

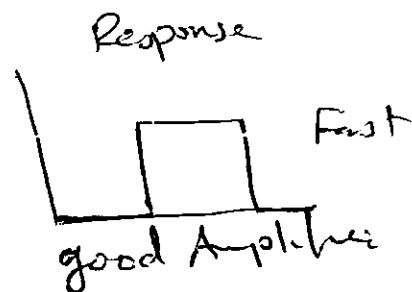
①

Today's lect.

Frequency Response

- * Objective motivation
- * Circuit theory concept.

Objective:-



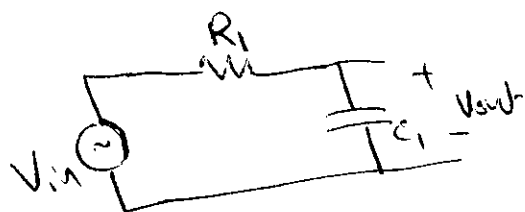
Here, we have to answer why that happens?



* In device delay, we have high speed circuit

- Ex:
- Wifi $\rightarrow$ 2.4 GHz — 5 GHz
 - Bluetooth $\rightarrow$ 2.4 GHz
 - GPS $\rightarrow$ 1.5 GHz
 - Radar $\rightarrow$ 26 GHz — 75 GHz
 - Microprocessor $\rightarrow$ 4 — 5 GHz

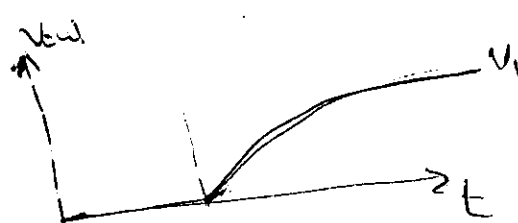
Ex:-



$$V_{out} = V_1 \left[1 - \exp^{-t/\tau} \right]$$

To improve the speed,
we must minimize $\tau = RC$

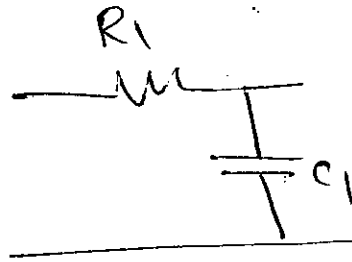
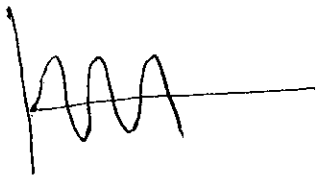
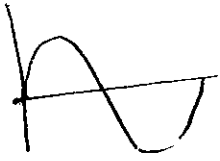
$\Rightarrow$ Called time domain Response



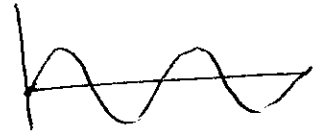
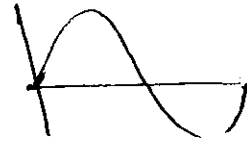
Frequency domain Analysis

(2)

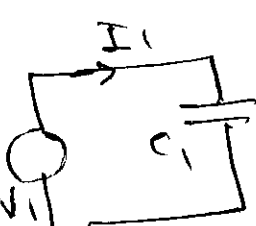
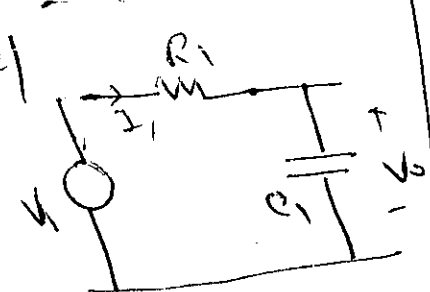
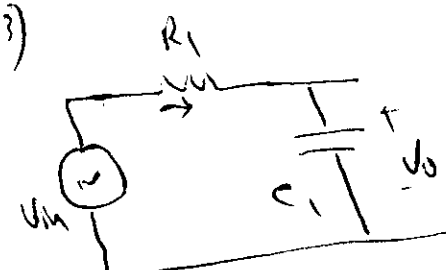
input



output



Circuit theory Concepts:

