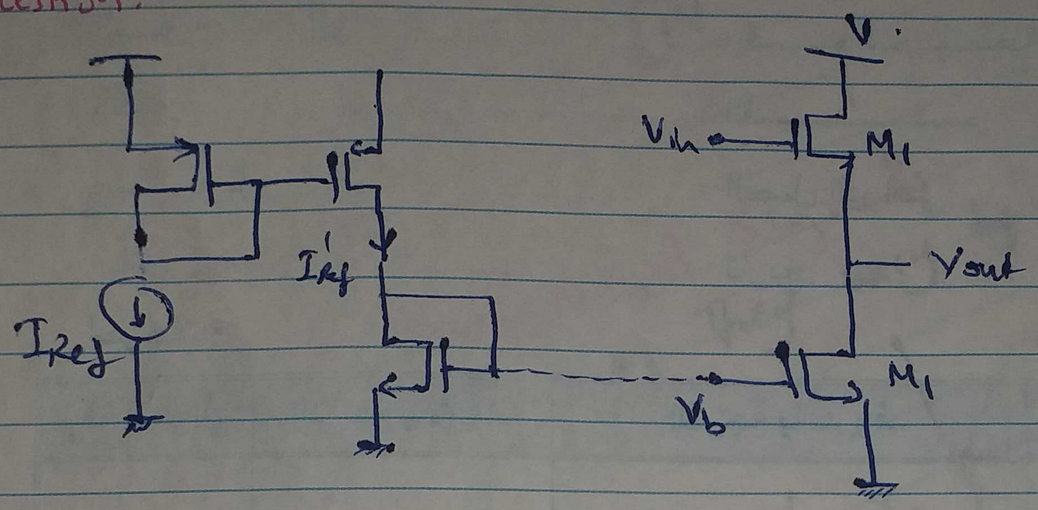
Today's lect:-
- Problem of Noise Coupling
- Differential Signals
- Differential pair

Question:-

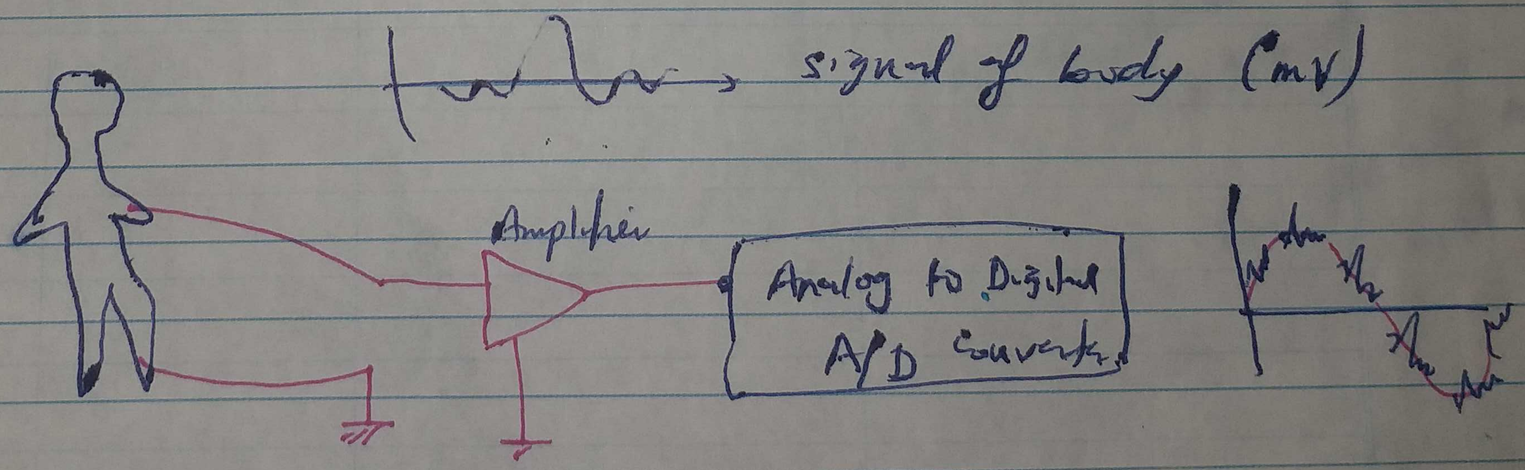


How to bias NMOS from PMOS current Source

* Problem of Noise Coupling:-

Noise: is something that we don't want

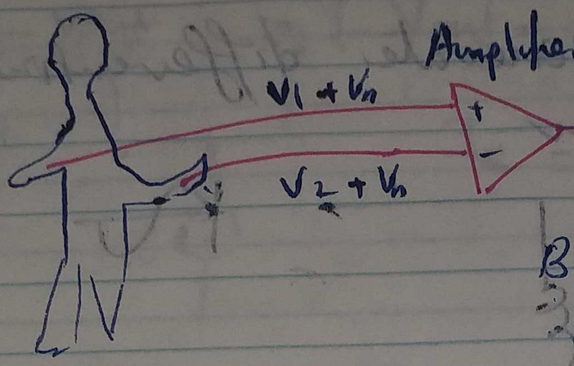
Electrocardiogram System (ECG)
measure heart activities:



- Here we have one measure problem the electrical wire around create signal with frequency (50-60) Hz

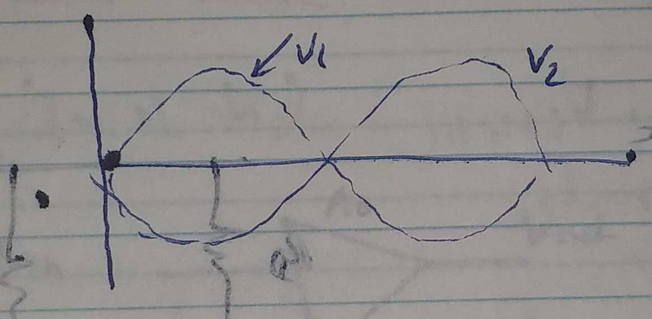
- As a result the signal of body is corrupted by this noise

In practical:-
they connect another line to second arm

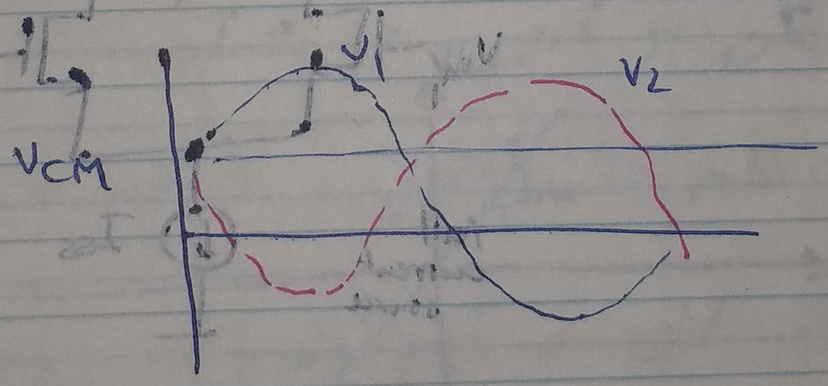


with No noise
Both lines carry the disired signal.

- Differential Signals:
  a) They vary by equal and opposite amount.



b) They have the same average (dc value) V_{CM}
Common mode level



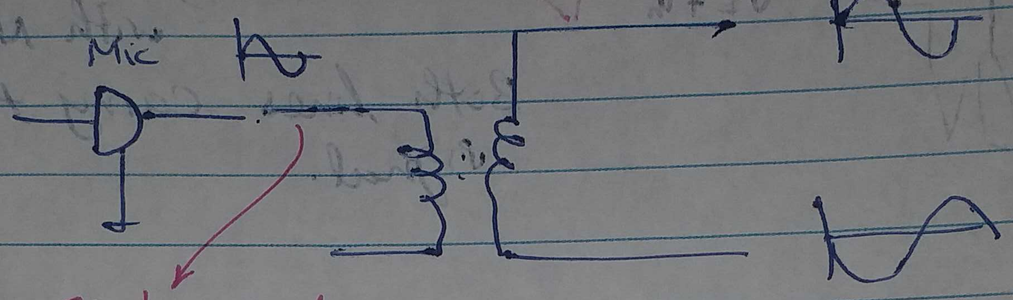
$V_1(t) = V_0 \sin\omega t + V_{CM}$
$V_2(t) = -V_0 \sin\omega t + V_{CM}$

Example: Determine the peak and peak-to peak amplitude of $V_1(t)$, $V_2(t)$ and $V_1 - V_2$

	V_1	V_2	$V_1 - V_2$
Peak amplitude	V_0	V_0	$2V_0$
peak-peak amplitude	$2V_0$	$2V_0$	$4V_0$

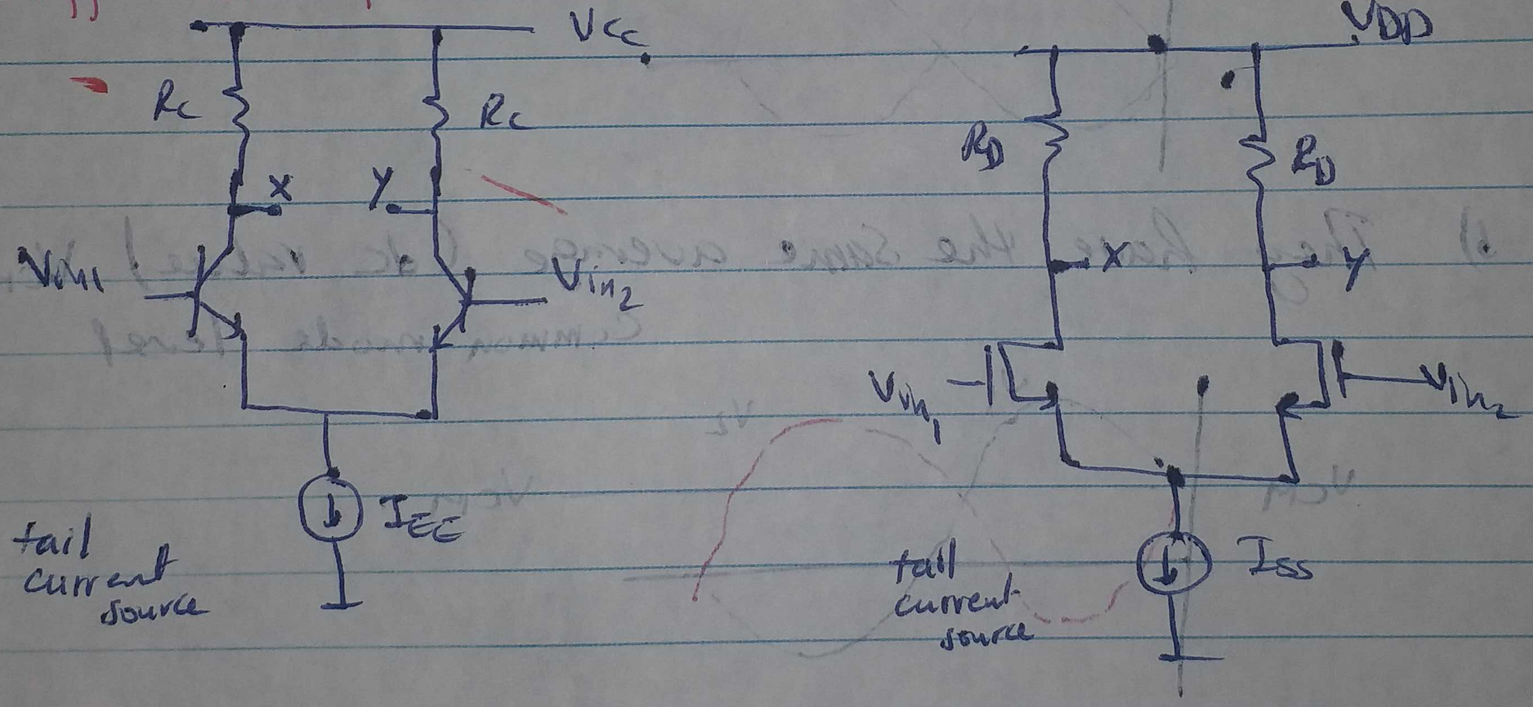
Question: How do we generate differential signals

using transformer



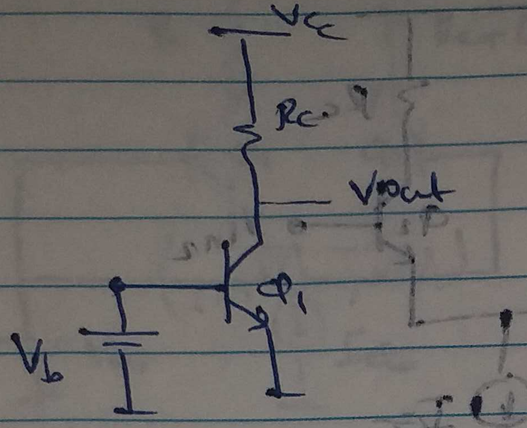
Quiz:-

Differential pair:-



Both V_{in_1}, V_{in_2} are two differential signals

Review of CE stage:



Q_1 operates at active Region

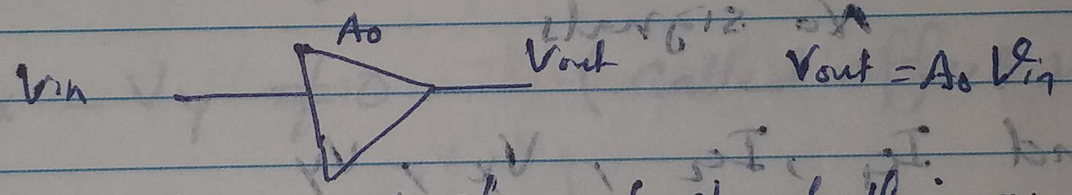
$$V_{out} = V_{CC} - I_C R_C$$

$$I_C = I_S \exp \frac{V_b}{V_T}$$

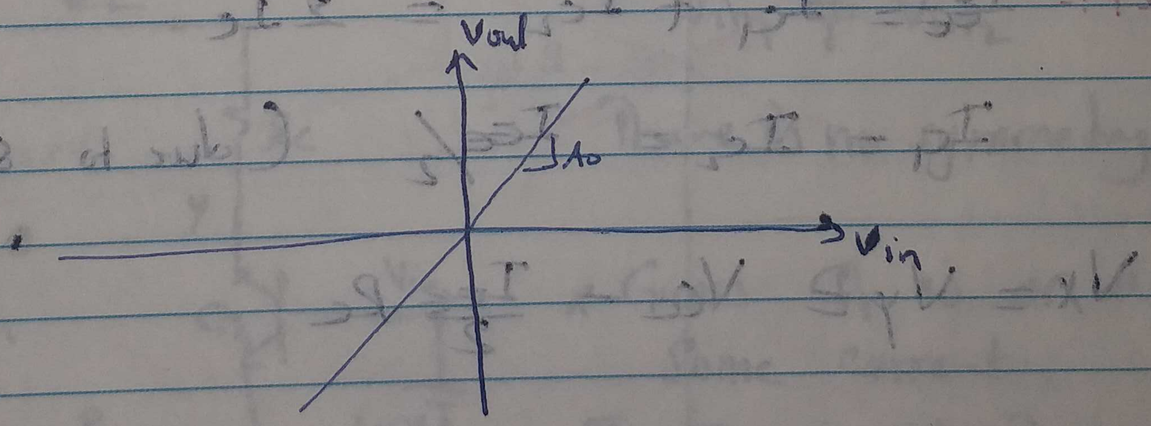
Today's lec.

- Intuitive study of Bipolar Differential pair
- Common Mode and Differential characteristics

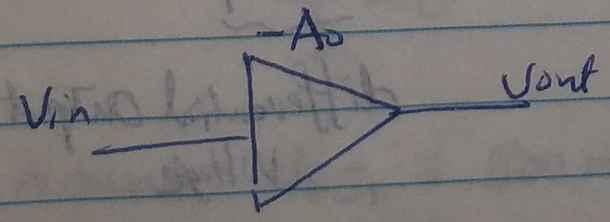
Example:-

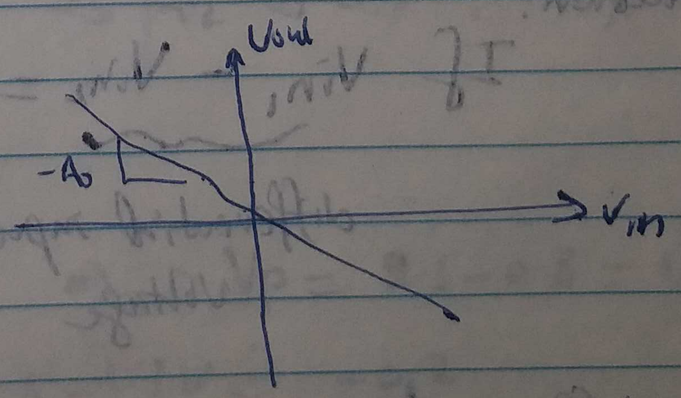


Construct the input-output characteristic of this circuit



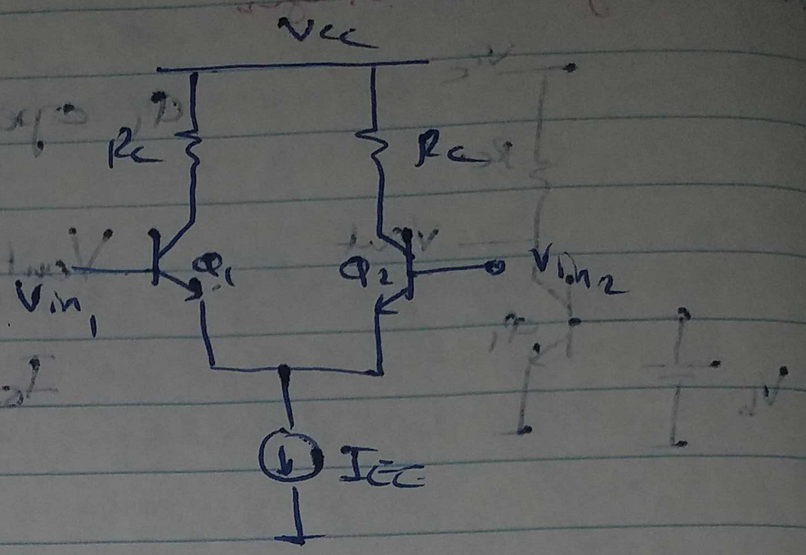
Example 2:-





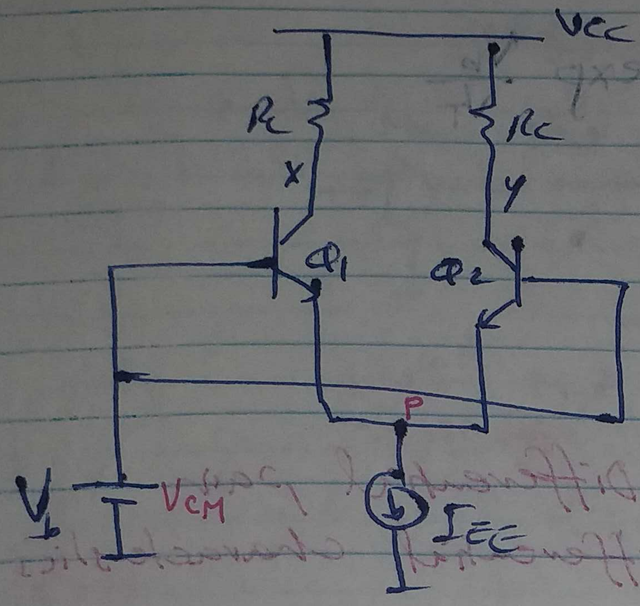
Slope = gain

* Bipolar Differential Pair:



Case I:-

assume both $V_{in_1} = V_{in_2} = V_b = V_{CM}$



V_b: is called V_{CM}, in (dc level for both Q_1 & Q_2)
No signals.

Find I_{C_1}, I_{C_2}, V_x, V_y

KCL @ node P:- $I_{EE} = I_{C_1} + I_{C_2} = 2I_C$

$I_{C_1} = I_{C_2} = I_{EE}/2$ (due to symmetry)

$V_x = V_y = V_{CC} - \frac{I_{EE}}{2} R_C$

Conclusion:

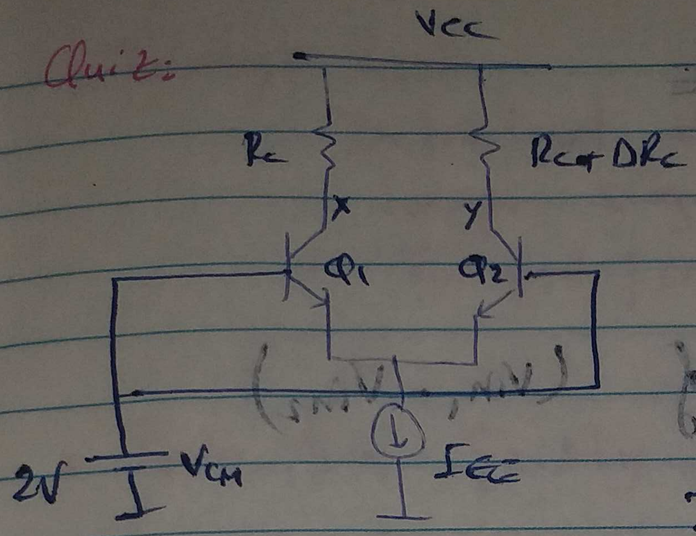
If $V_{in_1} - V_{in_2} = 0$, then $V_x - V_y = 0$

$V_{in_1} - V_{in_2}$: differential input voltage

$V_x - V_y$: differential output voltage

* The differential pair rejects perturbations in the input CM level.

