = 2\pi fT$$

$$T = \frac{P}{\omega} = \frac{P}{2\pi f}$$

$$P = T\omega$$

$$P = 2\pi fT$$

$$\tau_{max} = \frac{TC}{J}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{T}{\tau_{max}} \quad (\text{solid shafts})$$

$$\frac{J}{C_2} = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{T}{\tau_{max}} \quad (\text{hollow shafts})$$

$$P = \frac{\tau_{max} J}{C} = \frac{\tau_{max} \pi C^3}{2}$$

$$\tau_{max} = \frac{TC}{J}$$

$$\tau_{max} = \frac{k T C}{J}$$

$$\tau = \frac{T r}{J}$$

$$\phi = \frac{T L}{G J}$$

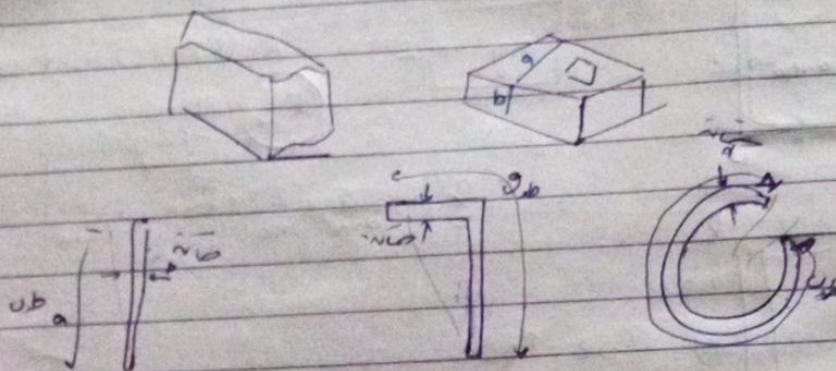
Part-3

stress concentrations

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau = k \frac{Tc}{J}$$

Torsion of noncircular Member

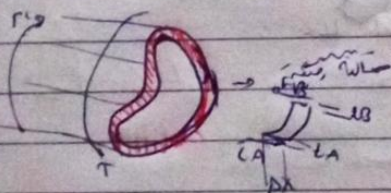


a/b

a/b	C ₁	C ₂
1	0.208	0.1406
1.2	0.219	0.1601
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.32	0.312
∞	0.333	0.333

$$\tau_{max} = \frac{T}{C_1 a b^2} < \phi_s = \frac{TL}{C_2 a b^3 G}$$

Thin walled Hollow shafts.



Summing forces in the x-direction on AB

$$\sum F_x = 0 = tA(\tau_A \Delta V) - tB(\tau_B \Delta V)$$

$$\tau_A tA = \tau_B tB = \tau t = q = \text{shear flow}$$

Shear stress varies inversely with thickness

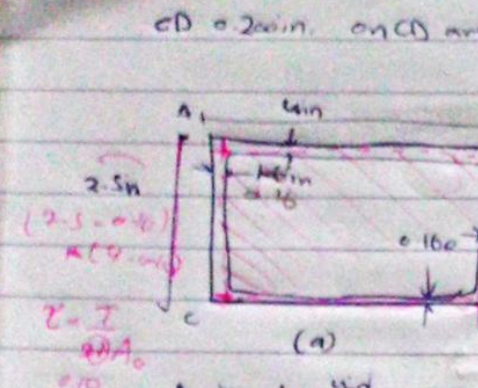
$$\phi = \frac{TL}{4A^2 G} \int \frac{ds}{t}$$

$$\tau = \frac{T}{2tA}$$

Max shear stress

example 3.10 // Extruded aluminum tubing with a rectangular cross section has a torque loading of 24 kip-in. Determine the shearing stress in each of the four walls with (a) uniform wall thickness of 0.16 in and wall thickness of (b) 0.12 in on AB and CD 0.20 in on AD and BD

shear $q = \frac{T}{2A}$

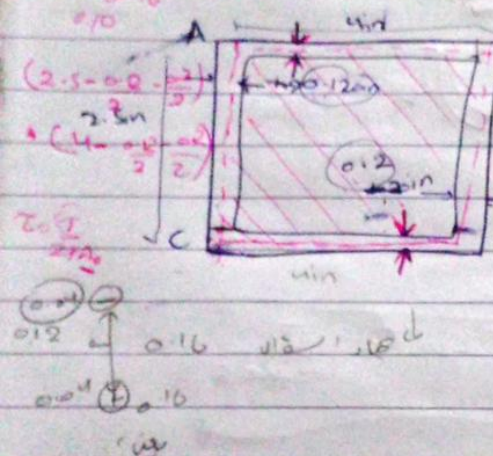
(a) 

$4 - 0.16 = 3.84$
 $2.5 - 0.16 = 2.34$

$A = (3.84)(2.34) = 8.986 \text{ in}^2$

$q = \frac{T}{2A} = \frac{24 \text{ kip-in}}{2(8.986) \text{ in}^2} = 1.335 \text{ ksi}$

$\tau = \frac{q}{t} = \frac{1.335}{0.16} = 8.34 \text{ ksi}$

(b) 

$4 - 0.12 = 3.88$
 $2.5 - 0.12 = 2.38$

$A = (3.88)(2.38) = 9.2544 \text{ in}^2$

$q = \frac{T}{2A} = \frac{24 \text{ kip-in}}{2(9.2544) \text{ in}^2} = 1.298 \text{ ksi}$

$\tau_{AB} = \tau_{CD} = \frac{q}{t} = \frac{1.298}{0.12} = 10.82 \text{ ksi}$

$\tau_{BD} = \tau_{AD} = \frac{q}{t} = \frac{1.298}{0.2} = 6.49 \text{ ksi}$

(c) $q = \frac{T}{2A}$

$$q = \frac{T}{2A}$$

$$\tau = \frac{T}{2A \cdot t}$$

$$\phi = \frac{TL}{GJ}$$

$$r_1 \phi = r_2 \phi_1$$

local shear flow

$$q = \frac{T}{2A_m}$$

A_m = mean area
 centerline

$$A_m = (4 - 0.16) \times (2.5 - 0.16)$$

$$(4 - 0.16) \times (2.5 - 0.16)$$

$t = 0.2 \text{ in}$ BD, CD

$$L_{median} = L - \frac{t_1}{2} - \frac{t_2}{2}$$

$$3.8 + 2.3 +$$

61

Review:

Collision

Stress in shaft due to
(T) torque

shear stress

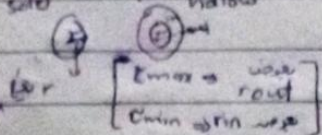
shear (T)

lets

r → shaft radius

$$\frac{\tau_{max}}{\tau_{min}} = \frac{r}{r_{min}}$$

solid hollow



J: Polar Moment of inertia

$$J_{solid} = \frac{\pi}{32} r^4$$

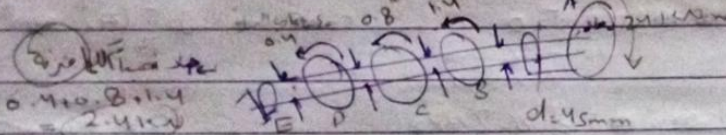
$$J_{hollow} = \frac{\pi}{32} (d_{out}^4 - d_{in}^4)$$

$$\tau = \frac{T \cdot r}{J}$$

$$J_{unit} = mm^4$$

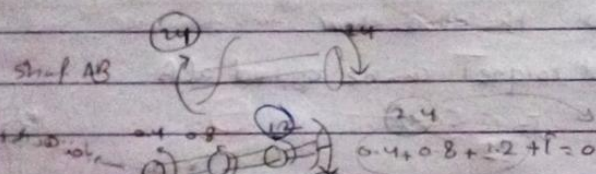
Q: under normal operation condition the electrical motor

2.4 kW, all shaft (solid) determine the max shear stress



$$\tau_{max} = \frac{T \cdot r}{J}$$

$$\tau_{max} = \frac{2.4 \times 10^3}{\frac{\pi}{32} (45)^4} = 17.62 \text{ MPa}$$



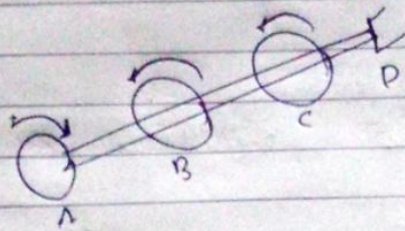
$$J_{solid} = \frac{\pi}{32} (27)^4 = 834.7857 \text{ mm}^4$$

$$\tau_{max} = \frac{T \cdot r}{J} = \frac{2.4 \times 10^3 \times 13.5}{834.7857} = 39.62 \text{ MPa}$$

$$J_{solid} = \frac{\pi}{32} (23)^4 = 277.62 \text{ mm}^4$$

$$\tau_{max} = \frac{T \cdot r}{J} = \frac{1.2 \times 10^3 \times 11.5}{277.62} = 25.9 \text{ MPa}$$

knowing that each of the AB and BC consist of solid cylindrical steel
determine the max shear stress occur



shaft AB $T = 148 \text{ N}\cdot\text{m}$ $J = 148 \times 10^3 \text{ N}\cdot\text{mm}$ $d = 15$ $r = 7.5 \text{ mm}$ $J = \frac{\pi (15)^4}{32}$

$$\tau_{\max} = \frac{T r}{J} = \frac{148 \times 10^3 \text{ N}\cdot\text{mm} \times 7.5 \text{ mm}}{\frac{\pi (15)^4}{32}} = 72.1 \text{ MPa}$$

shaft BC $T = 144 \text{ N}\cdot\text{m}$ $J = 144 \times 10^3 \text{ N}\cdot\text{mm}$ $d = 9$ $r = 4.5 \text{ mm}$ $J = \frac{\pi (9)^4}{32}$

$$\tau_{\max} = \frac{T r}{J} = \frac{144 \times 10^3 \text{ N}\cdot\text{mm} \times 4.5 \text{ mm}}{\frac{\pi (9)^4}{32}} = 83.8 \text{ MPa}$$

shaft CD $T = 156 \text{ N}\cdot\text{m}$ $J = 156 \times 10^3 \text{ N}\cdot\text{mm}$ $d_o = 21$ $d_i = 15$ $r_o = 10.5 \text{ mm}$ $J = \frac{\pi (21^4 - 15^4)}{32}$

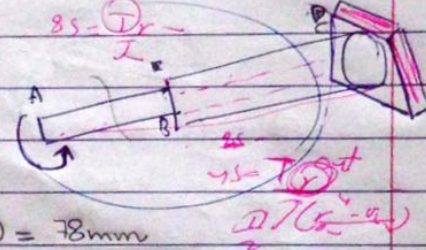
$$\tau_{\max} = \frac{T r_o}{J} = \frac{156 \times 10^3 \text{ N}\cdot\text{mm} \times 10.5 \text{ mm}}{\frac{\pi (21^4 - 15^4)}{32}} = 85.8 \text{ MPa}$$

Factor of safety in max field

max shear stress = shaft CD

problem (3-19) The rod AB diameter 60mm is made Prime steel for which the allow a shear stress 85MPa. shaft CD the out diameter 90mm and wall thickness of 6mm is made from Aluminum for which the allow shear inner diameter (outside diameter 2 thick) stress 45MPa. determine the largest torque T can be applied at A

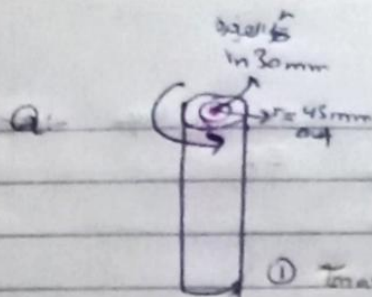
shaft AB = $\tau = 85 \text{ MPa}$ $r = 30 \text{ mm}$ $J = \frac{\pi (60)^4}{32}$



$$T = 3604977 \text{ N}\cdot\text{mm} = 3.61 \text{ kN}\cdot\text{m}$$

shaft BC = out $d_o = 90$ ind $d_i = 90 - 2(6) = 78 \text{ mm}$ $J = \frac{\pi (90^4 - 78^4)}{32}$ $r = 45$ $\tau = 45$

$$T = 3368762.5 \text{ N}\cdot\text{mm} = 3.37 \text{ kN}\cdot\text{m}$$



shift 45 T 30
hollow shaft
the max shear stress 45 MPa

$$\tau_{max} = T_{max} / J$$

$$45 = \frac{T}{\frac{\pi}{32} (45^4 - 30^4)}$$

$$T = 5.196 \times 10^3 \text{ Nm}$$

Simple

$$\tau = \frac{T}{J} (d_o^4 - d_i^4)$$

2) determine the max shear stress due T = 5.196 kNm
cross section area for hollow = solid

$$T = 5.196 \times 10^3 \text{ Nm} \Rightarrow 5.196 \times 10^6 \text{ Nmm}$$

(all in solid)

area
cross section

$$\text{Area hollow} = \text{Area solid}$$

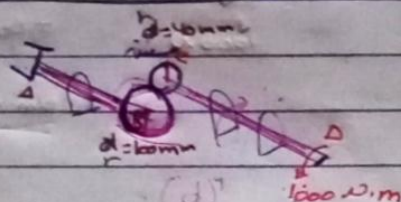
$$\frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} d^2$$

$$\frac{\pi}{4} (90^2 - 60^2) = \frac{\pi}{4} d^2$$

$$d = 67.08 \text{ mm}$$

$$J = \frac{\pi}{32} (d^4)$$

$$\tau_{max} = \frac{T}{J} = \frac{5.196 \times 10^6}{\frac{\pi}{32} (67.08^4)} = 87.2 \text{ MPa}$$



determine shear stress in AB, CD

$$d_{AB} = 20 \text{ mm}$$

$$d_{CD} = 10 \text{ mm}$$

shaft

$$CD: \tau_{max} = \frac{T}{J} = \frac{1000 \times 10^3}{\frac{\pi}{32} (10^4)} = 1599 \text{ MPa}$$

AB: - GSub CD = 1000 Nm

shear stress

shear stress

$$T_{CD} = 1000$$

$$\frac{T}{J} = \frac{T}{J}$$

$$\frac{1000}{40} = \frac{T_{AB}}{100}$$

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B}$$

$$T_{AB} = 2500 \text{ Nm}$$

shaft = 20

shear stress

$$CAB = \frac{2500 \times 10^3}{\frac{\pi}{32} (20^4)} = \text{MPa}$$

dis. ϕ



$$\tau_{BC} = 7 \text{ MPa}$$

$$d_{BC} = 16 \text{ mm}$$

$$d_{EF} = 8 \text{ mm}$$

$$\frac{\pi (8)^4}{32}$$

AC torque up

shaft BC

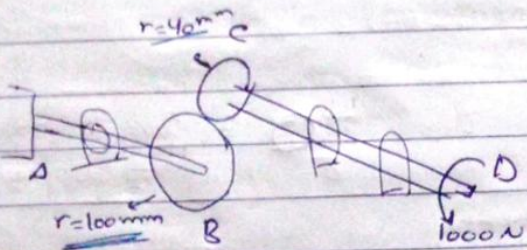
$$\tau = \frac{T r}{J} \rightarrow 7 \text{ MPa} = \frac{T (8)}{\frac{\pi (16)^4}{32}} \Rightarrow T = 5629.73 \text{ N}\cdot\text{mm}$$

shaft EF

$$\frac{T_{BC}}{r_{BC}} = \frac{T_{EF}}{r_{EF}}$$

$$\frac{5629.73}{8} = \frac{T_{EF}}{4} \Rightarrow T_{EF} = 22518.92 \text{ N}\cdot\text{mm}$$