	time domain	Frequency domain
1) 	$I_1 = C \frac{dV_1}{dt}$	$Z_C = \frac{1}{sC_1} \quad s = j\omega$ $I_1 = \frac{V_1}{Z_C} = \frac{V_1}{1/sC_1}$ $I_1 = sC_1 V_1$
2) 	$V_1 = I_1 R_1 + \frac{1}{C_1} \int I_1 dt$ Diff equation (difficult to solve)	$V_1 = R_1 I_1 + I_1 \times \frac{1}{sC_1}$ $= R_1 I_1 + \frac{I_1}{sC_1}$ Algebraic equation (easier)
3) 	$I_{R1} = I_{C1}$ $I_{C1} = C_1 \frac{dV_{out}}{dt}$ $V_{in} = R_1 C_1 \frac{dV_{out}}{dt} + V_{out}$ To find V_{out}	$V_0 = \frac{Z_C}{Z_C + R} V_{in}$ $= \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R} V_{in}$ $\frac{V_{out}}{V_{in}} = \frac{1}{1 + sC_1 R_1}$ Transfer function

4) s is generally a complex frequency

(3)

$$s = j\omega$$

$$s = j\omega$$

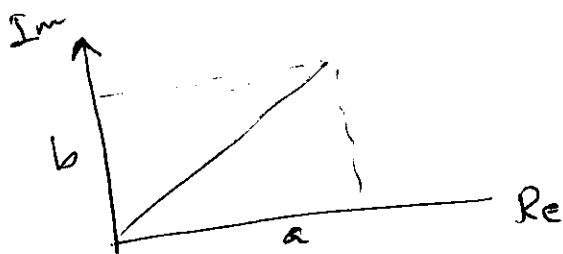
$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + sC_1R_1}$$

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{1}{1 + j\omega C_1R_1}$$

5) A complex number:

$$Z = a + jb$$

Can be represented in polar form



$$\text{magnitude} = \sqrt{a^2 + b^2}$$

$$\text{phase} = \tan^{-1} \frac{b}{a}$$

Ex: $H(j\omega) = \frac{1}{1 + j\omega R_1 C_1}$

$$\omega = 2\pi f$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 R_1^2 C_1^2}}$$

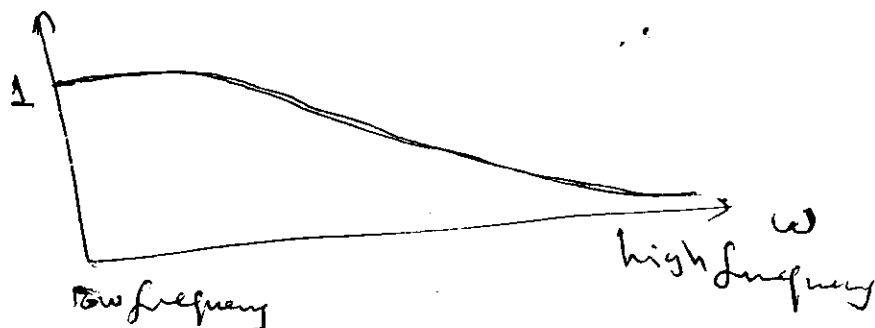
for $\omega = 0$

$$\Rightarrow |H(j\omega)| = 1$$

$$|H(j\omega)| = 0$$

$\omega \rightarrow \infty$

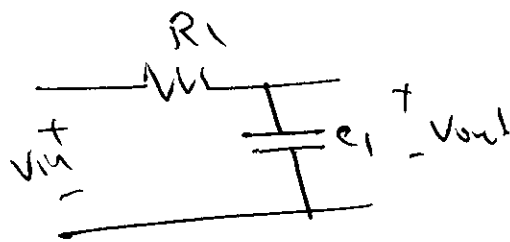
$\therefore |H|$



How to Compute the Frequency Response ④

- 1) Determine the transfer function $H(s)$
- 2) Replace s by $j\omega \rightarrow H(j\omega)$
- 3) Find the magnitude $|H(j\omega)|$
- 4) Draw $|H(j\omega)|$

