

chapter - 5

load  $\leftarrow$  concentrated load  
distributed

$\frac{1}{2} \rightarrow$  save at root

hinged roller  $\rightarrow$  simple

overhanging

cantilever

shear and Bending Moment diagram

internal

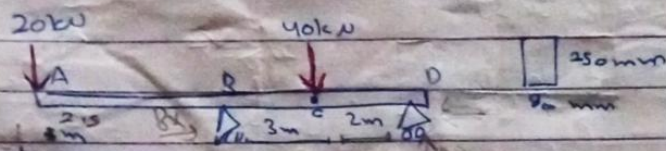
$V$  &  $M$

area

$$I_{xx} = \frac{bh^3}{12}$$

$$S = \frac{1}{6}bh^2$$

Sample Problem 51



For the timber and loading

shear draw the shear and

bending moment and determine

normal stress due to bending

$$\sum F_y = 0 = -20 - 40 + R_y + 14 = 0$$

$$\sum M = 0 = (+20 \times 2.5) + (-40 \times 5.5) + (14 \times 10) = 0$$

$$R_y = 14$$

$$R_y = 14$$

$$\sum F_y = 0 = -20 - V = 0$$

$$V = -20 \text{ kN}$$

$$\sum M = 0 = -20(2.5) + M$$

$$M = 50$$

$$\sum F_y = 0 = -20 - 20 - V = 0$$

$$\sum M = 0 = -20(2.5) - 20(5) + M$$

$$M = -50$$

$$\sum F_y = 0 = -20 - 40 + V = 0$$

$$V = 26 \text{ kN}$$

$$\sum M = 0 = -20(2.5) - 40(5.5) + M$$

$$M = -50$$

$$\sum F_y = 0 = -20 - 40 + 14 + V = 0$$

$$V = 26$$

$$\sum M = 0 = -20(2.5) - 40(5.5) + 14(10) + M$$

$$M = 28$$

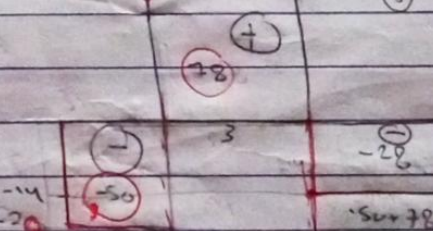
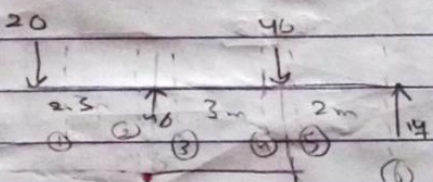
$$\sum M = 0 = -20(2.5) - 40(5.5) + 14(10) + M$$

$$M = 28$$

$$\sum M = 0 = -20(2.5) - 40(5.5) + 14(10) + M$$

$$M = 0$$

$$M = 0$$



max shear = 26

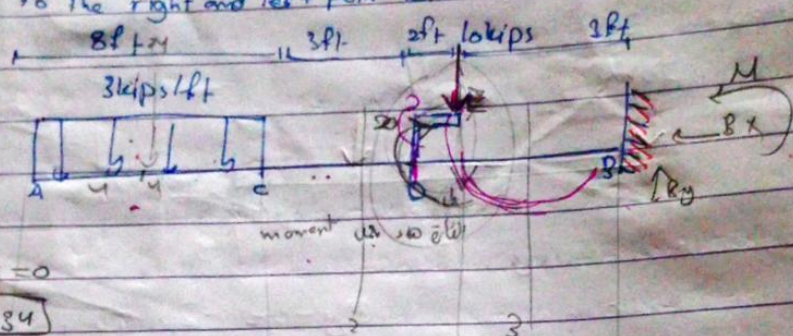
write

$$\frac{V}{I} = \frac{dM}{dx}$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(0.68)(0.25)^2 = 833.33 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{504 \times 10^3 \text{ Nmm}}{833.33 \times 10^{-6} \text{ m}^3} = 60410 \text{ Pa}$$

Sample Problem 5.2: The structure constructed of a 2010 #112 rolled steel beam.  
(a) draw the shear and bending moment diagrams (b) determine normal stress in sections just to the right and left of point D.



$$\sum F_y = -8(3) - 10 + R_D = 0$$

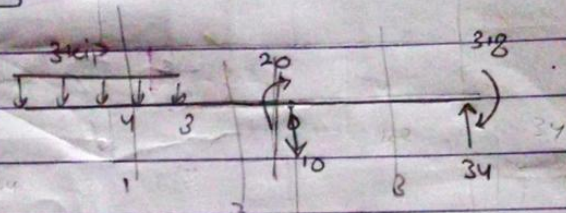
$$R_D = 34$$

$$\sum F_x = 0 = B_x \quad (3a)$$

(b) as  $\sum M = 0$  Moment - 1120 lb D line  $(10 \times 2) = 20 \times 10 = 200 \text{ lb}$  D line

$$\sum M_B = M_B + 30 + (8 \times 3)(12)$$

$$M_B = 318$$



$$0.25 \times 24 = 8 \times 3 \quad 3 \text{ kips/ft}$$

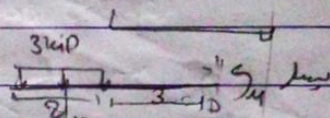
$$10 \times 2 = 20 \text{ kip-ft} \quad 10 \text{ kips/ft}$$

المساحة  $10 \times 2 = 20 \text{ kip-ft}$  والمساحة  $10 \times 2 = 20 \text{ kip-ft}$

$$\sum M = 0 = M + (3 \times 8)(7)$$

$$M = -168 \text{ kip-ft}$$

$$\sum F_y = 0 = -24 + 2 \quad V = 24 \text{ kip}$$



sec 1

$$\sum F_y = -24 - v$$

$$\therefore v = -3(x)$$

$$\sum M = +x \cdot 3 + M = 0$$

$$M = -1.5x^2$$

sec 2

$$\sum F_y = 0 = -24 - v \therefore v = -24 \text{ kip}$$

$$\sum M = M + 24(x-4) = 0$$

$$M = (16 - 24x)$$

sec 3

$$\sum F_y = -24 - 10 - v = 0 \therefore v = -34 \text{ kip}$$

$$\sum M = M - 20 + 10(x-11) + 24(x-4) = 0$$

$$-M = -20 + 10x - 110 + 24x - 96 =$$

$$24(7 + x - 11) - M = -226 + 34x$$

$$M = (226 - 34x) \text{ kip-ft}$$

(a/c)

$$-48 - 148$$

$$= -196$$

(-1) couple of

shear force diagram

$$(168) (48)(102)$$

shear

force

shear force diagram

S =

$$\sigma_m = \frac{|M|}{S} = \frac{2016}{126} \text{ kip/in}$$

$$S = 126$$

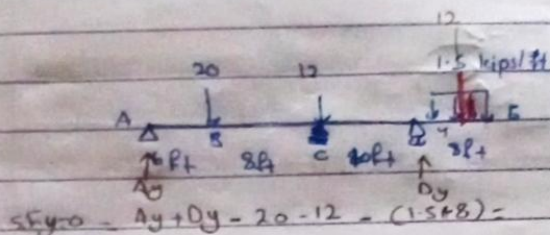
$$2016 \times 112$$

$$= 16 \text{ ksi}$$

$$\sigma_m = \frac{|M|}{S} = \frac{1776}{126} = 14.1 \text{ ksi}$$

# Relation Among load, shear, Bending Moment

Sample problem 5.3 a



$$\sum F_y = 0 \rightarrow A_y + D_y - 20 - 12 - (1.5 \times 8) = 0$$

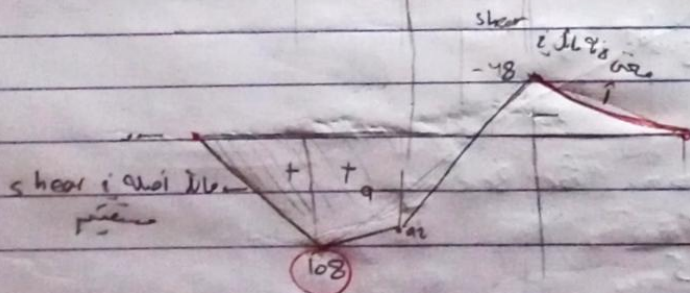
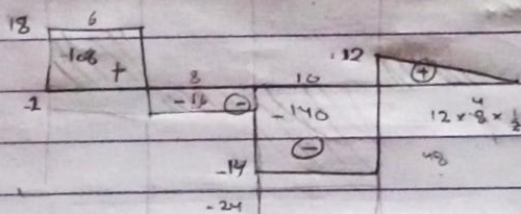
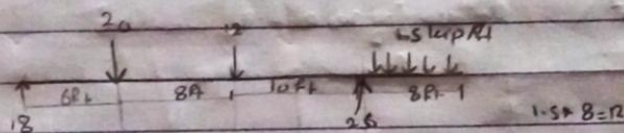
$$A_y + D_y = 44$$

$$\sum M_A = 0 \rightarrow (20 \times 6) + (12 \times 14) - D_y(24) + (1.5 \times 8) \times 28 = 0$$

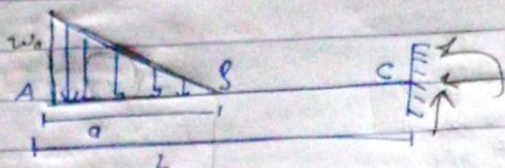
$$A_y = 18$$

$$D_y = 26$$

OK



# Sample Problem 5.5

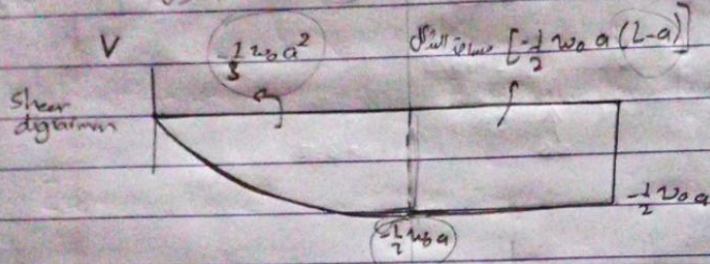


$$\sum F_y = 0 = -\frac{1}{2} w_0 a + R_C$$

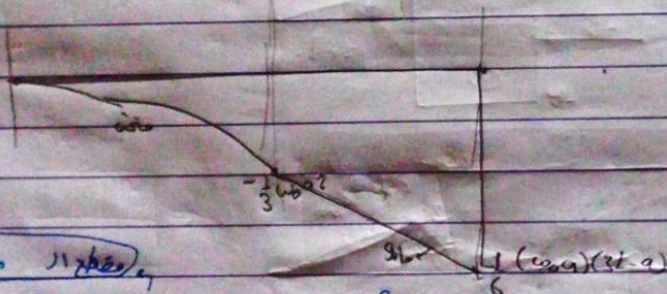
$$R_C = \frac{1}{2} w_0 a$$

(+) Clockwise  
(-) Counter-clockwise

$$\sum M_C = 0 = \frac{1}{2} w_0 a \left[ \frac{L-a}{3} \right] + M_C \quad \therefore M_C = -\frac{1}{6} w_0 a \left[ \frac{L-a}{3} \right]$$



