

Strength of materials - chapter 7

Stress

11/9/2020

$$\begin{bmatrix} \tau_{xy} & \tau_{xz} & \tau_{yz} \\ \tau_{yx} & \tau_{zx} & \tau_{zy} \end{bmatrix} \quad \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z \end{bmatrix}$$

مكونات الإجهاد

في حالة دوران الجسم - (يكون الوضع متساوي الإجهاد) $\sigma_x = \sigma_y = \sigma_z$ (في حالة التواء) (يتم تحليله بشكل منفصل فيما بعد).

1- **Plane stress** - حالة الإجهاد في مستوى واحد

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$



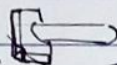
مكونات الإجهاد

في حالة الإجهاد في مستوى واحد (Plane stress) - حالة الإجهاد في مركز العنبر



State of plane stress

center of the element



Shaft

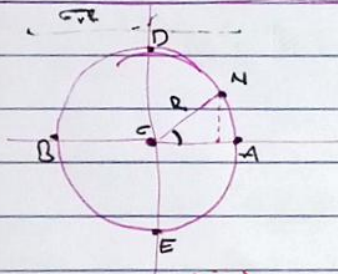
Internal stress distribution

$$(1) \quad \sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad \text{Plane stress: } \sigma \text{ (normal)}$$

$$(2) \quad \sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \quad \text{Plane stress: } \sigma \text{ (normal)}$$

$$(3) \quad \tau_{xy}' = -\left[\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \right]$$

مكونات الإجهاد في كل نقطة



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2} \right]^2 + \tau_{xy}^2}$$

Principle stress: σ_{max} and σ_{min} are the principal stresses.

R = Radius

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left[\frac{\sigma_x - \sigma_y}{2} \right]^2 + \tau_{xy}^2}$$

Angle of rotation

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Max shearing

E = 200 GPa

$$\tau_{max} = \tau = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

or

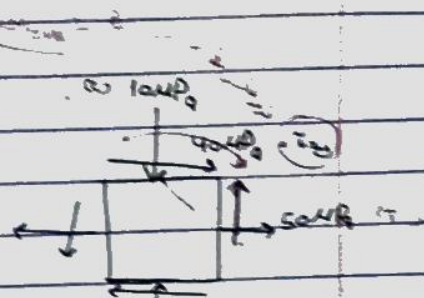
Example 7.01

For the state of plane stress shown

(a) the principle planes angles θ_p

(b) the principle stresses $\rightarrow \sigma_{max, min}$

(c) the max shear stress and corresponding normal stress



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

equation for principle stress

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

principle stress

$$\tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

max shear

$$\sigma' = \frac{\sigma_x + \sigma_y}{2}$$

or multiply

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(40)}{50 - (-10)} = 1.333$$

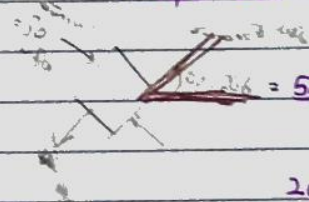
$$2\theta_p = 53.1^\circ, 233.1^\circ$$

$$\sigma_x = 50, \sigma_y = -10$$

$$\theta_p = 26.5^\circ, 116.5^\circ$$

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

Principle stress



$$\sigma_{max, min} = \frac{50 + (-10)}{2} \pm \frac{\sqrt{(50 - (-10))^2 + 4(40)^2}}{2}$$

$$20 \pm \sqrt{(30)^2 + (40)^2}$$

$$\sigma_{max} = 70 \text{ MPa}$$

$$70 + 10 = 80$$

$$\sigma_{min} = -30 \text{ MPa}$$

$$50 - 10 = 40$$

$$\tau_{xy} = +40$$

$$\sigma_x = 50$$

$$\sigma_y = -10$$

{ 60 MPa and 20 MPa principal stresses }
directions principal

$$\sigma_x = 50 \quad \sigma_y = 10$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

$$\theta_s = \theta_{p-45}$$

$$\theta_s = -18.4^\circ, 71.6^\circ$$

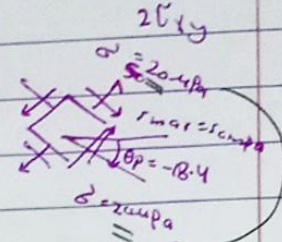
shear stress

plane stress

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 10}{2}$$

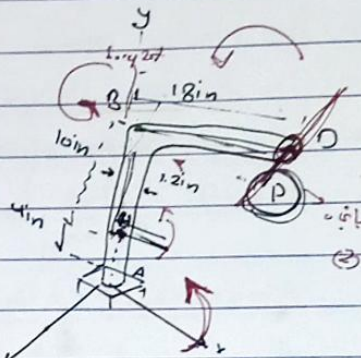
$$\sigma' = 30 \text{ MPa}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$



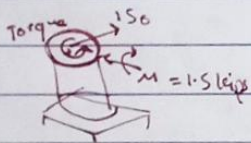
Sample Problem 7.1

A single horizontal force P of 150 lb magnitude is applied to end D of lever ABD



(a) normal and shearing stresses on an element at point H having sides parallel to the x and y axes

(b) the principle planes and principle stresses at the point H .



moment $M = P \cdot L = 150 \cdot 18 = 2700 \text{ lb-in}$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1) (18)^3 = 486 \text{ in}^4$$

$$M_x = 150 \cdot 18 \text{ in} = 2700 \text{ lb-in}$$

$$S_x = \frac{I}{c} = \frac{486}{9} = 54 \text{ in}^3$$

shear force $V = P = 150 \text{ lb}$

$$Q = 0$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{150 \cdot 0}{486 \cdot 1} = 0$$

$$\sigma_x = \frac{M_y}{I} = \frac{2700}{486} = 5.56 \text{ ksi}$$

normal stress σ_x

(1) $\sigma_x = 0$

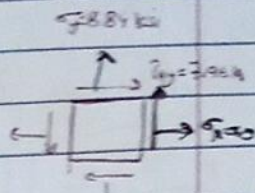
$$\frac{I_y}{I_x} = 9$$

$$\sigma_x = 0$$

$$\frac{M_x}{I} = \frac{15 \text{ kip-in}}{\frac{1}{4} \pi (6 \text{ in})^4}$$

$$\sigma_{xy} = \tau_{xy} = \frac{24 \text{ kip-in}}{\frac{1}{2} \pi (6 \text{ in})^4}$$

$$\sigma_x = 0 \quad \sigma_y = 12.81 \text{ ksi} \quad \tau_{xy} = 12.96 \text{ ksi}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(12.96)}{0 - 12.81} = -1.8 \quad -61.0^\circ = 119^\circ$$