Example:



$$H(j\omega) = \frac{1}{1 + j\omega C_1 R_1}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}}$$

when $\omega = 0$

$$|H(j\omega)| = 1$$

$$|H(j\omega)|_{dB} = 20 \log |H(j\omega)|$$

$$= 20 \log \frac{1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}}$$

$$= 20 \log 1 - 20 \log (1 + \omega^2 C_1^2 R_1^2)^{1/2}$$

$$= 0 - 20 \log (1 + \omega^2 C_1^2 R_1^2)^{1/2}$$

when $\omega = 0$

$$|H(j\omega)|_{dB} = 0$$

when $\omega = \frac{1}{C_1 R_1}$

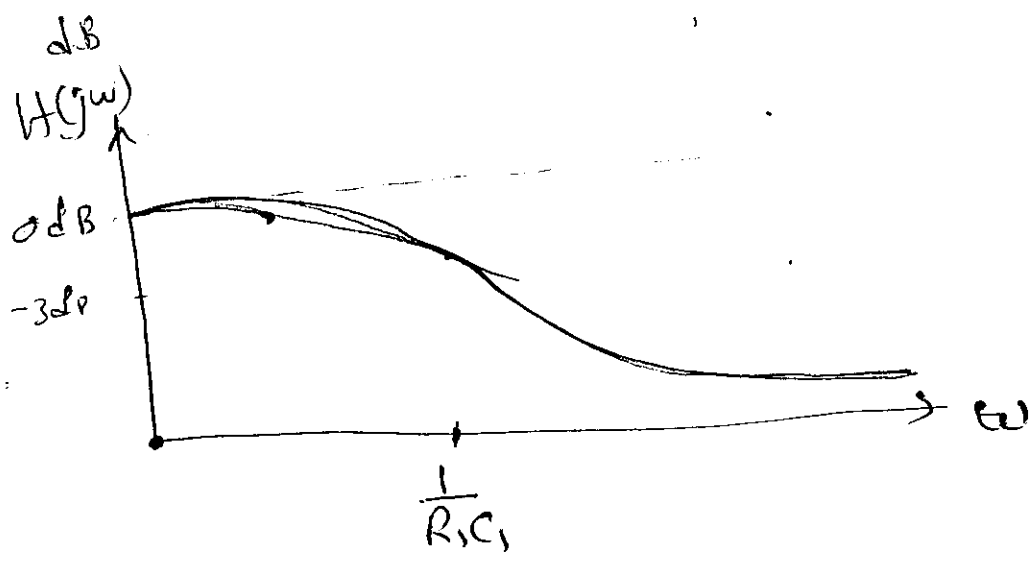
$$|H(j\omega)|_{dB} = -20 \log \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

$\omega \rightarrow \infty$

$$|H(j\omega)|_{dB} = 0$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{Z_C}{Z_C + R_1} \\ &= \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R_1} \\ &= \frac{1}{1 + sC_1 R_1} \\ s &\rightarrow j\omega \end{aligned}$$

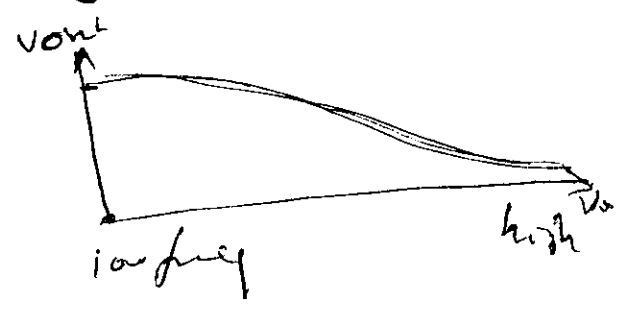
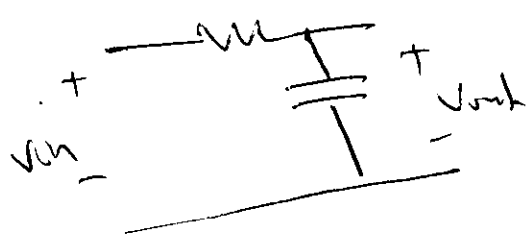
(5)