Quiz:



Suppce that $R_{c1} \neq R_{c2}$

$R_{c1} = R_c$

$R_{c2} = R_c + \Delta R_c$

find I_{C1}, I_{C2}, V_x, V_y:

$$I_C = I_s \exp \frac{V_{BE}}{V_T} \quad \text{(Ignore early effect)}$$

$$I_{EE} = I_{C1} + I_{C2} = 2I_C$$

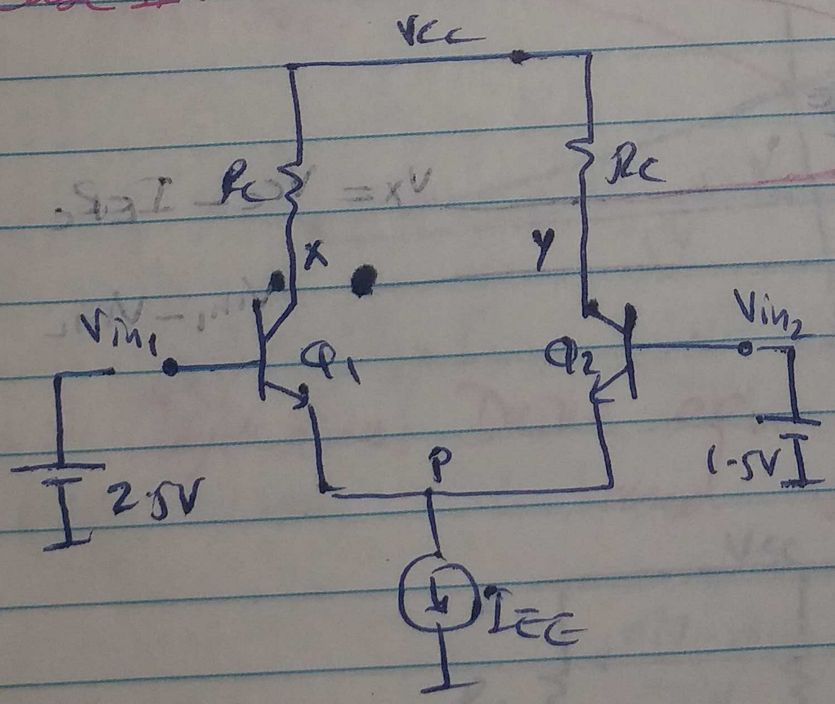
$$I_{C1} = I_{C2} = I_{EE}/2$$

$$V_x = V_{cc} - \frac{I_{EE}}{2} R_c$$

$$V_y = V_{cc} - \frac{I_{EE}}{2}(R_c + \Delta R_c)$$

$\Rightarrow V_x - V_y \neq 0$ (called offset voltage)

Case II:-



$V_{in1} \neq V_{in2}$ $(V_{in1} > V_{in2})$

There is no symmetry

* Guess Q_1 is on and has some current.

Is Q_2 is ON ?

- Assume $V_{BE1} \cong 800\,mV \Rightarrow V_p = 2.5 - 0.8 = 1.7V$

so that $V_{BE2} = 1.5 - 1.7V = -0.2$

$\Rightarrow Q_2$ becomes is OFF

$I_{C_2} = 0$, $I_{C_1} = I_{EE}$

$V_X = V_{CC} - I_{EE} R_C$

$V_Y = V_{CC}$

$\Rightarrow$ $V_X - V_Y$ is a function of $(V_{in_1} - V_{in_2})$

Case III :

$V_{in_1} = 1.5V$, $V_{in_2} = 2.5V$

no symmetry

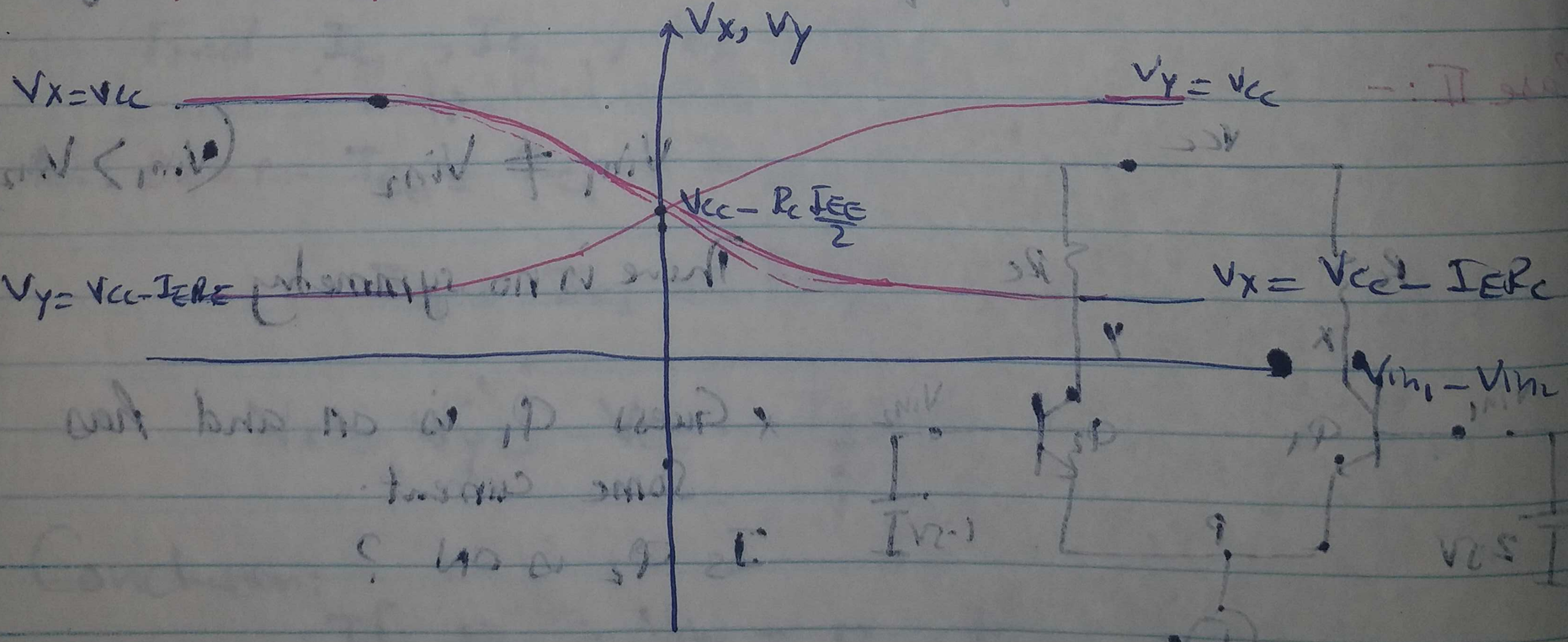
Q_2 becomes ON, Q_1 is OFF

$I_{C_1} = 0$, $I_{C_2} = I_{EE}$

$V_X = V_{CC}$

$V_Y = V_{CC} - I_{EE} R_C$

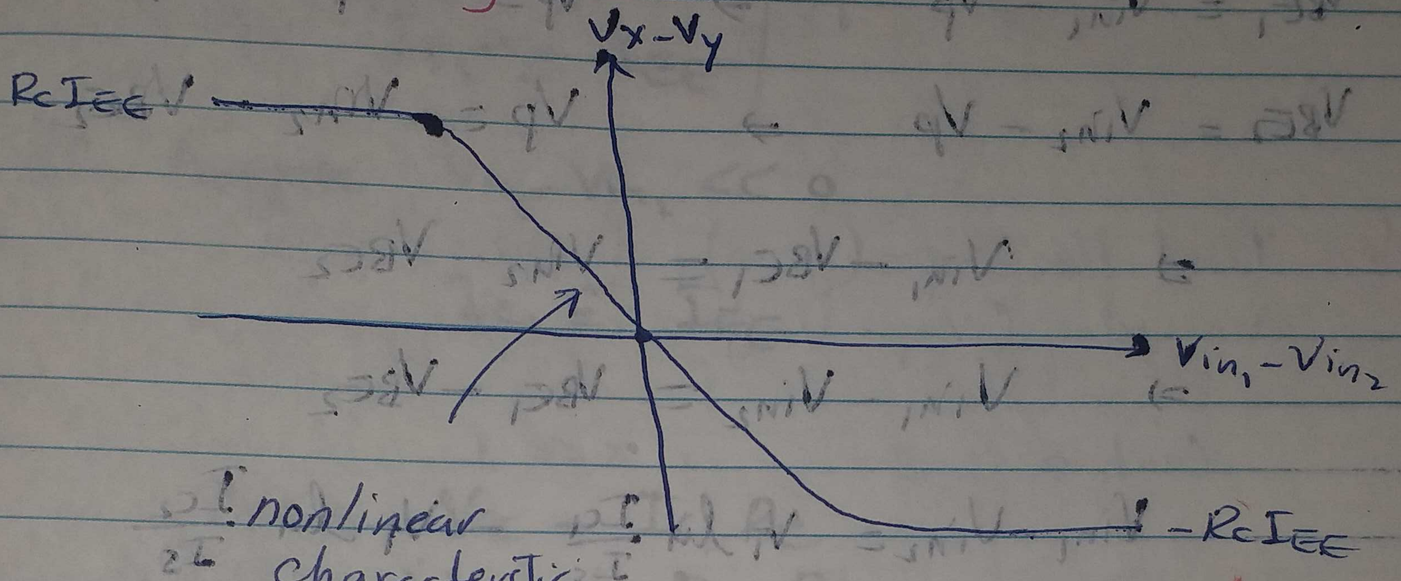
Input / output characteristic



Today's Lect.

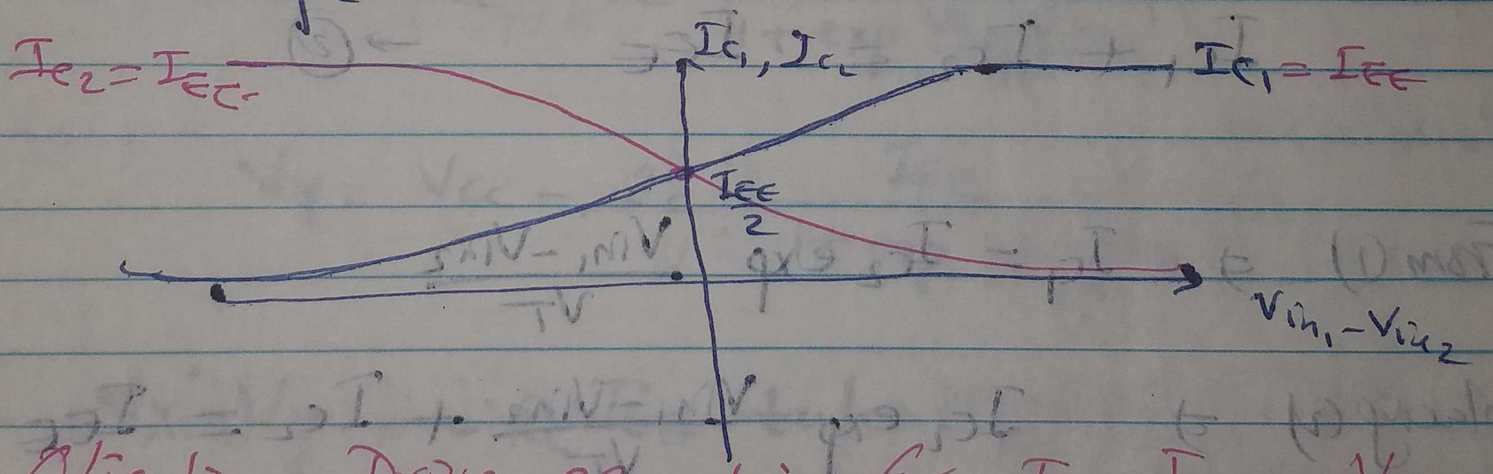
- Detailed Analysis of Bipolar Pair
- Large Signal Response
- (Input / output charact.)

* Intuitive Study

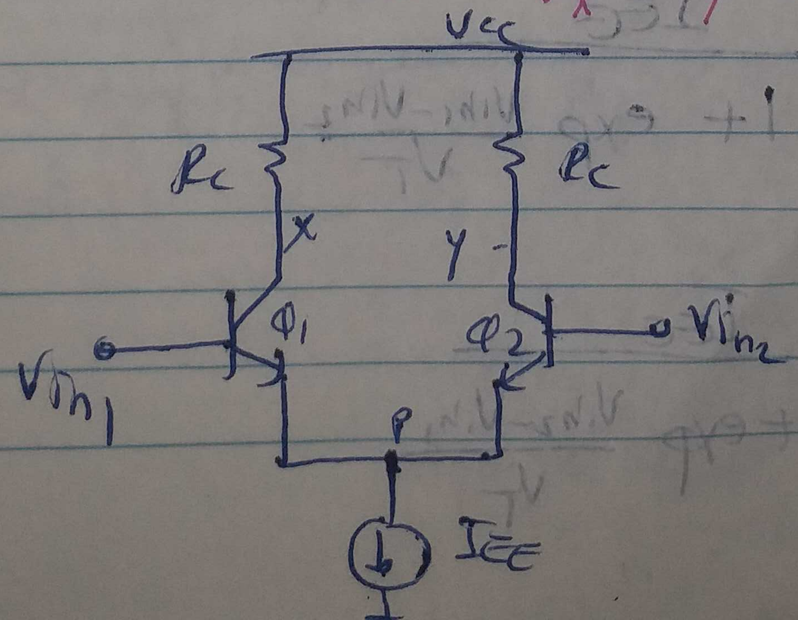


nonlinear characteristic

As we know, the gain of amplifier is the slope.



* Objective: Drive equation for I_{C_1}, I_{C_2}, V_x, V_y and $V_x - V_y$ as a function of $V_{in_1} - V_{in_2}$



In active Region

$$I_C = I_S \exp \frac{V_{BE}}{V_T}$$

$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$V_{BE_1} = V_{in_1} - V_p \Rightarrow V_p = V_{in_1} - V_{BE_1}$$

$$V_{BE_2} = V_{in_2} - V_p \Rightarrow V_p = V_{in_2} - V_{BE_2}$$

$$\Rightarrow V_{in_1} - V_{BE_1} = V_{in_2} - V_{BE_2}$$

$$\Rightarrow V_{in_1} - V_{in_2} = V_{BE_1} - V_{BE_2}$$

$$V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_S} - V_T \ln \frac{I_{C_2}}{I_S}$$

$$= V_T \ln \frac{I_{C_1}}{I_{C_2}} \rightarrow (1)$$

$$I_{C_1} = I_{C_2} \exp \frac{V_{in_1} - V_{in_2}}{V_T}$$

$$I_{C_1} + I_{C_2} = I_{EE} \rightarrow (2)$$

From (1) $\Rightarrow$ $I_{C_1} = I_{C_2} \exp \frac{V_{in_1} - V_{in_2}}{V_T}$

Replacing (2) $\Rightarrow$ $I_{C_2} \exp \frac{V_{in_1} - V_{in_2}}{V_T} + I_{C_2} = I_{EE}$

$$\Rightarrow I_{C_2} = \frac{I_{EE}}{1 + \exp \frac{V_{in_1} - V_{in_2}}{V_T}}$$

Similarly

$$I_{C_1} = \frac{I_{EE}}{1 + \exp \frac{V_{in_2} - V_{in_1}}{V_T}}$$

as shown:-

1) If $V_{in_1} = V_{in_2} \Rightarrow I_{C_1} = I_{C_2} = \frac{I_{EE}}{2}$

2) Suppose $V_{in_1} - V_{in_2}$ positive ~~neg~~ enough

$\Rightarrow V_{in_1} - V_{in_2} \gg 0 \Rightarrow \exp \frac{V_{in_1} - V_{in_2}}{V_T}$ very small

$\Rightarrow I_{C_2} \cong I_{EE}$

3) Suppose $V_{in_1} - V_{in_2} \ll 0$

$I_{C_1} \cong I_{EE}$

$V_X = V_{CC} - I_{C_1} R_C$

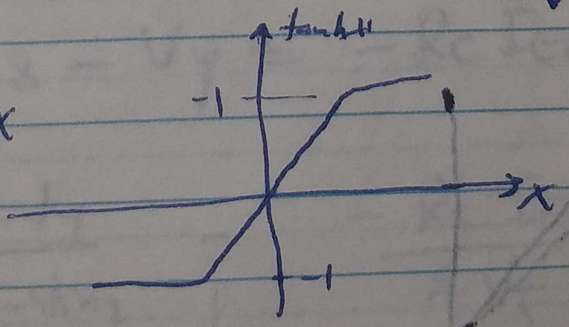
$V_Y = V_{CC} - I_{C_2} R_C$

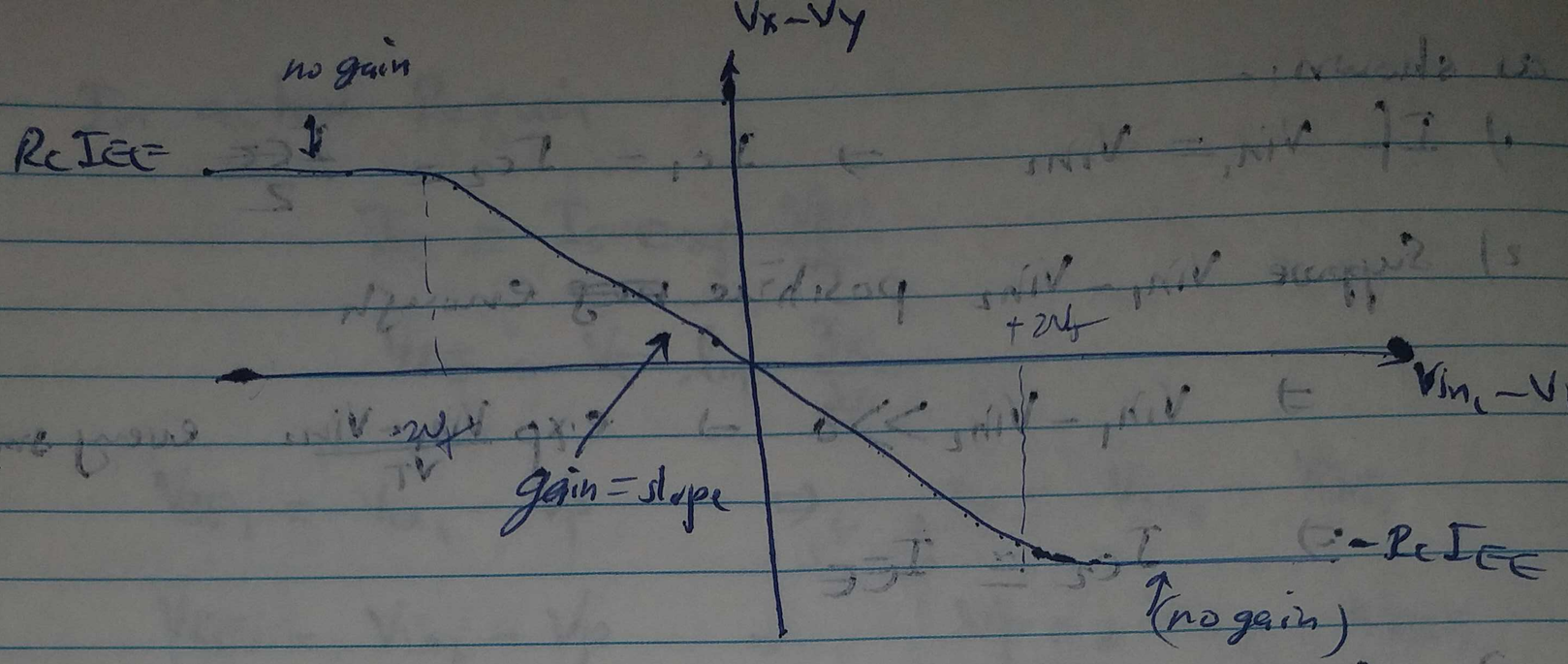
$\Rightarrow V_X = V_{CC} - R_C \frac{I_{EE}}{1 + \exp \frac{V_{in_2} - V_{in_1}}{V_T}}$

$V_Y = V_{CC} - R_C \frac{I_{EE}}{1 + \exp \frac{V_{in_1} - V_{in_2}}{V_T}}$

$\therefore V_X - V_Y = R_C I_{EE} \tanh \frac{V_{in_1} - V_{in_2}}{2V_T}$

MATH $\tanh x$





Example:- Find the slope (gain) of the characteristic around $(V_{in_1} - V_{in_2}) = 0$

$$\tanh x \cong x \qquad x \ll 1$$

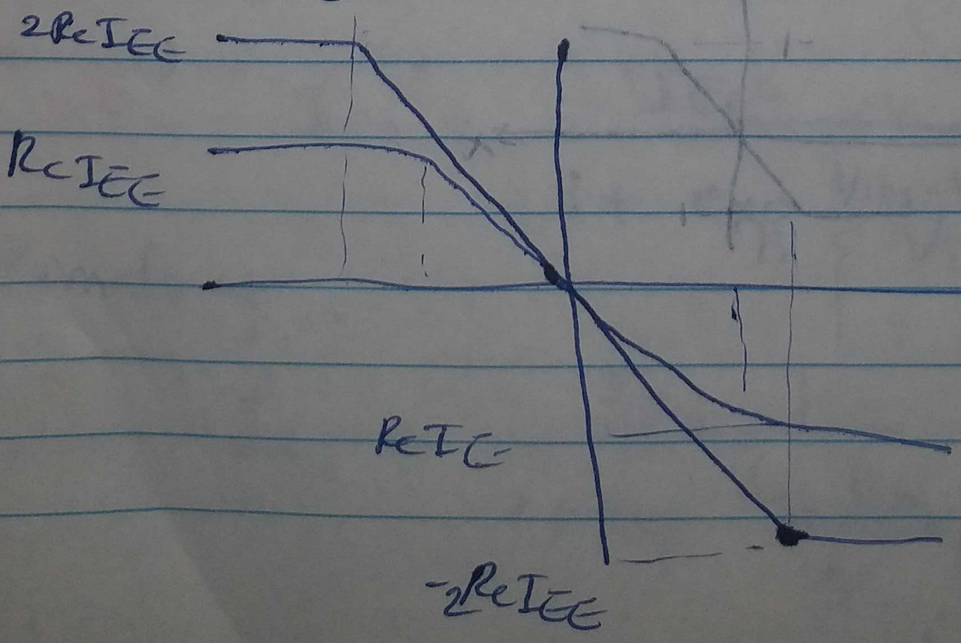
If $\frac{V_{in_1} - V_{in_2}}{2V_T} \ll 1 \Rightarrow \tanh\left(\frac{V_{in_1} - V_{in_2}}{2V_T}\right) \cong \frac{V_{in_1} - V_{in_2}}{2V_T}$

$$\Rightarrow V_x - V_y = -R_c I_{EE}\left(\frac{V_{in_1} - V_{in_2}}{2V_T}\right)$$

Slope = gain

$$\frac{V_x - V_y}{V_{in_1} - V_{in_2}} = -\frac{R_c I_{EE}}{2V_T}$$

Example: What happens to the characteristic if I_{EE} is doubled.



gain
Slope = because is doubled

Example:

What happens if R_C is doubled?