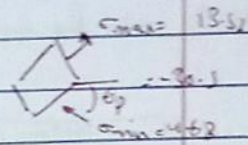
$$2\theta_p = -61.0^\circ, 119^\circ$$

$$\theta_p = -30.5^\circ, 59.5^\circ$$

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

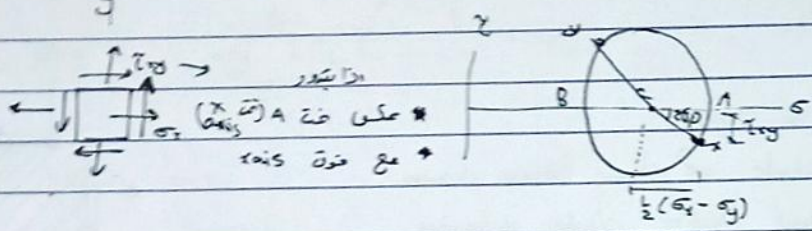
$$\frac{0 + 12.81}{2} \pm \sqrt{\left(\frac{0 - 12.81}{2}\right)^2 + 12.96^2}$$

$$\sigma_{max} = 13.57 \text{ ksi} \quad \sigma_{min} = -4.68 \text{ ksi}$$



part 2-

Mohr's Circle for plane stress

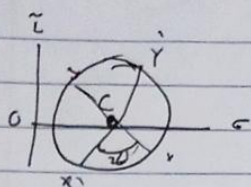


Mohr's Circle for plane stress

$$\sigma_{max, min} = \sigma_{ave} \pm R$$

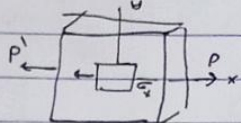
Mohr's Circle

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

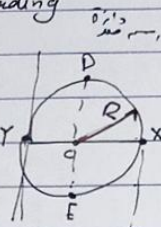


centric

Mohr's circle for axial loading

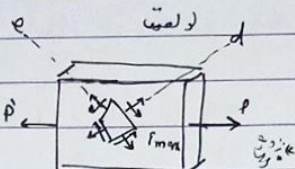


axial loading, $\sigma_x = \frac{P}{A}$ $\sigma_y = \tau_{xy} = 0$



circles are on sigma axis, $\sigma_x = P/A$

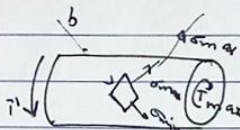
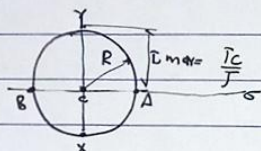
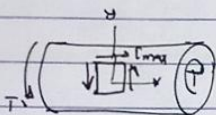
principle axis



$\sigma_x = \sigma_y = \tau_{xy} = \frac{P}{2A}$

max shear stress

Mohr's circle for torsional loading

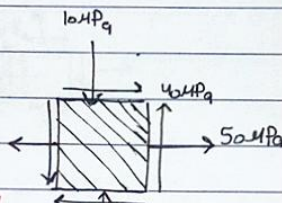


$\sigma_x = \sigma_y = \frac{Tc}{J}$ $\tau_{xy} = 0$

normal stresses also shear stresses

example 7.02:-

For the state of plane stress shown.

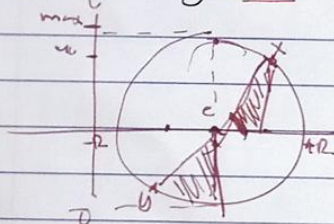


(a) Construct Mohr's circle, determine

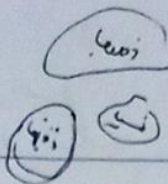
(b) the principle planes $\tan 2\theta_s = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{8}{10 - 5} = 1.6$ $\theta_s = 45^\circ$

(c) the principle stresses $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

(d) the max shearing stress and the cross bonding normal stress



max stress, min stress



$$\sigma_x = 50$$

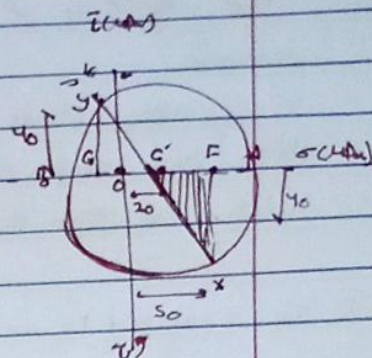
$$\sigma_y = -10$$

$$\tau_{xy} = 40 \text{ MPa}$$

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + (-10)}{2} = 20$$

$$CF = 50 - 20 = 30 \text{ MPa} \quad Fx = 40 \text{ MPa}$$

$$R = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$



B is 40

A is 70 MPa

G is 40 MPa

$$\sigma_{max} = 70 \text{ MPa} = 20 + 50$$

$$\sigma_A = \sigma_C + CA$$

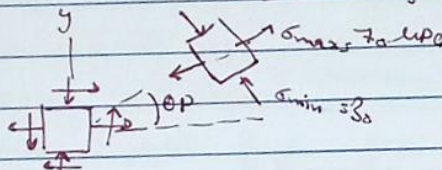
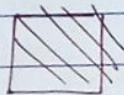
$$\sigma_{max} \text{ at } OB = \sigma_C - BC = 20 - 50 = -30 \text{ MPa}$$

$$\tan 2\theta_p = \frac{F_x}{\sigma_p} = \frac{40}{30}$$

$$\theta_p = 26.6^\circ$$

angle between sigma and tau

Shear stress (sigma)



max shear stress

max shear stress

$$\theta_s = \theta_p + 45^\circ$$

$$\tau_{max} = R$$

$$\sigma' = \sigma_{av}$$

$$\theta_s = 71.6^\circ$$

$$\tau_{max} = 50 \text{ MPa}$$

$$\sigma' = 20 \text{ MPa}$$

Sample Problem 7.2

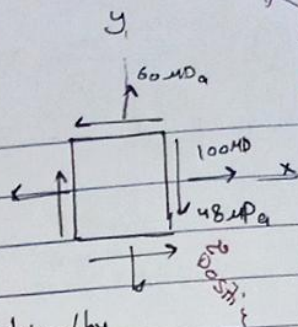
For the state of stress shown

determine

(a) the principle planes stress

(b) the stress components exerted on the element obtained by rotating the given element counterclockwise through 30 degrees

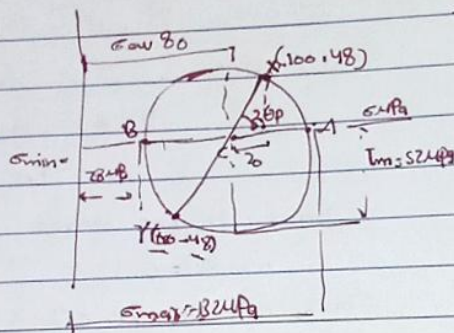
$$\sigma_{x'} = \sigma_{xx} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + 2\sigma_{xy} \sin \theta \cos \theta$$



$$\sigma_{av} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{80 + 100}{2} = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{\left(\frac{80 - 100}{2}\right)^2 + 48^2} = 52 \text{ MPa}$$