Moment



Prob 5.5 beam 11/2/2019

## Design of Prismatic Beams for Bending

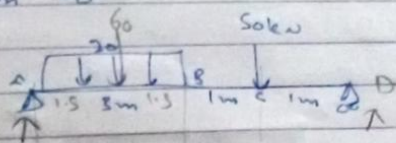
$$\sigma_m = \frac{M}{I} = \frac{M}{I} \quad \sigma_s = \frac{M}{I}$$

Si OIL allow stress on design soft is less than

$$\sigma_m \leq \sigma_{all}$$

$$\sigma_{min} = \frac{M_{max}}{\sigma_{all}}$$

# Sample Problem - 5.8 -



A simply supported steel beam is to carry the distributed and concentrated loads shown knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, select the wide flange shape should be used

$$\sum F_y = A_y - 60 - 50 + D_y = 0$$

$$A_y + D_y = +110$$

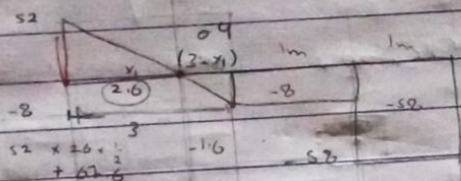
$$\sum M_A = (60 \times 1.5) + 50(4) - D_y(8) = 0$$

$$D_y = 58$$

$$A_y = 52$$

$$\frac{52}{+8} = \frac{x}{3-x}$$

$$x = 2.6$$



$$S_{min} = \frac{M_{max}}{\sigma_{all}}$$

$$= \frac{67.6 \text{ kN.m}}{160 \text{ MPa}}$$

$$= 422.5 \times 10^{-6} = 422.5 \times 10^{-3} \text{ m}^3$$

Choose the best standard section which meets this criteria

$$W = 360 \times 32.9$$

| Shape        | $S_{min}$ |
|--------------|-----------|
| W 410 x 38.8 | 637       |

|              |     |
|--------------|-----|
| W 360 x 32.9 | 474 |
|--------------|-----|

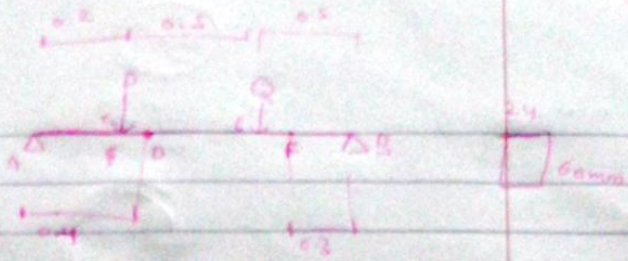
|              |     |
|--------------|-----|
| W 310 x 38.7 | 549 |
|--------------|-----|

|              |     |
|--------------|-----|
| W 250 x 44.8 | 535 |
|--------------|-----|

|              |     |
|--------------|-----|
| W 200 x 46.1 | 448 |
|--------------|-----|

① W 360 x 32.9

Ex 5.64



$$\sum F_y = 0 = A_y - (3.5 \times 3) + B_y = 0$$

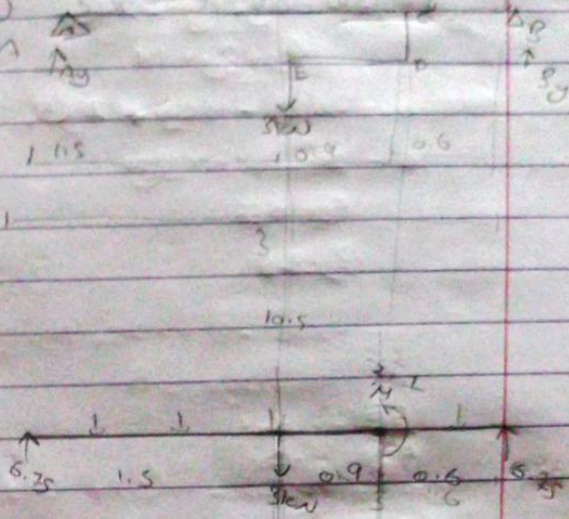
$$A_y + B_y = 10.5$$

$$\sum M_A = 0 = (0.5)(1.5) + 3(1.5) - B_y(3)$$

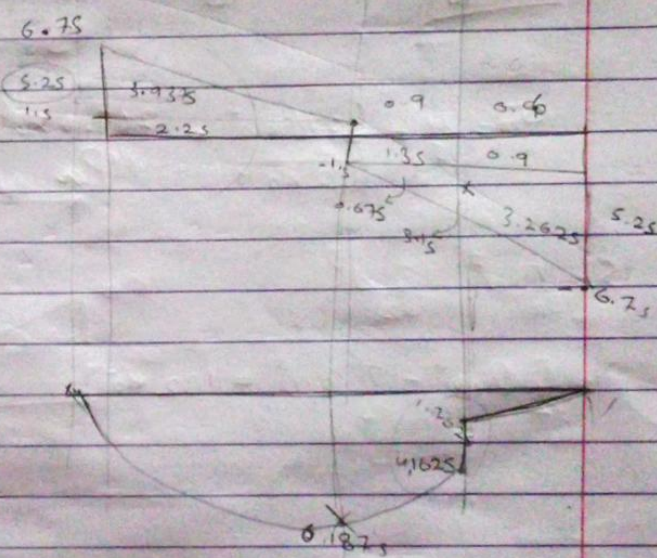
$$B_y = 6.75$$

$$A_y = 3.75$$

$$3 \times 0.9 = 2.7$$



$$\frac{5.25}{x} = \frac{1.5}{0.9}$$

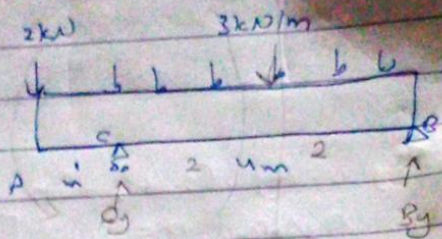
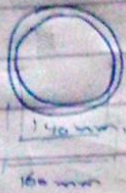


$\Pi_r^4$

12

$$I = \frac{\pi d^4}{32}$$

$$I = \frac{\pi d^4}{32}$$



$$\sum F_y = 0 \rightarrow C_y + B_y = 2 + 12$$

$$C_y + B_y = 14$$

$$\sum M_B = 0 = 12(2) - 4C_y + 2(5)$$

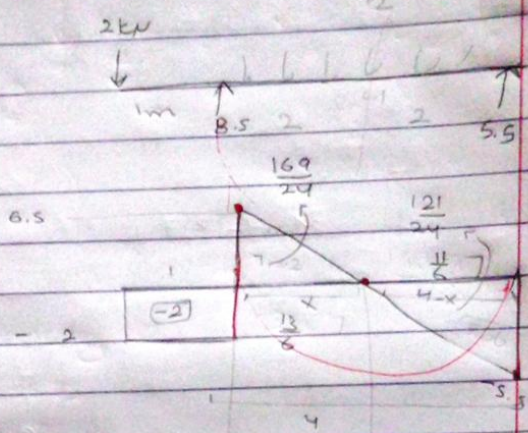
$$-24 + 10 - 4C_y = 0$$

$$C_y = 8.5$$

$$B_y = 5.5$$

$$\frac{5.5}{0.5} = \frac{4-x}{x}$$

$$x = \frac{13}{6}$$



$$\sigma_m = \frac{M}{S} = \frac{121 \cdot 10^3}{1.32 \cdot 10^{-4}} = 299.16 \cdot 10^6 \text{ Pa}$$

$$S = \frac{\pi d^3}{32} - \frac{\pi d_i^3}{32}$$

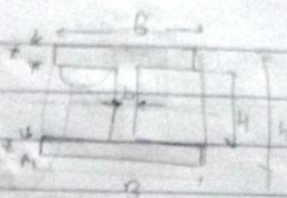
$$= \frac{\pi}{32} (0.16)^3 - \frac{\pi}{32} (0.14)^3 = 1.32 \cdot 10^{-4}$$

2  
12

$$\frac{I_s}{I_r} \frac{b^3}{h^3} \frac{\pi d^4}{32}$$

$$S = \frac{I}{C}$$

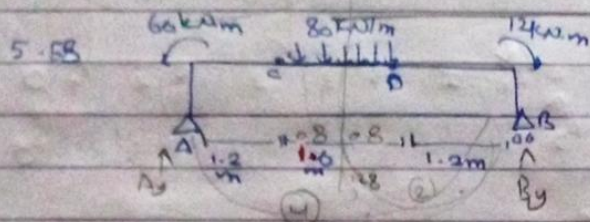
$$S = \left[ \frac{b \cdot h^3}{12} - \frac{(B-b) \cdot (H-2t)^3}{12} \right]$$



$$I = \frac{1}{12} b h^3 + 2 \left( \frac{1}{12} b t^3 \right)$$

$$I = \frac{1}{12} b h^3 + 2 \left( \frac{1}{12} b t^3 \right)$$

المساحة الكلية = 14400 سم<sup>2</sup>



$$W_{250 \times 80} S = 535 \text{ cm}^3$$

$$I = 10000 \text{ cm}^4$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - 128 = 0 \quad [A_y + B_y = 128]$$