Problem (3.21)



$$T_{CD} = 1000 \text{ N}$$

shear stress of shaft is 60 MPa

determine shaft CD, AB

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B}$$

$$\frac{1000}{40} = \frac{T_{AB}}{100} \Rightarrow T_{AB} = 2500 \text{ N}\cdot\text{mm}$$

CD

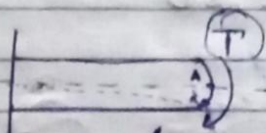
$$\tau = \frac{T r}{J} \Rightarrow 60 = \frac{1000 \times r}{\frac{\pi (r)^4}{32}} \Rightarrow r_{CD} = 40.2 \text{ mm}$$

AB

$$60 = \frac{2500 \times r}{\frac{\pi (r)^4}{32}} \Rightarrow r_{AB} = 40.2 \text{ mm}$$

40.2

Part 2 - Review T is Elong deformation $\rightarrow$ twist



deformation angle = ϕ ang of twist

static $\phi = \frac{T L}{G J} \text{ rad} \rightarrow \text{deg by } \frac{180}{\pi}$

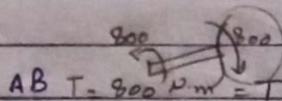
moduli - rigid pattern GJ

Q: an aluminum rod AB $G = 27 \text{ GPa}$ is bonded to the brass BD $G = 39 \text{ GPa}$. knowing that CD hollow out = 60 ind = 40 mm

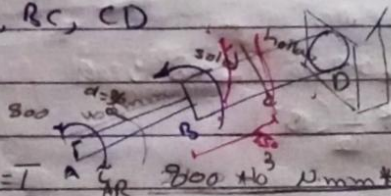
① T_{AB}, BC, CD

③ angle of twist at A, B, C, D

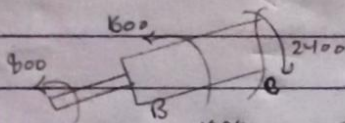
⑤ shear stress AB, BC, CD



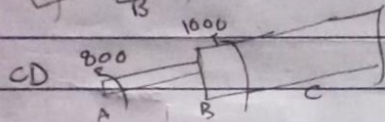
AB $T = 800 \text{ N.m} = T$



BC = $800 + 1600 = 2400 = T$ $\frac{800 + 1600}{\pi (30)^2} \text{ N.m} = \text{[]}$



$T_{BC} = \frac{2400 \times 10^3 \text{ N.m} \times 30 \text{ mm}}{\pi (60)^4}$ $\leftarrow \text{out (max)}$



$T_{CD} = \frac{2400 \times 10^3 \times (30 \text{ mm})}{\pi (60^4 - 40^4)}$

$800 \times 1600 = 2400 \text{ T N.m}$

③ Angle AB $\phi = \frac{T L}{G J} = \frac{T_{AB} L_{AB}}{J G} = \frac{800 \times 10^3 \times 400 \text{ mm}}{\frac{\pi}{32} (30)^4 \times 27 \times 10^3} = 0.0719 \text{ rad}$ 4.06°

$\phi_{BC} = \frac{T_{BC} L_{BC}}{G J} = \frac{2400 \times 10^3 \text{ N.m} \times 375 \text{ mm}}{\frac{\pi}{32} (60)^4 \times 39 \times 10^3 \text{ Pa}} = 0.081 \text{ rad}$ 4.64°

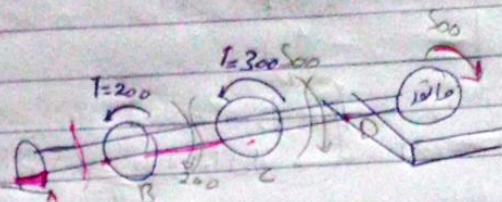
$\phi_{CD} = \frac{T_{CD} L_{CD}}{G J} = \frac{2400 \times 10^3 \times 250}{\frac{\pi}{32} (60^4 - 40^4) \times 39 \times 10^3} = 0.015 \text{ rad}$ 2.9°

$\phi_{\text{total}} = \phi_{AB} + \phi_{BC} + \phi_{CD} = \text{rad}$ angle twist at A

$\theta_{BC}, \theta_{CD} \quad \epsilon_{\theta} \quad \theta_B \quad \theta_C$

the directly motor $T = 500 \text{ Nm}$ $G = 27 \text{ GPa}$
 determine the angle of twist BC, BD

$$\phi = \frac{TL}{GJ}$$



$$T_{AB} = 0$$

$$T_{BC} = 200 \times 10^3$$

$$T_{CD} = 500 \text{ Nm}$$

$$\phi_{AB} = 0$$

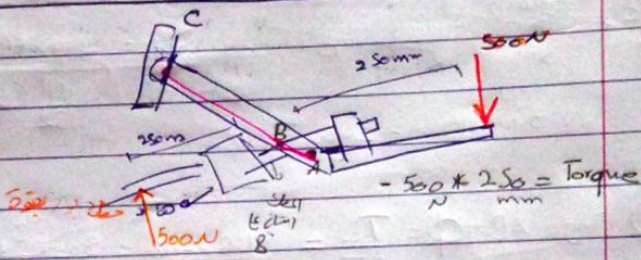
$$\phi_{BC} = \frac{T_{BC} \cdot L_{BC}}{GJ} = \frac{200 \times 10^3 \times 2}{27 \times 10^3 \times \frac{\pi}{32} (44^4)} = 0.0016 \text{ rad}$$

0.0016 rad
 0.09 degree

$$\phi_{CD} = \frac{T_{CD} \cdot L_{CD}}{GJ} = \frac{500 \times 10^3 \times 2}{27 \times 10^3 \times \frac{\pi}{32} (44^4)} = 0.008 \text{ rad}$$

$$\phi_{BD} = \phi_{BC} + \phi_{CD} = 0.0096 \text{ rad} = 0.547 \text{ degree}$$

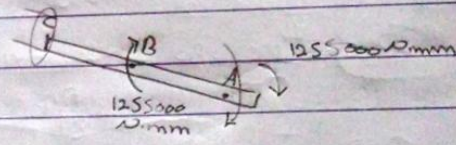
max shear stress in the shaft
 Torque = 500 Nm



- max shear stress in the shaft
 max shear stress AB, BC

hollow = outd. 60 in 30

$$\tau_{AB} = \frac{T_{AB} \cdot r}{J} = \frac{125000 \times 30}{\frac{\pi}{32} (60^4 - 30^4)} = 314 \text{ MPa}$$



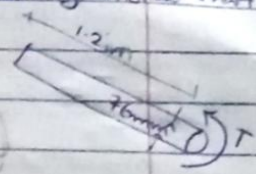
$$T = 125000 + 125000 = 250000 \text{ Nm}$$

$$\tau_{BC} = \frac{T \cdot r}{J} = \frac{250000 \times 30}{\frac{\pi}{32} (60^4 - 30^4)} = 6.287 \text{ MPa}$$

$G = 70 \text{ GPa}$ angle of twist

$$\phi = \frac{TL}{GJ}$$

- 3.1 (a) determine the maximum shearing stress caused by a 46 kN·m torque T in the 76mm diameter solid aluminum shaft shown.
- (b) solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same ^{out} diameter and 24mm inner diameter



$$\tau_{max} = \frac{T \cdot r}{J} = \frac{46 \times 10^6 \text{ N}\cdot\text{mm} \cdot 38 \text{ mm}}{\frac{\pi}{32} (76)^4} = 533.68 \text{ MPa}$$

(2) d_{out} (76mm) d_{in} (24mm)

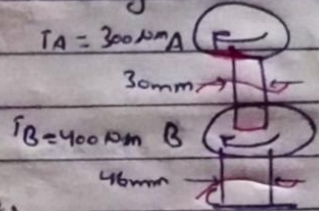
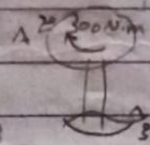
$$\tau_{max} = \frac{T \cdot r_{max}}{J} = \frac{46 \times 10^6 \text{ N}\cdot\text{mm} \cdot 38 \text{ mm}}{\frac{\pi}{32} (76^4 - 24^4)} = 539.04 \text{ MPa}$$

3.9 - The torques shown are exerted on pulleys A and B. knowing that both shafts are solid, determine the maximum shearing stress in (a) in shaft AB

(a) shaft BC.

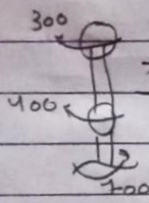
(1) $T_{internal}$

$$= 300 \text{ N}\cdot\text{m}$$



$$\tau_{AB} = \frac{T \cdot r}{J} = \frac{300 \times 10^3 \text{ N}\cdot\text{mm} \cdot 15 \text{ mm}}{\frac{\pi}{32} (30)^4} = 56.58 \text{ MPa}$$

$$\tau_{BC} = \frac{700 \times 10^3 \text{ N}\cdot\text{mm} \cdot 23 \text{ mm}}{\frac{\pi}{32} (46)^4} = 36.62 \text{ MPa}$$



3.23: under normal operating conditions a motor exerts a torque of magnitude $T_F = 1200 \text{ lb}\cdot\text{in}$ at F. knowing that $r_D = 8 \text{ in}$, $r_G = 3 \text{ in}$ and the allowable shearing stress is 10.5 ksi in each shaft, determine the required diameter of shaft CDE, FGH

$$\frac{\tau_F}{r_G} = \frac{\tau_F}{r_D} \quad \tau = 10.5 = \frac{T}{J} \quad \frac{1200}{8} = \frac{T}{3}$$

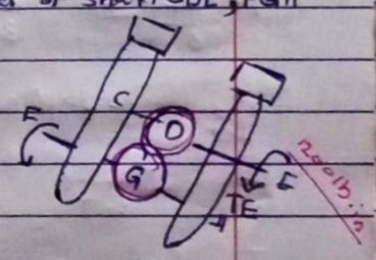
$$\frac{1200 \text{ lb}\cdot\text{in}}{3 \text{ in}} = \frac{T_F}{8 \text{ in}} \Rightarrow T_F = 3200 \text{ lb}\cdot\text{in}$$

$$\tau_{FGH} = 10.5 = \frac{T_F}{J}$$

$$\tau_{CDE} = \frac{T_F \cdot r}{J} \quad 10.5 \times 10^3 \text{ psi} = \frac{3200 \cdot r}{\frac{\pi}{2} (r)^4}$$

$$r = 1.72 \quad d = 3.45 \text{ in}$$

$$10.5 \times 10^3 \text{ psi} = \frac{1200 \text{ lb}\cdot\text{in} \cdot r}{\frac{\pi}{2} (r)^4} \quad r = 2.39 \text{ in} \quad d = 4.78 \text{ in}$$



- 3.31. For solid steel shaft shown ($G = 77 \text{ GPa}$), determine the angle of twist at A.
 (b) solve part a, assuming that the steel shaft is hollow with a 30mm outer diameter in 2mm diameter

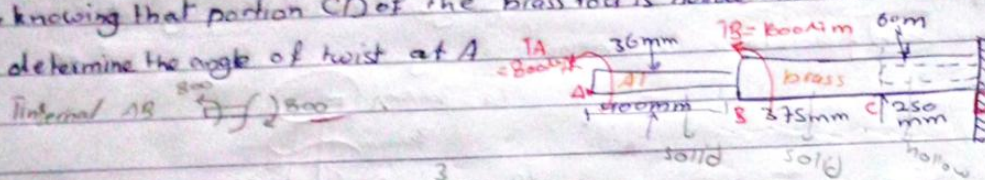
$$\phi = \frac{TL}{GJ} = \frac{250 \times 10^3 \text{ N} \cdot \text{mm} + 1.8 \times 10^3 \text{ N} \cdot \text{mm}}{77 \times 10^3 \text{ MPa} \times \frac{\pi (30)^4}{32}}$$

$\phi = 0.07 \text{ rad}$ 4.2°

$$\phi = \frac{250 \times 10^3 + 1.8 \times 10^3}{77 \times 10^3 \times \frac{\pi (30^4 - 20^4)}{32}} = 0.09 \text{ rad}$$

5.24°

- 3.39. The aluminum rod AB ($G = 27 \text{ GPa}$) is bonded to the brass rod BD ($G = 39 \text{ GPa}$). knowing that portion CD of the brass rod is hollow and has inner $d = 40 \text{ mm}$, determine the angle of twist at A.



$$\phi_{AB} = \frac{T_{AB} L_{AB}}{GJ} = \frac{800 \times 10^3 \text{ N} \cdot \text{mm} \times 1600}{27 \times 10^3 \text{ MPa} \times \frac{\pi (36)^4}{32}} = 0.071 \text{ rad}$$

4.0°

$$\phi_{BC} = \frac{T_{BC} L_{BC}}{GJ} = \frac{2400 \times 10^3 \text{ N} \cdot \text{mm} \times 375}{39 \times 10^3 \text{ MPa} \times \frac{\pi (36)^4}{32}} = 0.018 \text{ rad}$$

$$\phi_{CD} = \frac{T_{CD} L_{CD}}{GJ} = \frac{2400 \times 10^3 \text{ N} \cdot \text{mm} \times 2500}{39 \times 10^3 \text{ MPa} \times \frac{\pi (36^4 - 40^4)}{32}} = 0.015 \text{ rad}$$

$$\phi_A = \phi_{AB} + \phi_{BC} + \phi_{CD} = 0.071 + 0.018 + 0.015 = 0.104 \text{ rad}$$

5.9°

3.42: Two solid shafts are connected by gears as shown - knowing that $G = 77.2 \text{ GPa}$ for each shaft, determine the angle through shaft which end A rotates $T_A = 1200 \text{ N.m}$

$$\tau_C \phi_C = \tau_B \phi_B$$

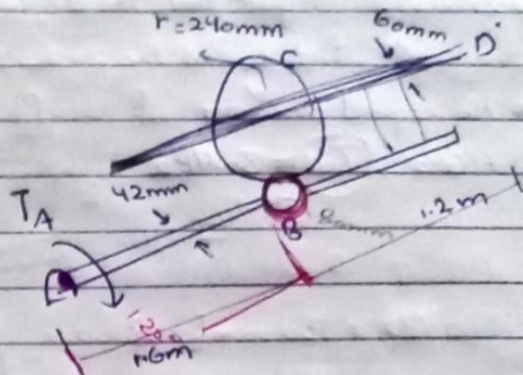
$$240 \text{ mm } \phi_C = 80 \text{ mm } \phi_B$$

$$\phi_B = 3 \phi_C$$

$$\frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C}$$

$$= \frac{1200 \times 10^3 \text{ N.m}}{80 \text{ mm}} = \frac{T_{CD}}{240 \text{ mm}}$$

$$T_{CD} = 3600 \text{ N.m}$$



$$\phi_{AB} = \frac{T_{AB}}{GJ} L_{AB} = \frac{1200 \times 10^3 \times 1.0 \times 10^3}{77.2 \times 10^9 \times \frac{\pi}{32} (42)^4} = 0.081 \text{ rad}$$

$$\phi_{DC} = \frac{T_{DC}}{GJ} L_{DC} = \frac{3600 \times 10^3 \times 1.0 \times 1.2}{77.2 \times 10^9 \times \frac{\pi}{32} (60)^4} = 0.043 \text{ rad}$$

$$\phi_B = 3 \phi_C = 3 \times 0.043 = 0.13 \text{ rad}$$

$$\phi_A = \phi_B + \phi_{AB} = 0.081 + 0.13 = 0.211 \text{ rad} \quad \boxed{12.08 \text{ rad}}$$