60°



• principle planes and stresses

$$\tan 2\theta_p = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{2(48)}{80 - 100} = -4.8$$

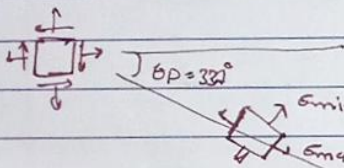
$\theta_p = 33.7^\circ$ clockwise

$$\sigma_{max} = \sigma_{xx} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + 2\sigma_{xy} \sin \theta \cos \theta$$

$$80 \cos^2 33.7^\circ + 100 \sin^2 33.7^\circ + 2(48) \sin 33.7^\circ \cos 33.7^\circ$$

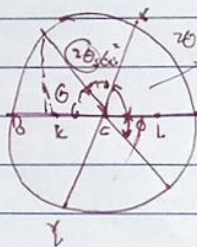
$$\sigma_{min} = \sigma_{xx} \sin^2 \theta + \sigma_{yy} \cos^2 \theta - 2\sigma_{xy} \sin \theta \cos \theta$$

$$80 \sin^2 33.7^\circ + 100 \cos^2 33.7^\circ - 2(48) \sin 33.7^\circ \cos 33.7^\circ$$



$$\phi = 170^\circ - 60^\circ = 110^\circ$$

$$\phi = 52.6^\circ$$



$$\phi = 170^\circ - 60^\circ = 110^\circ$$

$$\sigma_{x'} = \sigma_{xx} \cos^2 \phi + \sigma_{yy} \sin^2 \phi + 2\sigma_{xy} \sin \phi \cos \phi$$

$$\sigma_{y'} = \sigma_{xx} \sin^2 \phi + \sigma_{yy} \cos^2 \phi - 2\sigma_{xy} \sin \phi \cos \phi$$

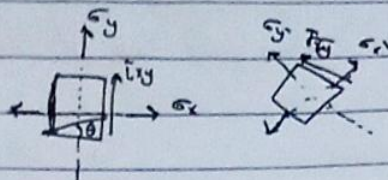
$$\tau_{x'y'} = (\sigma_{xx} - \sigma_{yy}) \sin \phi \cos \phi + \sigma_{xy} (\cos^2 \phi - \sin^2 \phi)$$

$$\sigma_{x'} = 80 \cos^2 52.6^\circ + 100 \sin^2 52.6^\circ + 2(48) \sin 52.6^\circ \cos 52.6^\circ$$

$$\sigma_{y'} = 80 \sin^2 52.6^\circ + 100 \cos^2 52.6^\circ - 2(48) \sin 52.6^\circ \cos 52.6^\circ$$

$$\tau_{x'y'} = (80 - 100) \sin 52.6^\circ \cos 52.6^\circ + 48 (\cos^2 52.6^\circ - \sin^2 52.6^\circ)$$

Review: stress transformation



positive w.r.t θ is plane is
negative θ

U.L.A. ω :-

$$\begin{aligned} \sigma_x' &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ \sigma_y' &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\ \tau_{xy}' &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \end{aligned}$$

principle planes in max stresses like 11 and 22, we call it

Principal Planes and Stresses $\rightarrow$ shear with principle planes = zero

11/19/

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \theta_{p2} = \theta_{p1} + 90^\circ \quad \sigma_{\text{principle}} \begin{cases} \text{max} \\ \text{min} \end{cases}$$

$\sigma_x - \sigma_y$: tension (positive) and compression (negative)
 τ_{xy} : shear stress (positive and negative)
 compression and tension

$$\sigma_{\text{max/min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

@ principal planes $\tau = 0$ $\rightarrow (\sigma_{\text{max}}, \tau = 0) \& (\sigma_{\text{min}}, \tau = 0)$

$\tau = 0$ $\therefore$ normal stress like (x) and (y) is

$\therefore$ Max-In Plane Shear & orientation: $\rightarrow$ (shear stress)

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}} \quad \theta_{s2} = \theta_{s1} + 90^\circ$$

$$\text{OR } \theta_{s1} = \theta_{p1} + 45^\circ, \quad \theta_{s2} = \theta_{p2} + 45^\circ$$

$$\tau_{\text{max, in-plane}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_{s1} =$ Max In-plane shear & principle plane is $\tau = 0$
 For the plane of maximum shear $\sigma = \sigma_{\text{avg}}$

11/19/

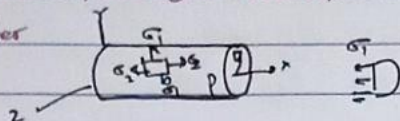
$$\left(\frac{\sigma_1 + \sigma_2}{2}, \pm \tau_{max} \right) \quad \left(\frac{\sigma_1 + \sigma_2}{2}, -\tau_{max} \right) \quad \left| \begin{array}{l} \theta_p = \frac{1}{2} \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \\ \theta_p = \pm 45^\circ \end{array} \right. \quad \text{Principal planes}$$

∴ Mohr's Circle, etc. questions. Ubbasi

We can use it to find σ_x, σ_y
 principle planes and stresses max in plane
 shear and its orientation (see example)

1. Thin-walled pressurized vessels

For cylinder



$$\sigma_x = \frac{P r_i}{t} \rightarrow \text{circumferential}$$

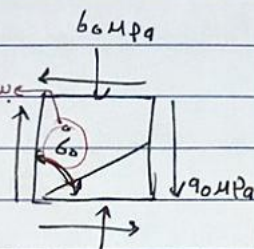
$$\sigma_z = \frac{P r_i}{2t}$$

For sphere: $\sigma_x = \sigma_z = \frac{P r_i}{2t}$

(All σ_x & σ_z Principle)

example P.7.2:-

For given state of stress determine the
normal shearing exerted on the oblique face of the
 shaded triangular element shown, use a method
 of analysis based on the equilibrium of that element as was done in the
 derivations of sec 7.1.1



$$\theta = -60^\circ$$

$$\sigma_x = 0$$

$$\sigma_y = -60 \rightarrow (2)$$

(σ_x, τ_{xy}) are required

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\frac{0 + (-60)}{2} + \frac{0 - (-60)}{2} \cos(-120) - 90 \sin(-120) = 32.94 \text{ MPa}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -\frac{(0 - 60)}{2} \sin(-120) - 90 \cos(-120) = 70.98 \text{ MPa}$$

$\begin{bmatrix} 84 & 42 \\ 42 & -42 \end{bmatrix}$ = Principle stress

7.4

(a) normal and shearing stress

use the method analysis

$$\sigma_x = -42 \text{ MPa}$$

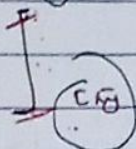
$$\sigma_y = 84 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\theta = -55^\circ$$