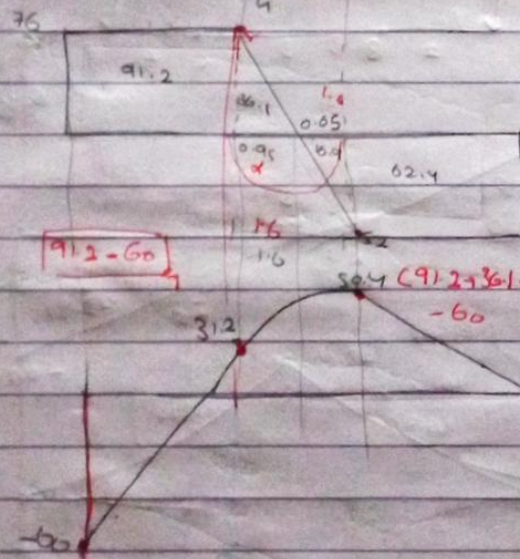
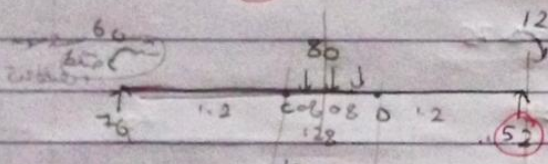
$$\sum M_B = 0 \Rightarrow -12 \cdot 28(7) + 60 \cdot 4 - A_y(4) = 0$$

$$A_y = 76$$

$$B_y = 52$$

$$\sigma = \frac{60 \times 10^3 \text{ N.m}}{535 \times 10^{-6} \text{ m}^3} = 112.14 \times 10^5 \text{ Pa}$$

MPa



$$\frac{76}{52} = \frac{x}{1.6 - x} \quad [x = 0.95]$$

48 نقطة

$$60 \cdot 12 = 48$$

-60 A curve

$$M_A + M_C = -60 \quad [C \text{ curve}]$$

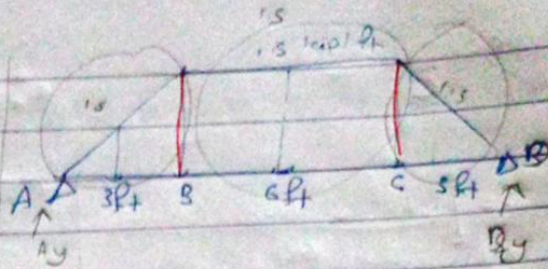
$$-60 + 91.2 = 31.2$$

$$M_A + M_C + M_D = 0 \quad [D \text{ curve}]$$

$$-60 + 91.2 + 30 + 69 = 50.4$$

$$50.4 = 48 \quad [B \text{ curve}]$$

A diagram of a rectangle. The top horizontal side is labeled 'sin' with a bracket above it. The right vertical side is labeled 'h'.



سورة طه ١٠٠  
١٠٠  
سورة طه ١٠٠  
سورة طه ١٠٠

A hand-drawn diagram of a number line from A to D. The line is divided into segments with lengths 1, 2, 3, 3, 2, and 1. Above the line, arrows point to the boundaries of these segments with labels 2.25, 4, and 7.25. Below the line, the segments are grouped into three pairs, each labeled with a '3'.

$$\sum F_y = 0 = -2.25 - 9 - 2.25 + A_y + D_y$$

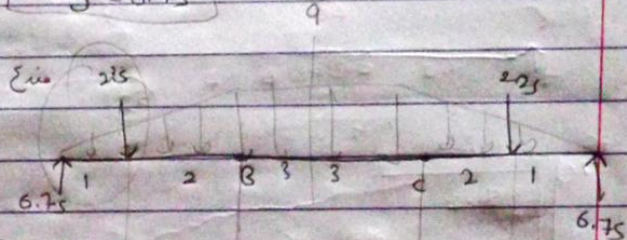
$$A_y + D_y = 13.5$$

$$+ \text{Ans } \sum M_D = 0 = 225(1) + 9(6) + (2.25 \times 11) - A_y(12)$$

$$\underline{81 = 12\Delta y}$$

$$A_y = 6.75$$

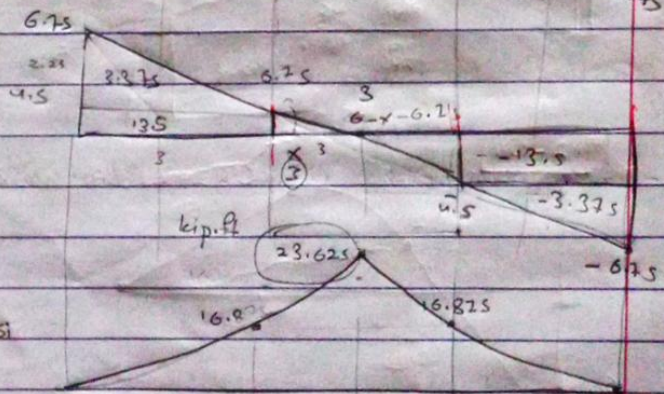
$$Dy = 6.75$$



$$\frac{x}{6-4} = \frac{4.5}{4.5} \text{ (1)}$$

$$6(-x) = x$$

$$2x = 6 \quad \boxed{x = 3}$$



$$\sigma = \frac{M}{S} = \frac{283.5 \text{ kip-in}}{(S)_{in^3}} = 1750 \text{ psi}$$

$$(s)_{in} = 283.5 \text{ kip/in}^2$$

$$1750 \text{ psi (lb) } \times 10^3$$

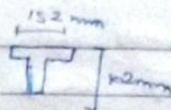
$$(S)_{in}^3 = 0.162$$

$$\frac{1}{6} \cdot 5 \cdot h^2 = 0.162 \quad h = 0.44$$

1 ft = 12 in  
lip 1 ft

(5.79)

Problem 5.79



Two L 102 x 7.5 rolled steel

$$\sigma_{all} = 140 \text{ MPa}$$

determine the min angle thickness that can be used

$$\Sigma F_y = -q - q + Ay + Cy = 0$$

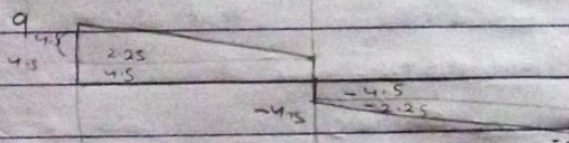
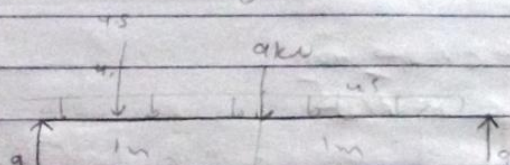
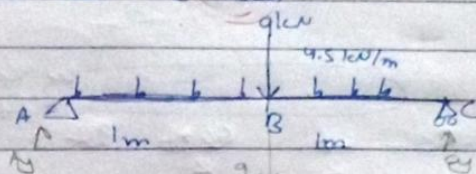
$$18 = Ay + Cy$$

$$\Sigma M_C = q(1) + q(1) - Ay(2) = 0$$

42.5

$$Ay = 9$$

$$Cy = 9$$



$$S = 48.21 \text{ m}^3 = \frac{I}{c}$$

$$\sigma = \frac{M \cdot c}{I}$$

max stress = M at midspan

$$\frac{6.75}{2} \cdot \frac{33.3}{\text{mm}} \cdot \frac{1}{\text{mm}^2}$$

$$3.375 \cdot 10^6 \text{ N} \cdot \text{mm}$$

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{3.375 \cdot 10^6 \text{ N} \cdot \text{mm}}{140 \frac{\text{N}}{\text{mm}^2}}$$

$$S = 24,107 \text{ mm}^3$$

Simplest, cheap

$$7.9 \text{ mm}$$

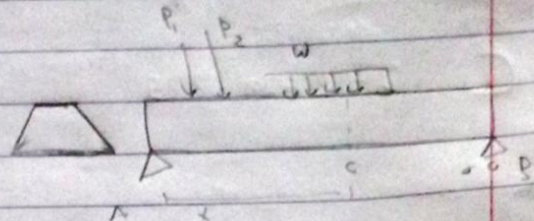
|                  |     |
|------------------|-----|
| $S \text{ mm}^2$ | 1   |
| 23100            | 6.4 |
| 28200            | 7.9 |
| 33300            | 9.5 |

# Chapter - 6 - Shearing stresses

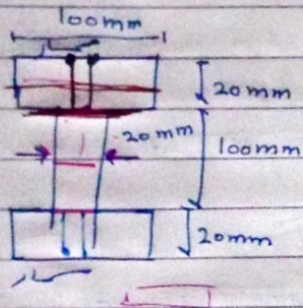
(internal shear force (V)  
vertical shear

$$q(\text{shear flow}) = \frac{VQ}{I} = \pm \tau$$

moment of inertia  
n.A. of plates



Example: 6.01



A beam is made of three planks nailed together. knowing that the spacing between nails is 25mm and that the vertical shear in the beam is  $V=500\text{N}$ , determine the shear force in each nail.

shear force in each nail

$$F = q \cdot s$$

$$q = \frac{VQ}{I}$$

(shear flow) (shear force) (shear flow)

I (shear flow)

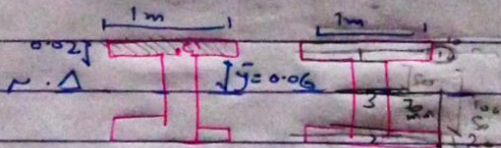
q (shear flow)

$$\text{shear} = \frac{VQ}{I} \cdot s$$

$$2. \quad q = \frac{VQ}{I} = \frac{500\text{N} (120 \times 10^{-6}\text{m}^3)}{16.20 \times 10^{-6}\text{m}^4} = 3704 \text{ N/m}$$

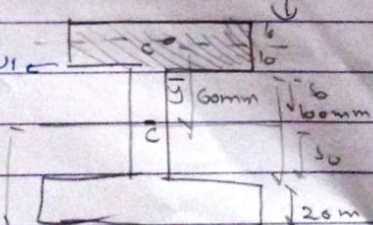
$$Q = A \bar{y}$$

$$(0.020\text{m} \times 0.1) \times (0.06\text{m}) = 120 \times 10^{-6}\text{m}^3$$



shear flow

70



|       | Area                             | $\bar{y}$ | $A\bar{y}$        |
|-------|----------------------------------|-----------|-------------------|
| $A_1$ | $100\text{mm} \times 20$<br>2000 | 120mm     | $240 \times 10^3$ |
| $A_2$ | $100 \times 20$<br>2000          | 10mm      | $20 \times 10^3$  |
| $A_3$ | $20 \times 100$<br>2000          | 70mm      | $140 \times 10^3$ |
|       | 6000                             |           | $400 \times 10^3$ |

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{400 \times 10^3}{6000} = 66.7 \text{ mm}$$

لا بد من

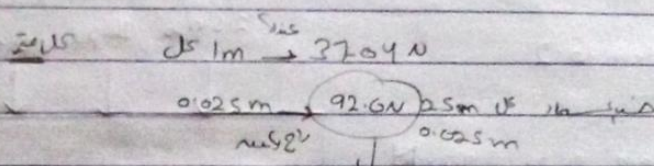
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.02 \text{ m}) (0.1 \text{ m})^3 + 2 \left[ \frac{1}{12} (0.1) (0.02)^3 + (0.02 \times 0.1) (0.05)^2 \right]$$

$$= 16.20 \times 10^{-6} \text{ m}^4$$

(Adg)