3.47:- $L = 2 \text{ m}$ solid circular $\phi = 3^\circ$ at $T = 9 \text{ kN.m}$ rad $\frac{180}{\pi} \text{ deg}$

allow shear stress 90 MPa $G, 77 \text{ GPa}$

allow shear stress 35 MPa $G, 42 \text{ GPa}$

$$\phi = \frac{T L}{GJ} \quad 0.052 = \frac{9 \times 10^6 \times 2 \times 10^3}{77 \times 10^9 \times \frac{\pi}{32} d^4}$$

1 rad $\approx 57.3 \text{ deg}$ $\frac{180}{\pi}$

$$\frac{\pi \text{ rad}}{180} = 180 \text{ deg} \quad 3^\circ \times \frac{\pi}{180} = 0.052 \text{ rad}$$

$$d = 82.2 \approx 83 \text{ mm}$$

$$\tau = \frac{T r}{J} \quad 90 = \frac{9 \times 10^6 \times r}{\frac{\pi}{32} (d)^4} \quad d = 39.92 \approx 40 \text{ mm}$$

83 is ok

bronze

$$0.052 = \frac{9 \times 10^6 \times 2 \times 10^3}{42 \times 10^9 \times \frac{\pi}{32} (d)^4}$$

$$d = 95.4 \approx 96 \text{ mm}$$

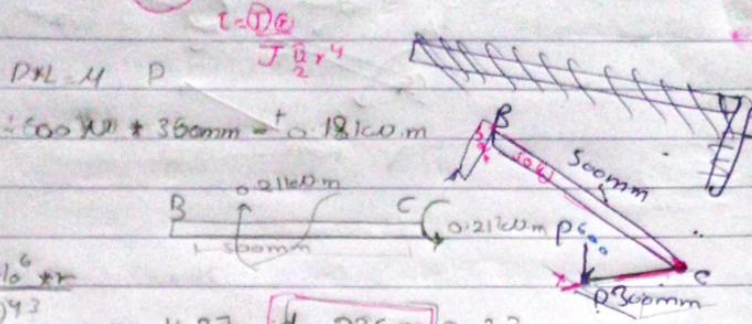
$$35 = \frac{9 \times 10^6 \times r}{\frac{\pi}{32} (r)^4} = 54.7 \text{ mm}$$

$$\phi = \frac{T L}{GJ} \quad 0.052 = \frac{9 \times 10^6 \times 2 \times 10^3}{77 \times 10^9 \times \frac{\pi}{32} d^4}$$

$$\delta = L \phi$$

$$15 \text{ mm} = 3 \phi$$

3.48: A hole is punched at A plastic sheet by apply force P to end D of lever CD, which is rigidly attached to the solid cylindrical shaft BC. Design specifications require that the displacement of D should not exceed 15mm. From the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77 \text{ GPa}$, $\tau_{all} = 80 \text{ MPa}$.



$$\tau = \frac{T r}{J}$$

$$80 = \frac{0.18 \times 10^6 \times r}{\frac{\pi}{32} (r)^4}$$

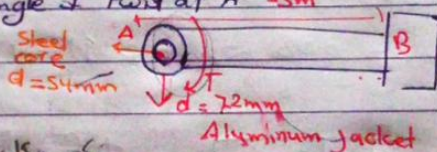
$$r = 11.27$$

$$d = 22.54 \text{ mm} \approx 23$$

$$\delta D = 15 \text{ mm}$$

$$PL = \delta AE$$

3.51: A torque of magnitude $T = 4 \text{ kNm}$ is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for steel and 27 GPa for aluminum, determine (a) maximum shearing stress in steel core, (b) the max shear stress in the aluminum jacket, (c) the angle of twist at A.



compatibility

الزاوية في الجزء الفولاذي = الزاوية في الجزء الألومنيوم

$$T = 4 \times 10^6 \text{ Nm}$$

$$4 \times 10^6 = T_s + T_a$$

$$\phi_s = \phi_a$$

$$\frac{T_s L_s}{G_s J_s} = \frac{T_a L_a}{G_a J_a}$$

$$\frac{T_s}{G_s J_s} = \frac{T_a}{G_a J_a}$$

$$\frac{T_s}{77 \times 10^9 \text{ Pa} \times \frac{\pi}{32} (54)^4} = \frac{(4 \times 10^6 - T_s)}{27 \times 10^9 \text{ Pa} \times \frac{\pi}{32} (72)^4}$$

$$27 \times (72^4) T_s = 77 \times (54^4) (4 \times 10^6 - T_s)$$

$$\frac{256 T_s}{231} = 4 \times 10^6 - T_s \quad 2.108 T_s = 4 \times 10^6$$

$$T_s = 1.89 \times 10^6 \text{ Nm}$$

$$1160 \text{ mm} \cdot \tau_s + T_{ol}$$

$$4 = 1.89 \cdot T_{ol}$$

$$T_{ol} = 2160 \text{ mm}$$

$$\text{shear stress at steel } \tau_{max} = \frac{T \cdot r}{J} = \frac{1.89 \cdot 10^6 \cdot 27 \text{ mm}}{\frac{\pi (54)^4}{32}} = 61.1 \text{ MPa}$$

$$\tau_{al} = \frac{2.16 \cdot 10^6 \cdot 36}{\frac{\pi (72)^4}{32}} = 28.65 \text{ MPa}$$

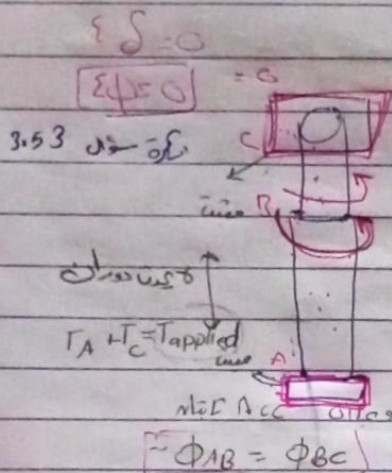
$$\text{angle of twist } \phi_A = \frac{T \cdot L}{GJ} = \frac{1.89 \cdot 10^6 \cdot 25 \cdot 10^3 \text{ mm}}{77 \cdot 10^9 \cdot \frac{\pi (54)^4}{32}} = 0.00073 \text{ rad}$$

angle of twist of steel is constant

$$\phi_B = \frac{T \cdot L}{GJ} = \frac{1.89 \cdot 10^6 \cdot 25 \cdot 10^3 \text{ mm}}{77 \cdot 10^9 \cdot \frac{\pi (54)^4}{32}} = 0.00073 \text{ rad}$$

$$\phi_{ol} = \frac{T_{ol} \cdot L}{GJ} = \frac{2.16 \cdot 10^6 \cdot 25 \cdot 10^3 \text{ mm}}{77 \cdot 10^9 \cdot \frac{\pi (72)^4}{32}} = 0.00073 \text{ rad}$$

$$0.042^\circ$$



$$\phi_1 - \phi_2 = 15^\circ \pm \frac{\pi}{180}$$

$$\phi = \frac{T \cdot L}{GJ}$$

$$\phi = 15 \text{ mm} = 0.05 \text{ rad}$$

$$0.05 = 15 \cdot 10^{-3} \cdot \frac{\pi}{180}$$

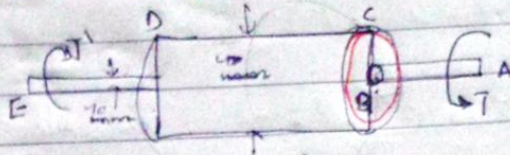
$$\phi = \frac{T \cdot L}{GJ} \quad J = \frac{T \cdot L}{G \phi} = \frac{\pi d^4}{32} = \frac{180 \cdot 10^3}{77 \cdot 10^9 \cdot 0.05}$$

$$d = 22.08$$

$$d = 22.5$$

$$\delta_{DC} = \delta_{DE} + \delta_{CE}$$

3.59. The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of right flange welded to the jacket and to the rod. The outer diameter of the jacket is 80mm and its wall thickness is 4mm. If 500 N.m torque are applied as shown, determine the max shear stress in the jacket



دستگاه را می بینیم. یک شفت 40 میلی و یک جکیت 80 میلی

در جکیت یک شفت 40 میلی و یک جکیت 80 میلی
در شفت یک شفت 40 میلی و یک جکیت 80 میلی
EA شفت و DC جکیت

$$\phi_{shaft}(CD) = \phi_{jacket}(CD) \quad \text{یک}$$

CD شفت و جکیت

$$T = 500 \text{ N.m} = T_1(CD) \text{ بر شفت} + T_2 \text{ CD jacket}$$

دش شفت = 40mm

dj = 80mm

$$\phi_s = \phi_j \quad (cd)$$

$$\frac{T_s L}{J_s G} = \frac{T_j L}{J_j G} \quad \left| \frac{T_s}{J_s} = \frac{T_j}{J_j} \right| \quad (1)$$

$$500 = T_s + T_j \rightarrow (2)$$

$$\frac{T_s}{J_s} = \frac{500 - T_j}{J_j}$$

$$J_j T_s = J_s (500 - T_j)$$

$$J_j T_s + J_s T_j = J_s (500)$$

$$T_s (J_j + J_s) = 500 J_s$$

$$T_s \left(\frac{\pi}{32} (80)^4 + \frac{\pi}{32} (40)^4 \right) = 500 * \frac{\pi}{32} (40)^4$$

$$T_s = 20411.76 \text{ N.mm}$$

$$29.4 \text{ N.m}$$

$$T_j = 470.58 \text{ N.m}$$

$$\tau_{\text{jacket}} = \tau_{\text{max}} = \frac{T_j \cdot r}{J} = \frac{4222 \pm 0.04}{1.385 \times 10^{-6}} = 12.22 \pm 0.04 \text{ Pa}$$

(3) 50-9
(2) 121-12

2.64. Determine the maximum shearing stress in a solid shaft of 12 mm diameter as it transmits 2.5 kw at frequency of (a) 25 Hz, (b) 50 Hz

$$2.9 \text{ kW}$$

$$P = 2\pi f T$$

$$P_{\text{shaft}} = 2\pi \text{ Hz } 10 \text{ m}$$

$$\text{kw } \frac{10}{10} \text{ W}$$

$$P = 2\pi f T$$

$$T = \frac{P}{2\pi f}$$

$$2.5 \pm 10^3 = 2\pi \cdot 25 \cdot T$$

$$\text{Frequency } 12 = \text{RPM } 60$$

$$T = 15.9 \text{ N.m}$$

$$\tau = \frac{15.9 \cdot 6}{\frac{\pi (12)^4}{32}} = 468 \text{ MPa}$$

$$P = 2\pi f T$$

$$\tau = \frac{T r}{J}$$

$$P_{\text{shaft}} = 2\pi \text{ Hz } 10 \text{ m}$$

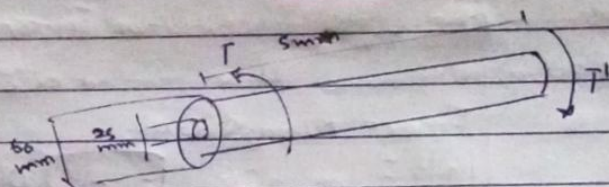
$$2.5 \pm 10^3 = 2\pi (50) T$$

$$T = 7.95 \text{ N.m}$$

$$\tau = \frac{7.95 \cdot 6}{\frac{\pi (12)^4}{32}} = 23.48 \text{ MPa}$$

3.70. The hollow steel shaft shown ($G = 77.2 \text{ GPa}$, $\tau_{\text{all}} = 50 \text{ MPa}$) rotates at 240 rpm. Determine (a) the maximum power that can be transmitted, (b) the corresponding angle of twist of the shaft

$$\frac{240 \text{ rpm}}{60} = 4 \text{ Hz}$$



$$\tau_{\text{max}} = 50 = \frac{T r}{J}$$

$$50 = \frac{T \cdot 30 \text{ mm}}{\frac{\pi (60^4 - 25^4)}{32}}$$

$$T = 2705 \text{ N.m}$$

$$2.05 \pm 10^6 \text{ W.m}$$

$$P = 2\pi f T = 2\pi (4) (2705) \pm 10^3 \text{ (W.m)}$$

$$P = 12880.5 \text{ watt}$$

$$12.8 \text{ kW}$$

$$\phi = \frac{T L}{G J} = \frac{2705 \cdot 5}{77.2 \pm 10^9 \text{ MPa} \cdot \frac{\pi (60^4 - 25^4)}{32}} = 0.10 \text{ rad}$$

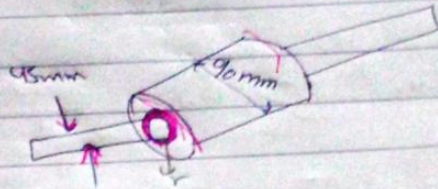
$$G = 16$$

$$P = 2\pi f T$$

$$50 = \frac{T \cdot 30}{\frac{\pi (60^4 - 25^4)}{32}}$$

$$P = 2\pi f T$$

3.86:- The stepped shaft shown must transmit 40 kW at a speed of 720 rpm. Determine the minimum r of the fillet if an allowable stress of 36 MPa is not to be exceeded.



$$P = 40 \times 10^3 \text{ watt}$$

$$f = \frac{720}{60} = 12 \text{ Hz}$$

$$40 \times 10^3 = T \cdot 2\pi f \cdot 0.09 \text{ m}$$

$$\omega = 2\pi f = 2\pi (12) = 75.4 \text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{40000}{75.4} = 530.52 \text{ N}\cdot\text{m}$$

(2) minimum shear stress $\tau_{nom} = \frac{T_c}{J} = \frac{530.52 \times 10^3}{\frac{\pi}{32} (45)^4} = 29.65 \text{ MPa}$
for shaft