⇒ the slope (gain) is doubled.

Example:

What happens if the ambient temperature drops considerably?

$$\text{slope} = -\frac{R_C I_{EE}}{2V_T}$$

$V_T = \frac{kT}{q}$ so if temp decreases

so the slope goes up.

Today's lect.

- Summery of Diff. pair operation
- Small Signal Analysis

Review:-

$$V_X - V_Y = -R_C I_{EE} \tanh \frac{V_{in_1} - V_{in_2}}{2V_T}$$

If $V_{in_1} - V_{in_2} \ll 2V_T$

$$\Rightarrow V_X - V_Y = -R_C I_{EE} \frac{V_{in_1} - V_{in_2}}{2V_T}$$

$$\text{slope } \frac{V_X - V_Y}{V_{in_1} - V_{in_2}} = -\frac{R_C I_{EE}}{2V_T} = \text{gain}$$

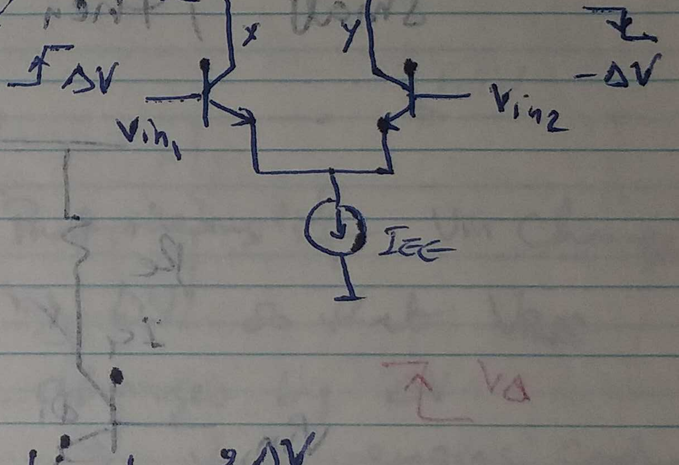
Summary of diff pair properties:-

1) If $|V_{in_1} - V_{in_2}|$ is Large, all of the tail current is "steered" to one side.
   ⇒ current steering circuits.

2) If $V_{in_1} = V_{in_2}$ and $V_{CM,in}$ goes up and down
   ⇒ the currents and voltages don't change.

3) If $|V_{in_1} - V_{in_2}| \ll 2V_T$ ⇒ the slope $= -\frac{R_C I_{EE}}{2V_T}$

Quiz:- Can this circuit amplify?



if the slope is greater than 1
so the circuit amplify

$$\text{slope} = -\left(\frac{R_C I_{EE}}{2V_T}\right) > 1$$

$$V_x - V_y = -R_C I_{EE} \tanh \frac{2\Delta V}{2V_T}$$

$$\cong -R_C I_{EE} \frac{2\Delta V}{2V_T}$$

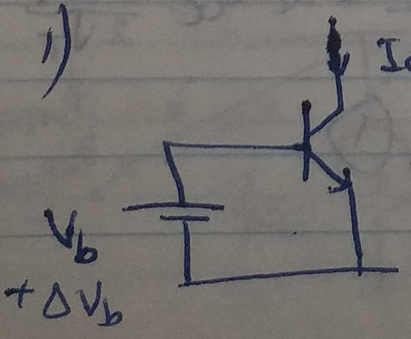
$$\frac{V_x - V_y}{2\Delta V} = -\left(\frac{R_C I_{EE}}{2V_T}\right) \leftarrow \text{if greater than 1}$$

⇒ amplify

Small Signal Analysis:-

Two basic Concepts:-

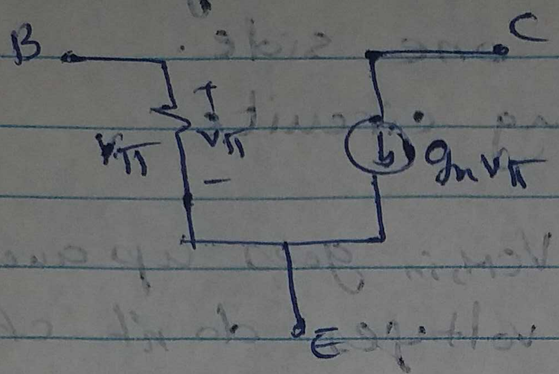
1)



$$\frac{\Delta I_C}{\Delta V_b} = g_m \text{ (transconductance)}$$

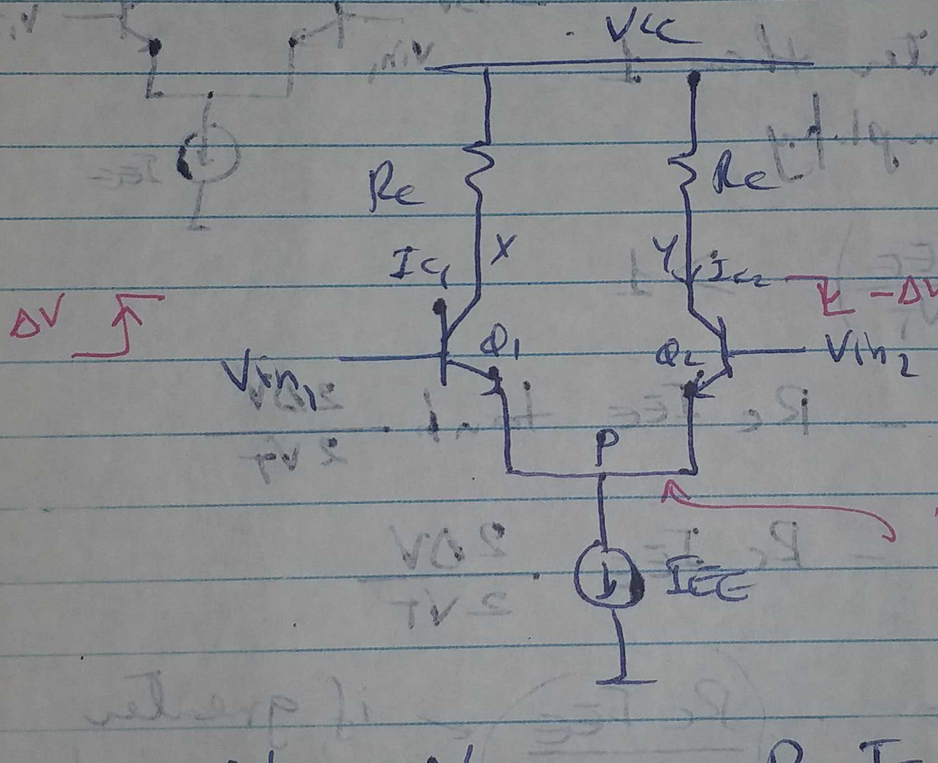
$$g_m = \frac{I_C}{V_T}$$

5) Small Signal of BJT



Lemma:-

If V_{in_1} and V_{in_2} change by equal and opposite amounts and the change is small, then the tail node voltage doesn't change



$$V_X - V_Y = -R_C I_{EE} \frac{2\Delta V}{2V_T}$$

$$V_X = V_{CC} - I_{C_1} R_C$$

$$V_Y = V_{CC} - I_{C_2} R_C$$

$$V_X - V_Y = -R_C (I_{C_1} - I_{C_2}) = -R_C I_{EE} \frac{2\Delta V}{2V_T}$$

$$I_{C_1} - I_{C_2} = I_{EE} \frac{\Delta V}{V_T} \rightarrow (1)$$

$$I_{C1} + I_{C2} = I_{EE} \quad (2)$$

Solving (1,2) ⇒

$$\Rightarrow I_{C1} = \frac{I_{EE}}{2}\left[1 + \frac{\Delta V}{V_T}\right]$$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} \Rightarrow \partial I_C = g_m \partial V_{BE}$$

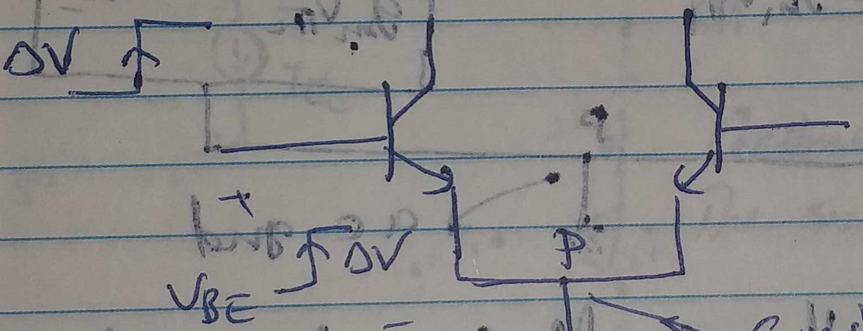
$$\Delta I_{C1} = \frac{I_{EE}}{2}\left[\frac{\Delta V}{V_T}\right] \Rightarrow \frac{I_{EE}}{2V_T}\Delta V$$

$$\Delta V_{BE1} = \frac{\frac{I_{EE}}{2V_T}\Delta V}{\left(I_{EE}/2V_T\right)} = \Delta V$$

ΔV_{BE1} ↗ ; g_m ↗

$$\Delta V_{BE} = \frac{\Delta I_D}{g_m}$$

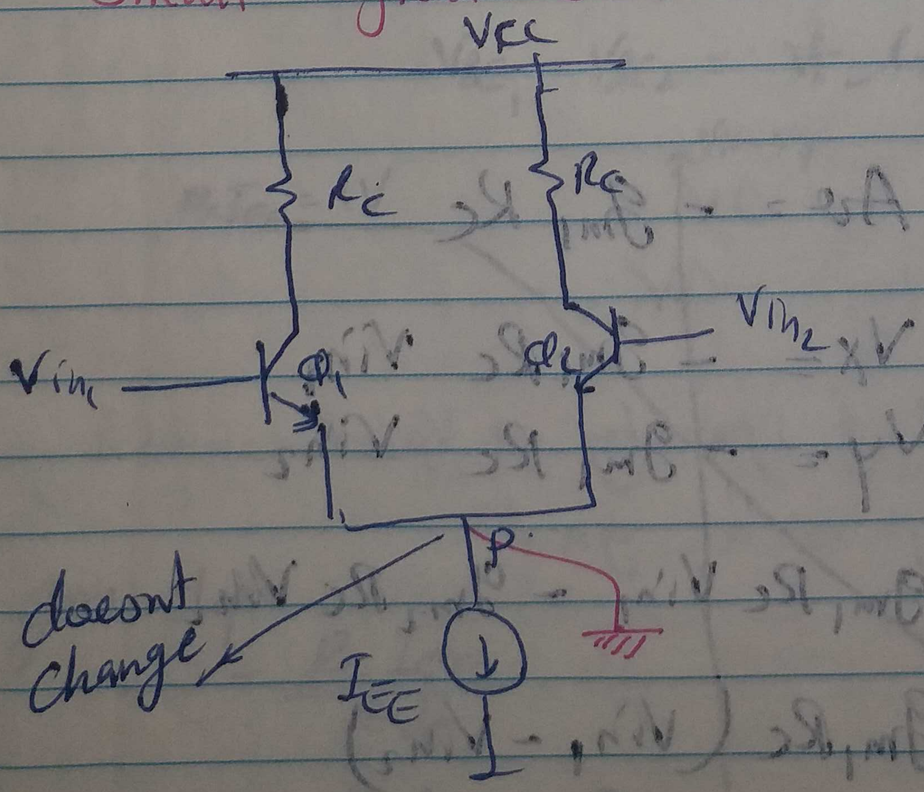
$$g_m = \frac{I_{EE}}{2V_T} = \frac{I_C}{V_T}$$



That means if V_{in} changes by ΔV so that V_{BE} changes by ΔV

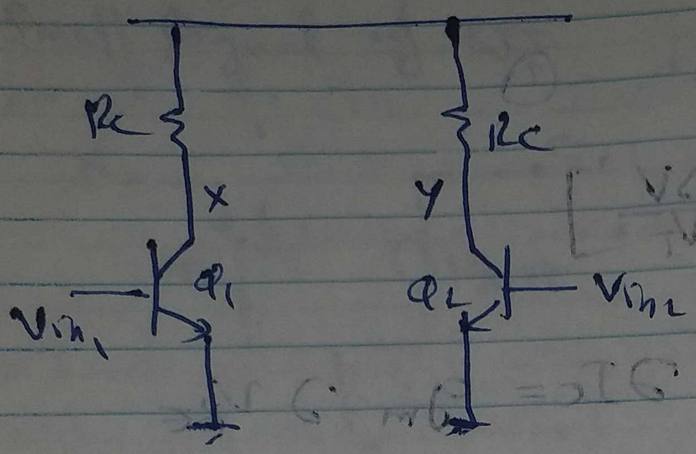
⇒ V_p ⇒ remains Constant

Small Signal Behaviour:-



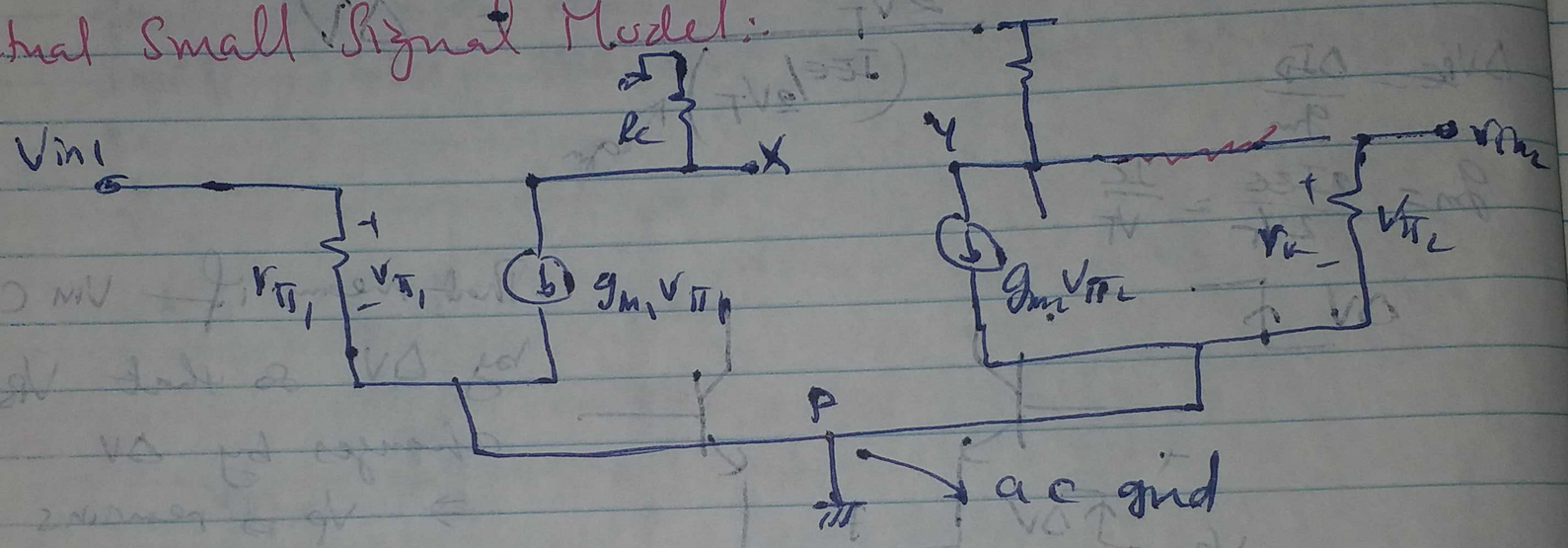
V_p: doesn't change with time...

So it is work as ac gnd (virtual gnd)



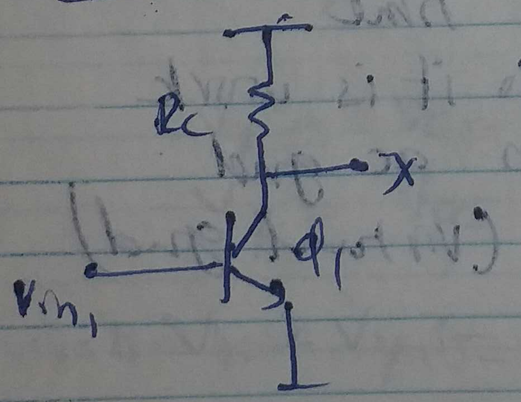
For small signal analysis V_p is constant so it work as ac gnd.

Actual Small Signal Model:



It clears that the small signal redues as two half circuits

Half circuit:



$$A_v = -g_{m_1} R_c$$

$$V_x = -g_{m_1} R_c V_{in_1}$$

$$V_y = -g_{m_1} R_c V_{in_2}$$

$$V_x - V_y = -g_{m_1} R_c V_{in_1} - g_{m_2} R_c V_{in_2}$$

$$= -g_{m_1} R_c (V_{in_1} - V_{in_2})$$

differential gain $A_d = -g_{m_1} R_c = \dfrac{V_x - V_y}{V_{in_1} - V_{in_2}}$

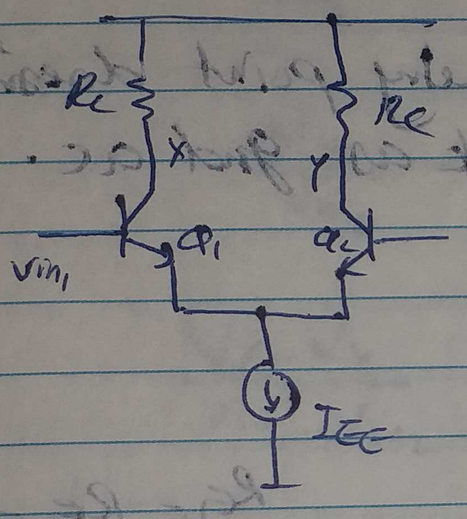
$g_m = \dfrac{I_{EE}}{2V_T}$

$$g_m = \frac{I_C}{V_T} = \frac{I_{EE}/2}{V_T}$$

$$A_v = -\frac{R_C I_{EE}}{2V_T} = -g_m R_C$$

Today's lect.
- Additional Examples on Bipolar Diff pair
- Introduction to MOS diff pair

Examples:



$R_C = 1\,k\Omega$ $\quad I_{EE} = 1mA$

$I_S = 2\times10^{-18}$ $\quad I_C = I_S \exp\frac{V_{BE}}{V_T}$

a) Determine the Bias Conditions of Q_1 and Q_2

b) Construct the input/output characteristic.