For Capacitor

low frequency $\rightarrow$ C is open
high frequency $\rightarrow$ C is shorted

$$H(j\omega) = \frac{1}{1 + j\omega C_1 R_1}$$

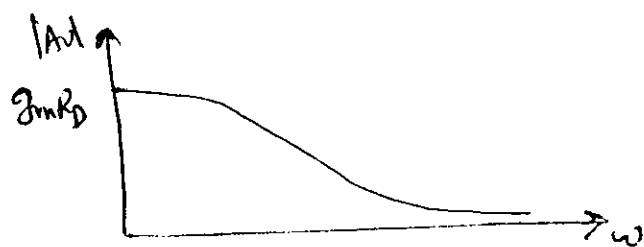
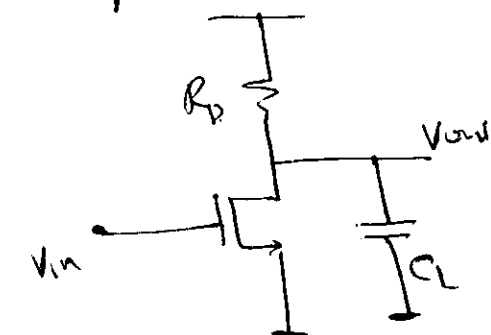


In general
Capacitor limits the BW
that means ~~to~~ to have
max BW, we have
to minimize $R_1 C_1$

Today's lect:

- Useful Frequency Response Concepts
- Poles and Zeros
- Bode Rules
- Find poles by inspection

Example:



* @ low frequency
C → open circuit
|Acl| = $g_m R_D$

* @ high frequency
C → short circuit
 $V_{out} = 0$
less current flows
into R_D .
⇒ gain Reduces

Poles and Zeros:

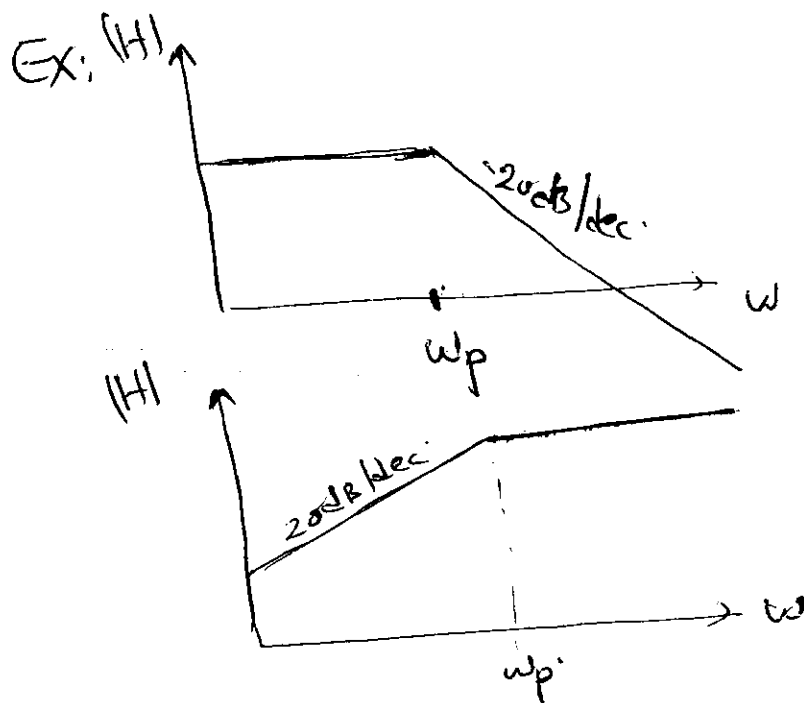
$$H(s) = \frac{N(s)}{D(s)}$$

- * The roots of $N(s)$ is zeros [$N(s) = 0$]
- * The root of $D(s)$ is poles [that makes by $D(s) = 0$]