(3) determine the stress concentration Factor (K)

the actual maximum τ_{max} must not exceed 36 MPa

$$\tau_{max} = K \tau_{nom} < 36 \text{ MPa}$$

$$K = \frac{\tau_{allow}}{\tau_{nom}} = \frac{36}{29.65} = 1.214$$

(4) Find the minimum radius (r)

$$D = 90 \quad d = 45 \quad \frac{D}{d} = \frac{90}{45} = 2$$

Using $\frac{D}{d} = 2$ $K = 1.21$

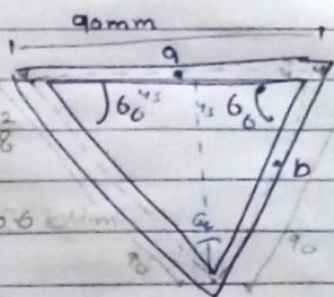
$$\frac{r}{d} = 0.1 \quad r = 0.1 \times 45 = 4.5 \text{ mm}$$

$$\frac{r}{d} = 0.1$$

$$\frac{r}{d} = 0.1, K = 1.2$$

$$\frac{r}{d} = 0.1$$

3.139. Torque $T = 750 \text{ N}\cdot\text{m}$ is applied to the hollow shaft shown that has uniform 8mm wall thickness. Neglecting the effect of stress concentration, determine the shearing stress at point a, and b



$$J_{max} = \frac{T}{C_{10} \theta} = \frac{750 \times 10^3 \text{ N}\cdot\text{mm}}{0.208 \times 90 \text{ mm} \times 8}$$

$$a/b = \frac{90}{90} = 1 \quad C_1 = 0.208 \quad C_2 = 0.1400$$

$$L_m = L_{out} - (b \times \sqrt{3}) \quad \text{with}$$

$$90 - (8 \times \sqrt{3}) = 76.14$$

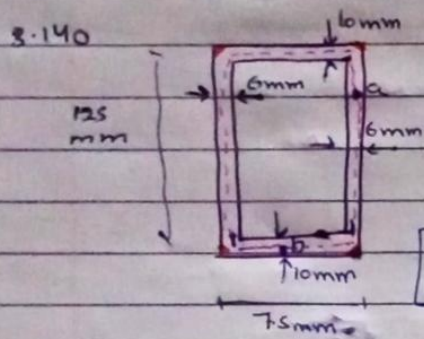
$$\text{area} = \sqrt{3} \times (\text{side})^2$$

$$= 0.433 \times (76.14)^2 = 2510 \text{ mm}^2$$

$$\tau = \frac{T}{2A_m t} = \frac{750 \times 10^3}{2(2510)(8)} = 18.675 \text{ MPa}$$

$$\tau = \frac{T}{2A_m t}$$

$$\frac{\sqrt{3}}{4} L^2$$



$T = 5 \text{ kN}\cdot\text{m}$
 $5 \times 10^3 \text{ N}\cdot\text{mm}$
hollow shaft

$$r_{avg} = \text{centroidal radius (or } r)$$

$$r_{avg} = \frac{75 - 6}{2} = 34.5 \text{ mm}$$

$$r_{avg} = \frac{125 - 10}{2} = 57.5 \text{ mm}$$

$$\text{area} = 69 \times 115 = 7.935 \text{ mm}^2$$

$$\tau_a = \frac{T}{2A_m t} = \frac{5 \times 10^3}{2 \times 7.935 \times 10} = 31.5 \text{ MPa}$$

$$\tau_b = \frac{5 \times 10^3}{2 \times 7.935 \times 10} = 31.5 \text{ MPa}$$

$$75 \times 125 - (10 \times 6 \times 2.5)$$

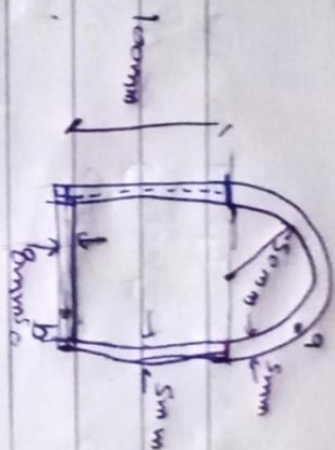
$$2760 \text{ mm}^2$$

$$(125 \times 75) - (69 \times 115)$$

3.142

I = 5610

50 mm



$$100 - 5 = 95 \text{ mm}$$

$$47.5 \text{ mm} \quad 50 - 2.5 = 47.5 \text{ mm}$$

$$A_m = (95 \times 100) + \frac{1}{2} \pi (47.5)^2$$

$$A_m = 130.486$$

$$C_g = \frac{I}{A_m} = \frac{5610}{130.486} = 42.944 \text{ Pa}$$

$$\frac{t_b}{2A_m} = \frac{5.6 \times 10^{-6}}{2 \times 130.486} = 2.684 \times 10^{-8}$$

- shear flow : Review - shear Torsion of non circular section

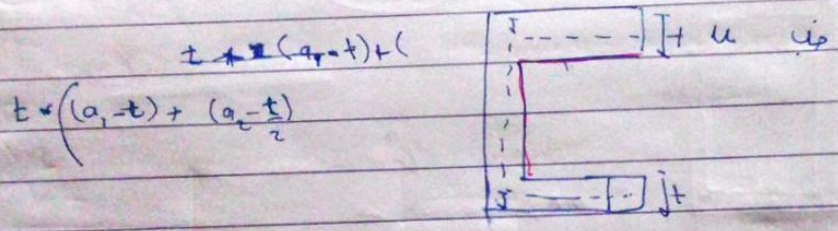
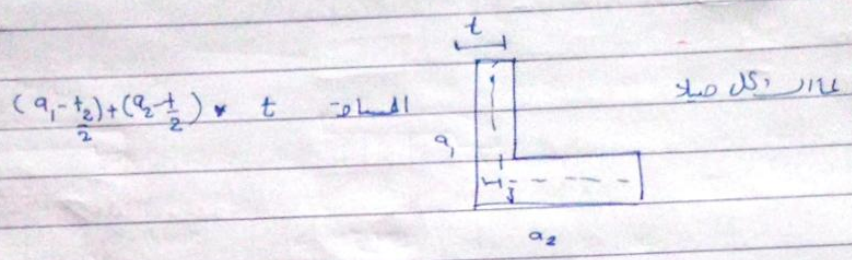
$T = \frac{M_x r}{J}$ *توزيع الجهد*

$\tau = \frac{M_x}{bc^2}$ *القانونية*

b, c أبعاد المقطع G البعد الصغير b البعد الكبير

الانحراف $\phi = \frac{M_x L}{\beta bc^3 G}$

$\frac{b}{c}$ β تتابع دالة على



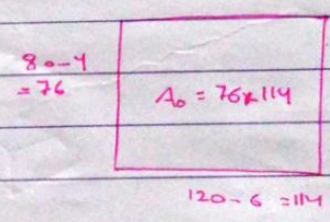
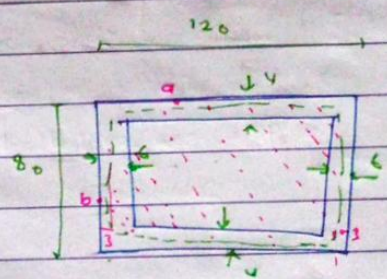
$\tau = \frac{T}{2A_0 t}$

$\tau = \frac{T}{2A_0 t}$

A_0 are enclosed by

$\tau = \frac{T}{2A_0 t}$

مساحة $a = 4mm$
 $b = 6mm$



Shear flow $(q) = \tau t = \frac{T}{2A_0}$ (const)

$$\tilde{I} = 40$$

$$\tilde{I} \approx 1$$

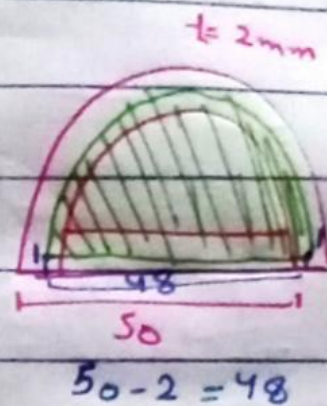
$$\tilde{I} = \frac{I}{2 + A_m}$$

$$\tilde{I} > \tilde{I}_{\text{req}}$$

$$A_m = \frac{1}{2} \pi r^2$$

$$\frac{1}{2} \pi (24)^2 = 904$$

$$40 = \frac{I}{2 + 2}$$



$$50 - 2 = 48$$

$$q = \tilde{I} = \frac{I}{2 + A_m}$$

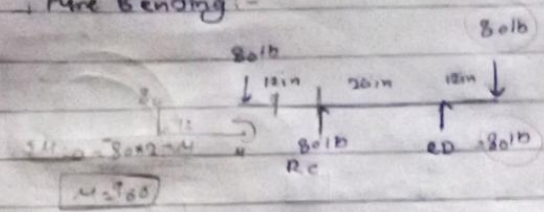
$$d > 20$$

$$\frac{50}{2}$$

$$25 > 20$$

$$[OK]$$

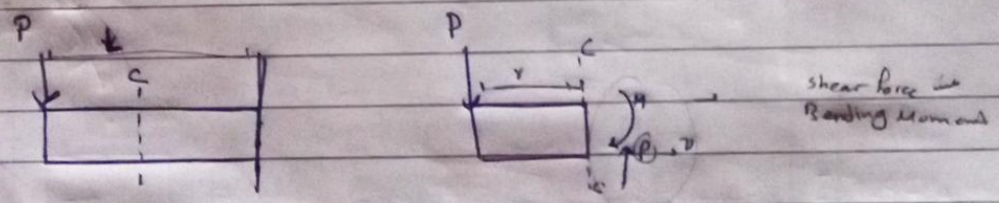
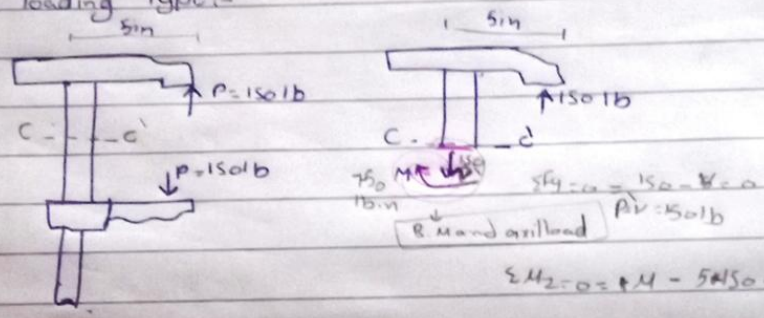
Chapter 4 - Pure Bending:-



قوة 80lb
 member length 50m
 $A = 960 \text{ lbm}$ $M = 960 \text{ lbm}$

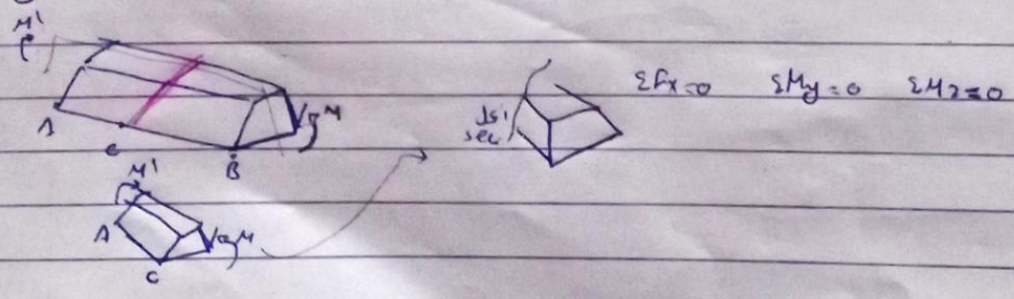
M ↓ ↓

- the loading type:-

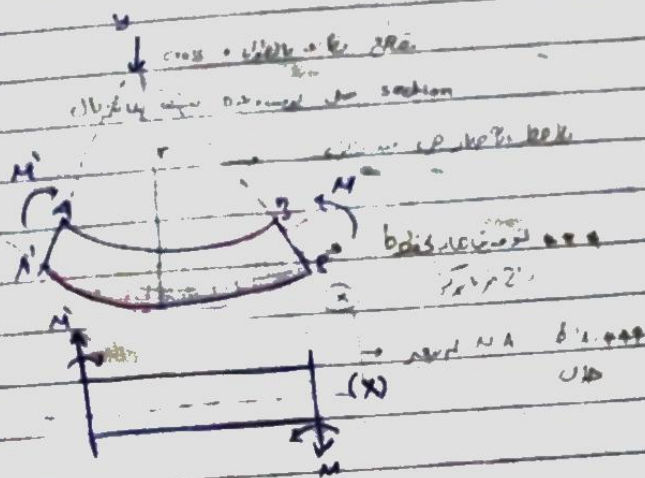


normal force shear force B.M. shear force & Bending moment

- Symmetric Member in Pure Bending



1. Bending deformation



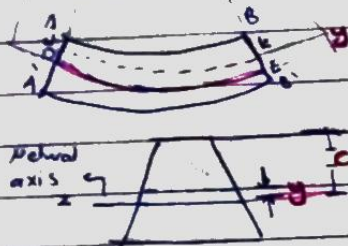
strain Due to Bending



$$s = r\theta = (r-y)\theta$$

$$s = L\theta = (r-y)\theta \quad \theta = \frac{s}{L} = \frac{y\theta}{r}$$

$$\epsilon_x = \frac{s}{L} = \frac{y\theta}{r} = -\frac{y}{r} \quad (\text{strain varies linearly})$$



$$\epsilon_{max} = \frac{c}{r} \quad \text{or} \quad \frac{c}{\epsilon_m}$$

$$\epsilon_x = -\frac{y}{c} \epsilon_{max}$$

strain = 0 at NA

$y=0$ at NA

NA is at the center of the beam

stress Due to Bending:

$$\sigma_{max} = \frac{Mc}{I} = \frac{M}{S}$$

$$\sigma_x = -\frac{y}{c} \sigma_{max}$$

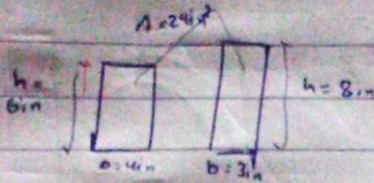
$$\sigma_x = -\frac{My}{I}$$

I → moment of inertia

$$I = \frac{bh^3}{12}$$

$$c = \frac{h}{2}$$

Beam Section Properties



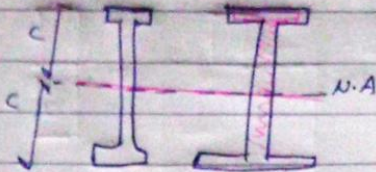
$$\sigma_m = \frac{M y}{I} = \frac{M}{S}$$

section moment of inertia

$$I = \frac{1}{12} b h^3$$

$$y = c = h/2$$

$$S = \frac{I}{c} = \frac{\frac{1}{12} b h^3}{h/2} = \frac{1}{6} b h^2 = \frac{1}{6} A h$$



Deformation in a Transverse Cross section

$$\frac{1}{P} = \frac{\epsilon_m}{c} = \frac{\sigma_m}{E c} = \frac{1}{E c} \frac{M c}{I} = \frac{M}{E I}$$

تغير ميل المقطع العرضي =
تغير ميل المقطع العرضي

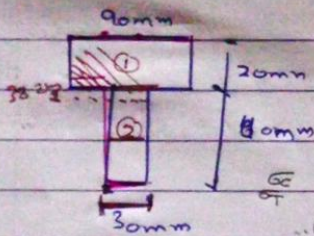
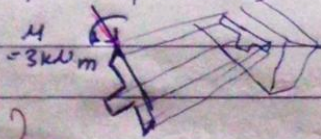
Poisson's ratio

$$\epsilon_y = -\nu \epsilon_x = -\frac{\nu}{P} \epsilon_x, \quad \epsilon_z = -\nu \epsilon_x = -\frac{\nu}{P} \epsilon_x$$

$$\frac{1}{P} = \frac{\nu}{P} = \text{anticlastic curvature}$$

$$P = \frac{E I}{R} \quad \dot{P} = P/\nu$$

Sample Problem (4.2)