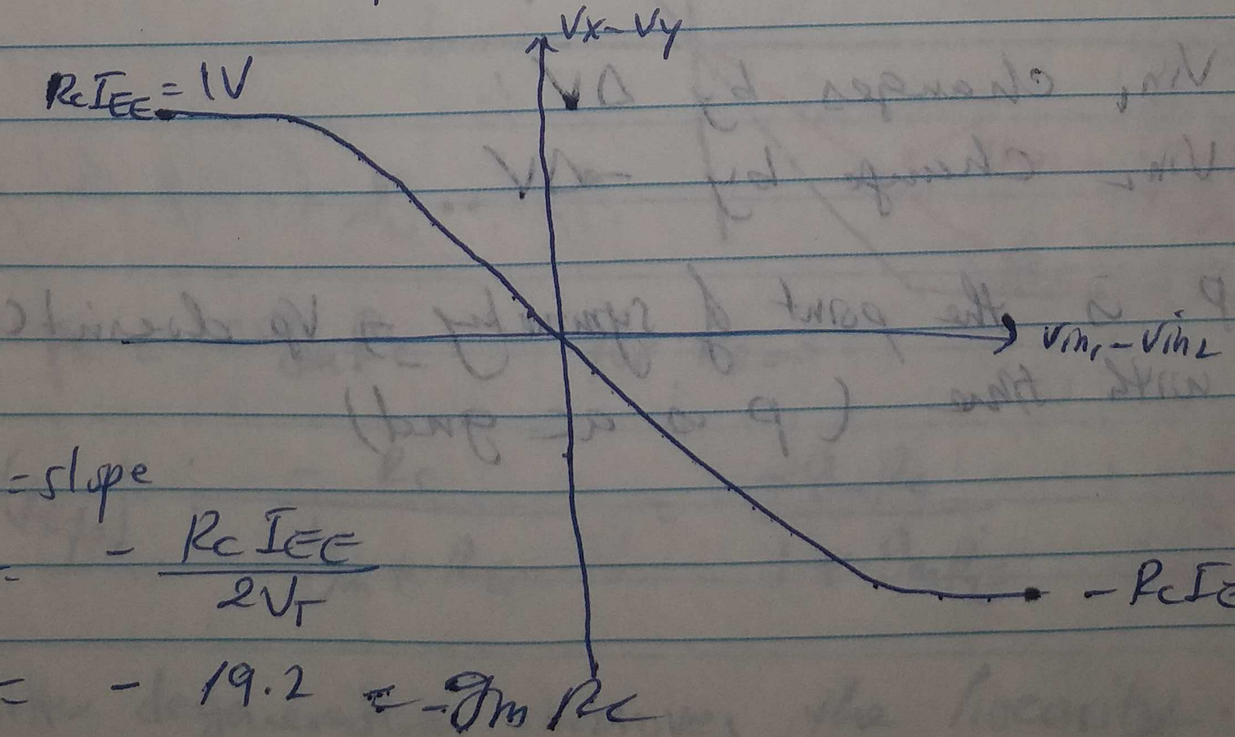
c) The small signal differential gain

a) Bias Conditions :- No signal $V_{in_1} = V_{in_2} = 0$

$$\Rightarrow I_{C_1} = I_{C_2} = \frac{I_{EE}}{2} = 0.5mA$$

$$V_{BE_1} = V_{BE_2} = V_T \ln\frac{I_C}{I_S} = 862\,mV$$

b)

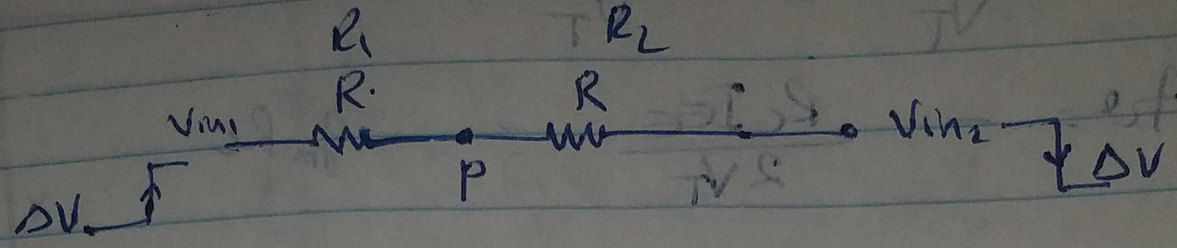


c) gain = slope

$$= -\frac{R_C I_{EE}}{2V_T}$$

$$= -19.2 = -g_m R_C$$

Example



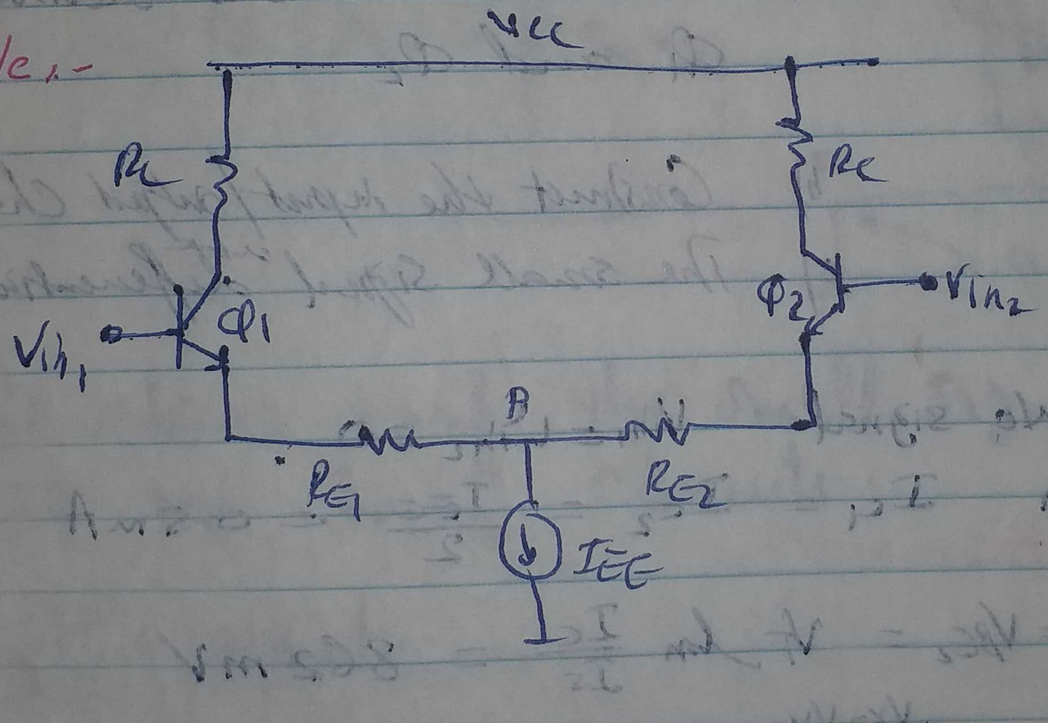
$$I = \frac{2\Delta V}{2R} = \frac{\Delta V}{R}$$

Voltage drop across $R_1 = \frac{\Delta V}{R} R = \Delta V$

that means V_p doesn't change (Constant)

That means the voltage of symmetry point doesn't change (remain Constant), work as gnd ac.

Example:-



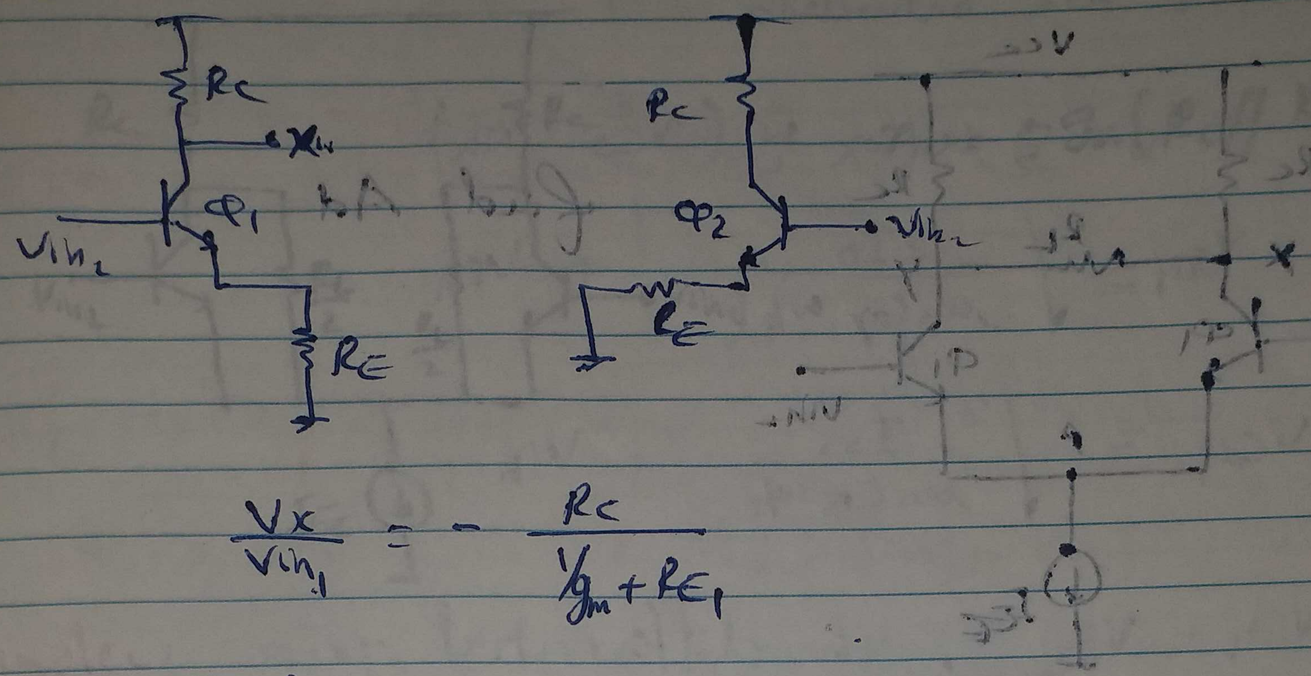
$R_{E1} = R_{E2} = R_E$

assume V_{in_1} changes by ΔV

V_{in_2} change by $-\Delta V$

$\Rightarrow$ P is the point of symmetry $\Rightarrow$ V_p doesn't change with time (p is ac gnd)

Using half circuit method ⇒ CE with ~~Degeneration~~

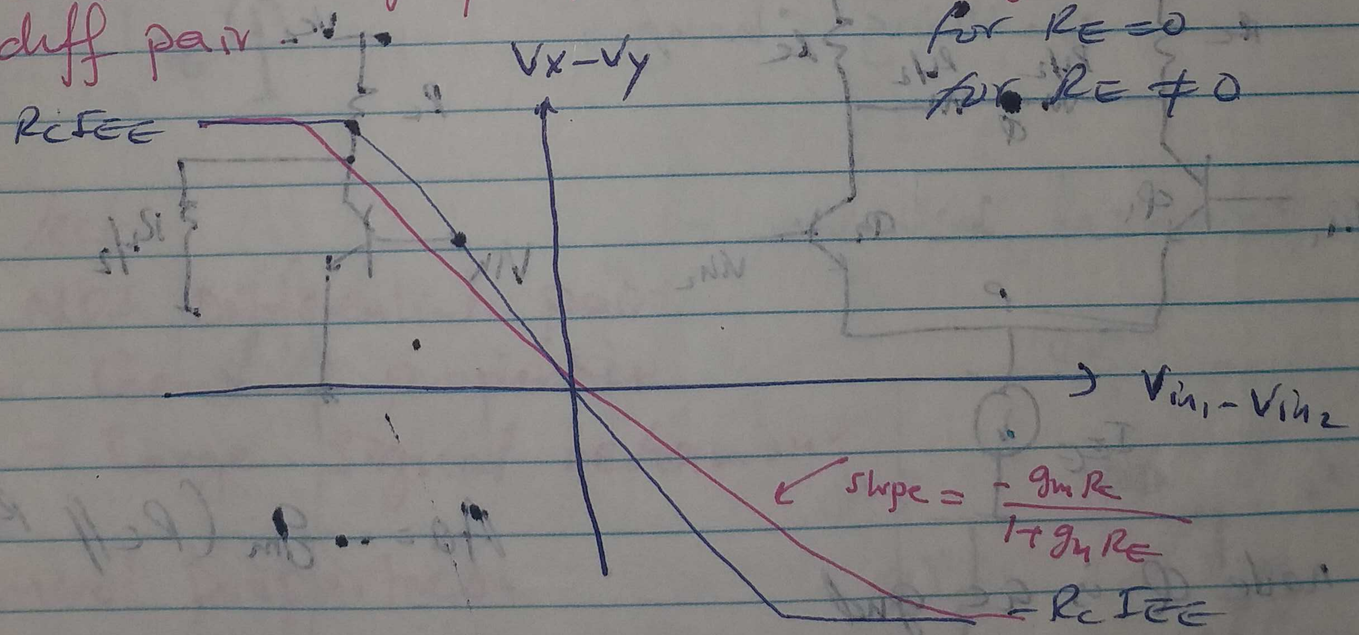


$$\frac{V_x}{V_{in_1}} = -\frac{R_C}{\frac{1}{g_m} + R_{E_1}}$$

$$\frac{V_y}{V_{in_2}} = \frac{-R_C}{\frac{1}{g_m} + R_{E_2}}$$

$$\frac{V_x - V_y}{V_{in_1} - V_{in_2}} = \frac{-R_C}{\frac{1}{g_m} + R_E}$$

Quiz: Construct the input/output characteristic of the degenerated diff pair



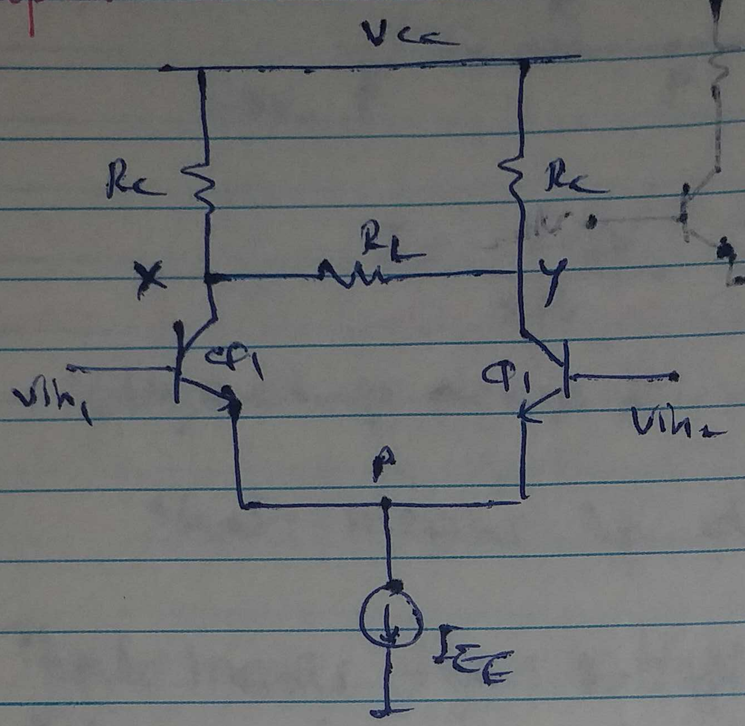
g_m = Slope = $-g_m R_C$ for $R_E = 0$

$$gain\ (R_E \neq 0) = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-g_m R_C}{1 + g_m R_E}$$

(has wide range more linearity

Emitter degeneration improves the linearity.

Example:-

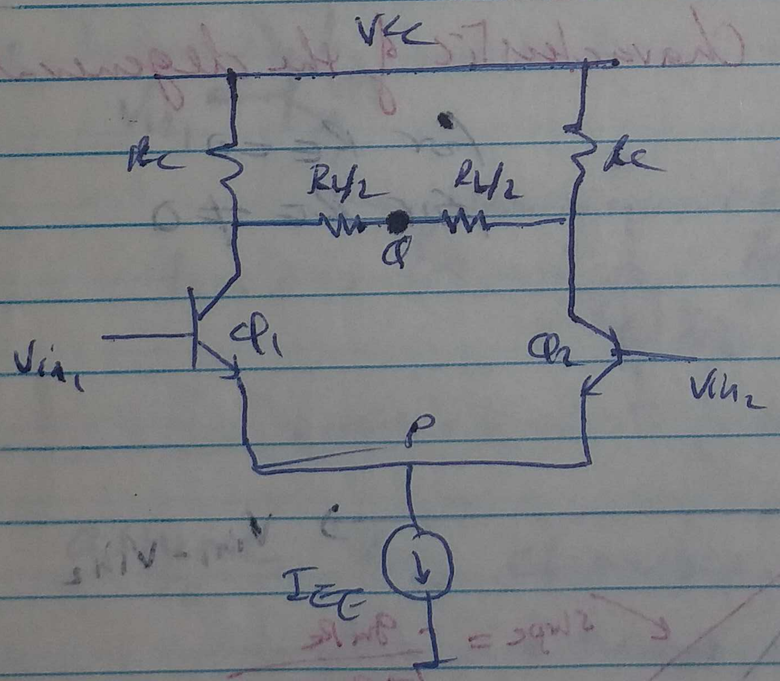


find Ad

If V_{in_1}, V_{in_2} change differentially

→ P is symmetrical point

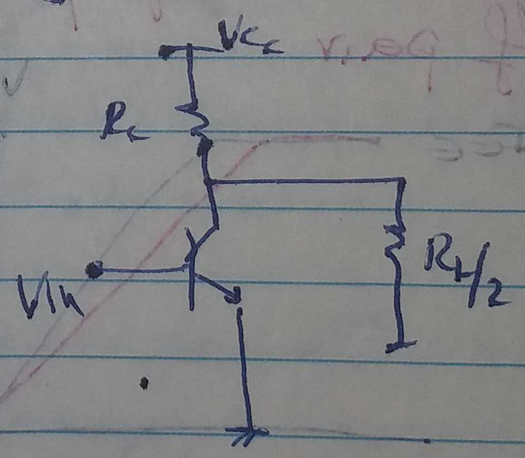
So X, y chang differentially



node Q is ac gnd

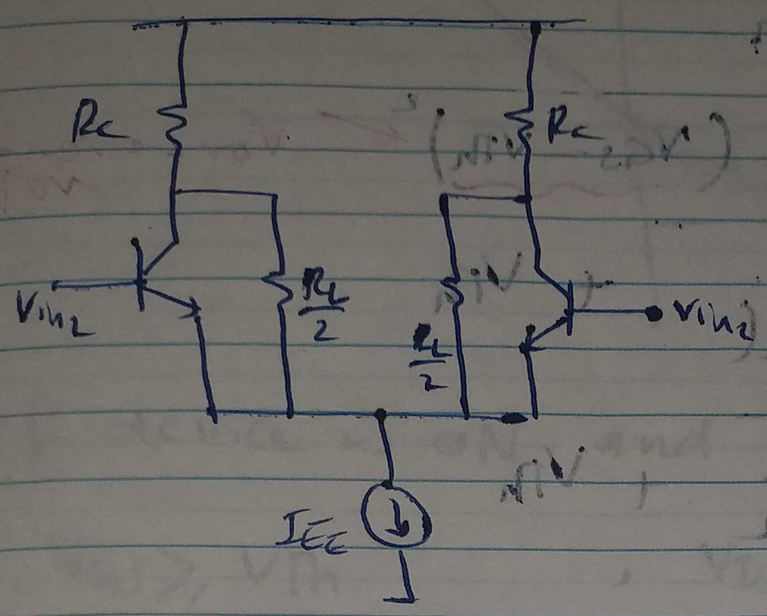
Using half circuit method

equivalent



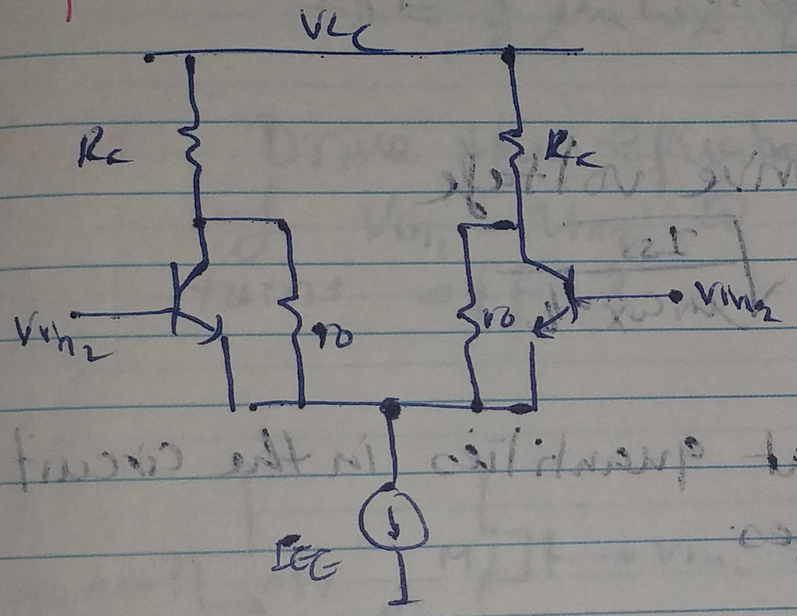
$A_0 = -g_m (R_c // R_L/2)$

Examples



$$A_v = \frac{1}{2} g_m (R_C // R_L/2)$$

Examples



$$A_v = -g_m (R_C // r_O)$$

Today's lect.

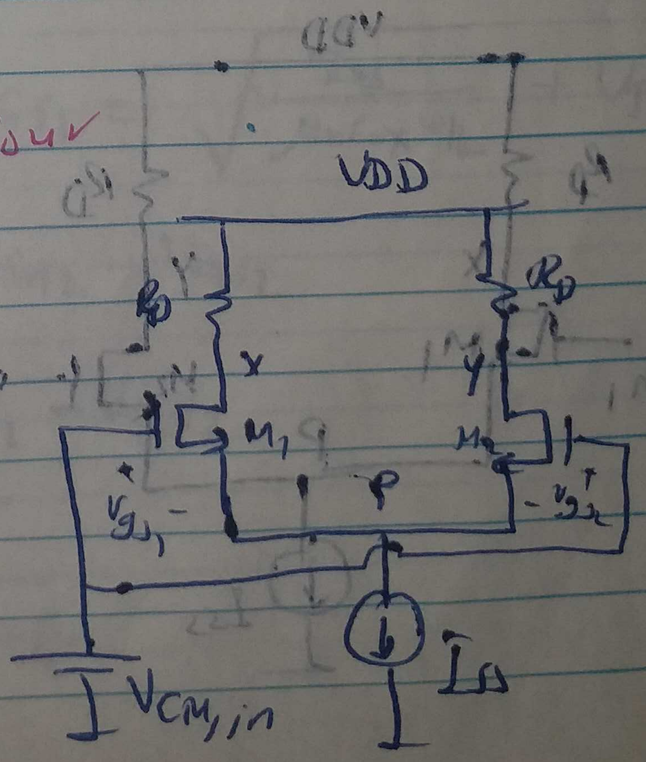
- MOS Differential pair
  - General properties
  - Large Signal behaviour

* General properties:-

1) $V_{in_1} = V_{in_2} = V_{CM}$

$$I_{D_1} = I_{D_2} = \frac{I_{SS}}{2}$$

$$V_X = V_{DD} - \frac{I_{SS}}{2} R_D = V_Y$$



For M_1, M_2 are Saturation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$$ ← V_{ov} = over drive voltage

$$V_{GS_1} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}} + V_{th}$$

$$V_{GS_1} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}} + V_{th}$$

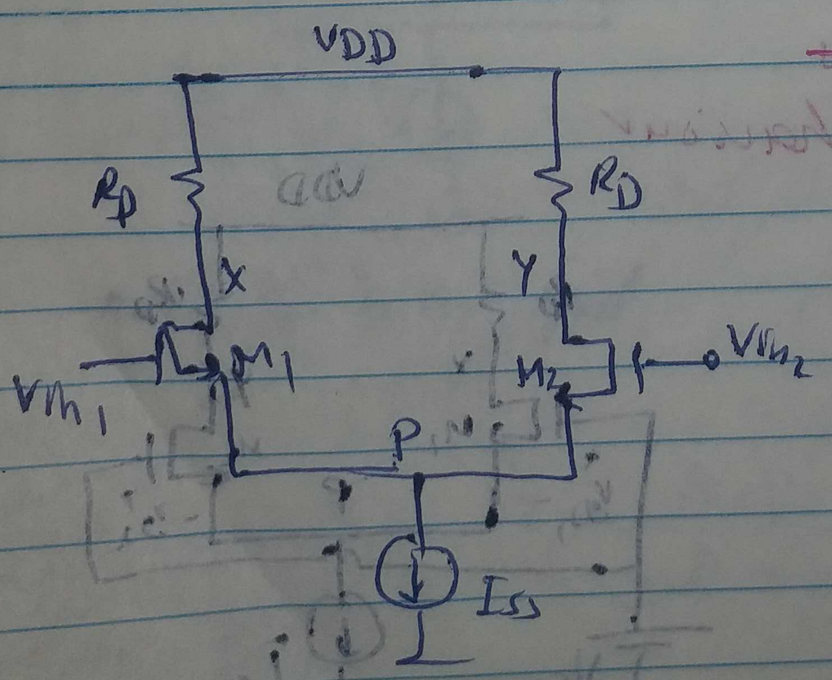
* If $V_{in_1} = V_{in_2}$, the differential pair is in equilbrium