مس - 450 ر. 1000  $\Delta_{\text{max}}$  في  $\Delta_{\text{max}}$  (التي هي القوة الواصلة التي لا تتجاوز 1000 ر.)

$$q = 3704 \text{ N/m}$$



$F = 0.025 \text{ kg} \times 3704 \frac{\text{m}}{\text{s}^2} = 92.6 \text{ N}$

horizontal shear stresses

$$\tau_{ave} = \frac{q}{t} = \frac{\Delta H}{\Delta A} = \frac{q \Delta y}{\Delta A} = \frac{VQ}{I} \frac{N}{\Delta H} = \frac{VQ}{I} \frac{1}{t}$$

ملاحظات: (1) دوائر طوع به ضيق اوتية يكن  
Shear stress =  $\tau$

٢٤.  $Q = \text{مقدار (دفتر) منسوب ارتفاع المساحة}$

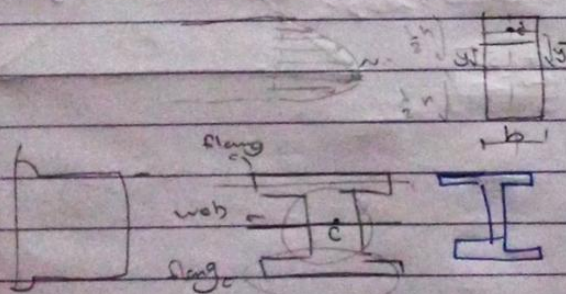
(initial axis) as  $\vec{a} \cdot \vec{a} = \vec{v}_x \cdot \vec{v}_x = \vec{v}_y \cdot \vec{v}_y = 0$  6

٥. لرفع عين المرء كرسماً أخضر في العنق بأخضر تكملة

### Shearing stresses $\tau_{xy}$ in Common Types of Beams

- For a narrow rectangular beam

$$I_{max} = \frac{3V}{2A}$$

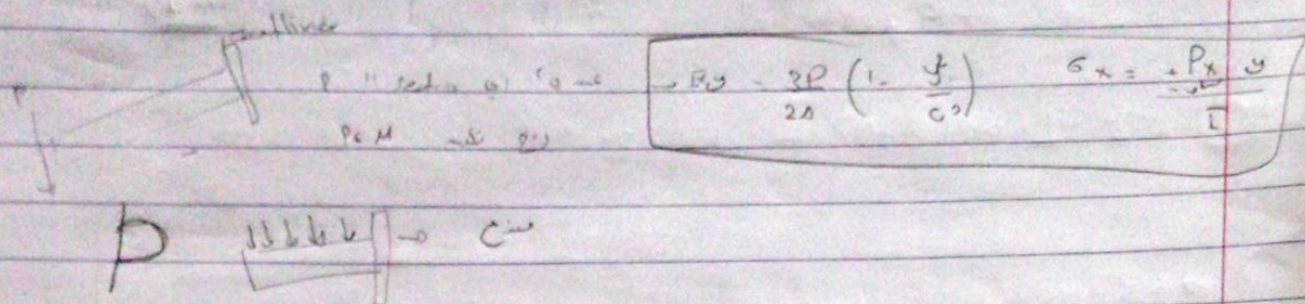


• I beam  $\tau_{\max} = \frac{V}{A_{web}}$

(بیتہ کے والدین کے لئے)

Shed Street

# Further Discussion of the Distributed distribution of stresses in a Narrow Rectangular Beam.



## Sample problem 6.2:

A timber beam is to support the three concentrated loads shown knowing that for the grade of the timber used

$\sigma_{all} = 1800 \text{ psi}$   $\tau_{all} = 120 \text{ psi}$   
determine the minimum required depth  $d$  of the beam.

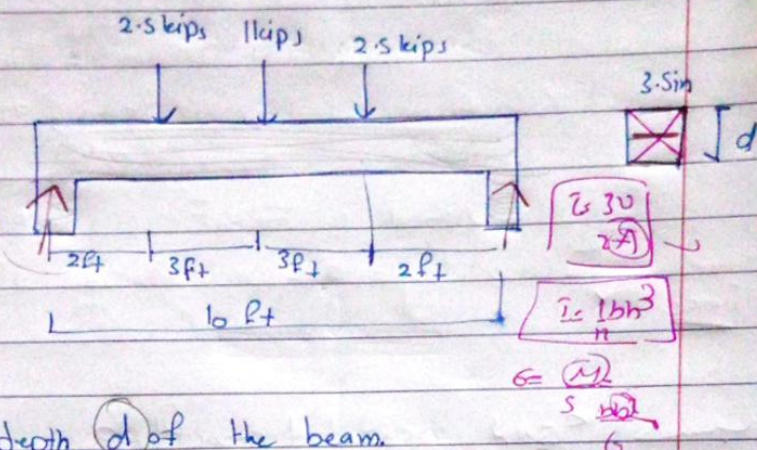


Diagram shear, moment

$$\sum F_y = A + B - 2.5 - 1 - 2.5 = 0$$

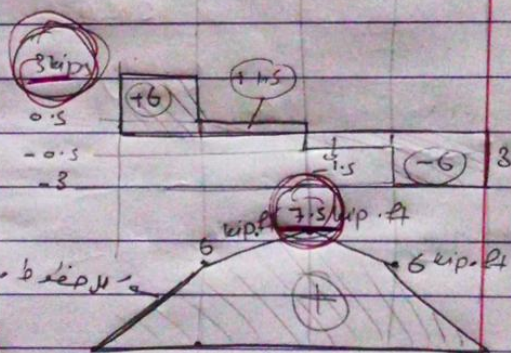
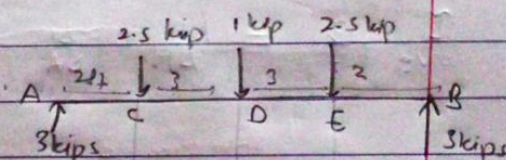
$$A + B = 6$$

$$\sum M_A = 0 = 2.5(2) + 1(5) + 2.5(8) - B(10)$$

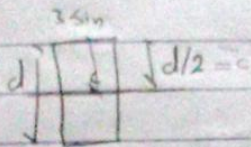
$$B = 3 \quad A = 3$$

$$V_{max} = 3 \text{ kips}$$

$$M_{max} = 7.5 \text{ kip} \cdot \text{ft} = 90 \text{ kip} \cdot \text{in}$$



$$\sigma_{max} = \frac{M}{I}$$



$$\tau_{all} = \frac{M_{max}}{S} = \frac{9000 \text{ lb} \cdot \text{in}}{0.5833 \text{ in}^3} = 15428 \text{ psi}$$

$$M_{all} = 4 \text{ k} \cdot \text{ft} \quad d = 9.26 \text{ in}$$

$$S = \frac{I}{c} = \frac{1}{6} b d^2 = \frac{1}{6} (3.5) d^2 = (0.5833 \text{ in}) d^2$$

$$I = \frac{1}{12} b d^3$$

⇒ determine the allow d on allowable shear stress.

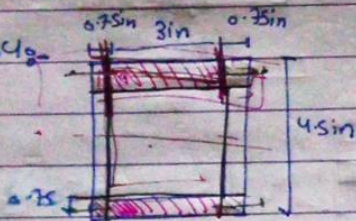
$$\tau_{all} = \frac{3}{2} \frac{V_{max}}{A} = \frac{3}{2} \cdot \frac{3000 \text{ lb}}{(3.5 \text{ in}) \cdot d} = 120 \text{ psi}$$

$$d = 10.71 \text{ in} \quad \text{shear stress}$$

$$10.71 \text{ in} \quad \text{shear stress}$$

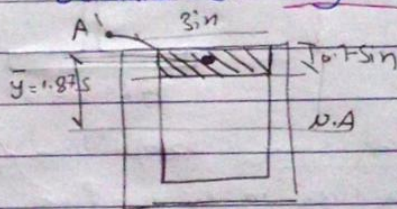
longitudinal shear on a Beam Element of Arbitrary shape:

Example: 604



A square box beam is constructed from four planks as shown, knowing that the spacing between nails is 1.5 in, and the beam is subjected to a vertical shear of magnitude  $V = 600 \text{ lb}$ , determine the shearing force in each nail.

$$Q = A' \bar{y} = (0.75 \cdot 3) \text{ in} \cdot 1.875 \text{ in} = 4.22 \text{ in}^3$$



$$I = \frac{1}{12} b h^3 =$$

lower cross section

0.6028

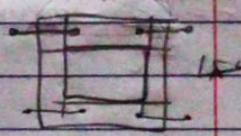
$$I = \frac{1}{12} (4.5) \text{ in}^4 + \frac{1}{12} (3) \text{ in}^4 = 27.42 \text{ in}^4$$

$$q = \frac{V Q}{I} = \frac{600 \text{ lb} \cdot 4.22 \text{ in}^3}{27.42 \text{ in}^4} = 92.3 \frac{\text{lb}}{\text{in}}$$

$$F = \frac{q}{2} = 46.15 \frac{\text{lb}}{\text{in}}$$

edge force per unit length

$$F = 1.5 \cdot 46.15 = 80.8 \text{ lb}$$



## Shearing stress in Thin Walled Members

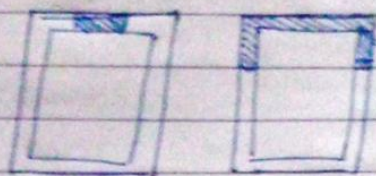


$$\tau = \frac{VQ}{It}$$

$\tau_{yz}$  = flange

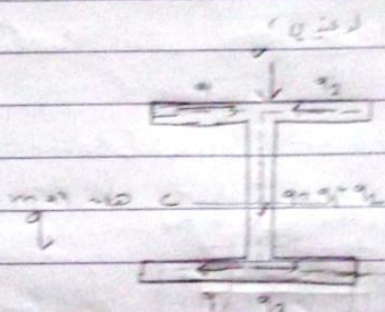
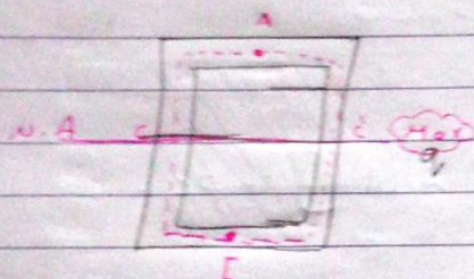
$\tau_{xz}$  = web

∴ profile is for the cross section

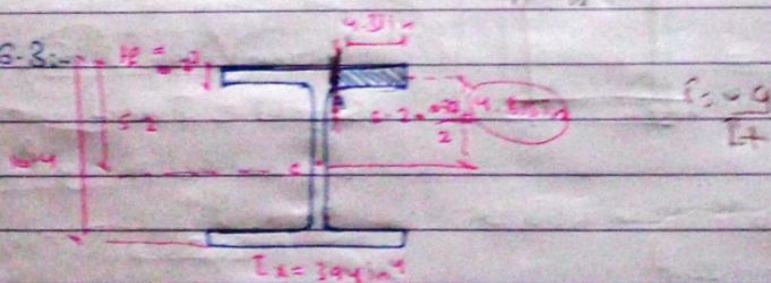


$$q = \tau t = \frac{VQ}{I}$$

shear flow



Sample problem 6.8:



Knowing that the vertical shear is 5 kips in a W10x68 rolled-steel beam, determine the horizontal shearing stress in the top flange at point a.