A cast-iron machine part is acted upon by a 3 kNm couple. knowing $E = 165 \text{ GPa}$ and neglecting the effects of fillets, determine

- (a) the maximum tensile and compressive stresses
(b) the radius of curvature

$$\frac{1}{P} = \frac{M}{E I}$$

$$P = \frac{E I}{R} \quad \dot{P} = P/\nu$$

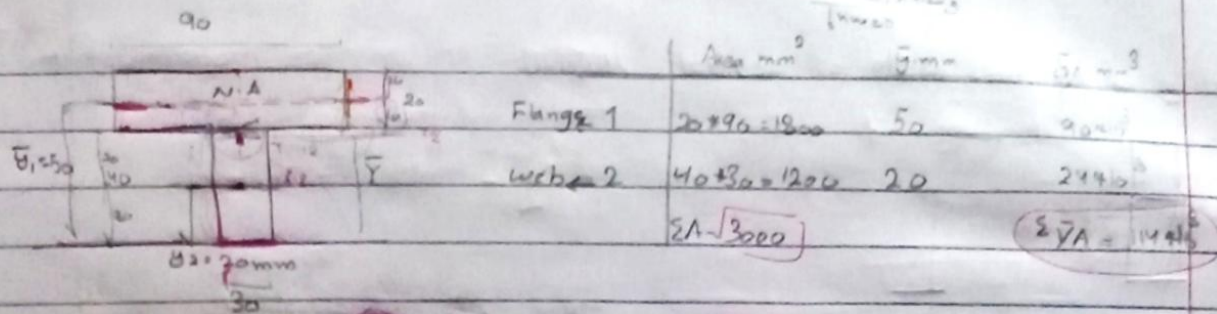
Section centroid

moment of inertia

$$\bar{Y} = \frac{\sum \bar{Y} A}{\sum A}$$

$$I_{\bar{Y}} = \sum (I + A d^2)$$

NA



centroid $\bar{Y} = \frac{\sum yA}{\sum A} = \frac{114000}{3000} = 38 \text{ mm}$

Diagram of the I-beam cross-section showing the centroid and the parallel axis theorem.

$$I_x = \sum (I + Ad^2) = \sum \left(\frac{1}{12} bh^3 + Ad^2 \right)$$

$$= \left(\frac{1}{12} \times 96 \times 20^3 + 1800 \times (12)^2 \right) + \left(\frac{1}{12} \times 30 \times 40^3 + 1200 \times (18)^2 \right)$$

Diagram labels: flange, web, centroid, center of gravity.

$$I = 868 \times 10^3 \text{ mm}^4 = 868 \times 10^{-9} \text{ m}^4$$

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M c_A}{I} = \frac{3 \text{ kN.m} \times 0.022 \text{ m}}{868 \times 10^{-9} \text{ m}^4} = \sigma_A = +76.4 \text{ MPa}$$

$$\sigma_B = \frac{M c_B}{I} = \frac{3 \text{ kN.m} \times 0.038 \text{ m}}{868 \times 10^{-9} \text{ m}^4} = \sigma_B = -131.3 \text{ MPa}$$

$$\phi = \frac{M}{EI} = \frac{3 \text{ kN.m}}{165 \text{ GPa} (868 \times 10^{-9} \text{ m}^4)} = \phi = 47.7 \text{ m}$$

$$1 \text{ mm} = 10^{-3} \text{ m}$$

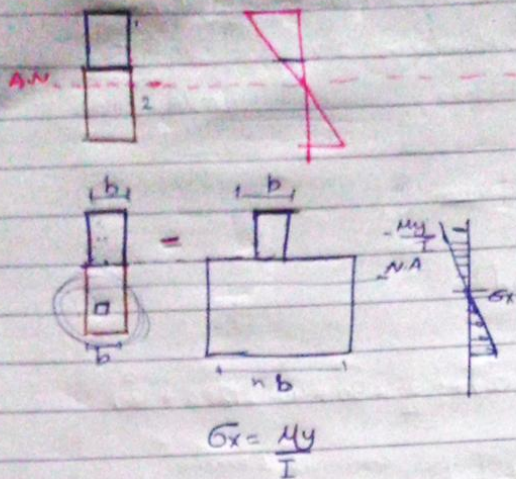
$$1 \text{ mm}^4 = 10^{-12} \text{ m}^4$$

$$868 \times 10^3 \times 10^{-12} = 868 \times 10^{-9} \text{ m}^4$$



center of curvature

Bending of Members Made of several Material



$$\epsilon_x = -\frac{y}{\rho}$$

$$\sigma_1 = E_1 \epsilon_x = -\frac{E_1 y}{\rho}$$

$$\sigma_2 = E_2 \epsilon_x = -\frac{E_2 y}{\rho}$$

$$dF_1 = \sigma dA = -\frac{E_1 y}{\rho} dA$$

$$n = \frac{E_s}{E_b} \frac{E_2}{E_1}$$

$$\sigma_x = \frac{My}{I}$$

$$\sigma_1 = \sigma_x \quad \sigma_2 = n \sigma_x$$

مثال 4:3

example problem

4:3



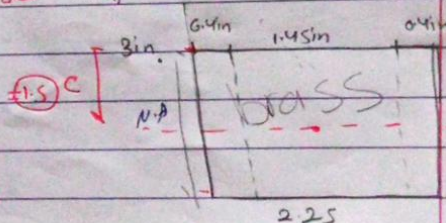
Bar is made from bonded pieces of steel ($E_s = 29 \times 10^6$ psi) and brass ($E_b = 15 \times 10^6$ psi). Determine the max stress in the steel and brass when

moment of 40 kip-in is applied

المثال 4:3

$$n = \frac{E_s}{E_b} = \frac{29 \times 10^6 \text{ psi}}{15 \times 10^6 \text{ psi}} = 1.933$$

$$b_{total} = 0.4 + 1.933 \times 0.75 = 2.25 \text{ in}$$



$$I = \frac{1}{12} b_{total} h^3 = \frac{1}{12} (9.25 \text{ in}) (3 \text{ in})^3 = 5.063 \text{ in}^4$$

$$\sigma_{max} = \frac{M}{I} = \frac{40 \text{ kip-in} (15 \text{ in})}{5.063 \text{ in}^4} = 11.85 \text{ ksi}$$

$$(\sigma_b)_{max} = \sigma_{max} = 11.85 \text{ ksi}$$

$$(\sigma_s) = n \sigma_b = 1.933 \times 11.85 \text{ ksi} = 22.906 \text{ ksi}$$

part 2

Reinforced Concrete Beam



$$(b_x) \frac{x}{2} - n A_s (d-x) = 0$$

$$\sigma_c = -\frac{4y}{I}$$

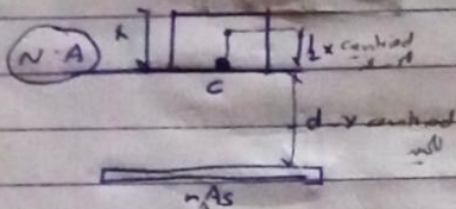
$$\sigma_s = n \sigma_c$$

$$\sigma_c = \sigma_s$$

$$\frac{1}{2} b x^2 - n A_s x - n A_s d = 0$$

3rd Eqn. 3rd degree eqn. 1st

x is the neutral axis



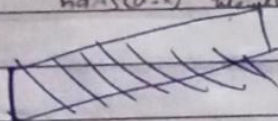
N.A. is at a distance 'x' from the top

the 3rd degree eqn. 1st

d = 1/2 x

n A_s (d-x) = 0

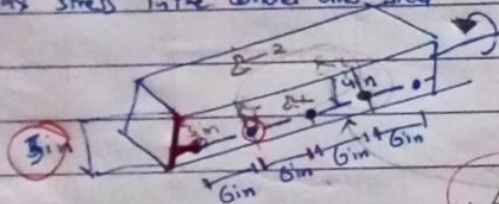
Sample Problem 4.4



A concrete floor slab is reinforced with 5/8 in diameter steel rod

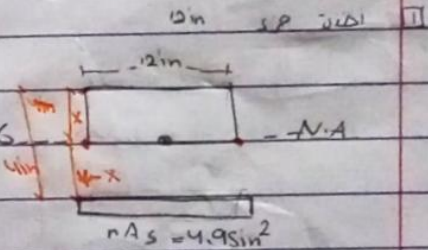
E_s for steel 29×10^6 psi, E_c for concrete 3.6×10^6 psi.

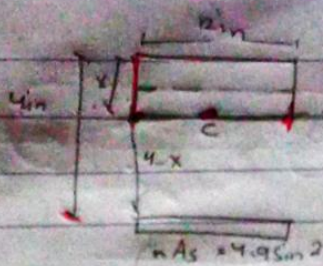
With an applied bending moment of 40 kip-in for 1-ft width of the slab, determine the max stress in the concrete and steel



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.6 \times 10^6} = 8.06$$

$$n A_s = 8.06 \left[\frac{1}{4} \left(\frac{5}{8} \right)^2 \right] \times 2 = 4.95 \text{ in}^2$$





calculate the geometric properties of the

transformed section C_s

$$A_s \cdot C_s - n A_s \left(\frac{h}{2} - x \right) = 0$$

$$12 \times x - 4.95 (4 - x) = 0$$

$$2x = -2.2$$

$$x = 1.45$$

$$I = \frac{1}{3} b x^3 + n A_s (d - x)^2$$

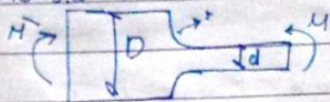
$$= \frac{1}{3} (12)(1.45)^3 + 4.95 (4 - 1.45)^2 = 44.38 \text{ in}^4$$

calculate max stresses

$$\sigma_c = \frac{M c}{I} = \frac{40 \text{ kip} \cdot \text{in} \times 1.45 \text{ in}}{44.38 \text{ in}^4} = 1.306 \text{ ksi}$$

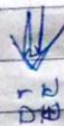
$$\sigma_s = n \frac{M c_s}{I} = 8.06 \frac{40 \text{ kip} \cdot \text{in} \times 2.55 \text{ in}}{44.38 \text{ in}^4} = 18.52 \text{ ksi}$$

stress Concentrations:



$$\sigma_m = K_t \frac{M c}{I}$$

(all) Factor
(change)



$$\text{①} \frac{r}{d} \text{ or } \frac{r}{D}$$

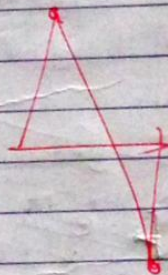
$$\frac{r}{d}$$

$$\frac{r}{D}$$

plastic Deformations

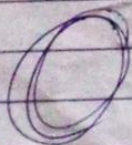
Residual stresses

$$\frac{s}{\sqrt{2}}$$



$$I = \frac{1}{3} b h^3 + n A_s (d - x)^2$$

$$= \frac{1}{3} (12)(1.45)^3 + 4.95 (4 - 1.45)^2$$



$$= \frac{\pi}{4} d^4$$



$$= \frac{\pi}{4} d^4$$

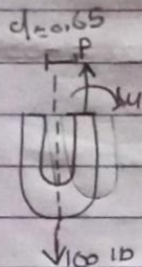
Eccentric Axial loading in a Plane of Symmetry :-

axial load $\sigma = \frac{P}{A}$ stress $E + \text{stress} \Delta \text{moment} = \text{linear}$ stress distribution

Example 4.07

An open link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine max tensile and compressive stresses

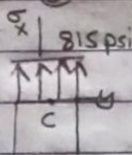
(a) distance between section centroid and natural axis.



internal force (1)

properties Parameter (2)

loads (3)



normal stress due to a centric load

$$A = \pi r^2 = \pi (0.25)^2 = 0.1963 \text{ in}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{160}{0.1963 \text{ in}^2} = 815 \text{ psi}$$

$$P = 160 \text{ lb}$$

$$M = Pd = 160 \text{ lb} \times 0.65 = 104 \text{ lb-in}$$

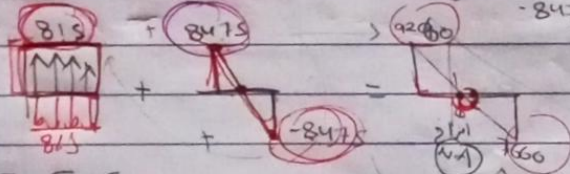
normal stress due to bending moment

$$I = \frac{1}{4} \pi r^4 = \frac{1}{4} \pi (0.25)^4 = 3.068 \times 10^{-3} \text{ in}^4$$

$$\sigma_m = \frac{Mc}{I} = \frac{(104 \text{ lb-in})(0.25 \text{ in})}{3.068 \times 10^{-3}} = 8475 \text{ psi}$$

Total stress = normal stress due to centric load

+ normal stress due to bending



$$\sigma_t = \sigma_0 + \sigma_m$$

$$815 + 8475 = 9290$$

$$\sigma_c = \sigma_0 - \sigma_m$$

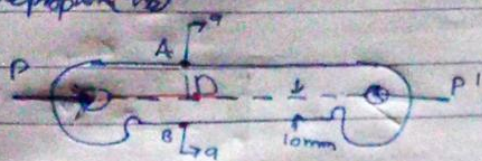
$$815 - 8475 = -7660$$

natural axis location

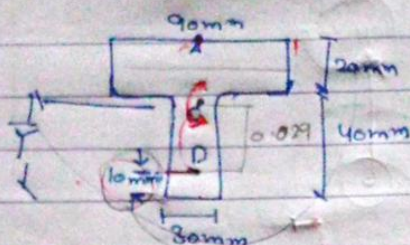
$$e = \frac{P}{A} = \frac{My_0}{I}$$

$$y_0 = \frac{PI}{AM} = \frac{815 (3.068 \times 10^{-3})}{104 \text{ lb-in}} = 0.024 \text{ in}$$

Sample problem (4.8)



The largest allowable stresses for the link are 30 MPa in tension and 120 MPa in compression. Determine the largest force P which can be applied to the link.



internal force

$$A = 3 \times 10^{-3} \text{ m}^2$$

$$\bar{Y} = 0.038 \text{ m}$$

$$I = 868 \times 10^{-9} \text{ m}^4$$

Determine an equivalent centric and bending load

$$d = 0.038 - 0.01 = 0.028 \text{ m}$$

Centric load

$$M = Pd = 0.028P \text{ - bending moment}$$

$$\sigma_A = \frac{-P}{A} + \frac{Mc}{I} = \frac{-P}{3 \times 10^{-3}} + \frac{(0.028P)(0.022)}{868 \times 10^{-9}} = +377P$$

$$\sigma_B = \frac{-P}{A} - \frac{Mc}{I} = \frac{-P}{3 \times 10^{-3}} - \frac{(0.028P)(0.022)}{868 \times 10^{-9}} = -1559P$$