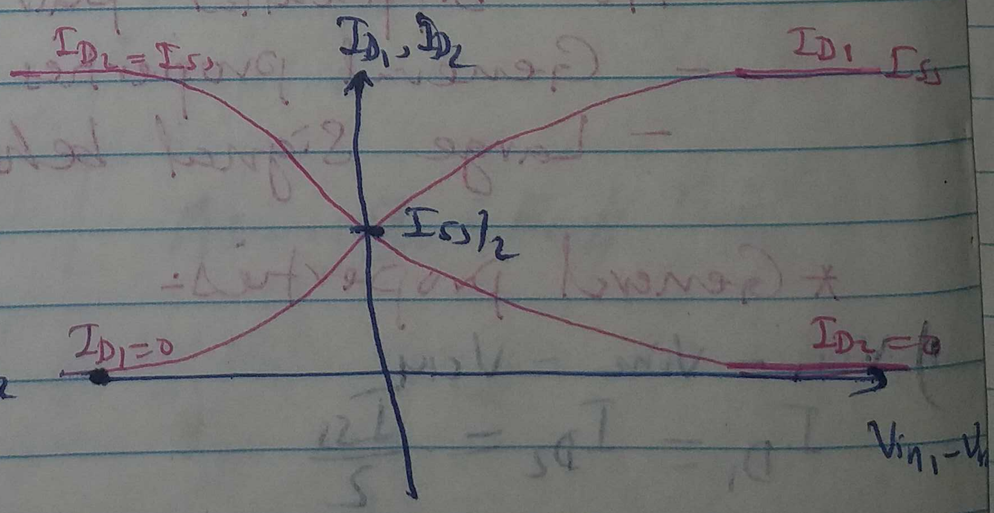
* The equilbrium over drive voltage

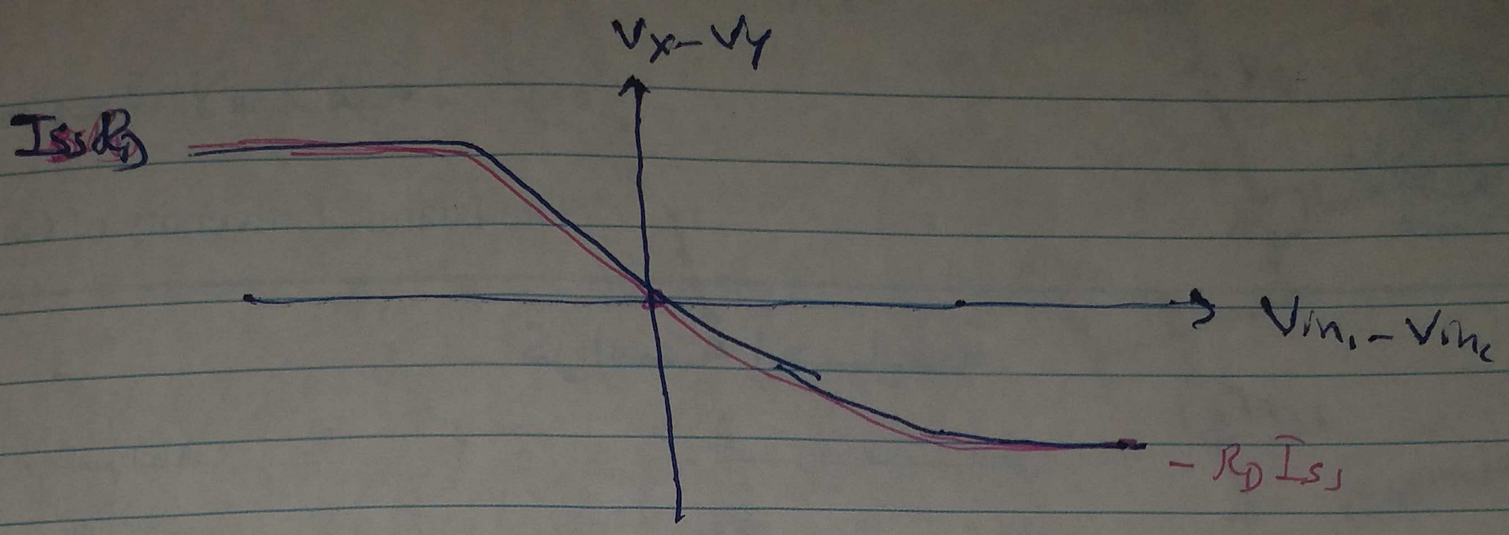
$$V_{ov_1} = V_{gs_1} - V_{th} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}}$$

* If $V_{CM,in}$ changes, what quantities in the circuit change?
  ⇒ Nothing changes.

(2) Differential behavior:-





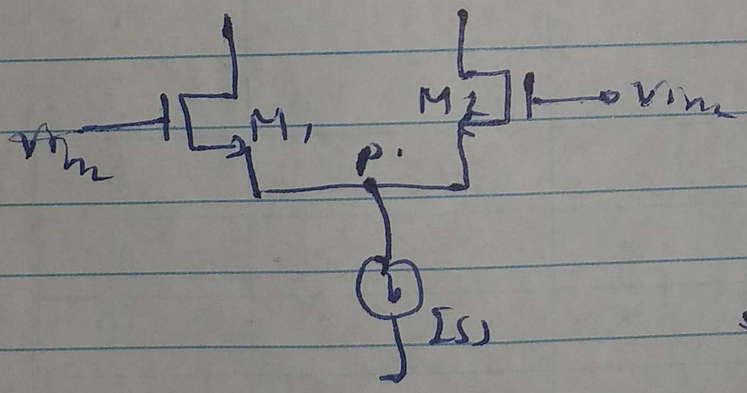


If device is ON and M @ saturation

$V_{GS} > V_{TH}$, $V_{DS} > (V_{GS} - V_{TH})$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$

Example.- Drive the equation of minimum value of $V_{in_1} - V_{in_2}$ @ which one transistor turns off.?



assume M_1 is ON and M_2 is OFF

$\Rightarrow I_{D_1} = I_{SS}$, $I_{D_2} = 0$

$\Rightarrow V_{GS_2} = V_{TH}$

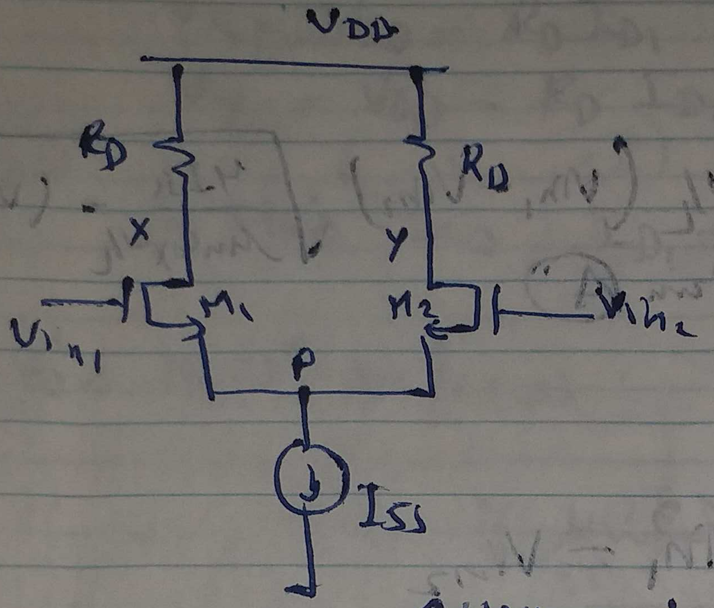
$$\Rightarrow V_{GS_1} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}} + V_{TH}$$

$$\Rightarrow V_P = V_{in_1} - V_{GS_1} = V_{in_2} - V_{GS_2}$$

$$\Rightarrow V_{in_1} - V_{in_2} = V_{gs_1} - V_{gs_2}$$

$$= \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}}$$

Large Signal Analysis:- Find $(V_x - V_y)$ in function of $(V_{in_1} - V_{in_2})$



$$V_p = V_{in_1} - V_{gs_1}$$
$$= V_{in_2} - V_{gs_2}$$

$$\Rightarrow V_{in_1} - V_{in_2} = V_{gs_1} - V_{gs_2}$$

assume both M_1 and M_2 are ON

$$V_{in_1} - V_{in_2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} W/L}} + V_{TH} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} W/L}} - V_{Th}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} W/L}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} W/L}} \rightarrow (1)$$

$$V_x = V_{DD} - R_D I_{D1}$$
$$V_y = V_{DD} - R_D I_{D2}$$

$$\Rightarrow V_x - V_y = -R_D (I_{D1} - I_{D2})$$

and we have

$$I_{D1} + I_{D2} = I_{SS} \rightarrow (2)$$

Solving (1) (2)

$$(V_{in_1} - V_{in_2})^2 = \frac{2I_{D1}}{\mu_n C_{ox} W/L} + \frac{2I_{D2}}{\mu_n C_{ox} W/L} - \frac{2\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox} W/L}$$

$$(V_{in1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} W/L} \quad \Rightarrow \quad -V_{in1} - V_{in2} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} W/L}}$$

Solving gives

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} W/L} - (V_{in1} - V_{in2})^2}$$

equation (A)

Observation :-

1) $I_{D1} - I_{D2} = 0$ if $V_{in1} = V_{in2}$

2) Equation A is valid if and only both M_1 and M_2 are ~~turning~~ ON

or at the edge of turning off up to

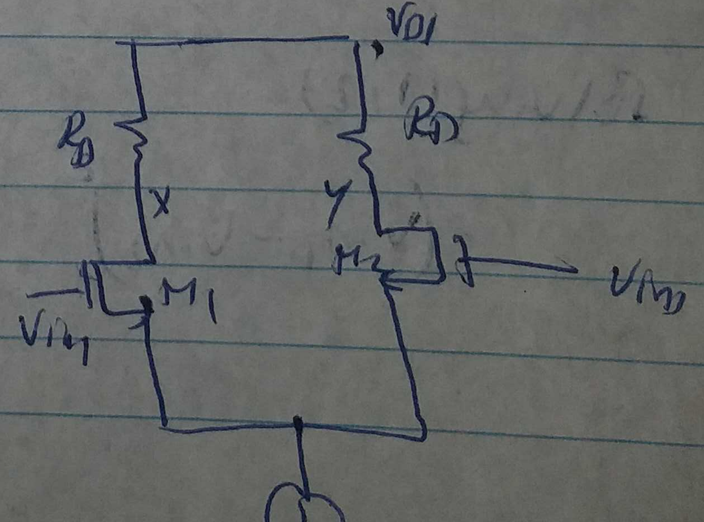
$$|V_{in1} - V_{in2}| = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} W/L}}$$

Today's lect.

- Detailed Analysis of MOS Diff pair
  Large signal Response
- Overall Input / output charact.
- Dependencies various parameters

Summary of the Results

1) $V_{in1} = V_{in2} = V_{CM,in}$



$$I_{S1} = I_{S2} = I_{D1} = I_{D2}' = I_{SS}/2$$

$$V_X = V_{DD} - R_D I_{D1}$$
$$V_Y = V_{DD} - R_D I_{D2}$$
$$\Rightarrow V_X - V_Y = -R_D(I_{D1} - I_{D2})$$

in this Cas $I_{D1} = I_{D2} \Rightarrow V_X - V_Y = 0$

- for M1 sat:

$$I_D = \frac{1}{2}\mu_n C_{ox}' \frac{W}{L}(V_{GS} - V_{TH})$$

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}} + V_{TH}$$

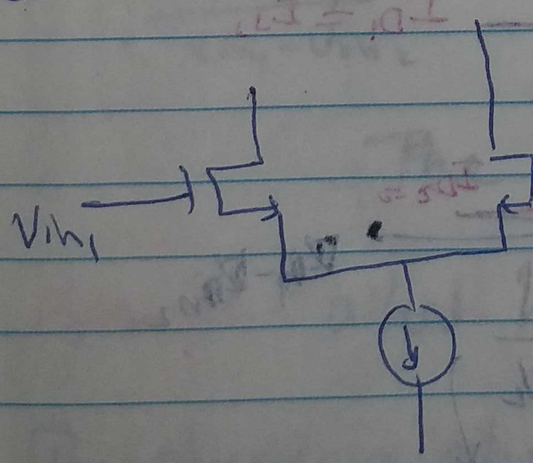
in this Case: $(V_{in1} = V_{in2} = V_{CM/M})$

$$V_{GS1} = V_{GS2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}} + V_{TH}$$

Equilbnium over drive:-

$$\Rightarrow V_{GS1} - V_{TH} = V_{GS2} - V_{TH} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} W/L}} \quad (\text{over drive voltage})$$

② Unbalanced inputs: $V_{in1} > V_{in2}$

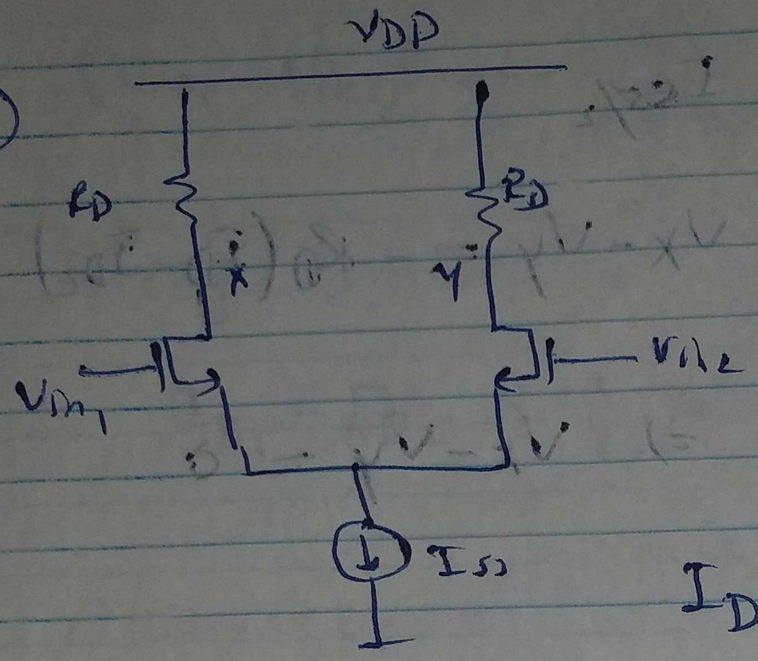


$\Rightarrow$ Minimum $(V_{in1} - V_{in2})$ necessary to turn off one side

$$= \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} W/L}}$$

③

(3)



Unblance Case:-
find all currents and voltages

$(I_{D1} - I_{D2})$ and $(V_X - V_Y)$

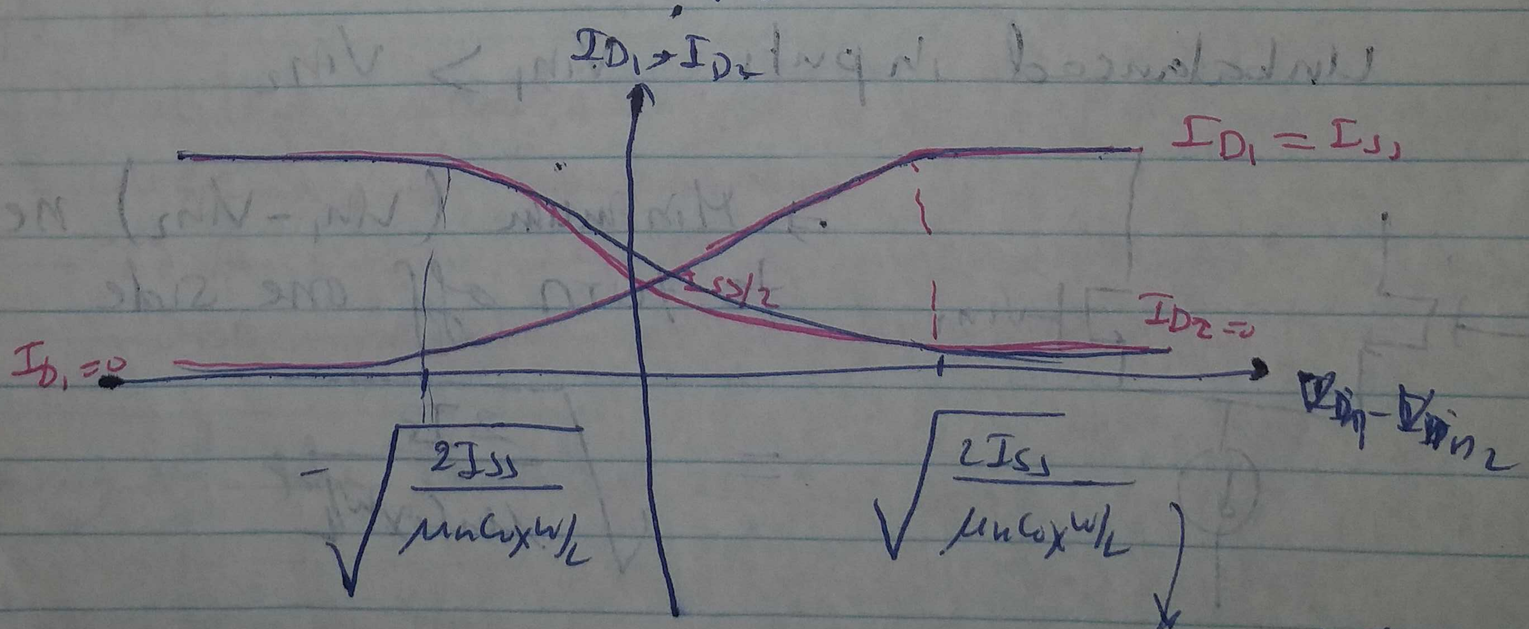
$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2}) * \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} W/L} - (V_{in1} - V_{in2})^2}$$

This equation is valid if both M_1, M_2 are ON or near turn off $(V_{GS} \geq V_{TH})$

⊗ If $V_{in1} - V_{in2} = 0 \Rightarrow I_{D1} = I_{D2}$

⊗ also this valued if (only)

$$|V_{in1} - V_{in2}| \leq \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} W/L}}$$

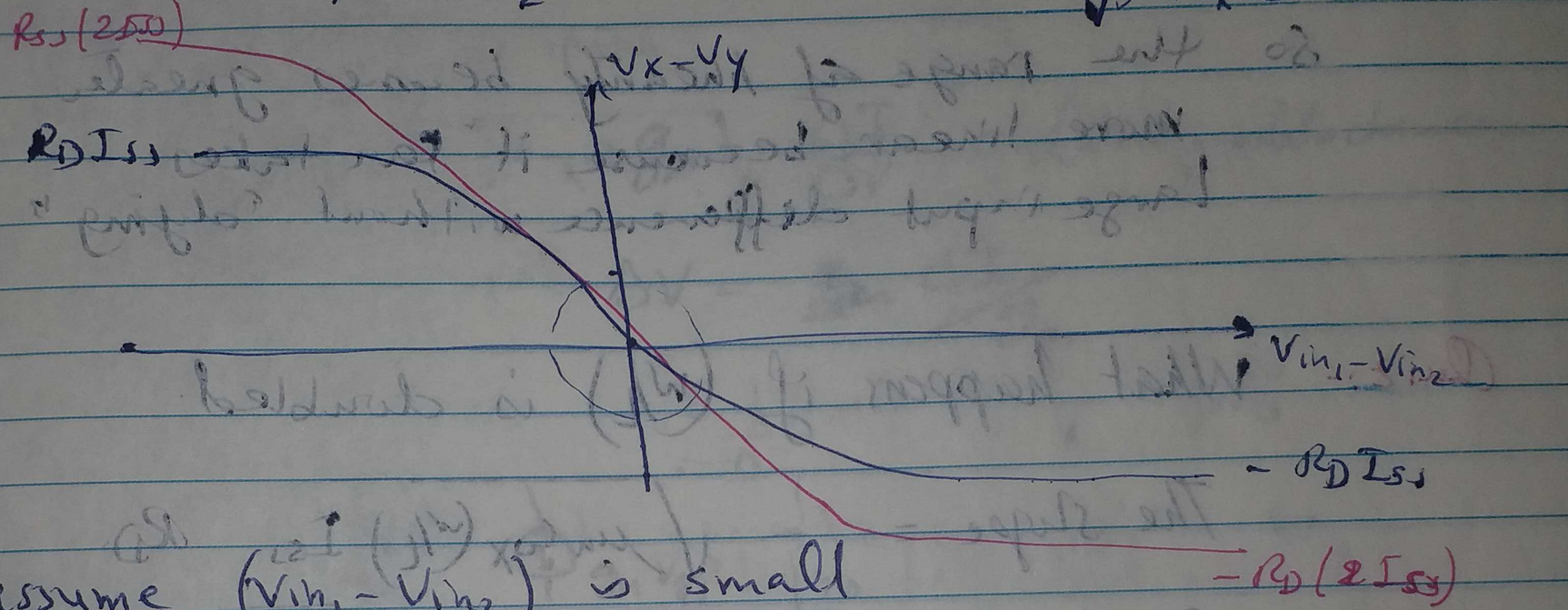


the value of $(V_{in} - V_{in2})$ where one M_1 is ON M_2 is OFF

- Minimum $|V_{in_1} - V_{in_2}|$ to turn off one side

$= \sqrt{2} *$ equilibrium overdrive voltage

4) $$V_x - V_y = -\frac{R_D}{2} \mu_n C_{ox} \frac{W}{L} (V_{in_1} - V_{in_2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} W/L} - (V_{in_1} - V_{in_2})^2}$$



assume $(V_{in_1} - V_{in_2})$ is small

If $(V_{in_1} - V_{in_2})^2 \ll \frac{4 I_{SS}}{\mu_n C_{ox} W/L}$

$$\Rightarrow V_x - V_y = -\frac{R_D}{2} \mu_n C_{ox} \frac{W}{L} (V_{in_1} - V_{in_2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} W/L}}$$

$$\frac{V_x - V_y}{V_{in_1} - V_{in_2}} \cong -\sqrt{\mu_n C_{ox} W/L \, I_{SS}} \, R_D$$

The slope is $-\sqrt{\mu_n C_{ox} W/L \, I_{SS}} \, R_D$ = gain

Example:- What happens if I_{SS} is doubled.