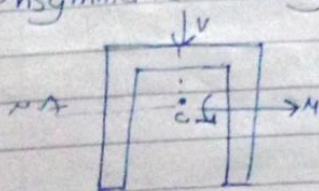
Q = area above + V/I

$$Q = 4.5 \text{ in} \times 0.27 \text{ in} \times 4.8 \text{ kips} = 15.98 \text{ in}^3$$

The shear stress at a,

$$\tau = \frac{VQ}{It} = \frac{50 \text{ kips} \times 15.98 \text{ in}^3}{394 \text{ in}^4 \times 0.27 \text{ in}} = 2.63 \text{ ksi} \quad \left( \frac{\text{kips}}{\text{in}^2} \right)$$

# Unsymmetric loading of Thin-walled Members



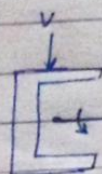
$$\tau_{xy} = \frac{VQ}{It}$$

$$\sigma_x = \frac{My}{I}$$

$$\tau_{xy} = \frac{VQ}{It}$$



- shear flow  
torsion

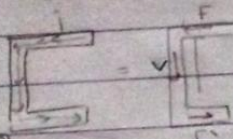


$$\tau_{xy} = \frac{VQ}{It}$$

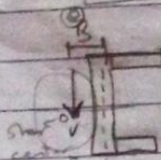
$$\sigma_x = \frac{My}{I}$$

$$\tau_{xy} = \frac{VQ}{It}$$

shear flow

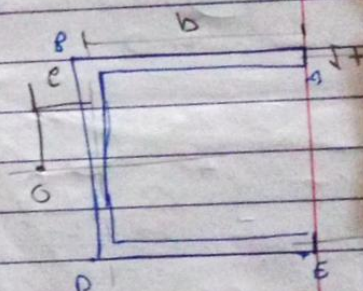
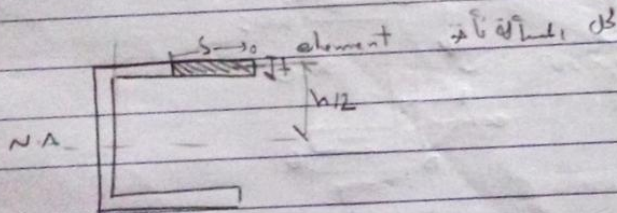


$$\tau_{xy} = \frac{VQ}{It}$$



$$F_h = VQ \rightarrow \text{shear flow}$$

6.05  
- examples - determine the shear center O of a channel section of uniform thickness, knowing the  $b=4\text{in}$ ,  $h=6\text{in}$ ,  $t=0.5\text{in}$



$$q = \frac{VQ}{It} = \frac{Vst}{2I}$$

$$F = \int_0^b q ds = \int_0^b \frac{Vst}{2I} ds = \frac{Vth}{2I} \int_0^b s ds$$

$$F = \frac{Vthb^2}{4I}$$

$$e = \frac{Fh}{V} = \frac{Vthb^2}{4I} \frac{h}{V} = \frac{th^2b^2}{4I}$$

$$I = I_{web} + 2I_{flange} \\ = \frac{1}{12} th^3 + 2 \left[ \frac{1}{12} bt^3 + bt \left( \frac{h}{2} \right)^2 \right]$$

neglecting the term containing  $t^3$

$$I = \frac{1}{12} th^3 + \frac{1}{2} th^2b = \frac{1}{12} th^2(6b+h)$$

$$e = \frac{3b^2}{6b+h} = \frac{b}{2+\frac{h}{3b}}$$

$$\frac{h}{3b} = \frac{6}{3 \times 4} = 0.5$$

$$e = \frac{4}{2+0.5} = 1.6 \text{ in}$$

For the channel section of example 6.05 determine the distribution of the shearing stresses caused by a 2.5 kip vertical shear  $V$  applied at the shear center  $O$

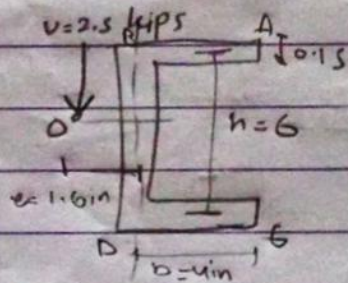
① shear stress in flange

$$\tau = \frac{q}{t} = \frac{VQ}{It} = \frac{Vh}{2I}$$

$$\tau_B = \frac{Vhb}{2 \left( \frac{1}{12} th^3 \right) (6b+h)} = \frac{6Vb}{th(6b+h)}$$

$$\tau_B = \frac{6(2.5 \text{ kips})(4 \text{ in})}{(0.15 \text{ in})(6 \text{ in})(6 \times 4 \text{ in} + 6 \text{ in})} = 2.22 \text{ ksi}$$

$$\tau_A = 0$$

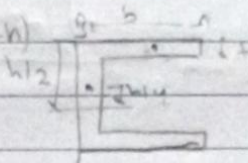


problem no 10

① Shear stress

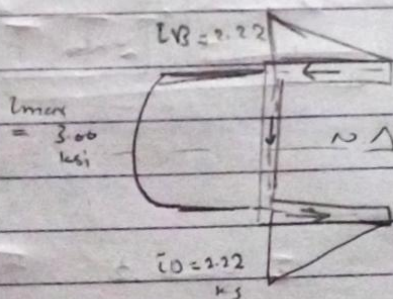
max at neutral axis

$$Q = bt \left(\frac{1}{2}h\right) + \frac{1}{2}ht \left(\frac{1}{4}h\right) = \frac{1}{8}ht(4b+h)$$



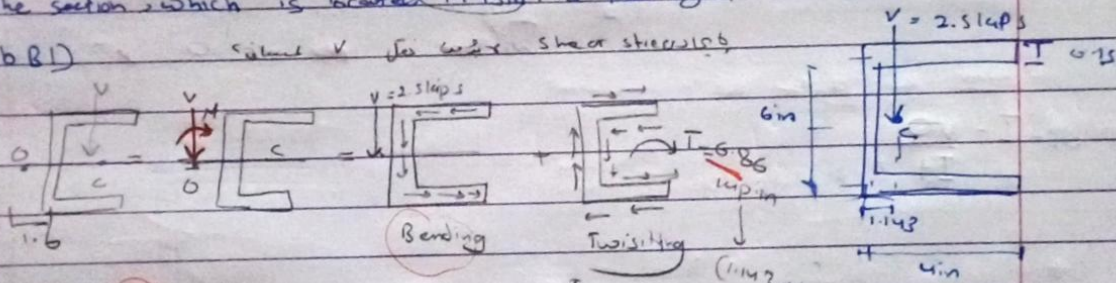
$$I_{max} = \frac{VQ}{t} = \frac{V \left(\frac{1}{8}ht(4b+h)\right)}{\frac{1}{2}ht} = \frac{3V(4b+h)}{2h(6b+h)}$$

$$\tau_{max} = \frac{3(2.5 \text{ kips})(4 + 4 \text{ in} + 6 \text{ in})}{2(0.15 \text{ in})(6 \text{ in} + 6 \times 4 \text{ in} + 6 \text{ in})} = 3.06 \text{ ksi}$$



example 6.7

For the channel section and neglecting stress concentration, determine the max shear stress caused by a 2.5 kip vertical V applied at the centroid C of the section, which is located 1.143 in. to the right of the centerline of the web (D).



1) stress due to bending example 6.6 =  $\tau_{max} = 3.06$

2) stress due to twisting

$$a = 4 + 4 + 6 = 14 \text{ in}$$

$$h = b = 0.15$$

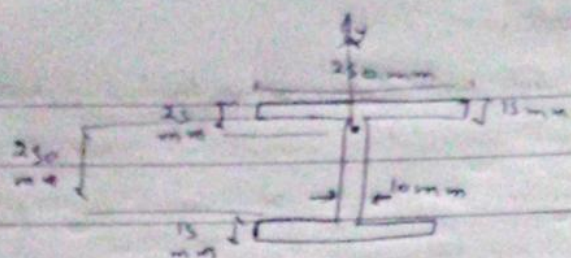
$$b/a = 0.0107$$

$$c = \frac{1}{3}(1 - 0.03b) = 0.331$$

$$(\tau_{max})_{twisting} = \frac{T}{c_1 ab^2} = \frac{68.6 \text{ kip-in}}{0.331 \times 14 \text{ in} \times 0.15 \text{ in}^2} = 65.8 \text{ ksi}$$

$$\tau_{max} = 3.06 + 65.8 \text{ ksi} = 68.9 \text{ ksi}$$

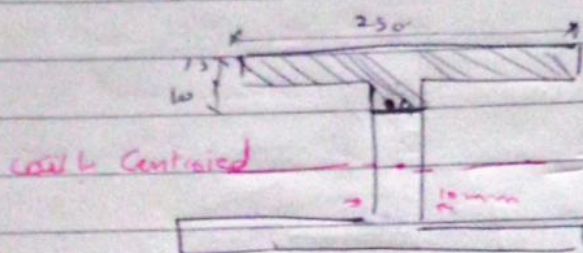
Review:



$$I_{xx} = 0.0001 \text{ m}^4$$

$$I_{yy} = 0.0001 \text{ m}^4$$

shear stress  $\rightarrow$   $\Delta$  is shear stress



Reaction values

I values

Centroidal axis

(it gets like a beam) ... shear stress ...

shear stress ...

...  $Q$  ...