** Evaluate critical loads for allowable stress

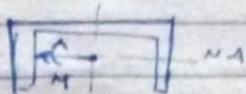
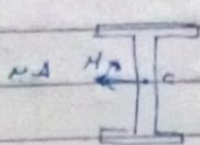
$$\sigma_A = +377P = 30 \text{ MPa} \quad P = 79.6 \text{ N}$$

$$\sigma_B = -1559P = -120 \text{ MPa} \quad P = 79.6 \text{ kN}$$

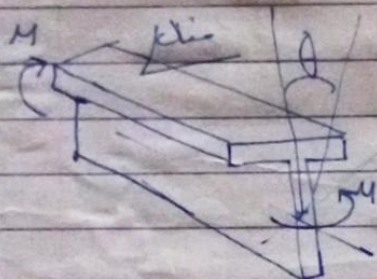
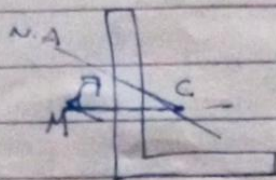
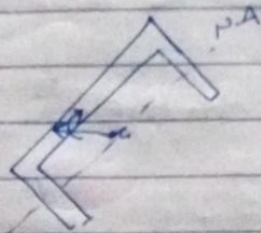
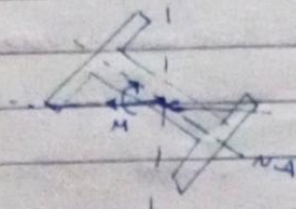
The largest allowable load

$$P = 79.6 \text{ kN}$$

Unsymmetric Bending



نورالست زفر N.A

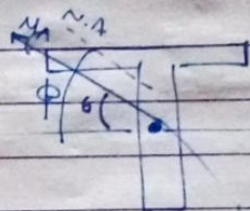


$$M_x = M \cos \theta \quad M_y = M \sin \theta$$

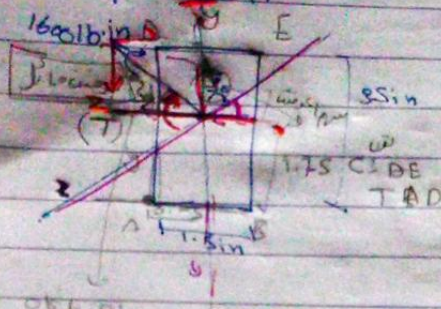
$$\sigma_x = -\frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$\sigma_x = 0 = -\frac{M_x y}{I_x} + \frac{M_y x}{I_y} = -\frac{(M \cos \theta) y}{I_x} + \frac{(M \sin \theta) x}{I_y}$$

$$\tan \phi = \frac{y}{x} = \frac{I_x}{I_y} \tan \theta$$



Example 4.08:



A 1600 lb.in couple is applied to a rectangular wooden beam in a plane forming an angle with the vertical. Determine

(a) max stress in the beam

(b) the angle that the natural axis forms with the horizontal plane.

$$M_x = M \cos \theta \quad M_y = M \sin \theta$$

$$\sigma_x = -\frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$\tan \theta = \frac{y}{x} = \frac{I_x}{I_y} \tan \phi$$



$$M_x = 1600 \text{ lb.in} \cos 30 = 1386 \text{ lb.in}$$

$$M_y = 1600 \text{ lb.in} \sin 30 = 800 \text{ lb.in}$$

$$I_x = \frac{1}{12} (1.5)(3.5)^3 = 53.59 \text{ in}^4$$

$$I_y = \frac{1}{12} (3.5)(1.5)^3 = 0.9844 \text{ in}^4$$

$$\sigma_1 = \frac{M_x y}{I_x} = \frac{(1386 \text{ lb.in})(1.75 \text{ in})}{53.59 \text{ in}^4} = 452.6 \text{ psi}$$

$$\sigma_2 = \frac{M_y x}{I_y} = \frac{(800 \text{ lb.in})(0.75 \text{ in})}{0.9844 \text{ in}^4} = 609.5 \text{ psi}$$

Stress is compressive
C is at the top
T is at the bottom

$$\sigma_{\max} = \sigma_1 + \sigma_2 = 452.6 + 609.5 = 1062 \text{ psi}$$

$$\tan \phi = \frac{I_x \tan \theta}{I_y} = \frac{53.59 \tan 30}{0.9844} = 72.4^\circ$$

$$\sigma_x = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \quad \text{Comp / Ten}$$

$$\frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{P}{A}$$

$\frac{dy}{dx} = \tan \phi$



دائرة Pure Bending

Pure Bending

$$\sigma = \frac{My}{I}$$

$y: m$

$$\sigma = N/m^2 \text{ (Pa)} \quad I: m^4$$

$$I = \frac{1}{12} b h^3 \text{ (solid)}$$

$$\text{hollow} = \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$

$$I_{circle} = \frac{1}{4} \pi r^4 \text{ (solid)}$$

hollow
is

$$\frac{1}{EI} = \frac{M}{EI}$$

$\frac{1}{EI}$ moment

position ratio

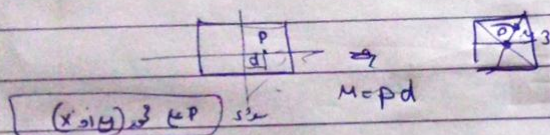
$$\rho = P/\bar{U}$$

$\bar{U}$ transverse radius of curvature

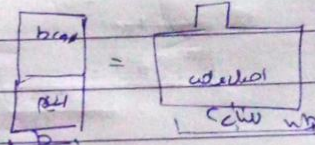
ρ radius of curvature

Eccentric Force

center of mass

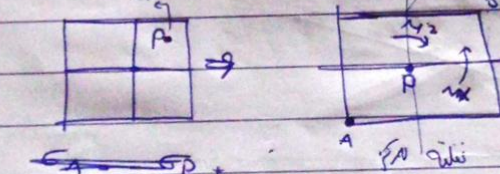


composite materials



(مركبة المواد)

Max stress



$$E_2 > E_1$$

$$n = \frac{E_2}{E_1} > 1$$

$$\sigma_A = -\sigma_P + \sigma_{mx} + \sigma_{my}$$

$$\sigma_B = -\sigma_P - \sigma_{mx} - \sigma_{my}$$

Max stress compression

$$b'_2 = n b_2$$

$$\sigma = My/I$$

$$\frac{62}{n} \leq \frac{My_2}{I}$$

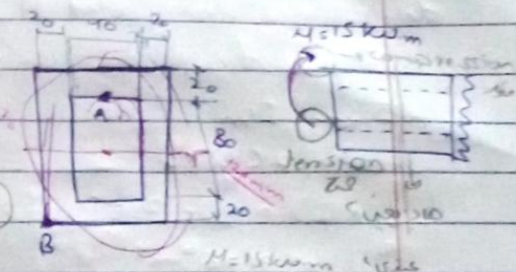
4.1 - knowing that the couple shown acts in a vertical plane, determine the stress of coupon A, B

$$I = \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$

$$= \frac{1}{12} (0.08) m (0.12)^3 - \frac{1}{12} (0.04) (0.08)^3$$

$$= 9.81 \times 10^{-6} m^4$$

hollow



$$\sigma = \frac{My}{I}$$

$$\sigma_A = \frac{(15 \times 10^3) N \cdot m (0.04) m}{9.81 \times 10^{-6} m^4} = -61.2 MPa$$



$$\sigma_B = \frac{(15 \times 10^3) (0.06)}{9.81 \times 10^{-6} m^4} = 91.7 MPa$$



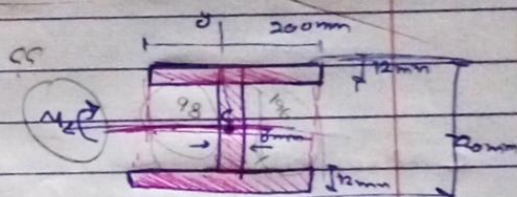
4.3 - Using an allowable stress of 155 MPa, determine the largest bending moment M that can be applied to the wide flange beam shown. Neglect the effect of fillets

$$\sigma = \frac{My}{I} \rightarrow 155 \times 10^6 = \frac{M}{I}$$

$$I = \frac{1}{12} b_1 h_1^3 - 2 \left(\frac{1}{12} b_2 h_2^3 \right)$$

$$= \frac{1}{12} (0.2) (0.22)^3 - 2 \left(\frac{1}{12} (0.096) (0.196)^3 \right)$$

$$= 5.7 \times 10^{-5}$$



$$155 \times 10^6 = \frac{M}{5.7 \times 10^{-5}} \rightarrow M = 8.835 \times 10^6 N \cdot m$$

$$M = 8.835 \times 10^6 N \cdot m$$

$$155 = \frac{M}{5.7 \times 10^{-5}} \rightarrow M = 8.835 \times 10^6 N \cdot m$$

$$220 - 24 = 169$$

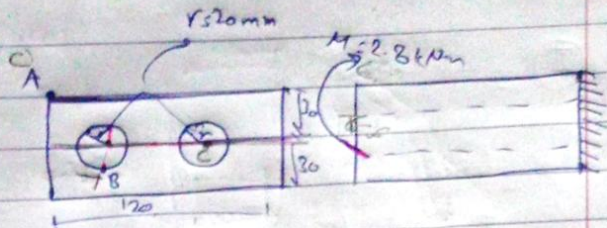
4.6, knowing that the couple shown acts in a vertical plane, determine the stress point A, B

$$I_{total} = I_{rect} + I_{circ}$$

$$= \frac{1}{12} b h^3 + 2 \left(\frac{\pi r^4}{4} \right)$$

$$= \frac{1}{12} (0.12)(0.06)^3 + 2 \left(\frac{\pi (0.02)^4}{4} \right)$$

$$= 1.91 \times 10^{-6} m^4$$

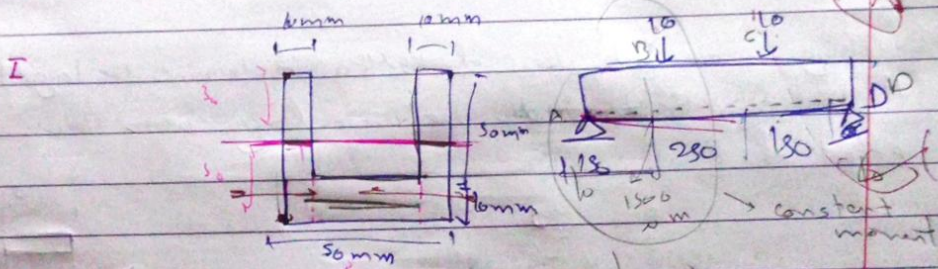


$$\sigma = \frac{M y}{I}$$

$$\sigma_A = \frac{(2.8 \times 10^3)(0.03)}{1.91 \times 10^{-6}} = -44 \text{ MPa} \quad (\ominus)$$

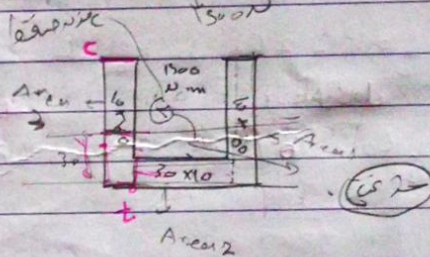
$$\sigma_B = \frac{2.8 \times 10^3 (0.02)}{1.91 \times 10^{-6}} = 29.8 \text{ MPa}$$

4.11 Two vertical forces are applied to a beam of the cross section, determine the max tensile and compressive stresses in portion BC of the beam



Centroid of the cross section

Area	$\bar{y}$	Area $\bar{y}$
600	30	18000
300	5	1500
600	30	18000
1500		37500



$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = \frac{37500}{1500} = 25 \text{ mm}$$

Centroid is 25 mm from the bottom of the horizontal rectangle.

$$I = \left(\frac{1}{12} (0.01)(0.06)^3 + (0.01)(0.06)(0.005)^2 \right) \times 2 + \left(\frac{1}{12} (0.03)(0.01)^3 + 0.03(0.01)(0.02)^2 \right) \times 2$$

$$= 512 \times 10^{-9} m^4$$

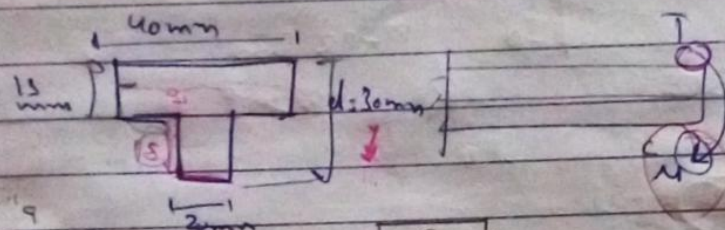
$$\sigma_t = \frac{(1500)(25 \times 10^{-3})}{512.5 \times 10^{-9}} = 73.2 \text{ MPa (T)}$$

$$\sigma_c = \frac{(1500)(25 \times 10^{-3})}{512.5 \times 10^{-9}} = 73.2 \text{ MPa (C)}$$

allow 10 MPa

allow 10 MPa in T & C

Q.15 The beam shown is made of nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



Part	Area A	y	Ay
1	1500	7.5	2250
2	600	22.5	13500
	2100		15750

$$\bar{y} = \frac{\sum Ay}{\sum A} = 7.5 \text{ mm}$$

$$I = \frac{1}{12} (40)(15)^3 + 300 (7.5 - 17.5)^2 + \frac{1}{12} (15)(22.5)^3 + 600 (22.5 - 17.5)^2 = 61.875 \times 10^3 \text{ mm}^4$$

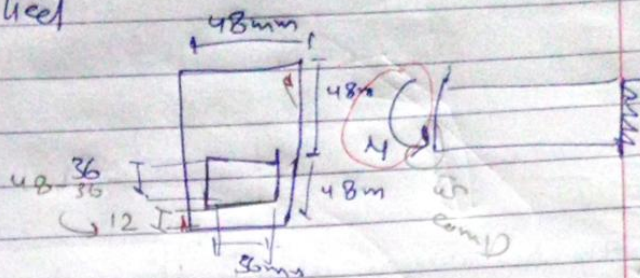
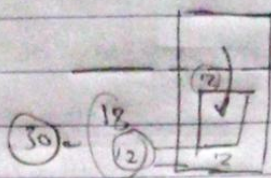
$$\sigma_t = \frac{M \bar{y}_t}{I} = 24 \times 10^6 = \frac{M (12.5 \times 10^{-3})}{61.875 \times 10^{-9}} \quad M_t = 119 \text{ N.m}$$

$$\sigma_c = \frac{M \bar{y}_c}{I} = 30 \times 10^6 = \frac{M (17.5 \times 10^{-3})}{61.875 \times 10^{-9}} \quad M_c = 106 \text{ N.m}$$

So the beam will fail in compression. The largest couple M is 106 N.m.