(σ_y, τ_{xy}) are require

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

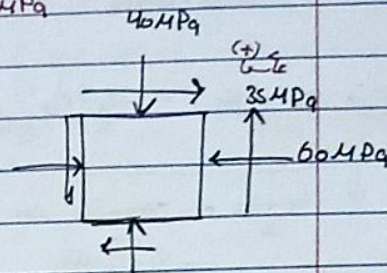
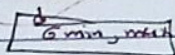


$$= \frac{-42 + 84}{2} + \frac{-42 - 84}{2} \cos -110 + 0 = -0.547 \text{ MPa}$$



7.5 (a) principle planes, (b) the principle stresses

$\sigma_x = 60 \text{ MPa}$, $\sigma_y = -40 \text{ MPa}$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 35}{60 - (-40)} =$$

$$\theta_{p1} = -37^\circ$$

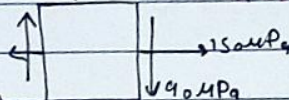
$$\theta_{p2} = \theta_{p1} + 90^\circ = 53^\circ$$

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{60 + (-40)}{2} \pm \sqrt{\left(\frac{60 - (-40)}{2}\right)^2 + (35)^2} = -50 \pm 36.4$$

$$\sigma_{max} = -13.6 \text{ MPa}$$

$$\sigma_{min} = -86.4 \text{ MPa}$$



7.7 (a) principle planes

(b) principle stress σ_{max} , σ_{min}

$$\sigma_x = 150 \text{ MPa}, \sigma_y = 30 \text{ MPa}, \tau_{xy} = -90 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-90)}{150 - 30} = \frac{-180}{120} = -1.5$$

$$\theta_{p1} = -28.2^\circ$$

$$\theta_{p2} = 61.8^\circ$$

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{150 + 30}{2} \pm \sqrt{\left(\frac{150 - 30}{2}\right)^2 + (-90)^2} = 90 \pm 108$$

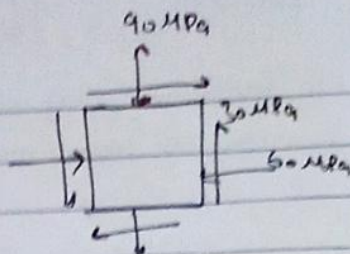
$$\sigma_{max} = 208 \text{ MPa}$$

$$\sigma_{min} = -18 \text{ MPa}$$

$$\sigma_{\pm R} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

2.11 (a) normal and shearing stress

to counter-clockwise 25° clockwise
11) 12)



$$\sigma_{xx} = 90 \quad \sigma_{yy} = -60 \quad \sigma_{xy} = +30 \quad (\text{MPa})$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_x' = \frac{90 + (-60)}{2} + \frac{90 - (-60)}{2} \cos(-50^\circ) + 30 \sin(-50^\circ)$$

$$= -36.18 \text{ MPa}$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{90 + (-60)}{2} - \frac{90 - (-60)}{2} \cos(2\theta) - 30 \sin(2\theta)$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_y' = \frac{90 + (-60)}{2} - \frac{90 - (-60)}{2} \cos(-50^\circ) - 30 \sin(-50^\circ)$$

$$= 86.18 \text{ MPa}$$

$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{90 - (-60)}{2} \sin(-50^\circ) + 30 \cos(-50^\circ)$$

$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{xy}' = -\frac{90 - (-60)}{2} \sin(-50^\circ) + 30 \cos(-50^\circ)$$

$$= -38.17 \text{ MPa}$$

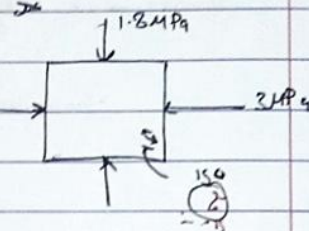
$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{90 - (-60)}{2} \sin(2\theta) + 30 \cos(2\theta)$$

2.12: (a) the in plane shearing stress parallel to the grain (b) the normal stress perpendicular to the grain

$$\sigma_x = -3 \quad \sigma_y = -1.8 \quad \tau_{xy} = 0$$

(σ_x, τ_{xy}) clockwise



$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -\frac{(-3) - (-1.8)}{2} \sin(-30^\circ) + 0 = -0.3 \text{ MPa}$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = \frac{-3 + (-1.8)}{2} + \frac{-3 - (-1.8)}{2} \cos(-30^\circ) + 0$$

$$= -2.42 \text{ MPa}$$

الضغط العمودي على الحبوب
الضغط القصوي الموازي للحبوب

$$\sigma_y' =$$

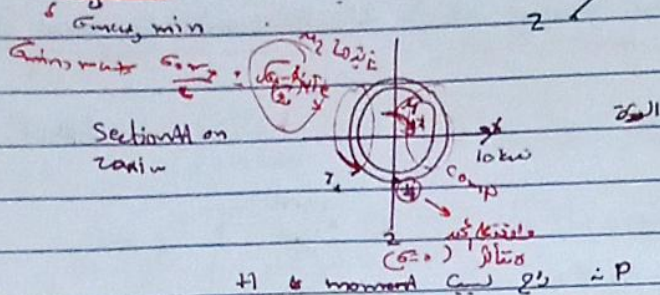
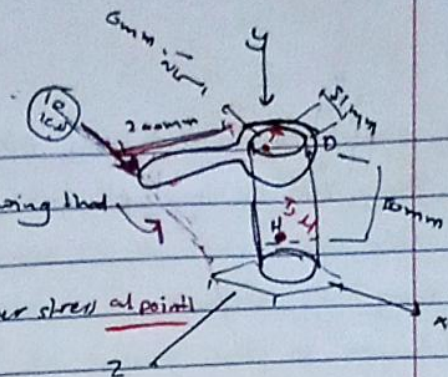
normal stress

7.26 A pipe AB has a 102 mm outer

diameter and a 6 mm wall thickness. Knowing that

arm CD is rigidly attached to the pipe

determine the principal stress and the max shear stress at point



$V = 10 \text{ kN}$ → load for τ_{xy}

$$M_z = (10 \text{ kN})(200 \text{ mm}) = 2000 \text{ N}\cdot\text{m}$$

$$\text{Torque } 10 \text{ kN}(200 \text{ mm}) = 2000 \text{ N}\cdot\text{m}$$

Find J and J_0

$P + D \& M =$

Find τ_{xy} and σ_x at point C

$$\sigma_{xz} = \frac{M_z}{I} = 0$$

$$I = 1500 \text{ cm}^4$$



$$\tau_{xy} = \frac{VQ}{I} = \frac{2000 \text{ N}\cdot\text{m}}{1500 \text{ cm}^4} = 11.05 \text{ MPa}$$

$$\frac{11.05}{24.37} = 35.42$$

$$\tau_{xy} = \frac{VQ}{I} = \frac{2000 \text{ N}\cdot\text{m}}{1500 \text{ cm}^4} = 24.37 \text{ MPa}$$

$$\sigma_{max} = \tau_{vmax} + \tau_{xy} \quad \text{at point C}$$

$$(11.05) + (24.37) = 35.42 \text{ MPa}$$

$$\therefore \text{max } \sigma_{max} = 35.42 \text{ MPa}$$

$$\text{Shear } \frac{2V}{A}$$

7.33 solve 7.6 and 7.10 using Mohr's circle.