... Area

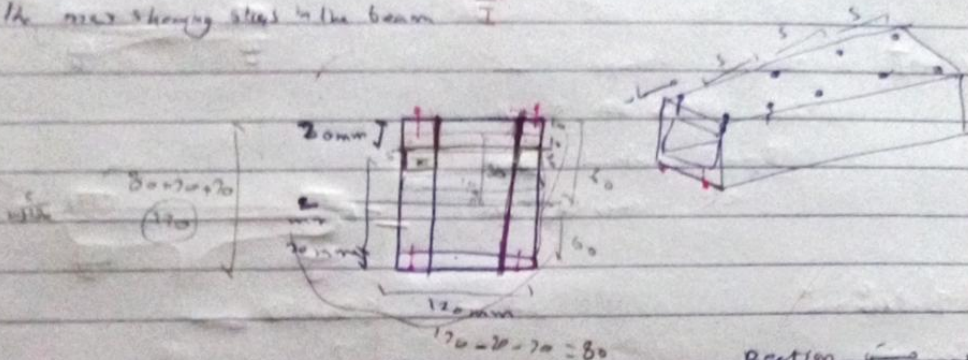
...  $Q$  ...

... center ...

apply  $\tau = \frac{VQ}{It}$

Q4. A square box beam made of two 20x80 mm planks and two 20x120 mm planks nailed together shown. knowing that the spacing between the nails is 30 mm and the vertical shear in the beam  $V = 1200 \text{ N}$ , determine

- (a) shearing force in each nail  $F = q \cdot s = \frac{VQ}{I} \cdot s$   
 (b) the max shearing stress in the beam  $\tau$

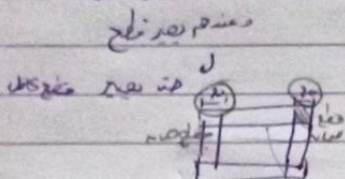


centroid : the centroid is at the center

the formula for the moment of inertia  $I$  is

$$I = \frac{1}{12} b h^3 = \frac{1}{12} b h^3 = \left( \frac{1}{12} (120) (120)^3 \right) + \left( \frac{1}{12} (80 + 80)^3 \right) + 120^2 = 13.8 \times 10^6 \text{ mm}^4$$

the distance between the centroids of the planks is  $20 + 20 = 40 \text{ mm}$



the distance between the centroids of the planks is 40 mm

Q → Area of the center of the beam  $= 120 \times 20 + 80 \times 20 = 120 \times 10^3 \text{ mm}^2$

$$\tau_{max} = \frac{VQ}{It} = \frac{1200 \text{ N} \times 120 \times 10^3 \text{ mm}^2}{13.8 \times 10^6 \text{ mm}^4 \times 40 \text{ mm} \times 10^{-3}} = 260 \text{ kPa}$$

$\tau_{max} = \tau_{min} + \frac{VQ}{It} (t + s)$   
 $= 260 (20 + 30 \times 10^3) = 1560 \text{ N}$

Q  $t = 40$

the center line is the shear max

$Q_{max} = Q_{center}$

the max value of  $Q_{max} = 120 \times 10^3 \times 2 (20 + 40 \times 20) = 152 \times 10^3$

$Q_{max} = \frac{1200 \times 152 \times 10^3}{13.8 \times 10^6 \times 40 \times 10^{-3}} = 329 \text{ kPa}$

the max value of  $\tau$  is 329 kPa

Q5 - The American standard rolled steel beam shown has been reinforced by attaching to it two  $16 \times 200$  mm plates, using 18 mm diameter bolts spaced longitudinally every 120 mm. Knowing that the average allow shear stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.



دیتا: No I : 36 ES2 S shape  
 $16 \times 200$  mm plates  
 18 mm bolts  
 CC shear force

$$I_s = 95.3 \times 10^6 \text{ mm}^4$$

$$d = 305 \text{ mm}$$

$$b_{\text{flange}} = 129 \text{ mm}$$

دیتا

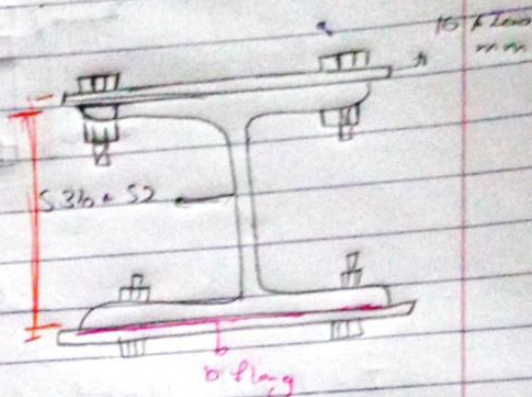


plate I

$$I_{\text{plate}} = \frac{1}{12} b h^3 + A y^2 = 82.5 \times 10^6 \text{ mm}^4$$

$$\frac{1}{12} \times 200 \times 16^3 + (200 \times 16) \times \left( \frac{305}{2} + \frac{16}{2} \right)^2$$

المركز عن مركز الـ I  
 distance between plates

$$I_{\text{total}} = I_s + 2 \times I_{\text{plate}} = 260.3 \times 10^6 \text{ mm}^4$$

دیتا

$$+ = b_{\text{flange}} = 129 \text{ mm}$$

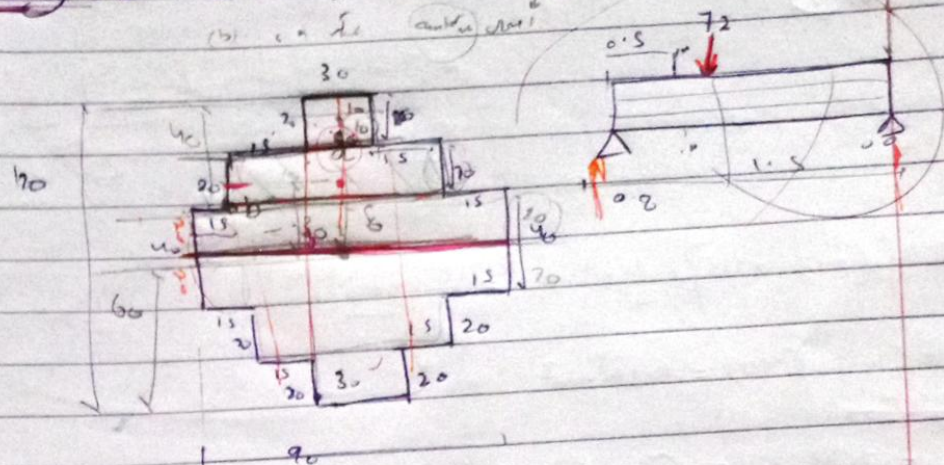
$$Q = \frac{V}{I} = \frac{16 \times 20 \times \left( \frac{305}{2} + \frac{16}{2} \right)}{260.3 \times 10^6} = 513.6 \times 10^{-6} \text{ m}^3$$

$$C = \frac{V}{I} = \frac{V \times (513.6 \times 10^{-6})}{260.3 \times 10^6 \times 129 \times 10^{-3}} = 15.3 \text{ V}$$

المركز عن مركز الـ I

6.9. For beam and loading shown consider section n-n and determine

- (a) largest shearing stress in that section, (b) the shearing stress point a.



$$\sum M_B = 0 \Rightarrow 72 \times 1.5 - A_y \times 2.3 \quad A_y = 47 \text{ kN} \quad \downarrow V$$

$$I = \frac{1}{12}(30)(120)^3 + 2 \left[ \frac{1}{12} 15 \times 60^3 \right] + 2 \left[ \frac{1}{12} 30 \times 40^3 \right] = 5.76 \times 10^{-6} \text{ m}^4$$

$$Q_a = (30 \times 20)(50) = 30 \times 10^3 \text{ mm}^3 \quad t = 3 \text{ mm}$$

$$Q_b = (60 \times 20)(30) + Q_a = 66 \times 10^3 \text{ mm}^3 + Q_a \quad t = 6 \text{ mm}$$

$$Q_c = Q_b + (90 \times 20 \times 10) = 84 \times 10^3 \text{ mm}^3 \quad t = 9 \text{ mm}$$

Area  $\bar{y}$

$$\tau = \frac{VQ}{I t}$$

$$\tau_a = \frac{47 \times 10^3 \times 30 \times 10^3}{5.76 \times 10^{-6} \times 30 \times 10^{-3}} = 8.2 \text{ MPa}$$

$$\tau_b = \frac{47 \times 10^3 \times 66 \times 10^3}{5.76 \times 10^{-6} \times 60 \times 10^{-3}} = 8.98 \text{ MPa (max)}$$

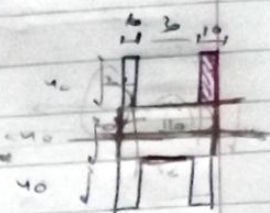
$$\tau_c = \frac{47 \times 10^3 \times 84 \times 10^3}{5.76 \times 10^{-6} \times 90 \times 10^{-3}} = 7.62 \text{ MPa}$$

G13:- For a beam having the cross section shown, determine the largest allowable

shear if the shearing stress is not to exceed 60 MPa

$V_1$

critical point



shear stress

$$I = 2 \left[ \frac{1}{12} (10) (40)^3 + \frac{1}{12} (30) (10)^3 \right] = 3.04 \times 10^{-6} \text{ m}^4$$

$$Q_1 = (10)(40) = 400 \text{ mm}^2$$

$$Q_2 = 80 \text{ mm}^2$$

$$Q_c = 2Q_1 + (30)(10) = 420 \text{ mm}^2$$

center

$$\tau = \frac{VQ}{It}$$

$$60 \times 10^6 = \frac{V_1 (10 \times 10^{-6})}{3.04 \times 10^{-6} (10 \times 10^{-3})}$$

$$V_1 = 114 \text{ kN}$$

$$60 \times 10^6 = \frac{V_c (420 \times 10^{-6})}{3.04 \times 10^{-6} (50 \times 10^{-3})}$$

$$V_c = 217 \text{ kN}$$

2 section  $\rightarrow 2 \times 114 \text{ kN}$   $\rightarrow 228 \text{ kN}$

G33:- The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the A, B

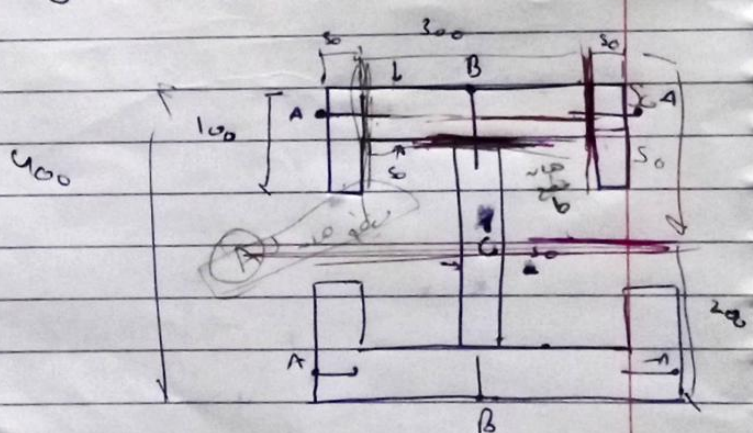
$$I_x = 1.5 \times 10^{-4} \text{ m}^4$$

$$F = \frac{Q}{I}$$

$$Q =$$

$$F = \frac{Q}{I} = \frac{8 \text{ kN}}{1.5 \times 10^{-4} \text{ m}^4} = 53333 \text{ N/m}$$

$$F = q \times s$$



point(A)

$$t = 50 \text{ mm}$$

$$Q = 50 \times 100 \times 150 = 750 \times 10^3 \text{ mm}^3$$

area of 1st Area

vertical shear

$$\tau = \frac{VQ}{It} = \frac{8 \times 10^3 \times 750 \times 10^3}{1.504 \times 10^8 \times 50 \times 10^{-3}} = 79.78 \text{ kPa}$$

$$\tau_{max} = 79.78$$

$$A \rightarrow F = \tau A = 79.78 \times 10^3 \times (50 \times 60 \times 10^{-6}) = 239.4 \text{ N}$$