Q20:- know that for the extended show, the allow stress is 120 MPa T , 150 MPa C , determine the largest couple M that can be applied

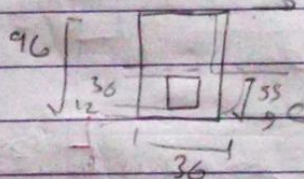
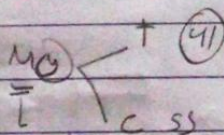
$$\frac{18}{2\sqrt{36}} = \frac{18}{12}$$



	Area	$\bar{y}$	$A \bar{y}$
1	$96 \times 48 = 4608$	48	221 184
2	-1296	30	-38 880
	3312		$182 304$

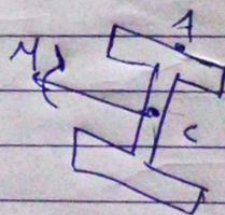
$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = 55 \text{ cm}$$

$$I = \left[\frac{1}{12} (48)(96)^3 + 4608(48-55)^2 \right] - \left[\frac{1}{12} (36)(36)^3 + 1296(30-55)^2 \right]$$



Q4.31:- A 200×31.3 rolled steel beam is subjected to a couple M of moment of 45 kN.m , knowing $E = 200 \text{ GPa}$ and $\nu = 0.29$, determine (a) the radius of curvature ρ , (b) the radius of curvature ρ of a transverse cross section

$$\rho = \frac{EI}{M} = \frac{(200 \times 10^9) (I)}{45 \times 10^3 \text{ N.m}} = 31.3816 \times 10^6 = 139 \text{ m}$$

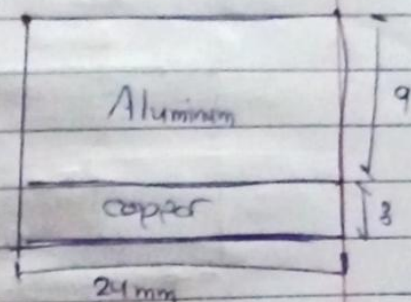
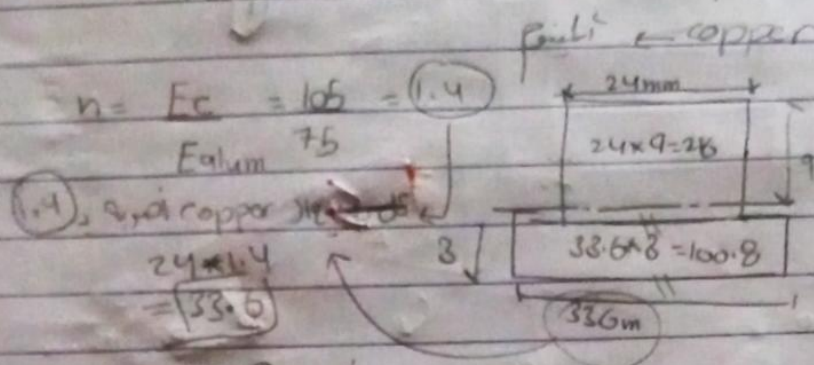


$$\rho = \frac{r}{\nu}$$

$$\frac{1}{\rho} = \frac{\nu}{r} \quad \frac{1}{\rho} = \frac{\nu}{EI}$$

4.40 A copper strip ($E_c = 105 \text{ GPa}$) and an aluminum strip ($E_a = 75 \text{ GPa}$) are bonded together to form the composite beam shown knowing that the beam is bent about a horizontal axis by a couple. If moment $M = 35 \text{ N}\cdot\text{m}$, determine the max stress in aluminum and copper strip.

(گروهی و دوگانه) 578 استرین



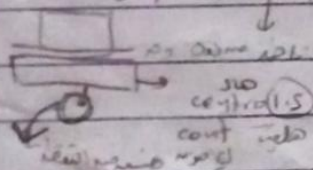
نقطه

نقطه از 24 میلی متر

دوبل centroid

A_{mm}	$\bar{y}$	$A\bar{y}$
100.8	15	151
216	7.5	162
317		1771

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = 5.59$$



$$I = \frac{1}{12} (33.6)(8)^3 + 100.8(15 - 5.59)^2 + \frac{1}{12} (24)(9)^3 + 216(7.5 - 5.59)^2$$

$$= 4008 \text{ mm}^4 = 4008 \times 10^{-12} \text{ m}^4$$

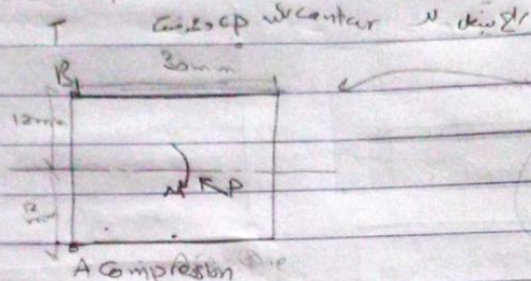
$$\sigma_{al} = \frac{35(12 - 5.59)10^{-3}}{4008 \times 10^{-12}} = 56 \text{ MPa}$$

$$\sigma_c = \frac{35(5.59 \times 10^{-3})}{4008 \times 10^{-12}} = 48.3 \text{ MPa}$$

499. knowing that the magnitude the horizontal force is 8 kN
determine the stress at point A, B

moment due to P force is $8 \times 0.45 = 3.6 \text{ kN}\cdot\text{m}$

at base $\uparrow$ compression



lets find the stress at point A, B



$$M = P \cdot d$$

centroid N

$$= 8 \times 10^3 (45 - 12) \times 10^{-3} = 264 \text{ N}\cdot\text{m}$$

24 mm

$$\sigma_A = \frac{-P}{A} - \frac{My}{I} = -\frac{8 \times 10^3}{24 \times 30 \times 10^{-6}} - \frac{264 (12 \times 10^{-3})}{\frac{1}{12} (30) (24)^3 \times 10^{-12}} = -102.8 \text{ MPa}$$

axial load moment

$$\sigma_B = \frac{-P}{A} + \frac{My}{I} = -\frac{8}{24 \times 30} + \frac{264 (12 \times 10^{-3})}{\frac{1}{12} (30) (24)^3 \times 10^{-12}} = 80.6 \text{ MPa}$$

axial load moment

axial load moment

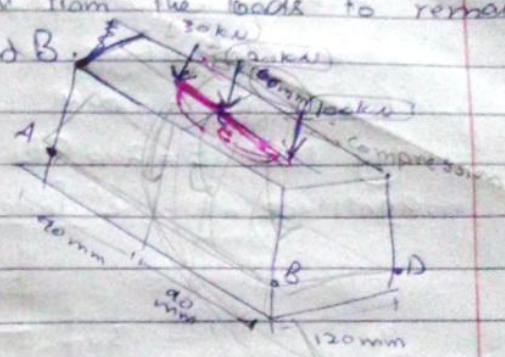
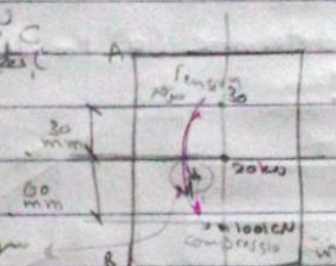
B, A beam

axial

4.102

A short 120 x 180 mm column supports three axial loads shown. knowing that the section ABD is sufficiently far from the loads to remain plane, determine the stress (a) over A and B.

Point 30, 20, 100 (150) kN
load into distance
pressure with
moment



$$M_{total} = 100(60) - 30(30) = 5100 \text{ Nmm}$$

$$\sigma_A = \frac{-P}{A} + \frac{My}{I} = \frac{-150 \times 10^3}{180(120)/10} + \frac{5100 \times 0.09}{\frac{1}{12} \times 0.12 \times 0.18^3} = 0.98 \text{ MPa}$$

$$\sigma_B = \frac{-P}{A} + \frac{My}{I} = \frac{-150 \times 10^3}{0.18 \times 0.12} - \frac{5100 \times 0.09}{\frac{1}{12} \times 0.12 \times 0.18^3} = -1184 \text{ MPa}$$

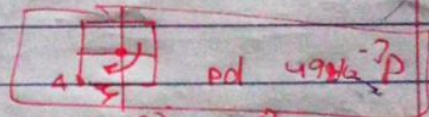
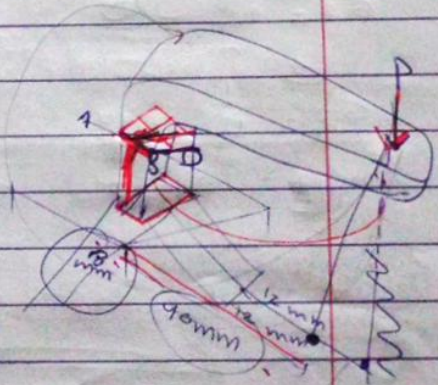
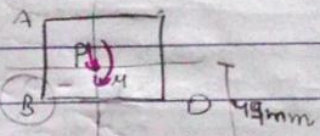
4.106 knowing that the allowable stress in section ABD is 80 MPa, determine the largest force P that can be applied to the bracket shown.

$$M_{BD} = 49 \times 10^{-3} P \text{ N.m}$$

$$\sigma_B = \frac{-P}{A} - \frac{My}{I}$$

$$-30 \times 10^6 = \frac{-P}{18(24)(10^{-6})} - \frac{(49 \times 10^{-3} P)(9 \times 10^{-3})}{\frac{1}{12}(24)(18^3 \times 10^{-12})}$$

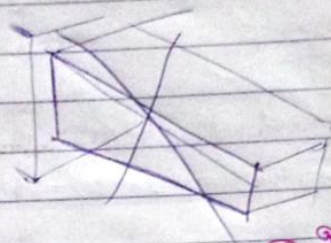
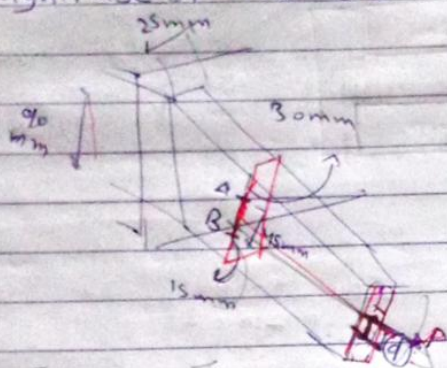
$$P = 1994 \text{ (2 kN)}$$



$$\sigma = 80 \text{ MPa} \times 10^6 = \frac{-P(1)}{18 \times 24(10^{-6})} + \frac{-49 \times 10^{-3} P \times 9 \times 10^{-3}}{\frac{1}{12} \times 18 \times 12^3 \times 10^{-12}}$$

4121 - An eccentric force P is applied as shown to a steel bar of 25x90 mm cross section. The strains at A and B have been measured and found to be $\epsilon_A = +350 \times 10^{-6}$ and $\epsilon_B = -70 \times 10^{-6}$.

Knowing that $E = 200 \text{ GPa}$, determine (a) the distance d , (b) the magnitude of the force P .



$$\sigma = E\epsilon$$

$$\sigma_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \text{ MPa (T)}$$

$$\sigma_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \text{ MPa (C)}$$

$$M = Pd$$

$$\sigma_A = \frac{P}{A} + \frac{My}{I} \quad 70 \times 10^6 = \frac{P}{25 \times 90 \times 10^{-6}} + \frac{(Pd)(45 \times 10^{-3})}{\frac{1}{12} 25(90^3) \times 10^{-12}}$$

$$\sigma_B = \frac{P}{A} - \frac{My}{I} \quad -14 \times 10^6 = \frac{P}{25 \times 90 \times 10^{-6}} - \frac{(Pd)(30 \times 10^{-3})}{\frac{1}{12} 25(90^3) \times 10^{-12}}$$

$$P = 94.5 \text{ kN}$$

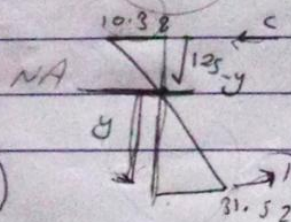
$$d = 80 \text{ mm}$$

$$\sigma_A = \frac{P}{A} + \frac{My}{I} = \frac{10.57}{10^{-6}} + \frac{70.95 \times 45 \times 10^{-3}}{\frac{1}{12} 25(90^3) \times 10^{-12}} = 31.52 \text{ MPa}$$

$$\sigma_B = \frac{P}{A} - \frac{My}{I} = \frac{10.57}{10^{-6}} - \frac{70.95 \times 30 \times 10^{-3}}{\frac{1}{12} 25(90^3) \times 10^{-12}} = -10.38 \text{ MPa}$$

$$\frac{125 - y}{10.38} = \frac{y}{31.52}$$

$$y = 94 \text{ mm (front)}$$



4.123 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown knowing that the cross section of the bar is a square of side 4 mm and the strain on the inner edge was measured and found to be 0.001. Determine P .

$E = 200 \text{ GPa}$

$\mu = 100 \text{ P} \quad - 0.1 \text{ P D.m}$

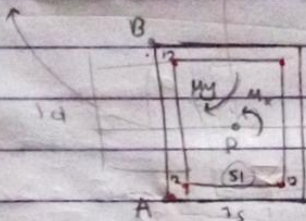
$$(450 \times 10^{-6}) (200 \times 10^9) = 90 \text{ MPa}$$

$$\sigma_a = \frac{P}{A} + \frac{M}{I} x$$

$$\frac{90 \times 10^6}{(0.04)^2} + \frac{0.1P(20810)^3}{0.04(0.04)^3}$$

$(P = 9 \text{ kN})$

1.144 The tube shown has uniform wall thickness of 12 mm. For the loading given, determine (a) the stress at point A, (b) the point where the normal neutral axis intersect line ABD.



$$A_{\text{area}} = 0.075 \times 0.125 - 0.051(0.101) = 4.224 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12} (0.075 \text{ m})^3 - \frac{1}{12} (0.051 \text{ m})^3 = 783 \times 10^{-6} \text{ m}^4$$

$$[y]_{\text{sub}} = \frac{1}{12} \times 0.125 \times 0.03^3 - \frac{1}{12} \times 0.01 \times 0.03^3 = 3.28 \times 10^{-6}$$

$$GP = \frac{P}{A} = \frac{70 \times 10^3}{4.224 \times 10^{-3}} = 16.57 \text{ MPa}$$

$$M_x = 56 \left(\frac{12}{2} \right) - 14 \left(\frac{22}{2} \right) = 2628 \text{ Nmm}$$

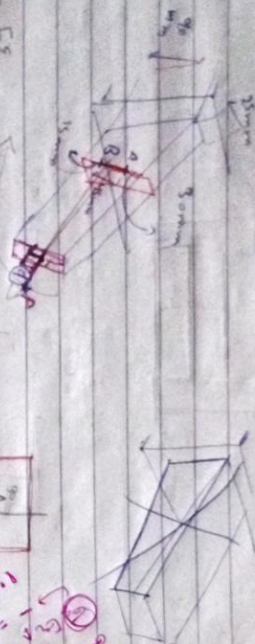
$14 \left(\frac{75}{2} \right) = 525$

$$\sigma_{my} = \frac{M_y}{I_x} = \frac{525.575 \times 10^3}{3.28 \times 10^{-6}}$$

$$\frac{M_{xy}}{I_x} = \frac{2625 (62.5 \text{ kPa}^{-2})}{1} = 20095 \text{ MPa}$$

Ans 1.1 - An eccentric force P is applied as shown to a steel bar of 25mm cross section. The stress at A and B have been measured and found to be $\sigma_A = 100 \text{ MPa}$ and $\sigma_B = 70 \text{ MPa}$.