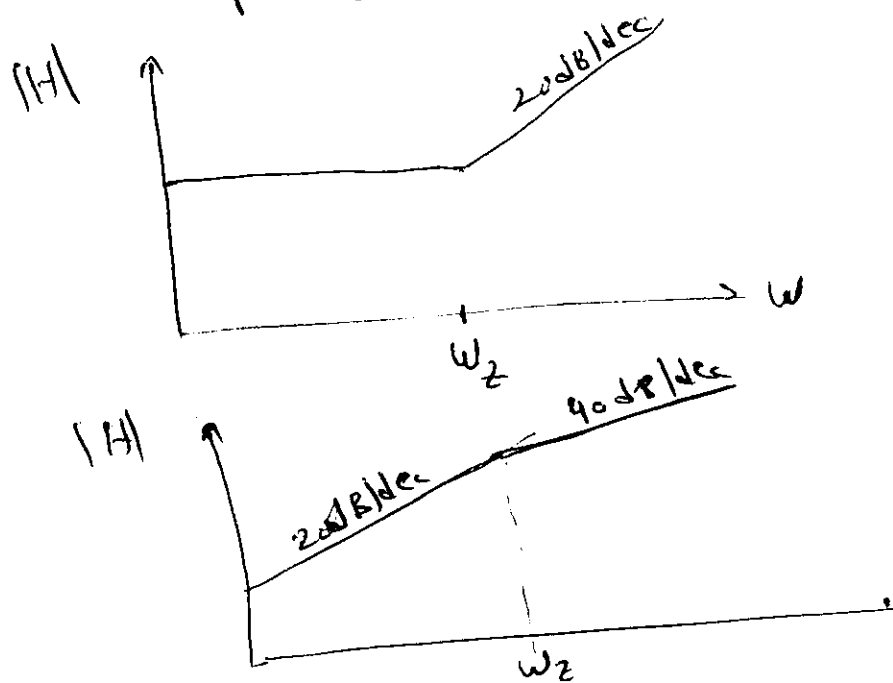
Bode's Rules:

(7)

- 1) As ω approaches (passes) the poles frequency, the slope of $|H|$ decreases by 20 dB/dec.

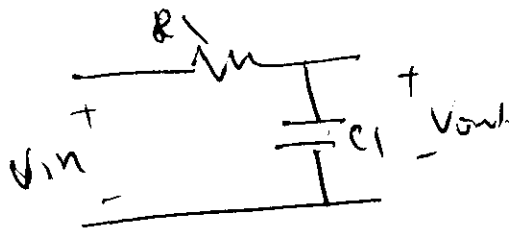


- 2) As ω passes a zero frequency, the slope of $|H|$ increases by 20 dB/dec.



Example

(2)



$$H(s) = \frac{1}{1 + sC_1R_1}$$

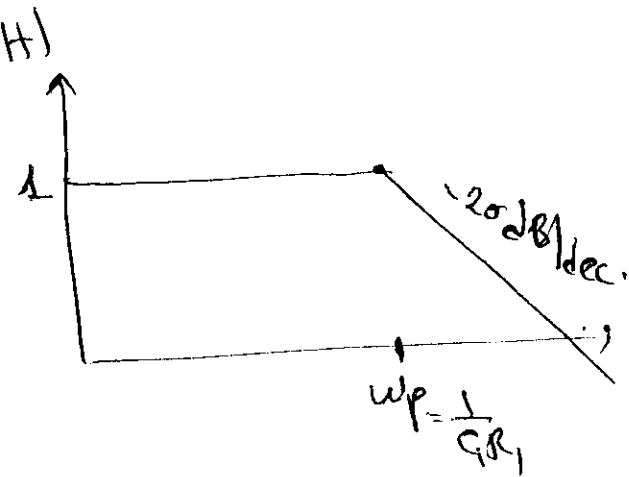
To find the pole:

$$1 + sC_1R_1 = 0$$

$$1 + j\omega C_1R_1 = 0$$

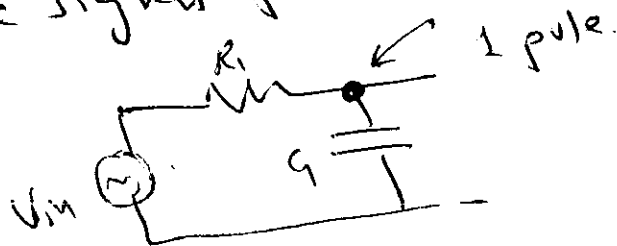
$$\omega_p = -\frac{1}{C_1R_1}$$

negative pole: stable



Finding pole by inspection

We can assume that each node in the signal path contributes a pole.