Point(B)

$$t = 50$$

$$Q = 2QA + QB \Rightarrow 300 \times 50 \times 175 + 2 \times 50 \times 100 \times 150$$

$$QB = 4.125 \times 10^8$$

$$C = \frac{VQ}{It} = \frac{8 \times 10^3 \times 4.125 \times 10^8}{1.504 \times 10^8 \times 50 \times 10^{-3}} = 439 \text{ kPa}$$

$$F = \tau A = (439 \times 10^3) \times (50 \times 25 \times 10^{-6}) = 548.75 \text{ N}$$

16.34: knowing that a  $360 \times 177$  rolled steel beam is subjected to a 250 kN

vertical shear, determine shear stress point A, C

$$I_x = 369 \times 10^6 \text{ mm}^4$$

(20)

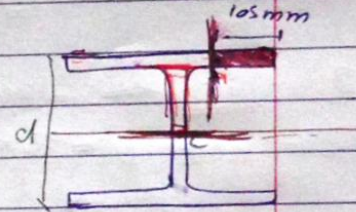
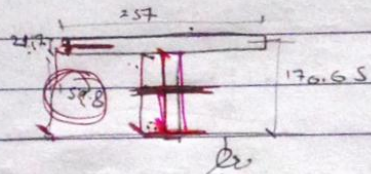
section is C

$$d = 363 \text{ mm}$$

$$b_{flange} = 21.7 \text{ mm}$$

$$b_{web} = 13 \text{ mm}$$

$$b_{flange} = 25.7$$



$$\tau = \frac{VQ}{It}$$

$$Q = (363 \times 21.7) \times \left( \frac{363}{2} - \frac{21.7}{2} \right) = 389 \times 10^3 \text{ mm}^3$$

$$C = 250 \times 10^3 + 389 \times 10^3 = 12.28 \text{ MPa}$$

$$363 \times 10^3 \times (21.7 \times 10^3)$$

flange area + web area

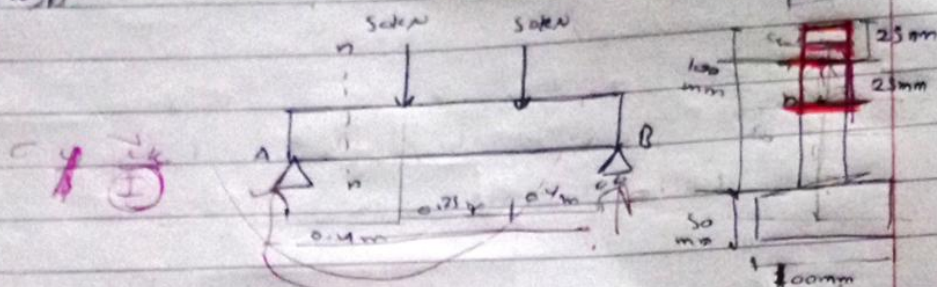
$$Q_c = Q_{flange} + Q_{web}$$

$$(25.7 \times 21.7 \times 170.65) + (13 \times 170.65 \times 170.65) =$$

$$\tau = \frac{250 \times 10^3 + 1.12 \times 10^3}{305 \times 10^3 \times 13 \times 10^3} = 59 \text{ MPa}$$

Q92. For the beam and loading shown, consider section n-n and determine the shear stress  $\tau$  at a/b

Q92  
14



Find the shear stress at point a/b

|          | Area                   | $\bar{y}$ | $A \cdot \bar{y}$ |
|----------|------------------------|-----------|-------------------|
| $A_1$    | $50 \times 100 = 5000$ | 25        | 125000            |
| $A_2$    | $25 \times 100 = 2500$ | 100       | 250000            |
| $\Sigma$ | 7500                   |           | 375000            |

$$\bar{Y} = \frac{\Sigma A \cdot \bar{y}}{\Sigma A} = \frac{375000}{7500} = 50 \text{ mm}$$

$$I_1 = \frac{1}{12} (100) \times 50^3 + 5000 (50 - 25)^2 = 4.17 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} 25 (100)^3 + 2500 (50 - 100)^2 = 8.33 \times 10^6 \text{ mm}^4$$

$$I_{\text{total}} = 12.5 \times 10^6 \text{ mm}^4$$

$$Q_a = 25 \times 25 \times 50 = 3125 \text{ mm}^3$$

$$Q_b = 25 \times 50 \times 75 = 9375 \text{ mm}^3$$

$$\tau = \frac{VQ}{Ib}$$

$$\tau = \frac{VQ}{Ib}$$

$$\tau_a = \frac{50 \times 10^3 \times 3125}{12.5 \times 10^6 \times 25 \times 10^{-3}} = 8.75 \text{ MPa}$$

$$\tau_b = \frac{50 \times 10^3 \times 9375}{12.5 \times 10^6 \times 25 \times 10^{-3}} = 15 \text{ MPa}$$

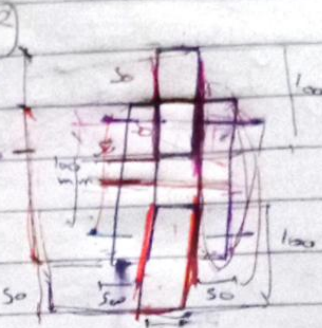
Q92  
14

6.93 - The built up timber beam is subjected given vertical shear knowing that the longitudinal spacing of the nails is  $s = 60 \text{ mm}$  and that each nail 9 mm long, determine the shearing force in each nail

$$I_1 = \frac{1}{12} \times 50 \times 150^3 = 11.46 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} \times 50 \times 100^3 + 50 \times 100 \times (75)^2 = 92.7 \times 10^6 \text{ mm}^4$$

$$I_{\text{total}} = 2I_1 + 2I_2 = 212.7 \times 10^6 \text{ mm}^4$$

$$Q = 100 \times 50 \times 75 = 375 \times 10^3 \text{ mm}^3$$


$$\tau = \frac{VQ}{I_t} = \frac{60 \times 10^3 \times 375 \times 10^3}{212.7 \times 10^6 \times 100 \times 10^{-3}} = 242.7 \text{ MPa}$$

$$F = \tau A$$

$$242.7 \times 10^3 \times 60 \times 50 \times 10^{-6} = 728.1 \text{ N}$$

donor a piece SD nail for shear joint

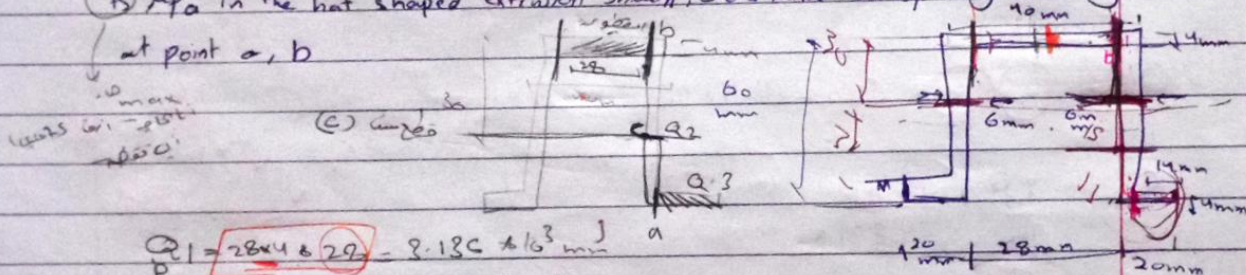
$$F = \tau A$$

shear joint (dial)

$$F = \tau A$$

6.94 - knowing that given vertical shear  $V$  causes a maximum shearing stress of

$75 \text{ MPa}$  in the hat shaped extension shown, determine corresponding shearing stress at point a, b



$$Q_1 = 28 \times 4 \times 29 = 3.136 \times 10^3 \text{ mm}^3$$

$$Q_2 = 30 \times 6 \times 15 = 2.7 \times 10^3 \text{ mm}^3$$

$$Q_3 = 14 \times 28 \times 14 = 1.568 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_2 + Q_3$$

$$\frac{Q_c}{t_c} = \frac{Q_3}{t_a} = \frac{Q_1}{t_b}$$

$$\frac{1.568 \times 10^3}{6} = \frac{1.568 \times 10^3}{4} = \frac{3.136 \times 10^3}{2 \times 4}$$

at point a, b

shear

FS max

$$0.711$$

$$0.392 \quad 0.392$$

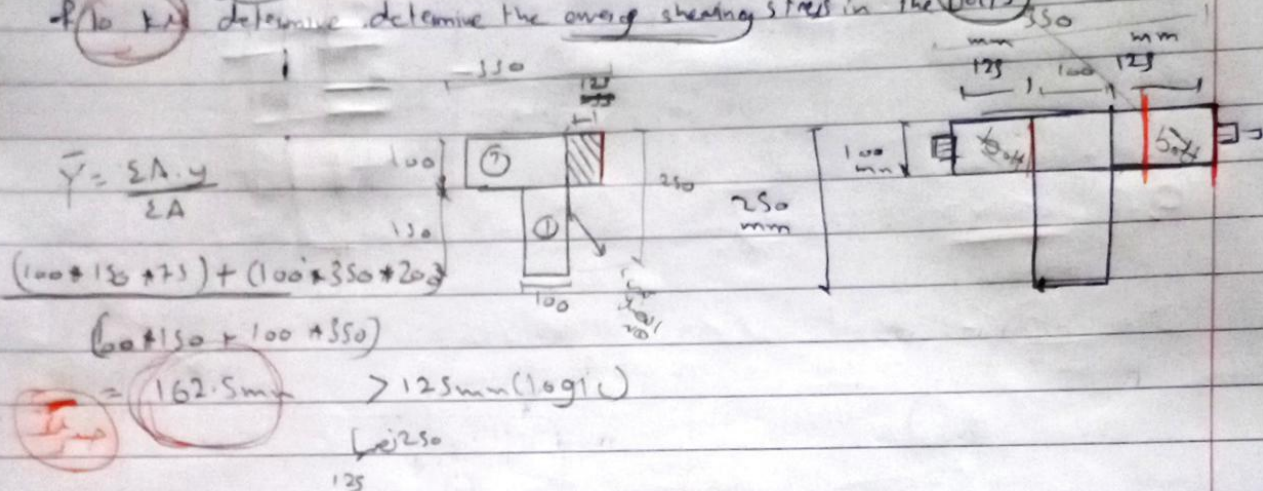
$$C = \frac{V Q}{I \tau}$$

$$0.711 : 0.392 : 0.392$$

$$75 \text{ MPa} : 41.4 : 41.4$$

$$\tau_a = \tau_b = 41.4$$

6.95- Three planks are connected as shown by bolts of 14mm diameter spaced every 150mm along the longitudinal axis of the beam. For a vertical shear of 10 kN determine the average shearing stress in the bolts.