7.5 through 7.8 For the given state of stress determine (a) principle stress plane (b) principle stress

7.9 through 7.12 determine (a) orientation of the planes of max in plane shearing stress (b) the max in plane shearing stress (c) the corresponding normal stress

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{84}{2}$$

$$R = \sqrt{\tau_{xy}^2} = 70.4 \text{ MPa}$$

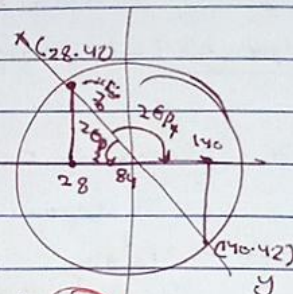
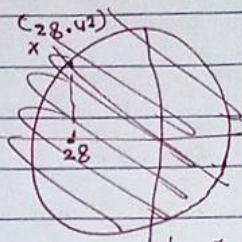
$$\sigma_{max} = \sigma_{avg} + R = 154 \text{ MPa}$$

$$\sigma_{min} = \sigma_{avg} - R = 14 \text{ MPa}$$

$$\theta_{p1} = -71.6^\circ \quad \text{normal s'}$$

$$\theta_{p2} = 18.8^\circ \quad \text{Principle axes}$$

0



7.63 For the shown is known that the normal stress and shearing stress directed as shown

$$\sigma_x = 98 \text{ MPa} \quad \tau_{xy} = 63 \text{ MPa}$$

$$\sigma_{min} = 35 \text{ MPa} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(a) the orientation of the principle planes $\rightarrow \theta_{p1} \theta_{p2} (\theta_p)$

(b) the principle stress σ_{max}

(c) the max in plane shearing stress

$$\sigma_x = 98 \quad \sigma_y = 63 \quad \sigma_{min} = 35 \quad \tau_{xy}$$

$$\tau_{xy} = \sqrt{\left(\frac{98 - 63}{2}\right)^2 + 42^2} = 45.5 \text{ MPa}$$

$$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$35 = \frac{98 + 63}{2} - \sqrt{\left(\frac{98 - 63}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = 42 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(42)}{98 - 63} = 33.69^\circ \quad \theta_{p1} \quad +90^\circ \Rightarrow \theta_{p2} = 127.7^\circ$$

$$(b) \sigma_{max} = \frac{98 + 63}{2} + \sqrt{\left(\frac{98 - 63}{2}\right)^2 + 42^2} = 126 \text{ MPa}$$

Position ration

299 | A spherical gas has outer D 5mm and a wall thickness 24mm made steel $E = 200 \text{ GPa}$ and $\nu = 0.29$, knowing that the gage pressure in the container is increased from zero to 1.8 MPa, determine (a) the max normal stress in the container

(b) the crosshounding increase in the diameter of the container

sphere eq

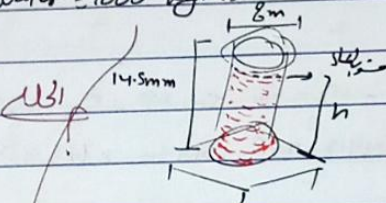
$$(a) \sigma_1 = \sigma_2 = \frac{P_{\text{inner}}}{2t} = \frac{1.8 (2.5 - 0.024)}{2 \times 0.024} = 92.85 \text{ MPa}$$

$$(b) \epsilon = \frac{\sigma}{E} - \nu \frac{\sigma}{E} = \frac{92.85 \times 10^6}{200 \times 10^9} - 0.29 \times \frac{92.85 \times 10^6}{200 \times 10^9} = 329.6 \times 10^{-6}$$

$$\epsilon = \frac{\delta}{d} = \frac{\delta}{5} \quad \delta = 1.65 \times 10^{-3} = 1.65 \text{ mm}$$

17.164 - The unpressurized cylindrical storage tank shown has a 5mm thick made steel having a 400 MPa ultimate strength in tension, determine the max height to which it can be filled with water if a factor of safety 4 is desired. (Specific weight of water = 1000 kg/m³)

400 MPa ultimate strength in tension
Factor of safety 4



$$\sigma_{\text{all}} = \frac{\sigma_{\text{ult}}}{F.S.} = \frac{400}{4} = 100 \text{ MPa}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{P \left(\frac{8}{2} \right)}{0.005} = 800P = 100$$

$$P = 0.125 \text{ MPa}$$

$$P = \gamma h = 9.81 \times h = 0.125 \text{ MPa}$$

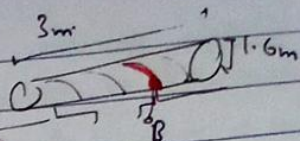
$$h = 12.74 \text{ m}$$

$$\sigma_{\text{aver}} = \frac{P_{\text{inner}}}{t}$$

$$P = \gamma h$$

2.17

Can the largest allowable pressure knowing the normal stress allow perpendicular to the weld is 120 MPa and allow shear stress parallel to the weld is 80 MPa

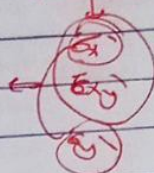


6mm thickness $\beta = 20^\circ$ with a transverse plane. For the pressure of 600 MPa (a) normal stress perpendicular to the weld (b) shear stress parallel to the weld

principle $\sigma_r = \frac{p r^2}{t^2} = \frac{P (792)^2}{8^2} = 99P$

$\sigma_z = \frac{p r^2}{2t^2} = 49.5P$

$\frac{p r}{t}$
 $\frac{r}{2t}$



$\sigma_{\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau \sin 2\theta$

$= \frac{49.5P + 99P}{2} + \frac{49.5P - 99P}{2} \cos(40^\circ) + 0 = 55.3P$

$\tau_{\theta} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$
 $= \frac{(49.5 - 99)P}{2} \sin 40^\circ = 15.4P$

$55.3P = 120 \text{ MPa} \rightarrow P = 2.17 \text{ MPa}$
 $15.4P = 80 \rightarrow P = 5.03 \text{ MPa}$
pol/ten
same $\frac{1}{2} \sigma$

7-122

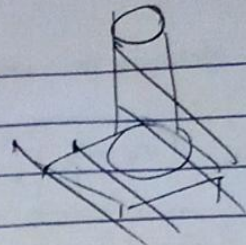
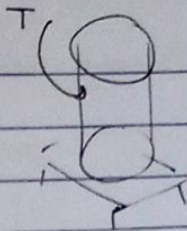
Torque $\rightarrow 12 \text{ kN}$