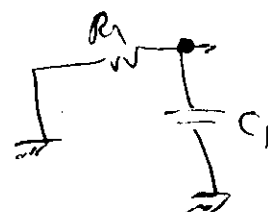


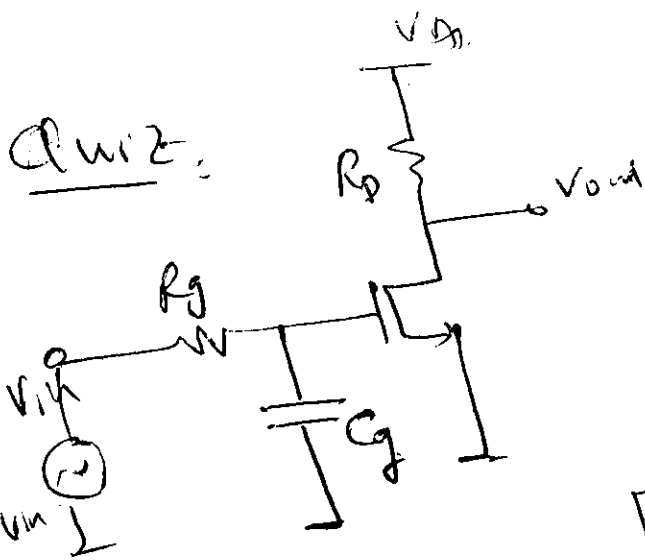
To Calculate the poles:

- 1) all dependent source $\rightarrow$ shorted
- 2) Find the resistance from that pole to ac gnd.
- 3) Find the capacitance from that pole to ac gnd.

$$\omega_p = \frac{1}{RC}$$

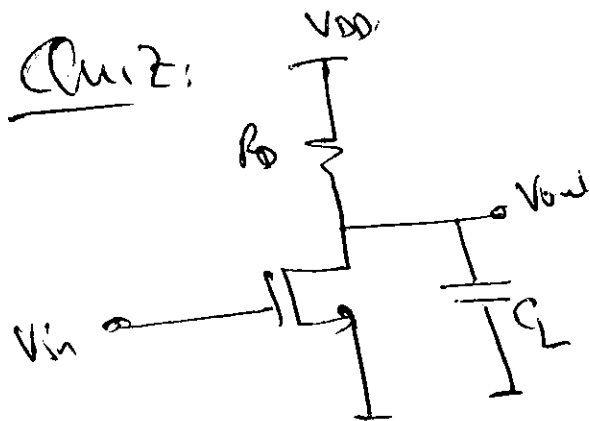
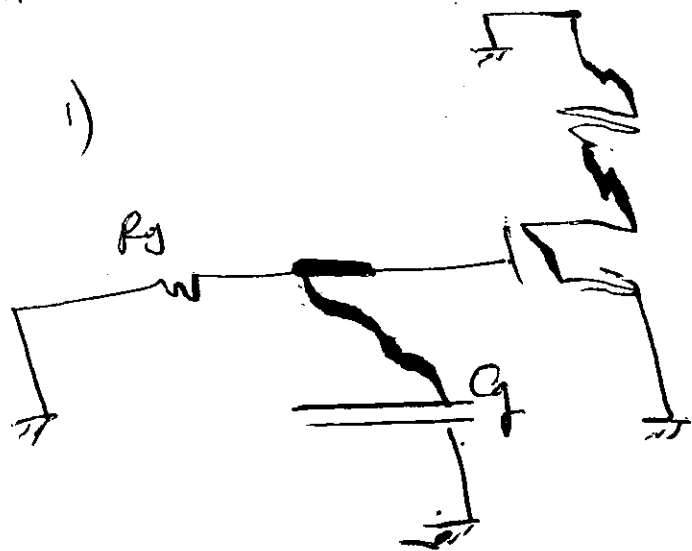
$$\omega_p = \frac{1}{R_1C_1}$$