$$\bar{Y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$(100 \times 150 \times 75) + (100 \times 350 \times 200)$$

$$(60 \times 150 + 100 \times 350)$$

$$= 162.5 \text{ mm} > 125 \text{ mm (logic)}$$

$$y_1 = 250$$

$$125$$

$$I_{total} = I_1 + I_2 = 221 \times 10^{-6} \text{ m}^4$$

$$I_1 = \frac{1}{12} 100 \times 150^3 + 100 \times 150 (75 - 162.5)^2 = 143 \times 10^{-6} \text{ m}^4$$

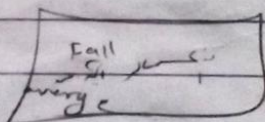
$$I_2 = \frac{1}{12} 350 \times 100^3 + 100 \times 350 (200 - 162.5)^2 = 78.4 \times 10^{-6} \text{ m}^4$$

$$Q_{bolt} = 25 \times 100 \times (200 - 162.5) \times 10^{-6} = 469 \times 10^{-6} \text{ m}^3$$

$$q_{bolt} = \frac{V Q}{I} = \frac{10 \times 10^3 \times 469 \times 10^{-6}}{221 \times 10^{-6}} = 21.2 \text{ kN/m}$$

$$F_{bolt} = q \times s = 21.2 \times 0.15 = 3.18 \text{ kN}$$

$$\tau = \frac{F}{A} = \frac{3.18 \times 10^3}{155} = 20.7 \text{ MPa}$$



$$\tau = \frac{V Q}{I \tau}$$

$$q = \frac{V Q}{I}$$

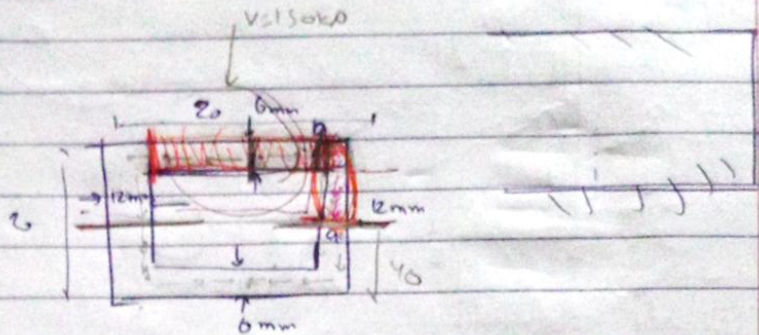
$$\tau = \frac{V Q}{I A}$$

$$\tau_s = \frac{F}{A}$$

6.35 An extended aluminum beam has shown cross section  
 Knowing the the vertical shear in the beam is 150kN determine  
 the shearing stress at point a/b

$$\tau = \frac{VQ}{It}$$

$$\tau = \frac{VQ}{It}$$



$$Q_b = Area \cdot d$$

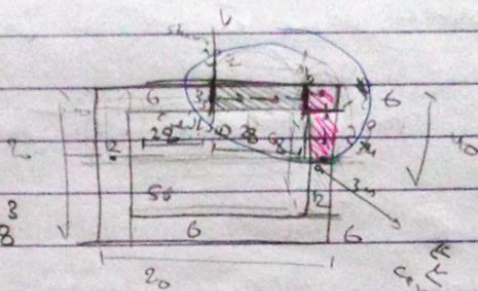
for Q<sub>b</sub>

$$Q_a = A_b d_b + A_{a'b'} d_{a'b'}$$

$$I = \frac{1}{12} b h^3 - \frac{1}{12} b h^3$$

$$= \frac{1}{12} \cdot 80 \cdot 80^3 - \frac{1}{12} \cdot 56 \cdot 68^3$$

$$= 1.945 \times 10^{-6} \text{ m}^4 = 1.945 \times 10^{-6} \text{ m}^4$$



$$Q_b = A_b d_b = \left( \frac{56}{2} \times 6 \right) \times 37 = 6.216 \times 10^3 \text{ mm}^3 \Rightarrow 6.216 \times 10^{-6} \text{ m}^3$$

$$Q_a = Q_b + Q_{a'}$$

$$= (6 \times 6 + 34) \times 6 \times 68$$

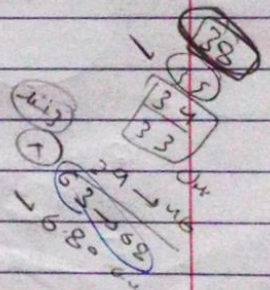
$$= 12 \times 68 \times 6 + 12 \times 40 \times 20 = 9.60 \times 10^3 \text{ mm}^3$$

$$9.6 \times 10^{-6} \text{ m}^3$$

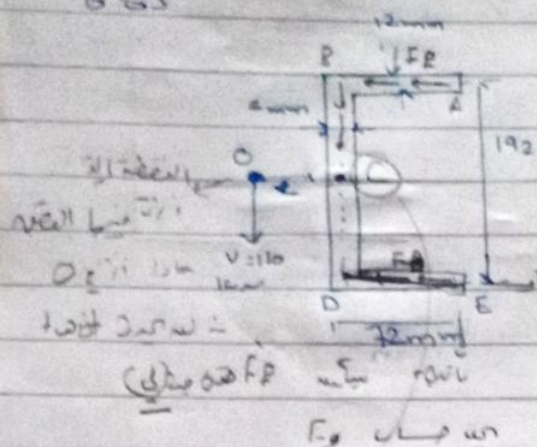
$$t_b = 6 \text{ mm} \quad t_a = 12 \text{ mm}$$

$$\tau_a = \frac{V Q_a}{I t_a} = \frac{150 \times 10^3 \times 9.6 \times 10^{-6}}{1.945 \times 10^{-6} \times 0.012}$$

$$\tau_b = \frac{150 \times 10^3 \times 6.216 \times 10^{-6}}{1.945 \times 10^{-6} \times 0.006}$$



6.63



vertical shear stress  $\tau$  is a force

$$b = 12 - \frac{6}{2} = 9 \text{ (69)}$$

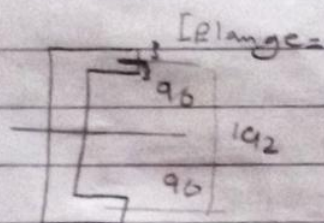
position

Force

Shear stress distribution is zero at the top and bottom edges

12 mm, 192 mm, shear stress  $\tau$  is a force

$$I_{web} = \frac{1}{12} b h^3 = \frac{1}{12} (12) (192)^3 = 3.53 \times 10^6 \text{ mm}^4 \Rightarrow 3.53 \times 10^{-6} \text{ m}^4$$



$$\text{Area Flang} = 69 \times 96 = 6624 \text{ mm}^2$$

$$I_{flang} = \frac{1}{12} b h^3 + \text{Area } d^2$$

$$= \frac{1}{12} (72 \times 6^3) + 6624 (96)^2 = 7.96 \times 10^8 \text{ mm}^4$$

$$I_{total} = I_{web} + 2 I_{flang} = 3.53 \times 10^6 \text{ mm}^4 + 2 (7.96 \times 10^8 \text{ mm}^4) = 1.59 \times 10^9 \text{ mm}^4$$

shear stress  $\tau$  is a force

$$q = \frac{V Q}{I}$$

$$Q = s \cdot t \cdot d = (5) (12) (96) = 1152 \text{ mm}^3$$

$$F_E = \int_0^b q ds = \int_0^b \frac{V}{I} 1152 (s) ds = \frac{V}{I} 1152 \frac{s^2}{2} \Big|_0^b$$

$$F_p = \int_0^b q \, ds = \frac{V}{I} + r \cdot \frac{d \cdot b^2}{2}$$

$$F_p = \frac{110 \times 10^3}{18.8 \times 10^{-6}} \times 0.012 + 0.096 \times \frac{(0.069)^2}{2} = 16.05 \text{ kN} = 16.05 \text{ kN}$$

C is F.M.I.C

$$V \cdot r = F_p \cdot h$$

$$h = 192 \text{ mm}$$

$$e = \frac{F_p \cdot h}{V} = \frac{16.05 \text{ kN} \times 192 \text{ mm}}{110 \text{ kN}} = 28 \text{ mm}$$

new value for  $V$  is 2

$$T = \frac{V \cdot Q}{I}$$

$$Q = 69 \times 12 \times 96 = 79.49 \times 10^3 \text{ mm}^3 \quad \text{Flang } \textcircled{1}$$

$$\tau = \frac{110 \times 10^3 \times 79.49 \times 10^3}{18.8 \times 10^{-6} \times 0.012} = 38.74 \text{ MPa} \quad \text{B is}$$

$$\tau_{\text{top of web}} = \frac{T \cdot t_p}{t_w} \quad \text{web } \textcircled{2}$$

$$= \frac{38.74 \times 12}{6} = 77.48 \text{ MPa}$$

$$Q_{\text{half web}} = 6 \times 96 \times \frac{96}{2} = 27648 \text{ mm}^3 \quad \text{in the } \textcircled{3}$$

$$Q_{\text{total in center}} = 79.488 + 27648 = 107.14 \times 10^3 \text{ mm}^3$$

$$\tau_{\text{max}} = \frac{110 \times 10^3 \times 107.14 \times 10^3}{18.8 \times 10^{-6} \times 0.006} = 104.5 \text{ MPa}$$