W $\rightarrow 60 \text{ mm}$ $\rightarrow 10 \text{ mm}$

under pressure 8 MPa

inner $\phi 120$ thick 12 mm

(a) max normal stress and the max shear stress in the torque



$$\sigma_1 = \frac{pr}{t} = \frac{8 \times 60}{12} = 60 \text{ MPa}$$

$$\sigma_2 = 30 \text{ MPa} = \frac{pr}{2t}$$

$$\tau = \tau_c = \frac{12 \times 10^3}{J} \times (90 \times 12) \times 10^{-3} = 18.28 \text{ MPa}$$

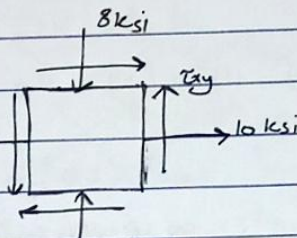
$$J = \frac{\pi}{2} (60^4 - 120^4) = 1.2 \times 10^{-6} \text{ m}^4$$

$$\sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 68.68 \text{ MPa}$$

$$\sigma_{\min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 34.34 \text{ MPa (out of plane)}$$

7-27 :- state plane

(1) largest value τ_{xy} which the max shear stress is equal to or less 12 ksi



$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{10 - (-8)}{2}\right)^2 + 12^2} = 12.8 \text{ ksi}$$

$$\tau_{xy} = 12.8$$

the corresponding (b) principle stress $\sigma_x = 10$ $\sigma_y = -8$ $\tau_{xy} = 12$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.33 \quad \text{shift tan } 1.33 = 53.1 = 2\theta$$

$$\theta_1 = 26.56$$

$$\theta_2 = 116.56$$

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{10 + (-8)}{2} \pm \sqrt{\left(\frac{10 - (-8)}{2}\right)^2 + 12^2}$$

$$= 1 \pm 15$$

$$= 16 \quad -14$$

max min

7.29:- determine σ_x with the max plan shearing stress is equal to or less than 10ksi

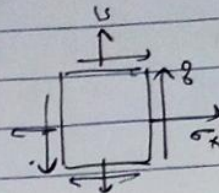
$$\tau_{max} \leq \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$10 \leq \sqrt{\left(\frac{\sigma_x - 15}{2}\right)^2 + 8^2}$$

$$\sigma_x \leq 3 \quad \sigma_x \leq 27$$

من أجل $\sigma_x = 3$ $\tau_{max} = 10$ $\sigma_x = 27$ $\tau_{max} = 10$

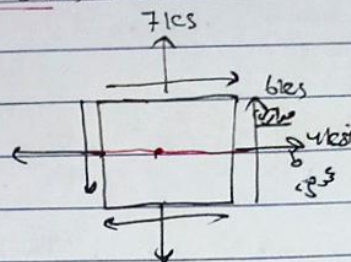
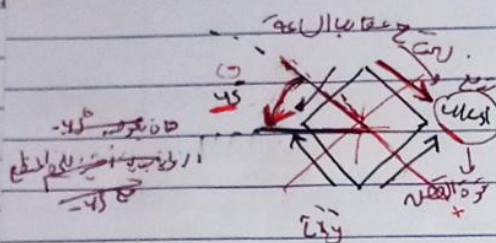
كل المصنفات
من البقية



$$\tau_{max} = 2\tau_{xy}$$

$$\sigma \pm R$$

7.54 - 7.55:- determine the principle plane and the principle stress for the state of plane stress resulting from the superposition the two state of stress shown



أما
في

المعادلة وصيغة

$$\theta = 45^\circ$$

المعادلة وصيغة σ_x σ_y τ_{xy} τ_{yx}

$$\theta = -45^\circ$$

$$\theta = 45^\circ$$

$$\sigma_x' = \sigma_x + \sigma_y + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_x' = \sigma_x + \sigma_y + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = (-4) + \sin 2(-45) = +4 \text{ ksi}$$

$$\sigma_y' = -\tau_{xy} \sin 2\theta = (-4) \sin 2(-45) = -4 \text{ ksi}$$

$$\tau_{xy}' = \tau_{xy} \cos 2\theta = -4 \cos 2(-45) = 0$$

$$\sigma_x' = \tau_{xy}' \sin \theta$$

$$\sigma_x = \tau_{xy}' \sin 2\theta$$

$$\tau_{xy} = 0$$

$$\sigma_x = 4 \quad \sigma_y = -4 \text{ ksi}$$

$$\sigma_x = 4 \quad \sigma_y = 7 \quad \tau_{xy} = 6 \text{ (ksi)}$$

المعادلة وصيغة

$$\sigma_x = \sigma_x^{(1)} + \sigma_x^{(2)} = 4 + 4 = 8 \text{ ksi}$$

$$\sigma_y = \sigma_y^{(1)} + \sigma_y^{(2)} = -4 + 7 = 3 \text{ ksi}$$

$$\tau_{xy} = \tau_{xy}^{(1)} + \tau_{xy}^{(2)} = 0 + 6 = 6 \text{ ksi}$$

المعادلة وصيغة

(C) Principle stress

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2 \times 6}{8-3} =$$

$$\therefore \theta_{p1} = 33.69^\circ \quad \theta_{p2} = 123.69^\circ$$

Principle stress

$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\frac{8+3}{2} \pm \sqrt{\left(\frac{8-3}{2}\right)^2 + 6^2} = 12 \text{ (max)} \quad -1 \text{ (min)}$$

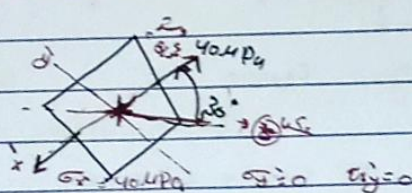
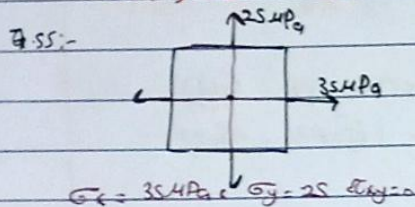
الضغط في عناصر في عنصر واحد :-