Knowing that $E = 200 \text{ GPa}$, determine (a) the distance d , (b) the magnitude of the force P .



$$\sigma = E \epsilon$$

$$\sigma_A = (200 \times 10^9) \epsilon_A$$

$$= 70 \text{ MPa}$$

$$\epsilon_B = (200 \times 10^9) \epsilon_B = -40 \text{ MPa}$$

$$M = P d$$

$$\sigma_A = \frac{P}{A} + \frac{M y}{I} = \frac{P}{A} + \frac{P d y}{I}$$

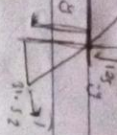
$$\sigma_B = \frac{P}{A} - \frac{M y}{I} = \frac{P}{A} - \frac{P d y}{I}$$

$$P = 94.5 \text{ kN}$$

$$\sigma_A = \frac{P}{A} + \frac{M y}{I} = 100 \text{ MPa}$$

$$\sigma_B = \frac{P}{A} - \frac{M y}{I} = 70 \text{ MPa}$$

$$P = 94.5 \text{ kN}$$



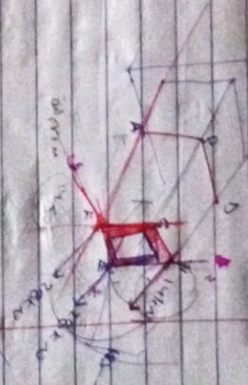
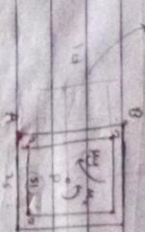
Ans 1.2 The C-shaped steel bar is used as a sign holder to determine the magnitude P of the force shown knowing that the cross section of the bar is a square of side 20 mm and the bar is fixed at the other end.



$$\sigma_A = \frac{P}{A} + \frac{M y}{I}$$

Using the known dimensions and the stress at A, the force P can be determined.

$$P = 10.2 \text{ kN}$$



$$P = 10.2 \text{ kN}$$

$$\sigma_B = \frac{P}{A} - \frac{M y}{I}$$

$$P = 10.2 \text{ kN}$$

4.34:

Two materials joined at ends

aluminum

Brass

Modulus of elasticity

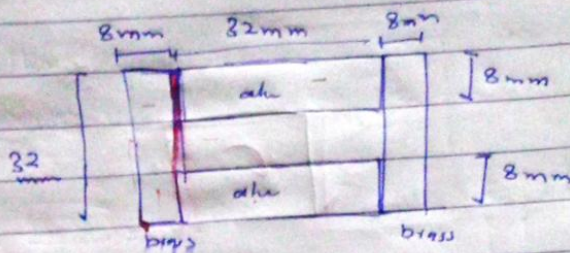
70 GPa

105 GPa

allow stress

100 MPa

60 MPa



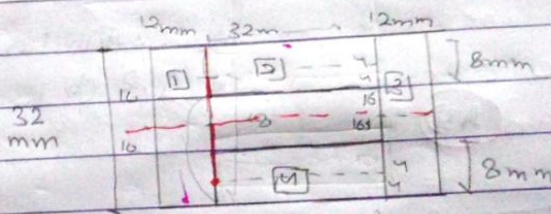
position brass on: $E_{al} < \text{brass } E$ will

$$n = \frac{E_b}{E_{al}} = \frac{105}{70} = 1.5$$

$$b_{al} \rightarrow b_{br} n = 8 \times 1.5 = 12 \text{ mm}$$

brass on the

$$\begin{matrix} [2] = [9] \\ [1] = [3] \end{matrix}$$



	Area	$\bar{y}$	Area $\bar{y}$
[1]	$12 \times 32 = 384$	16	6144
[3]	$12 \times 32 = 384$	16	6144
[4]	$8 \times 32 = 256$	4	1024
[2]	$8 \times 32 = 256$	28	7168

aluminum

$$\bar{Y} = \frac{\sum \text{Area} \bar{y}}{\sum \text{Area}} = \frac{20480}{1280} = 16$$

$$I = [I_1 + I_3] + [I_2 + I_4]$$

$$I = \frac{1}{12} b h^3 + A \bar{d}^2 = 2 \left[\frac{1}{12} (12)(32)^3 + 12 \times 32 (16-16)^2 \right] + \left[\frac{1}{12} (32)(8)^3 + 32(8)(16-4)^2 \right]$$

$$+ \frac{1}{12} (32)(8)^3 + 32 \times 8 (28-16)^2 = 1.408 \times 10^{-7} \text{ m}^4$$

Follow Alumin = $\frac{1 \times 10^6}{1.102 \times 10^{-7}} = M = 9.07 \times 10^{-4} \text{ M}$

$$\frac{V}{R} = \frac{\Delta \phi}{I}$$

$$\frac{\sigma_b}{r} = \frac{M y}{I_{xx}}$$

$$\frac{160 = 40.016}{1.5} \quad 1.408 \times 10^{-7}$$

$M = 9.28 \times 10^{-4}$ 15.00

8840-4

2160 M

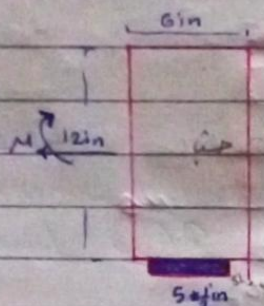
$$(4.41 - 4.42) (4.48 - 4.46)$$

4.41 (4.45) —

$M = 450$ kip-in $\bar{P}$ U.L.S.P. U.L.A.L.

$$\frac{1}{\rho} = \frac{M}{E t}$$

$$\rho = \frac{EI}{M}$$



3. $\frac{1}{2} \sin 2\theta$ I lost

عن عبد الجواد

$$E_v = 1.8 \times 10^6 \text{ psi}$$

$E_c = 29 \times 10^6 \text{ psi}$

Steel wood

$$n = \frac{E_s}{E_\infty} = \frac{29 \times 10^6}{1.8 \times 10^6} = 16.11$$

$$h' = n h = 16.11 \times \frac{1}{2} = 8.055$$

		Gin		
12 in	[2]		Area	9
6		[1]	6×12	4.0275
			72	289.98
4.0275	[1]	8.055	[2]	5 x 8.055
5			40.275	14.055
				566.06

$$\bar{Y} = \frac{\sum \text{Area} \cdot \bar{y}}{\sum \text{Area}} = 7.629$$

$$I = \left[\frac{1}{12} b h^3 + \text{Area } d^2 \right] + \left[\frac{1}{12} b h^3 + \text{Area } d^2 \right]$$

$$= \left[\frac{1}{12} \cdot 5 (8.055)^3 + 40.275 (14.055 - 7.62) \right]^2 +$$

$$\left[\frac{1}{12} 6 (12)^3 + 72 (4.025 - 7.62)^2 \right] = 3.67 \times 10^{-9}$$

$$P_{\text{work}} = \frac{18110}{450} \times 3.67 \times 10^{-9} = 1.468 \times 10^{-5}$$

$$P_{steel} = \frac{29 \times 10^6 \times 367 \times 10^9}{450} = 2.3 \times 10^4$$

$$\frac{1}{P_1} = \frac{1}{P_2} + \frac{1}{P_3}$$

4.47 R.F.C. $M_{positive} = 175 kNm$
 $E_c = 25 GPa$ $E_s = 200 GPa$

الطلب stress in steel
 max stress in concrete

نسبة التحويل
 $n = \frac{E_s}{E_c} = \frac{200}{25} = 8$

مقدار الحديد
 $n A_s = 8 \cdot \frac{d^2}{4} \cdot \frac{y}{d-x}$

$= 8 \cdot \frac{480^2}{4} \cdot \frac{y}{480-x} = 15707.96$

$A_c + C_c = n A_s (d-x)$

$\Sigma M_{N.A} = 0 \Rightarrow \frac{(300x) \cdot x}{2} - 15707.96 (480-x)$
 $d = 480$

$x = 177.86$

$I = \frac{1}{3} b x^3 + n (d-x)^2$

$\frac{1}{3} (300) (177.86)^3 + 8 (480 - 177.86)^2$

=

$= \frac{(bh) \left(\frac{h}{2} \right) + n A_s (d)}{b h + n A_s}$

مركز ثقل

$\bar{y} = \frac{(300 + 540) \left(\frac{540}{2} \right) + (15707.96) (480)}{300 + 540 + 15708} = 288.5 mm$

$I = \frac{1}{12} b h^3 = \frac{1}{12} (300) (540)^3 = 3.936 \times 10^9 mm^4$

معادلة التوازن $b \cdot \frac{x^2}{2} = A_s (d-x)$

مركز ثقل

b: العرض المثل

x: عمق التواء

A_s

d: العمق التواء من سطح التواء

$$b \cdot \frac{x^2}{2} = (n A_s) (d - x)$$

المعادلة الأولى (1)

$$n \frac{E_s}{E_c} = 200 = 8$$

$$(2) A_s = (n A_s) = 8 (1125^2) 4 = 15707.96$$

(3) المبدأ الثاني (X) المبدأ

$$b \frac{x^2}{2} = n A_s (d - x)$$

d = 480 mm
المبدأ الثاني

$$300 \cdot \frac{x^2}{2} = 15707.96 (480 - x)$$

$$x = 177.87$$

$$I = \frac{1}{3} b x^3 + n A_s (d - x)^2$$

$$= \frac{1}{3} (300) (177.87)^3 + 15707.96 (480 - 177.87)^2 = 1.99 \times 10^9$$

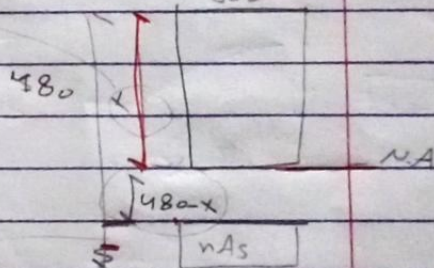
NA المبدأ

(5) $\sigma_c =$

$$M = 175 \text{ kN.m} \rightarrow 175 \times 10^6 \text{ Nmm}$$

$$\frac{175 \times 10^6 \text{ Nmm} \times 177.87 \text{ mm}}{1.99 \times 10^9 \text{ mm}^4}$$

$$15.64 \text{ MPa}$$



$$\sigma_s = \frac{M}{I} = \frac{175 \times 10^6}{1.99 \times 10^9} \times (480 - 177.87) = 212.8 \text{ MPa}$$

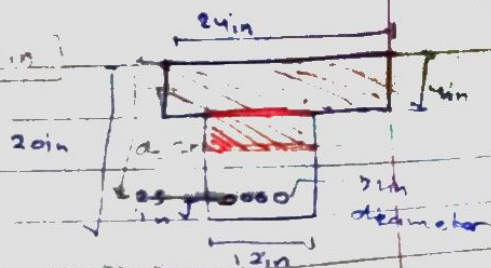
4.52

$M = +100 \text{ kip} \cdot \text{ft}$

$E_c = 3.625 \times 10^6 \text{ psi}$

$E_s = 29 \times 10^6 \text{ psi}$

و.ا.ل. (5c, 5s)



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6 \text{ psi}}{3.625 \times 10^6 \text{ psi}} = 7.97$$

(18)

$$A_s = \frac{M}{f_s (d - \frac{a}{2})} = \frac{100 \text{ kip} \cdot \text{ft}}{(20 \text{ in} - \frac{a}{2})} = 25.132$$

$$d = 20 - 2 = 18 \text{ in}$$

$$\textcircled{1} \quad \frac{b x^2}{2} = n A_s (d - x)$$

(x) in

$$= \frac{24 x^2}{2} = 25.13 (18 - x)$$

$$\textcircled{-} x - 5.10 \rightarrow \text{min}$$

$$(18 - 5.10) \times \text{slab area} = \dots$$

$$(24)(x - 2) + 12(x - 4)^2 = n A_s (d - x)$$

$$= 25.13 (18 - x)$$

$$x = 9.28$$

5.2 slip at I

$$\frac{1}{12} b h^3 + \frac{b (x - 2)^2}{2} = \frac{1}{12} (24)(4)^2 + \frac{24 (9.28 - 2)^2}{2} = 1.39$$

kip

$$\frac{b (x - 4)^2}{3}$$

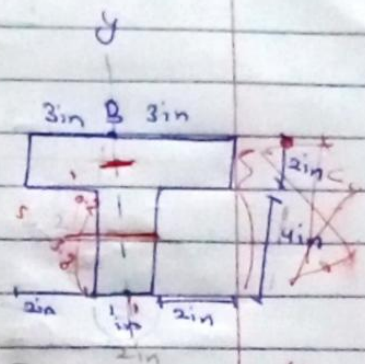
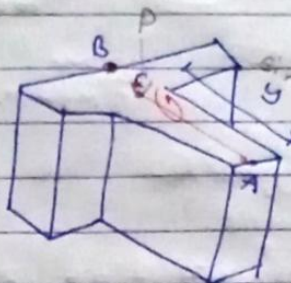
ل.ا.ل. 1.39

Total slip

4.117 a vertical (P) 20kip applied at point C

$$y = 5 \text{ in}$$

location natural axis.

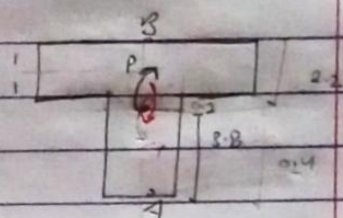


continued

	Area	$\bar{y}$	Area $\cdot \bar{y}$
[1]	12	5 in	60
[2]	8	2	16
	<u>20</u>		<u>76</u>

$$\bar{Y} = \frac{\sum \text{Area} \cdot \bar{y}}{\sum \text{Area}} = \frac{76}{20} = 3.8 \text{ in}$$

continued



$$M = P \cdot d = 20 (1.2) = 24 \text{ kip} \cdot \text{in}$$

$$I = \left[\frac{1}{12} b h^3 + \text{Area} (\bar{r} - \bar{y})^2 \right] + \left[\frac{1}{12} b h^3 + \text{Area} (\bar{r} - \bar{y})^2 \right] \\ = \left[\frac{1}{12} (6)(2)^3 + 12(5-3.8)^2 \right] + \left[\frac{1}{12} (2)(4)^3 + 8(2-3.8)^2 \right] = 57.8 \text{ in}^4$$

$$\sigma_B = \frac{-P}{A} + \frac{M y}{I} = \frac{-20}{20} + \frac{24(2.2)}{57.8} = -1.9125 \text{ comp}$$

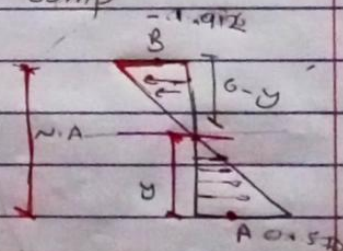
Location of cross section $[12 + 8] = 20$

$$\sigma_A = \frac{-P}{A} - \frac{M y}{I} = \frac{-20}{20} - \frac{24(3.8)}{57.8} = +0.576 \text{ comp}$$

$$x = \frac{1.912}{0.576} = 3.32 \text{ in}$$

$$x = 1.8$$

6 in



Stress at B is 1.9125 comp