The slope is change to $-\sqrt{\mu_n C_{ox} W/L \, (2I_{SS})} \, R_D$.

the slope goes up by $\sqrt{2}$

$\Rightarrow$ the gain is improved.

→ The range of linearity

$$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox} W/L}} \quad , \quad +\sqrt{\frac{2I_{SS}}{\mu_n C_{ox} W/L}}$$

So when the I_{SS} is doubled

So the range of linearity becomes greater more linear because it can takes Large input difference without "dying"

Quiz:- What happens if (W/L) is doubled.

$$\text{The slope} = -\sqrt{\mu_n C_{ox} (W/L)\, I_{SS}}\; R_D$$

$$\text{Range} \quad -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox} W/L}} \quad , \quad +\sqrt{\frac{2I_{SS}}{\mu_n C_{ox} W/L}}$$

slope becomes:-

$$\text{slope} = -\sqrt{\mu_n C_{ox} (2W/L)\, I_{SS}} \cdot R_D$$

$$= -\sqrt{2} \text{ old slope}$$

Range decrease by $-\frac{1}{\sqrt{2}}$, $+\frac{1}{\sqrt{2}}$

(-) The linearity becomes less (linear) because it can takes smaller input difference before (dying)

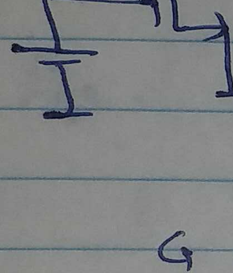
Today's lect.

- Applications of some circuit techniques
- Small Signal Analysis of MOS Diff. pair

Small Signal Behavior of MOS Diff. pair:-

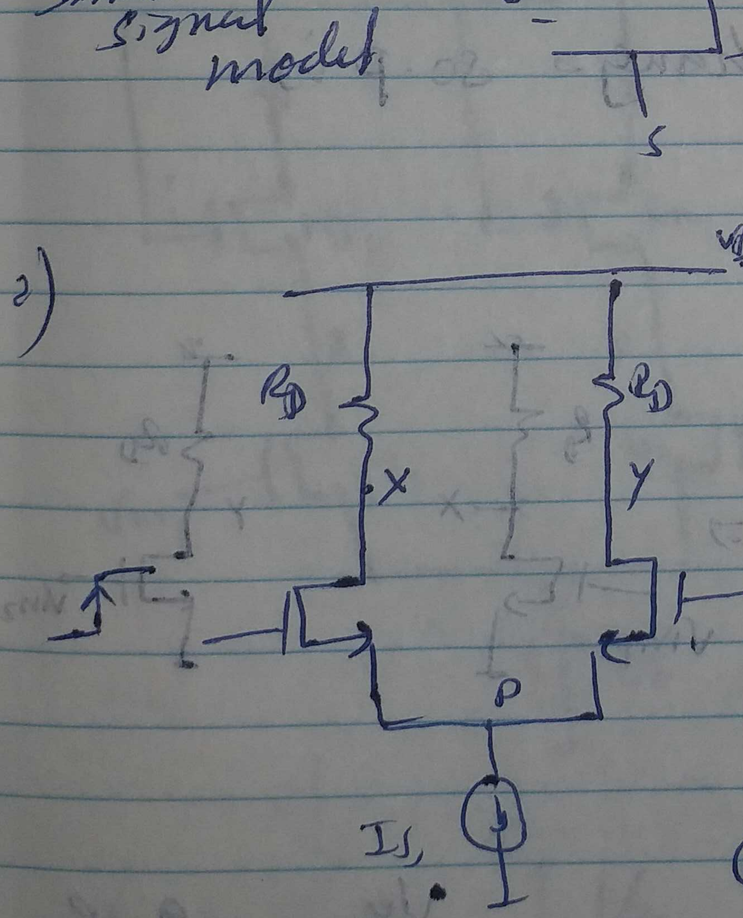
A Few points:-

1) * $g_m = \frac{\Delta I}{\Delta V}$ (transconductance)



$\Delta V = \frac{\Delta I}{g_m}$

2)



2)

Both V_{in_1}, V_{in_2} are differentials

$$\frac{V_X - V_Y}{V_{in_1} - V_{in_2}} = -R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}$$

$$A_v = -R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}$$

* the P-point is symmetrical point

$$V_X - V_Y = -R_D (I_{D_1} - I_{D_2}) = -R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} \quad (20V)$$

$$I_{D_1} - I_{D_2} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} \quad (20V)$$

$$I_{D1} + I_{D2} = I_{SS}$$

solving ⇒

$$I_{D1} = \frac{I_{SS}}{2} + \underbrace{\sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS} \Delta V}}_{\text{change in } I_{D1}}$$

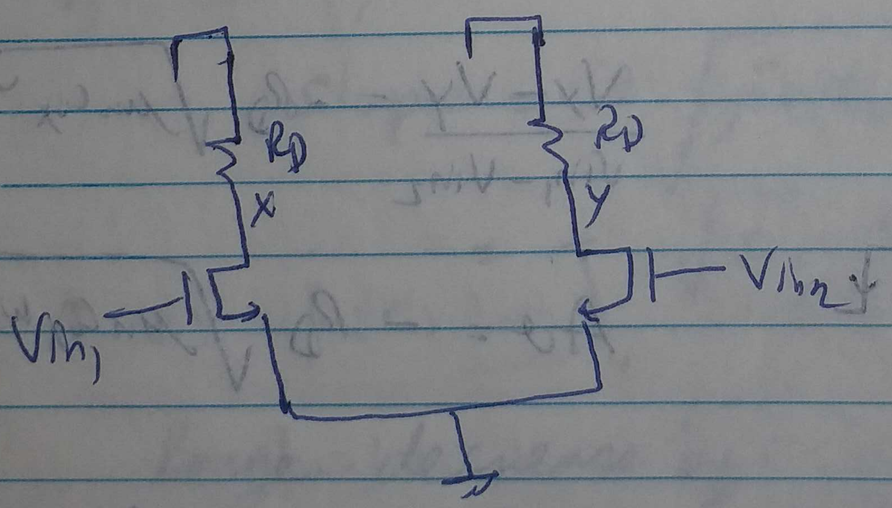
V_{GS1} changes by $\frac{\sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ \Delta V}{g_{m1}}$

$$g_{m1} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

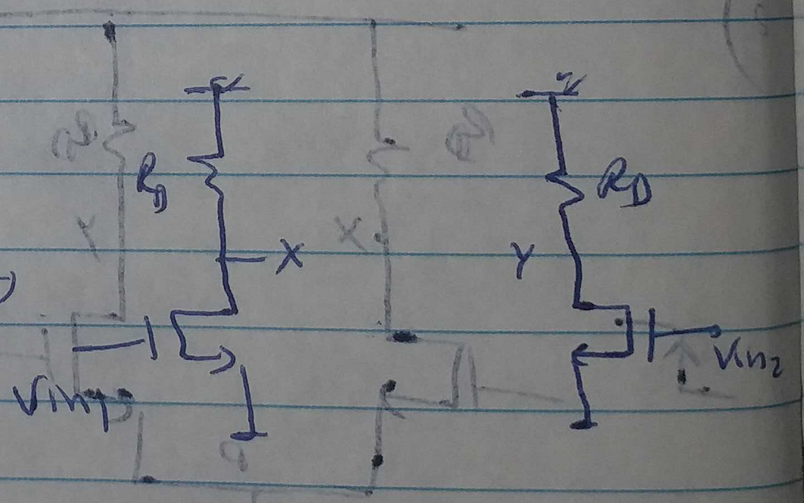
⇒ V_{GS1} is change by ΔV

That means V_p doesn't change so p is ac gnd.

Small signal operation:



⇒

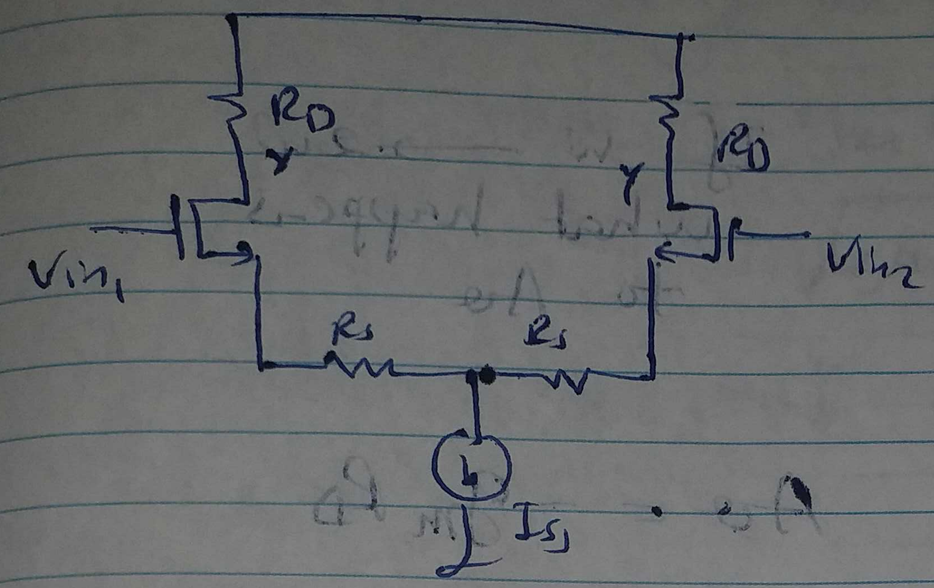


$$\frac{V_x}{V_{in}} = -g_m R_D$$

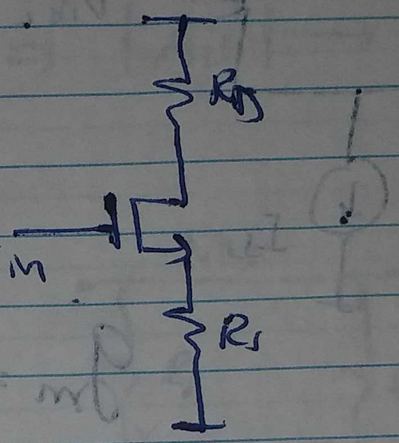
$$\frac{V_y}{V_{in2}} = -g_m R_D$$

$$\Rightarrow \frac{V_x - V_y}{V_{in1} - V_{in2}} = -g_m R_D$$

Example:-

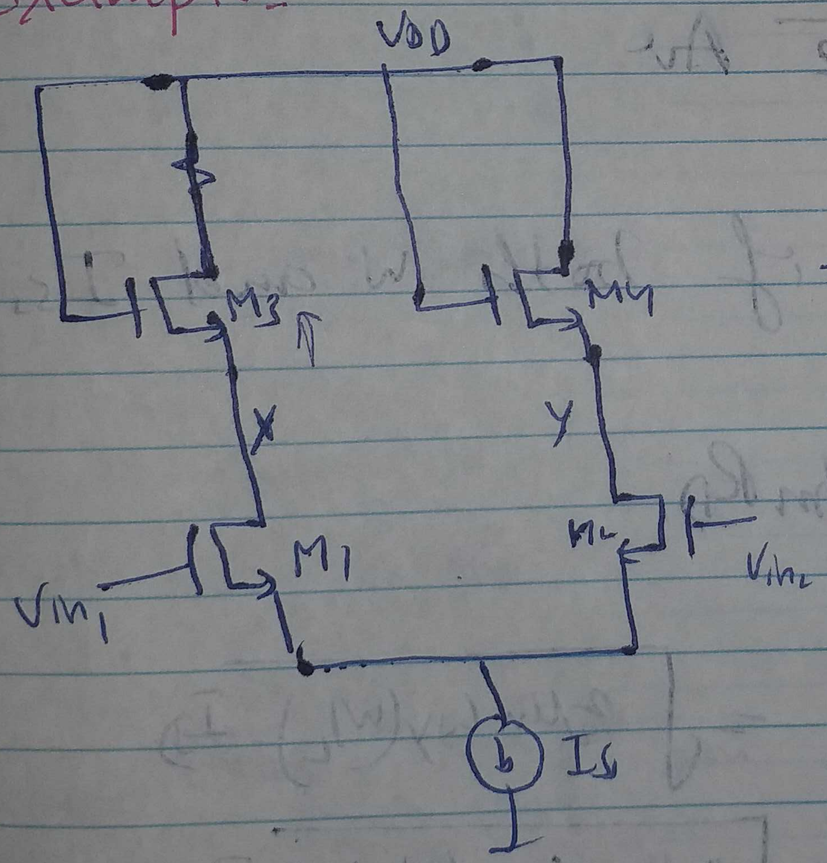


find A_v using half circuit

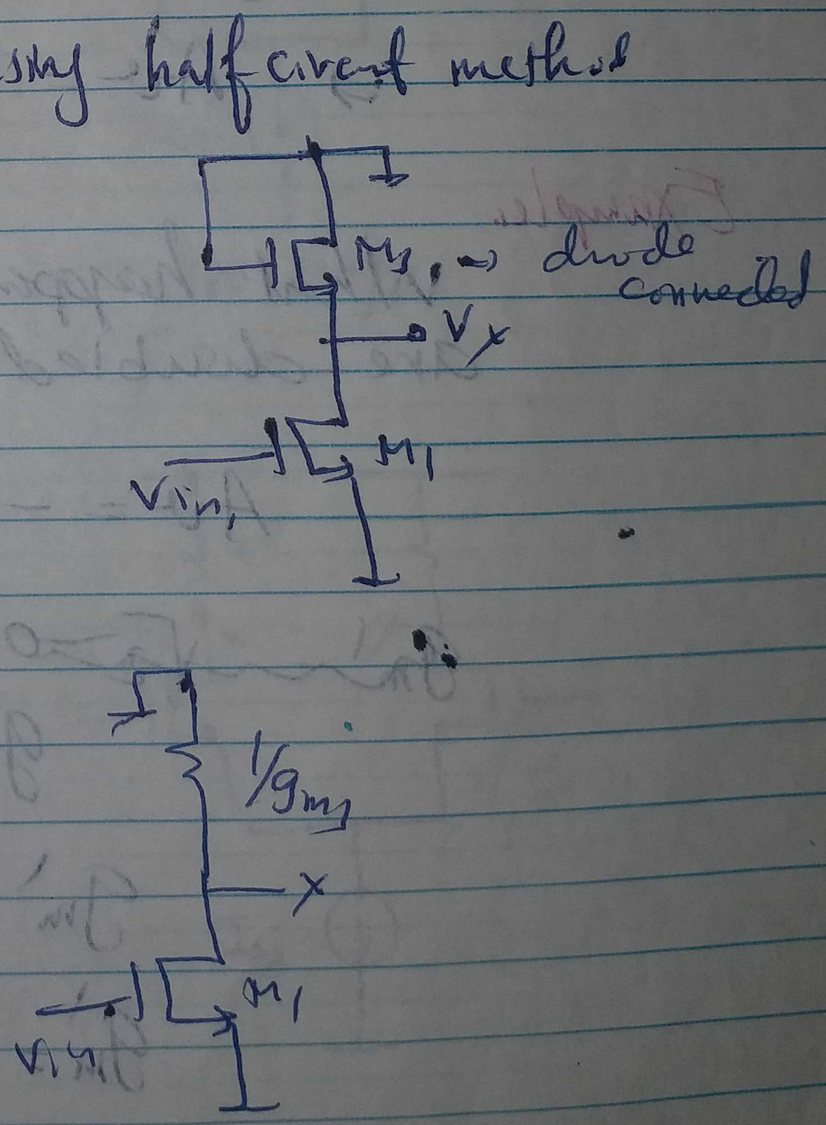


$$A_v = \frac{-R_D}{1/g_m + R_s}$$

Example:-



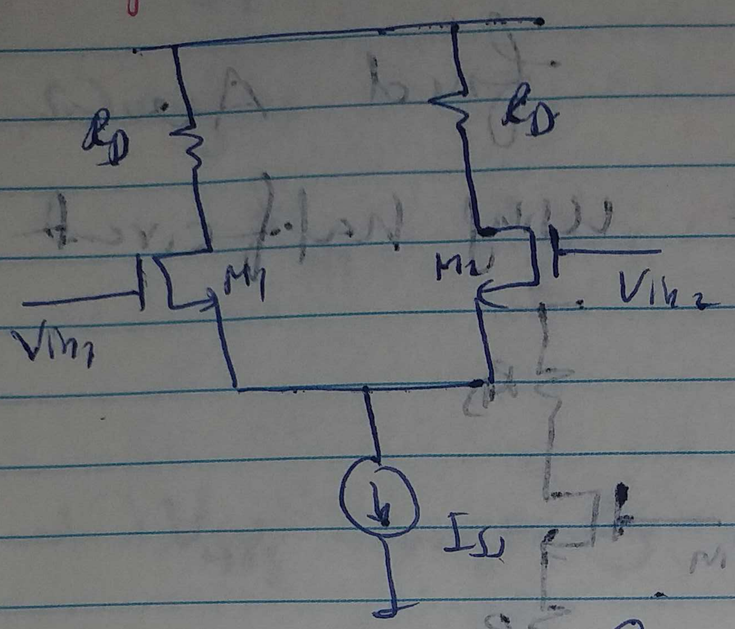
Using half circuit method



$$A_v = -g_{m_1}\left(1/g_{m_3}\right)$$

$$= -\frac{g_{m_1}}{g_{m_3}}$$

Example:



If $W \longrightarrow 2W$
what happens to A_v

$$A_v = -g_m R_D$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

doubling $g_m \Rightarrow$ $g_m' = \sqrt{2}\, g_m$

$\Rightarrow A_v' = \sqrt{2}\, A_v$

Example:

What happens if both W and I_{SS} are doubled.

$$A_v' = -g_m' R_D$$

$g_m' \sim \sqrt{2}$

$$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$$

$$g_m' = \sqrt{2\mu_n C_{ox} (2W/L) 2I_D}$$

$$g_m' = 2 g_m$$

$\Rightarrow A_v' = 2A_v$

Example:-

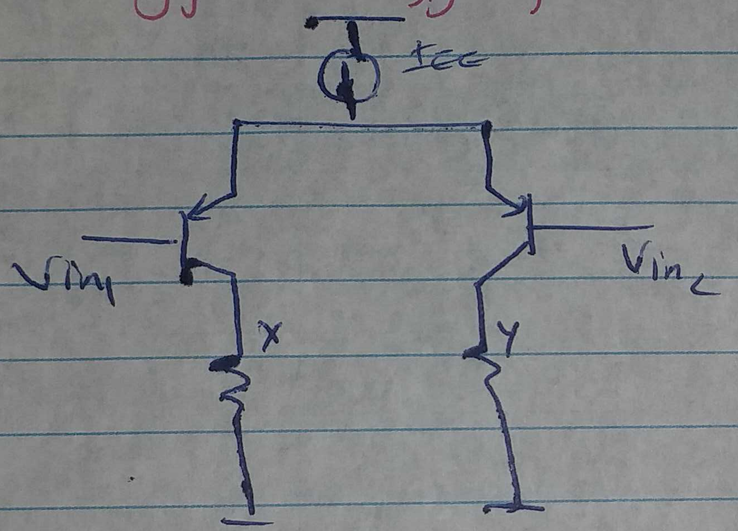
What happens if the temperture rises?

$$A_v = -g_m R_D$$

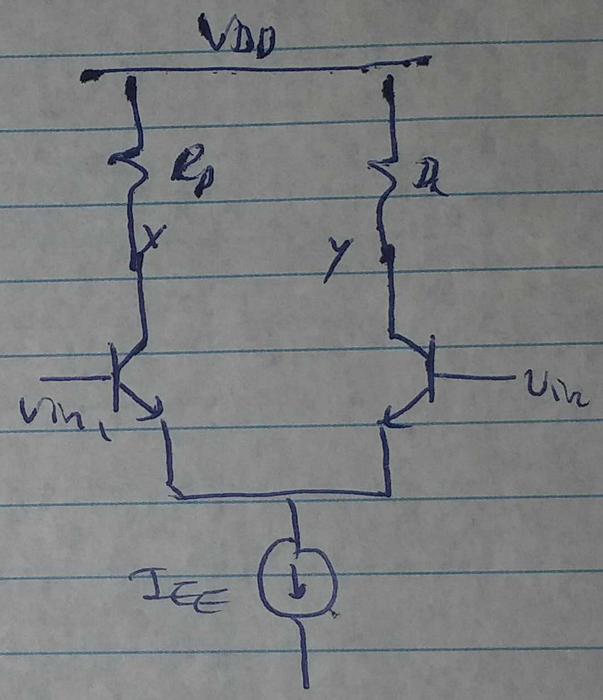
$$= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}\, R_D$$

with $T \uparrow \Rightarrow \mu_n \downarrow \Rightarrow |A_v| \rightarrow \downarrow$