بالإضافة إلى الإزاحة 45 - (الضغط في العناصر) (الضغط في العناصر)

$$\sigma_y = -\tau_{xy} \sin 2\theta$$

في حالة تقاطع

الضغط في حالة تقاطع



الضغط في حالة تقاطع

الضغط في حالة تقاطع

الضغط في حالة تقاطع

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_1 = 40 + 0 + 40 - 0 \cos 60 + 0 = 30 \text{ (30)}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta = \frac{40+0}{2} - \frac{40-0}{2} \cos 60 - 0 = 10$$

$$\tau_{xy} = -\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -\frac{(40-0)}{2} \sin 60 + 0 = 17.3 \text{ MPa}$$

$$\sigma_x = \sigma_{x1} + \sigma_{x2} = 35 + 30 = 65 \text{ MPa}$$

$$\tau_{xy} = \tau_{xy1} + \tau_{xy2} = 0 + 17.3 = 17.3 \text{ MPa}$$

$$\sigma_y = \sigma_{y1} + \sigma_{y2} = 25 + 10 = 35 \text{ MPa}$$

الضغط في حالة تقاطع

الضغط في حالة تقاطع

principle plane

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \cdot 17.32}{65 - 35}$$

$$\theta_1 = 24.5 \quad \theta_2 = 114.55$$

$\sigma_{max, min}$ principal stress

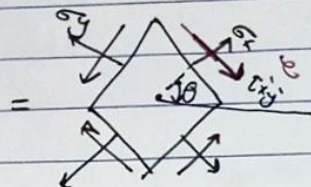
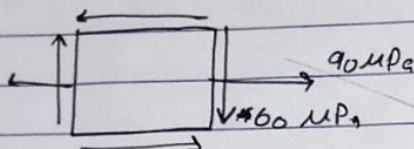
$$\sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{65 + 35}{2} \pm \sqrt{\left(\frac{65 - 35}{2}\right)^2 + 17.32^2}$$

$$= 50 \pm 22.9$$

$$\begin{matrix} 122.9 & -2.9 \\ \text{max} & \text{min} \end{matrix}$$

7.38:- determine the range of value of θ for which the magnitude of the shear stress τ_{xy} is equal to or less 8ksi



the value of τ_{xy} is 8ksi

the value of τ_{xy} is 8ksi

$$\sigma_x = 90 \quad \sigma_y = 0 \quad \tau_{xy} = 60$$

$$\sigma_y = ?? \quad \tau_{xy} = -8 \quad \sigma_x = ??$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{max, min} = 90 + 0 \pm \sqrt{\left(\frac{90 - 0}{2}\right)^2 + 60^2}$$

$$= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\begin{matrix} 120 & -75 \\ \text{max} & \text{min} \\ \theta & \theta \end{matrix}$$

$$-75 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 64}$$

$$120 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 64}$$

$$(-75)^2 \pm \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 64$$

$$\sqrt{689} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$120 = \frac{\sigma_x' + \sigma_y'}{2} + \sqrt{\left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2 + 64}$$

$$(120)^2 = \left(\frac{\sigma_x' + \sigma_y'}{2}\right)^2 + \left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2 + 64 \quad (1) \text{ و (2)}$$

$$-75 = \left(\frac{\sigma_x' + \sigma_y'}{2}\right)^2 - \left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2 + 64 \quad (1) \text{ و (2)}$$

$$\frac{8775}{2} = \frac{1}{4} \left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2$$

$$\sigma_x' - \sigma_y' = 66.23$$

$$\sigma_x' = 66.23 + \sigma_y'$$

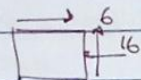
$$\sigma_y' = 52.81$$

$$\sigma_x' = 119.08$$

$$\sigma_{xy}' = -\sigma_x - \sigma_y \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$-8 = -\frac{16}{2} - \frac{90}{2} \sin 2\theta + -6 \cos 2\theta$$

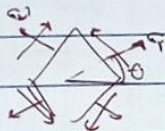
8/10/10



$$\sigma_x = -6$$

$$\sigma_y = 0$$

$$\tau_{xy} = 6$$



$$|\sigma_{xy}'| \leq 8 \text{ ksi}$$

$$-8 \leq -\frac{16}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$R =$$

$$-2 \leq 8 \sin 2\theta + 6 \cos 2\theta \leq 8$$

$$R = \frac{\sigma_x + \sigma_y}{2} = \frac{-16 + 0}{2} = -8$$

$$R = -8$$

$$R = \frac{\sigma_x + \sigma_y}{2} = \frac{-16 + 0}{2} = -8$$

$$R = \frac{\sigma_x + \sigma_y}{2} = \frac{-16 + 0}{2} = -8$$

$$2 \sqrt{8 \sin 2\theta + 6 \cos 2\theta} = 8$$

$$4 \sin 2\theta = 4 - 3 \cos 2\theta$$

$$16 \sin 2\theta = 16$$



Chapter-8-

(8.1)

$\Delta W10 \times 39$ $P = 4 \text{ kips}$ act in

$G_{all} = 18 \text{ ksi}$

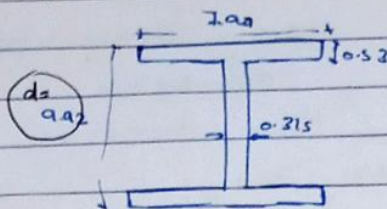
(a) max value σ_m
normal stress

(b) max value principle stress at the junction of the flange and web

(c) whether the specified shape is acceptable as far as these two stress are concerned

for

$s = 42.1$



max value σ_m at junction

$$\textcircled{1} \sigma_m = \frac{Mc}{I} = \frac{450 \times 10^3 \text{ lb} \times \frac{9.92}{2}}{209} = \frac{4}{5}$$

$$= 10.69 \text{ or } \frac{4 \times 450 \times 10^3}{5 \times 42.1} = 10.68 \text{ ksi}$$

(2) principle stress at junction

$$\sigma_p = \frac{Mc}{L}$$

$$= \frac{450 \times 10^3 \times (4.96 - 0.53)}{209}$$

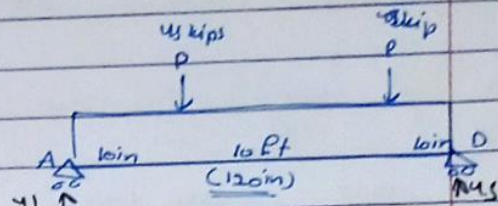
$$= 9.588 \text{ ksi}$$

$$\textcircled{1} VQ = \frac{45 \times 10^3 \times 19.88}{2 \times 209 \times 4.645} = 13.588 \text{ ksi}$$

$$Q = 7.99 \times 0.53 + \frac{(4.96 - 0.53)}{2} = 19.82 \text{ in}^3$$

$$\sigma_{max} = \sigma_m + \tau = \frac{9.588 + 0}{2} + \sqrt{\left(\frac{9.588 - 0}{2}\right)^2 + 13.588^2} = 19.17 \text{ ksi} > 18 \text{ ksi}$$

not safe



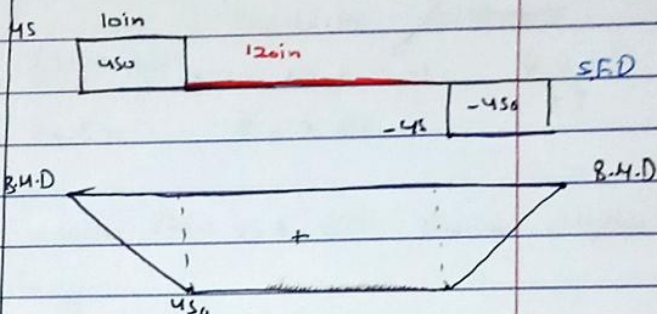
Free body diagram

$$\sum F_y = 0 = A_y + B_y - 45 - 45$$

$$A_y + B_y = 90$$

$$+\circlearrowleft \sum M_A = (45 \times 10) + 45(10 + 10) - B_y(20) = 0$$

$$B_y = 45 \quad A_y = 45$$



85, 8-6

$$G_{all} = 24 \text{ ksi}$$

$$\tau_{all} = 14.5 \text{ ksi}$$

(a) G_m, I_m

(b) principle stress at junction
20.81, 1.26, 1.63

$$G_m = \frac{H}{S_{min}}$$

$$S_{min} = \frac{I}{c} = \frac{1260 \times 10^3}{24 \times 10^3} = 52.5 \text{ in}^3$$

$$J_{3, 100} / 1018.85 \quad K5 = 57.6 \text{ in}^3$$

$$(a) G_m = \frac{H}{S} = \frac{1260 \times 10^3}{57.6} = 21.9 \text{ ksi} < 24 \text{ ksi}$$

$$\tau_m = \frac{V}{A} = \frac{16.25}{0.3 \times 17.7} = 3.06 \text{ ksi} < 14.5 \text{ ksi}$$

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(b) principle stress

$$\sigma = \frac{M \cdot c}{I} = \frac{1260 \times 10^3 \times \left(\frac{17.7}{2} - 0.425 \right)}{510} = 20.81 \text{ ksi}$$

$$\tau = \frac{V \cdot Q}{I \cdot t} = \frac{16.25 \times 6 \times 0.425 \times \left(\frac{17.7}{2} - 0.425 \right)}{510 \times 0.3} = 3.06$$

$$V = 8.75 \quad M = 1260$$

$$\sigma = \frac{1260 \times 10^3 \times \left(\frac{17.7}{2} - 0.425 \right)}{510} = 20.81$$

② τ

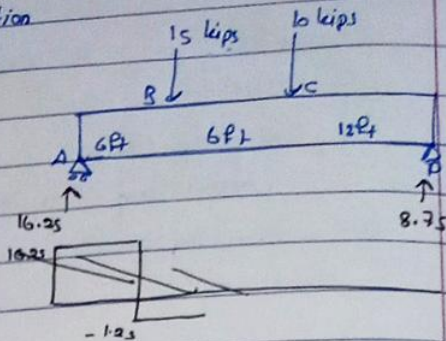
$$\tau = \frac{V \cdot Q}{I \cdot t} = \frac{8.75 \times 10^3 \times \left(\frac{17.7}{2} - 0.425 \right)}{510 \times 0.3} = 1.26 \text{ ksi}$$

$$\text{or } \left(\frac{V}{A} \right)$$

$$\text{① also } \sigma_m = \sigma_{av} + p = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = 19.81 \text{ ksi} < 24 \text{ ksi}$$

$$\text{② τ } \sigma_{av} + R = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = \frac{20.81 + 0}{2} + \sqrt{\left(\frac{20.81}{2} \right)^2 + 1.63^2}$$

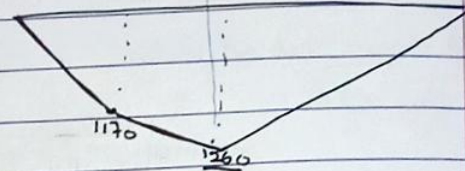
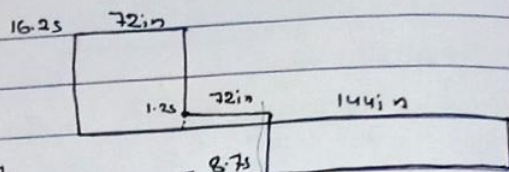
$$= 20.44 < 24 \text{ ksi}$$



$$\sum M_A = 0 \Rightarrow 15(6) + 10(12) - 24D_y = 0$$

$$D_y = 8.75 \text{ kips}$$

$$\sum F_y = 0 \quad A_y = 16.25 \text{ kips}$$



8.49.

principle stress

normal shear

principle plane

$$\sigma_H = \frac{P}{A} + \frac{Mx}{Ix}$$

$$= \frac{-120 \times 10^3}{0.1 \times 0.15} + \frac{9.375 \times 0.05}{\frac{1}{12} \times 0.1 \times 0.15^3} = 29.5 \text{ MPa}$$

$$\tau_{xy} = \frac{VQ}{Iy} = \frac{433 \times 10^3 \times 0.075 \times 0.1 \times 0.375}{\frac{1}{12} \times 0.1 \times 0.15^3 \times 0.1} = 4.37 \text{ MPa}$$

principle stress

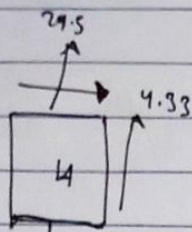
principle plane

max

min

shear

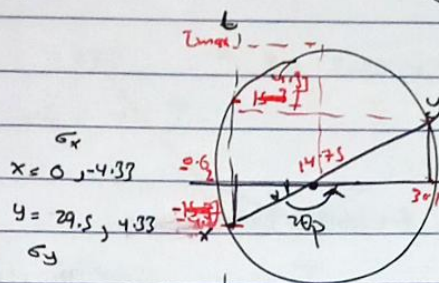
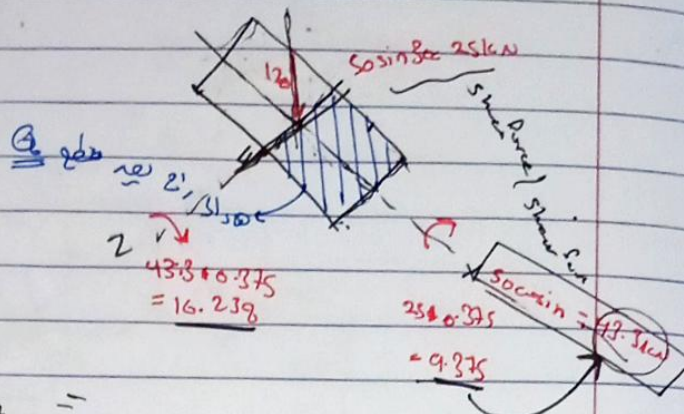
principle stress



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{29.5 + 0}{2} = 14.75$$

$$C = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{29.5 - 0}{2}\right)^2 + 4.37^2} = 15.37 \text{ MPa}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\sigma_x - \sigma_y} = \frac{4.37}{29.5}$$



$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{29.5 + 0}{2} = 14.75$$

$$C_{max} = R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.37^2} = 15.37 \text{ MPa}$$

$$\sigma_{max} = C + R = 14.75 + 15.37 = 30.1$$

$$\sigma_{min} = C - R = 14.75 - 15.37 = -0.62$$

$$\theta_p = \tan^{-1} \frac{4.37}{14.75} = 16.36^\circ$$

$$\theta_p = 190 - 16.36 = 173.64 \rightarrow \theta_p = 81.8^\circ$$