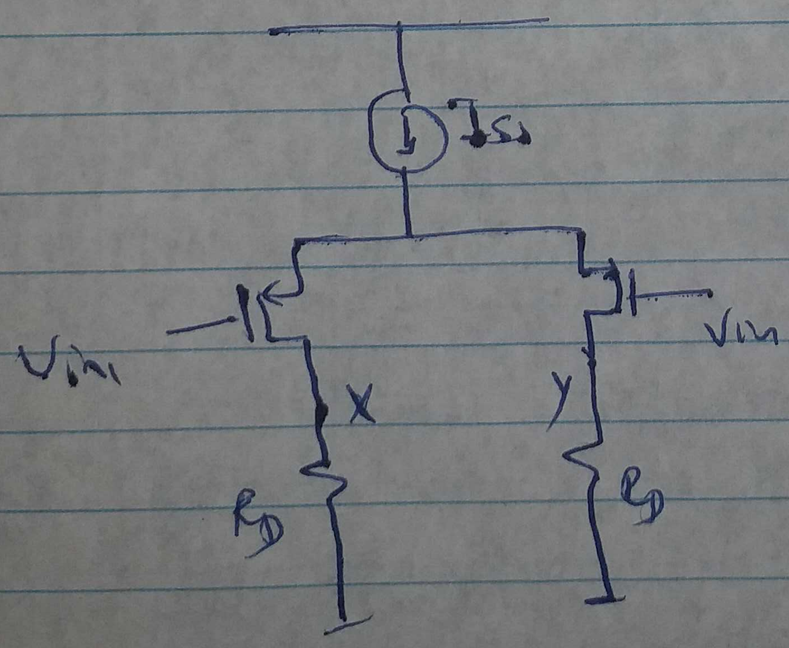
P-type diff pair:-

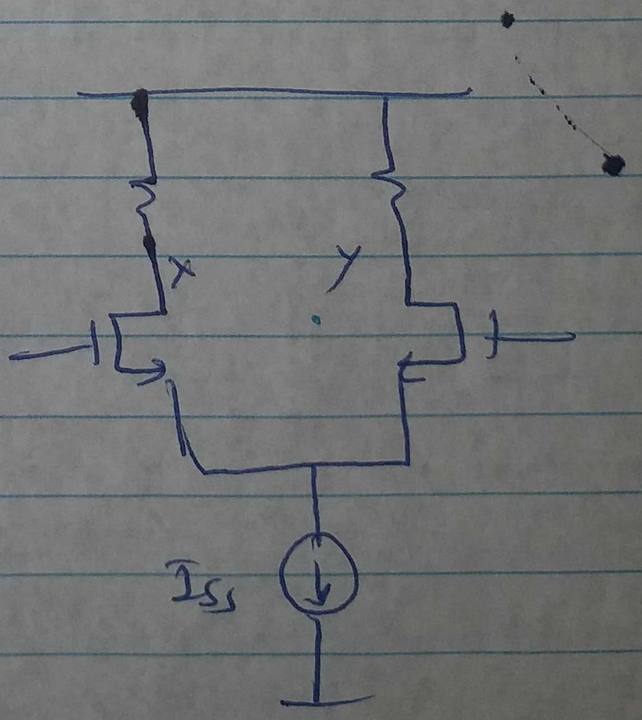


P-type

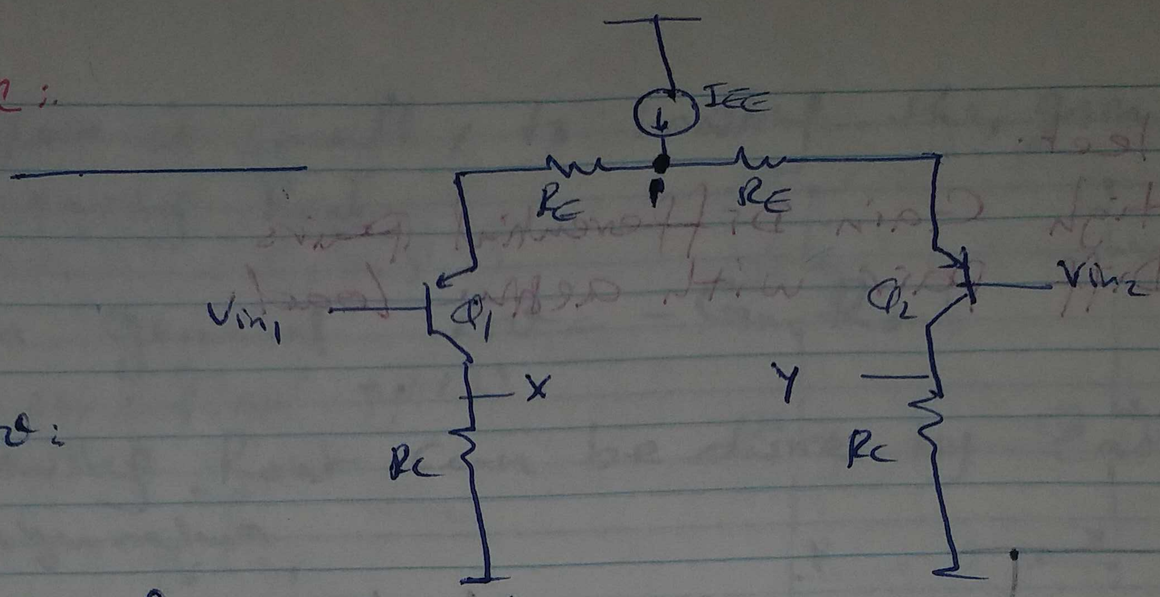


N-typ





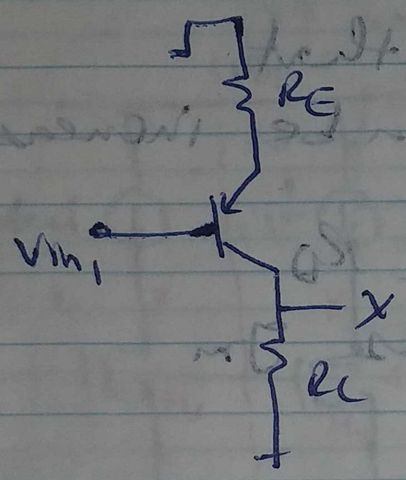
Example:



Find Av:

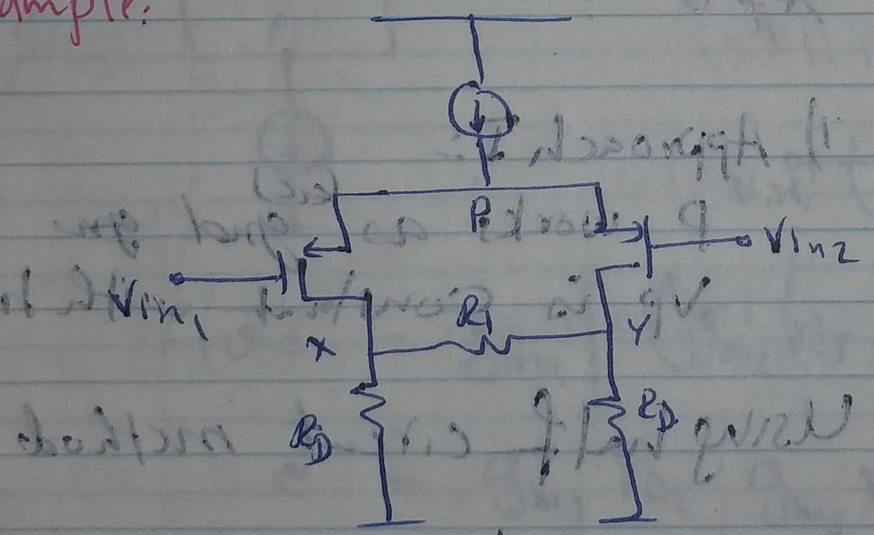
Using half circuit

degenerated Bipolar Transistor

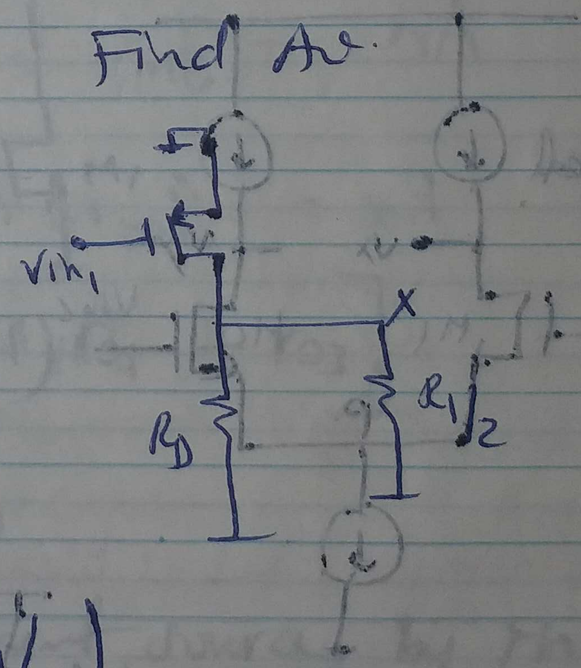


$$A_v = \frac{-R_C}{1/g_m + R_E}$$

Example:



Find Av.

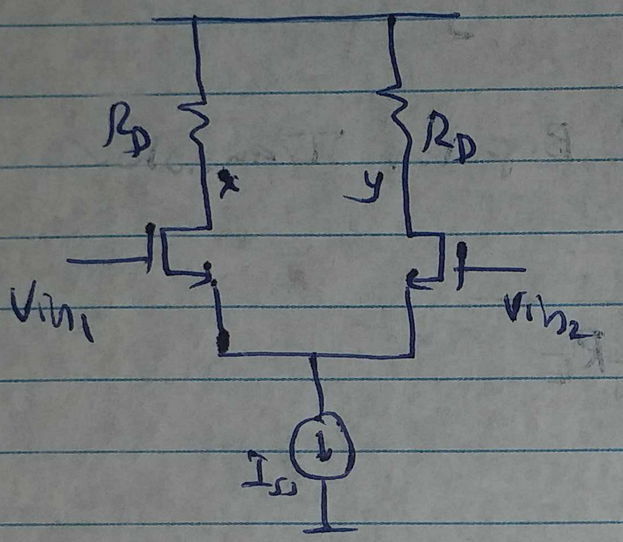


$$A_v = -[g_m (R_D \parallel R_L/2)]$$

Today's lect.

- High Gain Differential pairs
- Diff pair with active load.

Review:-



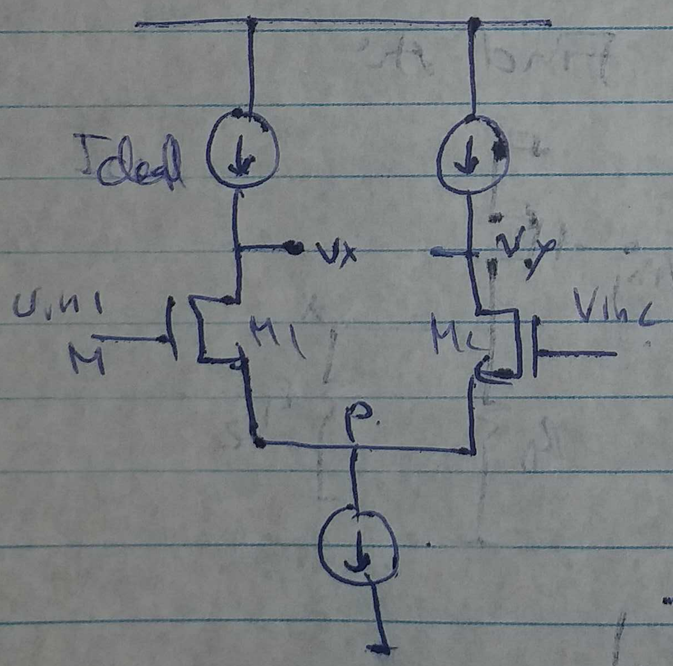
$$A_v = \frac{V_x - V_y}{V_{in_1} - V_{in_2}} = -g_m R_D$$

Here, we see that the gain can be increase by

1) increase R_D
2) increase g_m

* Diff Pair with current source load

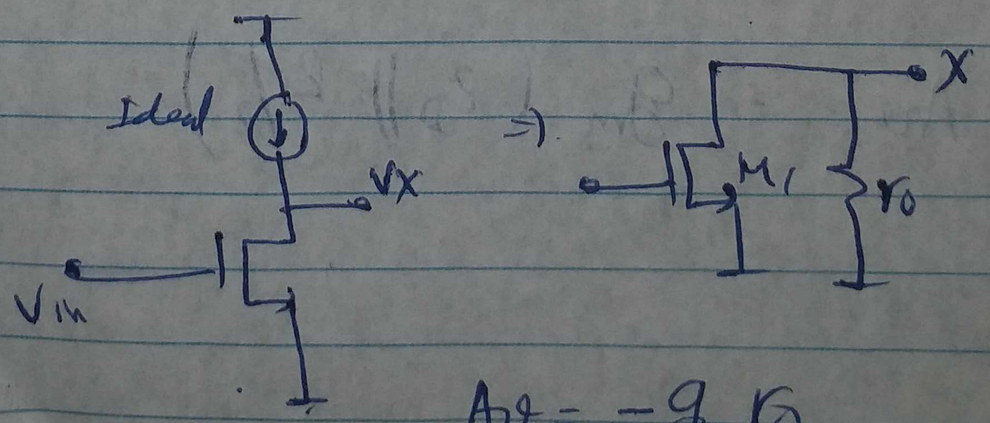
$\lambda \neq 0$:



i) Approach I:-

P works as (ac) gnd
V_p is constant with time

Using half circuit method



$$A_v = -g_m r_o$$

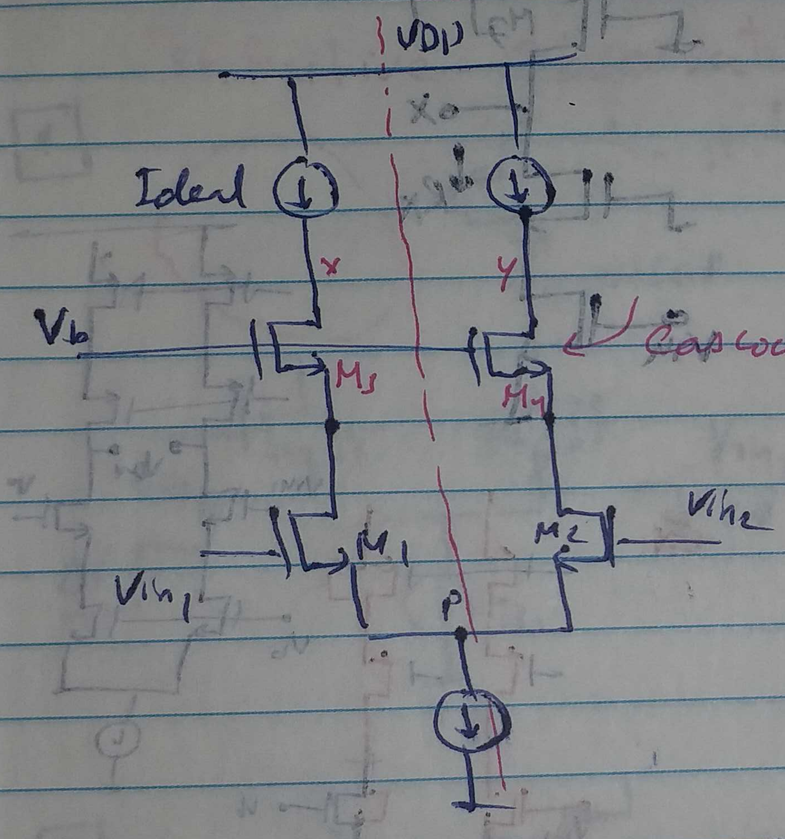
intrinsic gain

The gain is small, to boosting the gain by increasing Rout

In general $A_v = -G_m R_{out}$

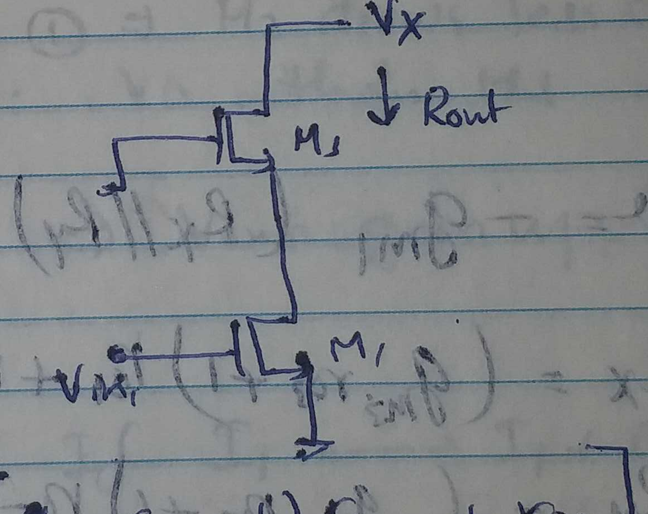
increasing Rout can be done by cascode configuration

Diff pair with Cascode:



Using line symmetry so we can assume p as ac gnd

Half circuit



$$A_v = -g_{m1}\left[(g_{m3} r_{o3} + 1) r_{o1} + r_{o3}\right]$$

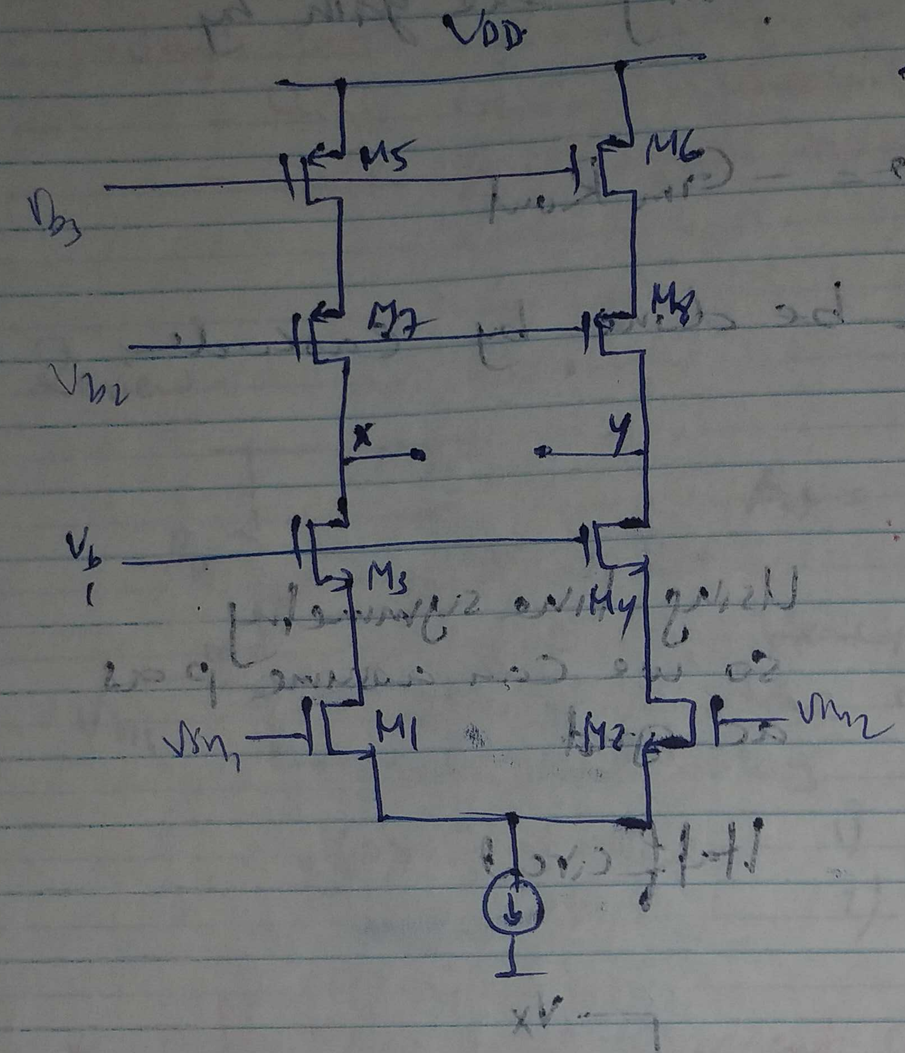
$$\approx -g_{m1} r_{o1} g_{m3} r_{o3}$$

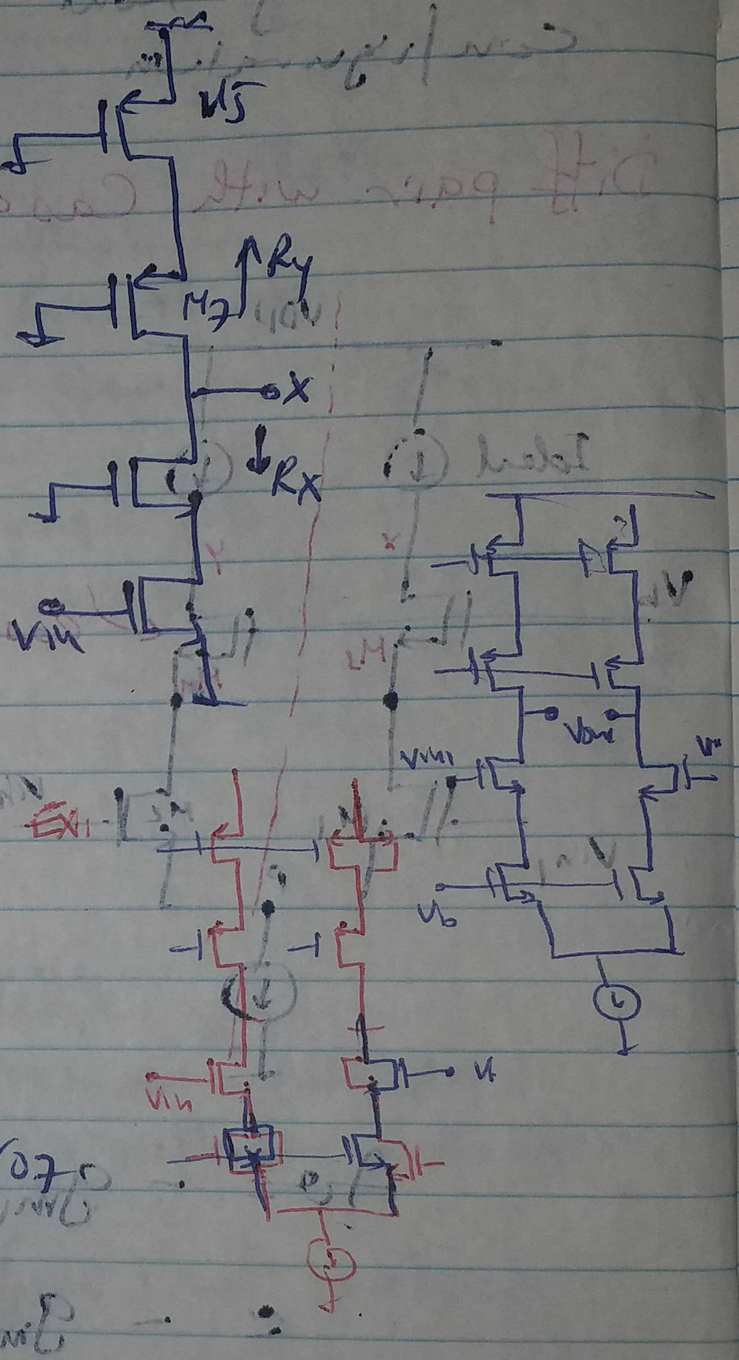
Now, Replacing the ideal current source by Mos current source

let implement current source by PMOS

Telescopic Cascode

Using half circuit



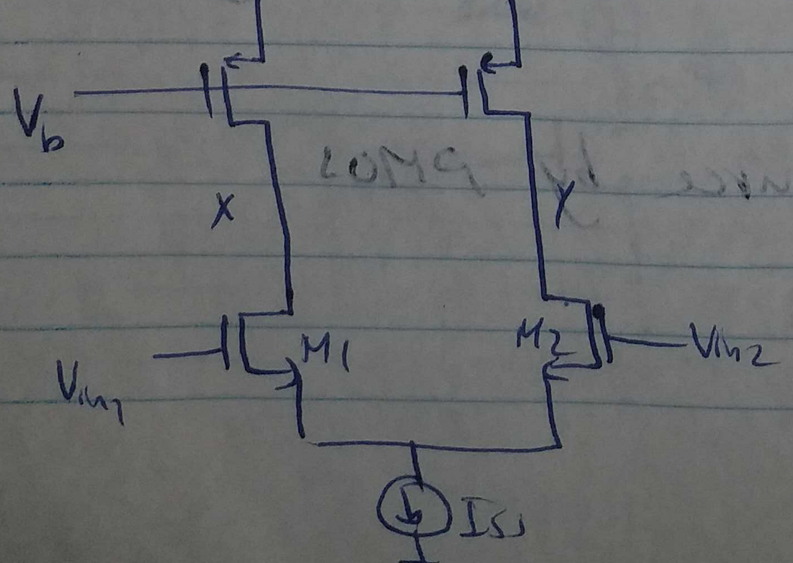


$$A_v = -g_{m1}(R_x \| R_y)$$

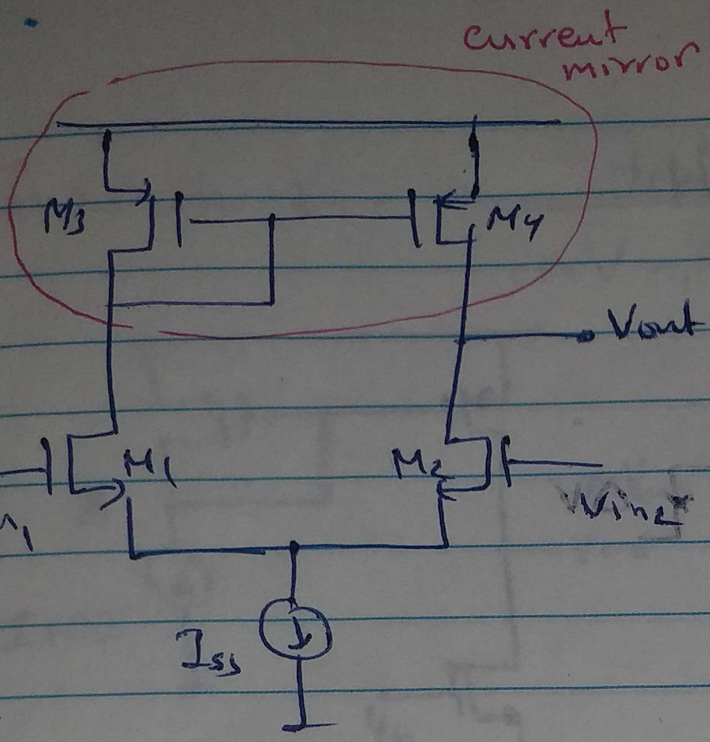
$$R_x = (g_{m3} r_{o3} + 1) r_{o1} + r_{o3}$$

$$R_y = (g_{m7} r_{o7} + 1)(r_{o5} \| r_{o7})$$

Diff pair with active load:-



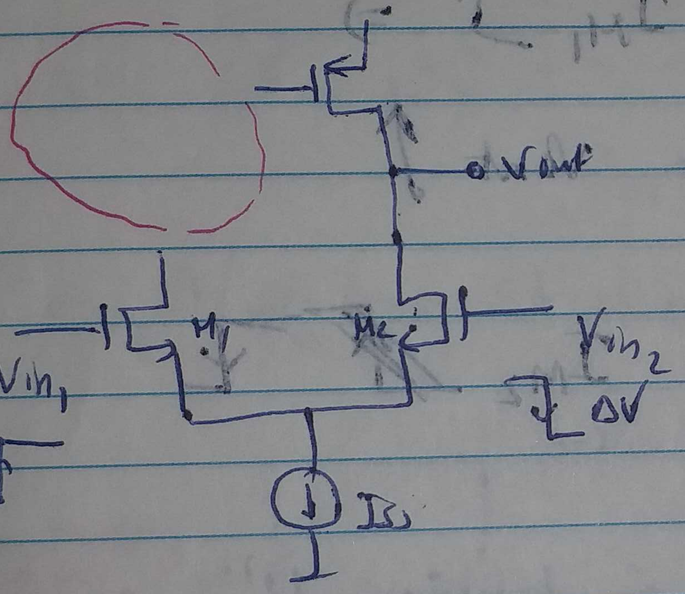
- Here the output is differential output
- But now if we want to have single output ended



The circuit must have the same orginal

Intuitive Observations :-

1



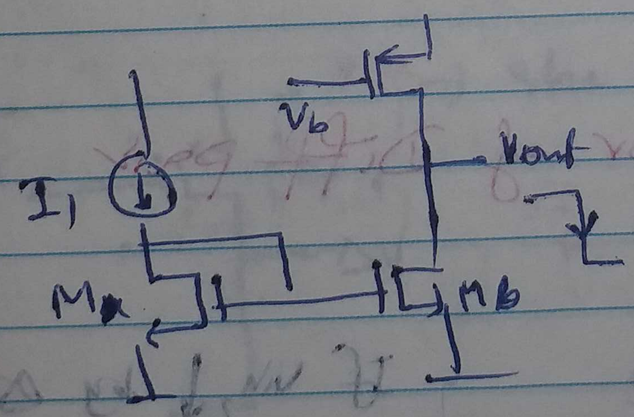
If V_{in_1} goes up by ΔV ,

V_{in_2} goes down by ΔV

① ⇒ M_2 draws less current than M_1

⇒ Vout goes up. ↑

2

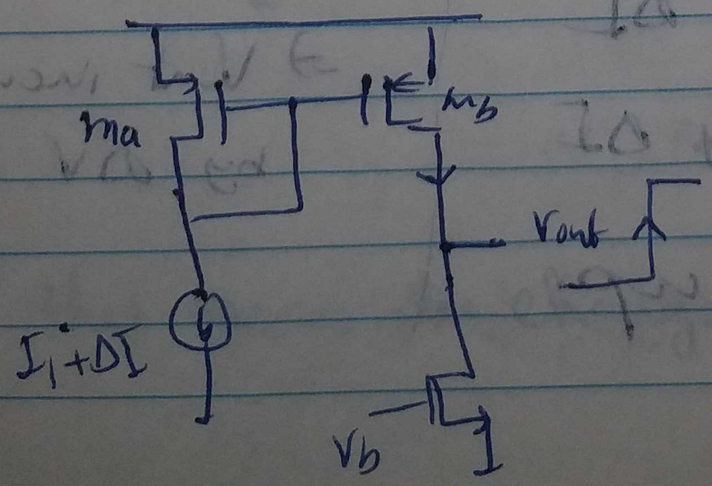


If $I_1 \longrightarrow I_1 + \Delta I$

So $I_{M_b} \longrightarrow$ increase by ΔI

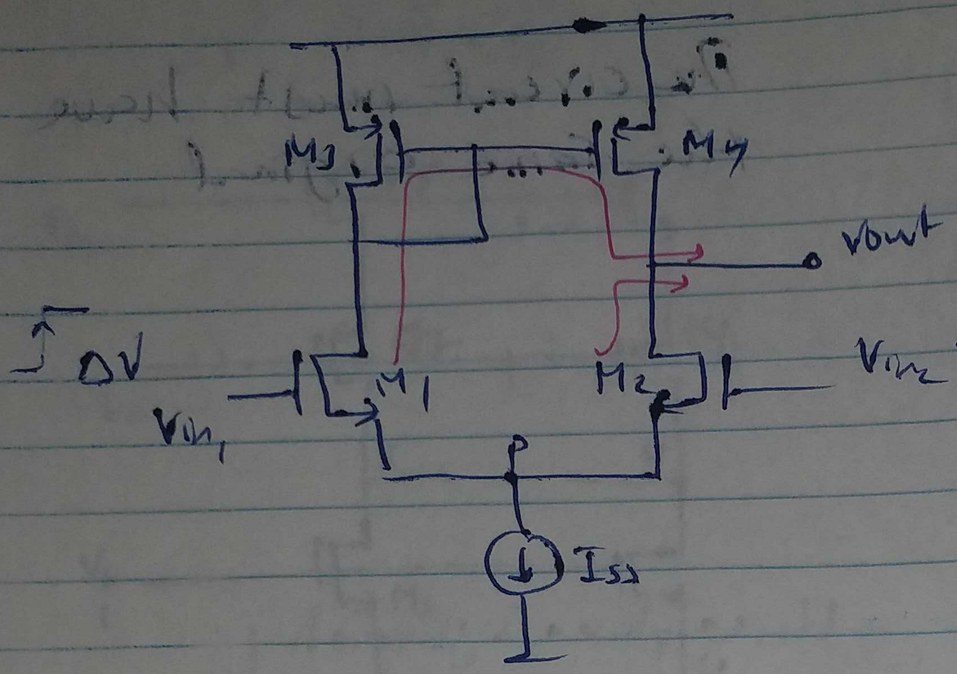
that means

Vout decreases



$I_1 \longrightarrow I_1 + \Delta I$

⇒ Vout goes up



(A) when V_{in_1} ↑ by ΔV ⇒ I_{M_1} ↑ ⇒

I_{M_3} ↑ ⇒ I_{M_4} ↑ ⇒ V_{out} ↓

(B) when V_{in_2} ↓ by ΔV ⇒ I_{M_2} ↓

⇒ V_{out} ↑

Today's lect.:-

Small Signal Behavior of Diff pair with active load:-

If V_{in} ⟶ changes by ΔV

I_{D_1} → change by ΔI

I_{D_3} → Change by ΔI

⇒ I_{D_4} ⇒ Change by ΔI

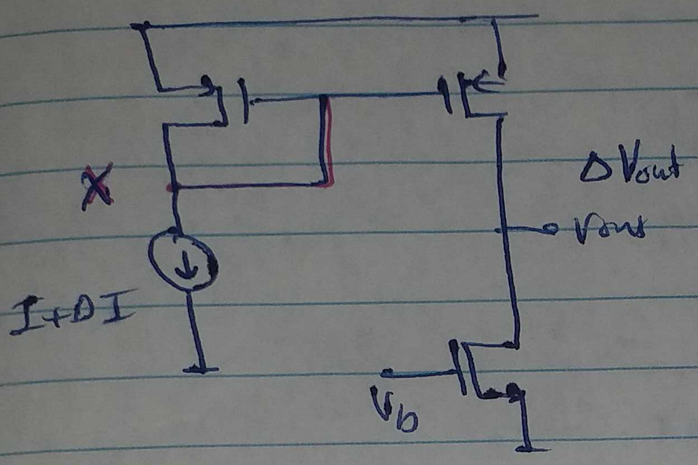
⇒ Vout changes up.

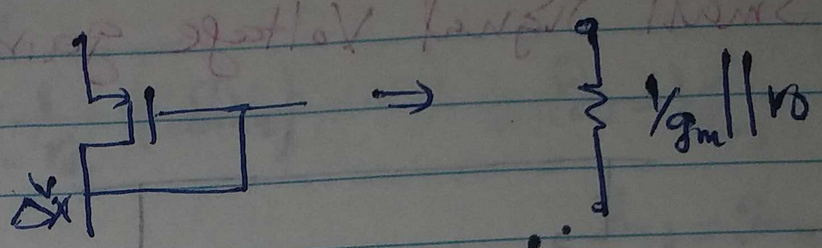
If V_{in_2} ↓ by ΔV

I_{D_2} → decrease by ΔI

⇒ Vout increases by ΔV

Question 1:- "How much the voltage of X changes?





$$\Delta V_{out} = \Delta I \left(\frac{1}{g_m} \| r_o \right)$$

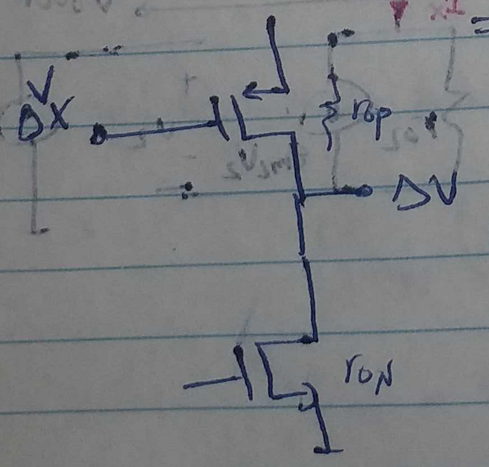
$$g_m r_o \gg 1$$

$$\Rightarrow \Delta V_{out} = \left(\frac{r_o}{1 + g_m r_o} \right) \Delta I$$

$$= \frac{1}{g_m} \Delta I$$

$$\Rightarrow \Delta V_x = \frac{\Delta I}{g_m}$$

Quiz:- "How much is the change in Vout?"



=) the configuration as common source

$$A_v = -g_{mp} (r_{ON} \| r_{op})$$

$$\Delta V_{out} = \Delta X \, A_v$$

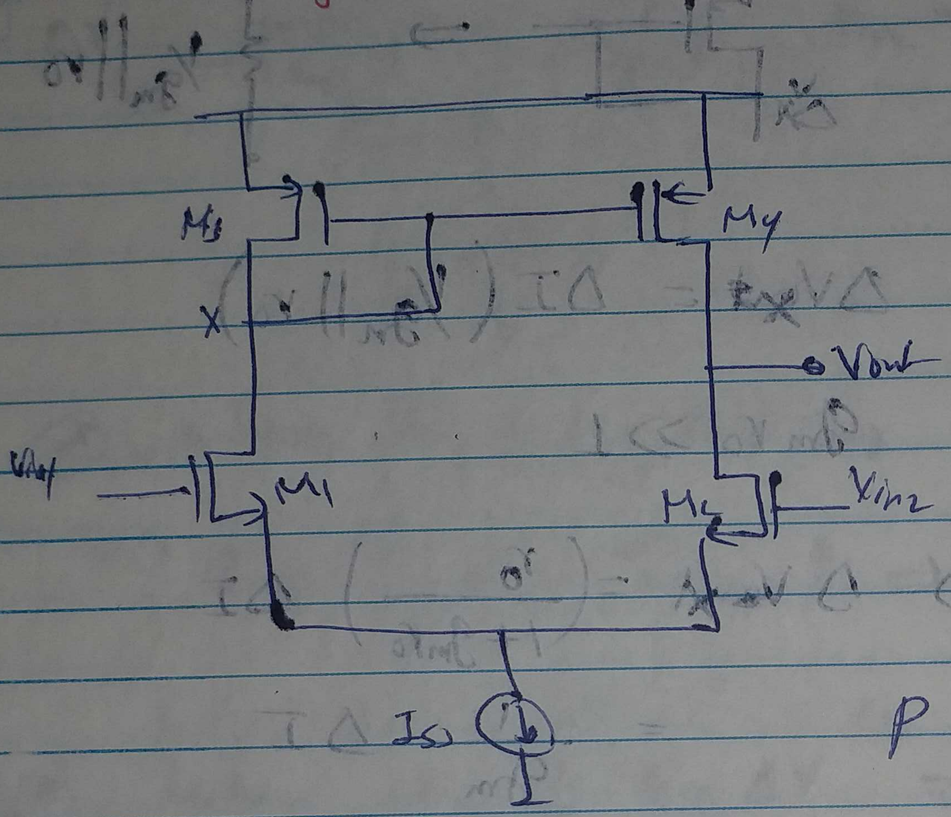
$$= -g_{mp} (r_{ON} \| r_{op}) \frac{\Delta I}{g_{mp}}$$

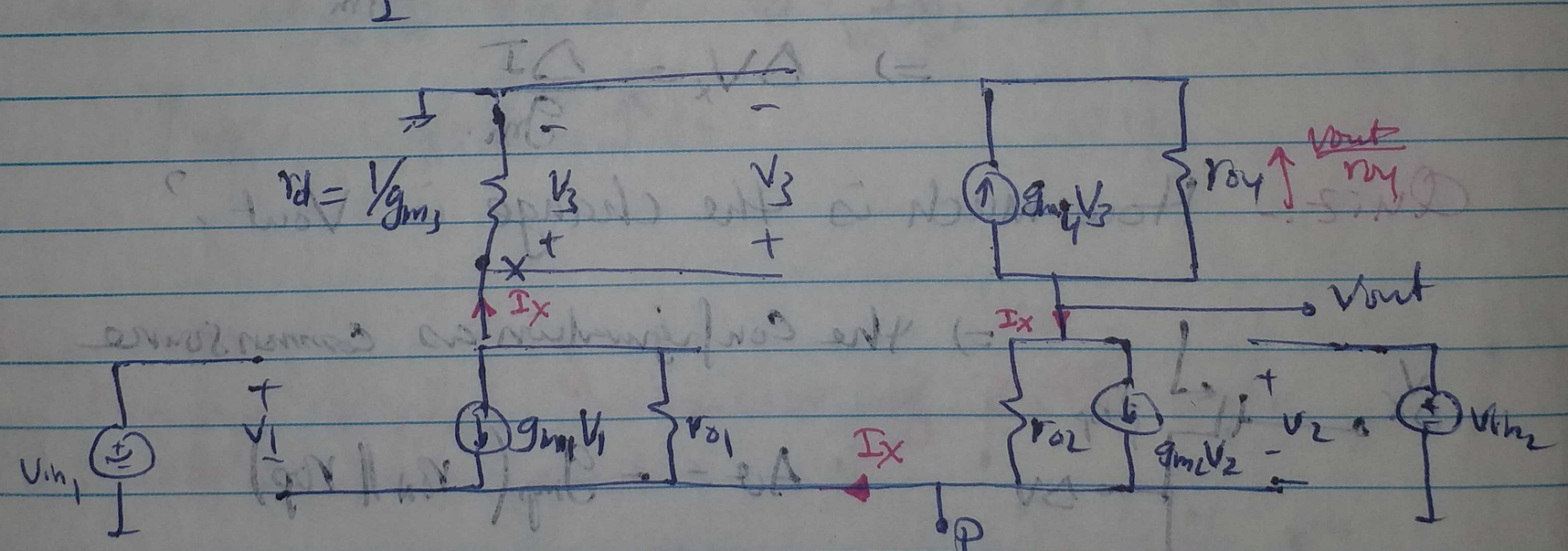
$$= -(r_{ON} \| r_{op}) \Delta I$$

that means the change of $|V_x| <$ | change of V_{out} |

The voltage gain = $\frac{V_{out}}{V_{in_1} - V_{in_2}}$

Small signal voltage gain:





$$I_X = \frac{V_3}{r_d}$$

KCL @ output node:

$$I_X + \frac{V_{out}}{r_{o4}} + g_{m_4} V_3 = 0$$

$$\frac{V_3}{r_d} + \frac{V_{out}}{r_{o4}} + g_{m_4} V_3 = 0$$

$$V_3 = -\frac{V_{out}}{r_{o4}\left(\frac{1}{r_d} + g_{m4}\right)}$$

$$= -\frac{V_{out}}{r_{o4}\left(\frac{1}{r_{d_3}} + g_{m4}\right)} \qquad r_{d_3} = 1/g_{m_3}$$

$$g_{m_3} = g_{m_4}$$

$$V_3 = -\frac{V_{out}}{r_{o4}\left(2 g_{mp}\right)}$$

KCL @ node X :-

$$g_{m1} V_1 + \frac{V_3 - V_p}{r_{o1}} + \frac{V_3}{r_d} = 0 \rightarrow (1)$$

KCL @ output

$$(g_{m2} V_2) + \frac{V_{out} - V_p}{r_{o2}} - \frac{V_3}{r_d} = 0 \qquad (2)$$

$r_{o_1} = r_{o2}$

$g_{m_1} = g_{m_2}$

after subtraction (1) − (2)

$$g_{mN}(V_1 - V_2) + \frac{V_3 + V_{out}}{r_{oN}} + \frac{2V_3}{r_d} = 0$$

$V_{in_1} - V_1 = V_p$

$V_{in_2} - V_2 = V_p$

$\Rightarrow V_1 - V_2 = V_{in_1} - V_{in_2}$

Replacing V_3

$$\Rightarrow g_{mN}(V_{in_1} - V_{in_2}) + \frac{V_3 - V_{out}}{r_{oN}} + \frac{2V_3}{r_d} = 0$$

$$V_3 = -\frac{V_{out}}{2 g_{mp} r_{op}}$$

$$\Rightarrow g_{m_N}(V_{in_1} - V_{in_2}) + \frac{V_{out}}{2 g_{mp} r_{op} r_{oN}} - \frac{V_{out}}{r_{oN}} - \frac{2 V_{out}}{2 g_{mp} r_{op} r_d} = 0$$

$$r_d = \frac{1}{g_{m_3}} = \frac{1}{g_{mp}}$$

$$g_{m_N}(V_{in_1} - V_{in_2}) = V_{out}\left[\frac{1}{2 g_{mp} r_{op} r_{oN}} + \frac{1}{r_{oN}} + \frac{1}{r_{op}}\right]$$

$$g_{m_N}(V_{in_1} - V_{in_2}) = V_{out}\left[\frac{1}{r_{oN}} + \frac{1}{r_{op}}\right]$$

$$\frac{V_{out}}{V_{in_1} - V_{in_2}} = -g_{m_N}(r_{oN} \| r_{op})$$