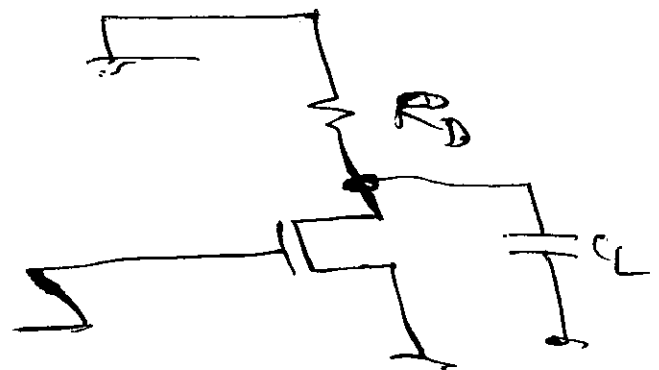


$$\omega_p = -\frac{1}{R_g C_g}$$

Find the pole



$$\omega_p = -\frac{1}{R_D C_s}$$

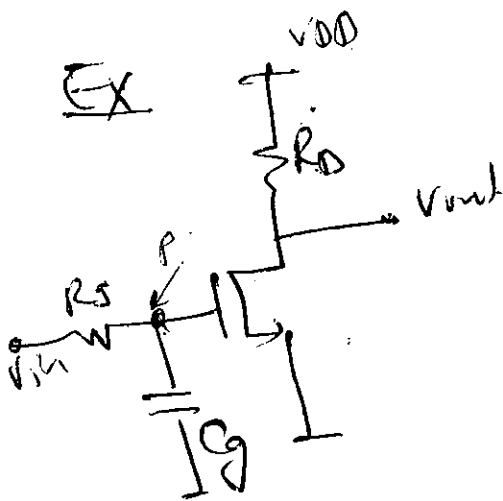


The general formulation of transfer function

$$H(s) = \frac{A_0}{(1 + s/\omega_{p1}) (1 + s/\omega_{p2}) \dots}$$

A_0 : the gain (small signal)

(10)

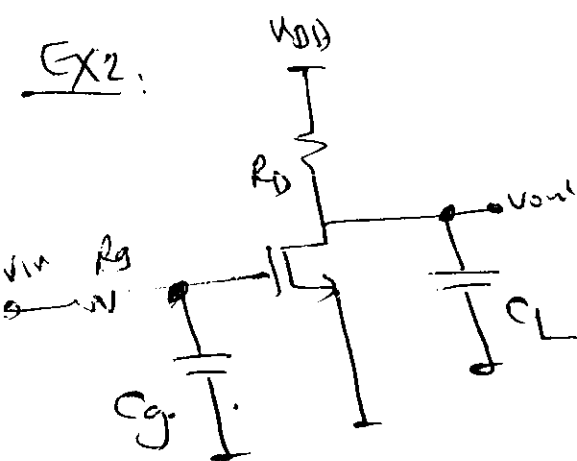
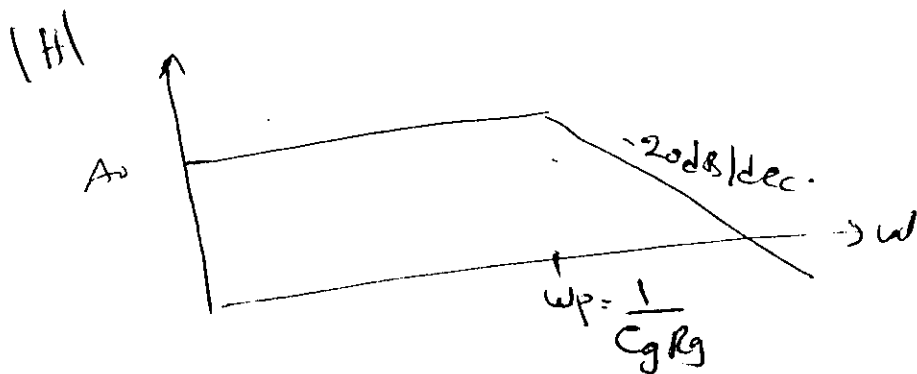


$$H(s) = \frac{A_0}{(1 + s/\omega_p)}$$

$$A_0 = -g_m R_D$$

$$\omega_p = -\frac{1}{R_g C_g}$$

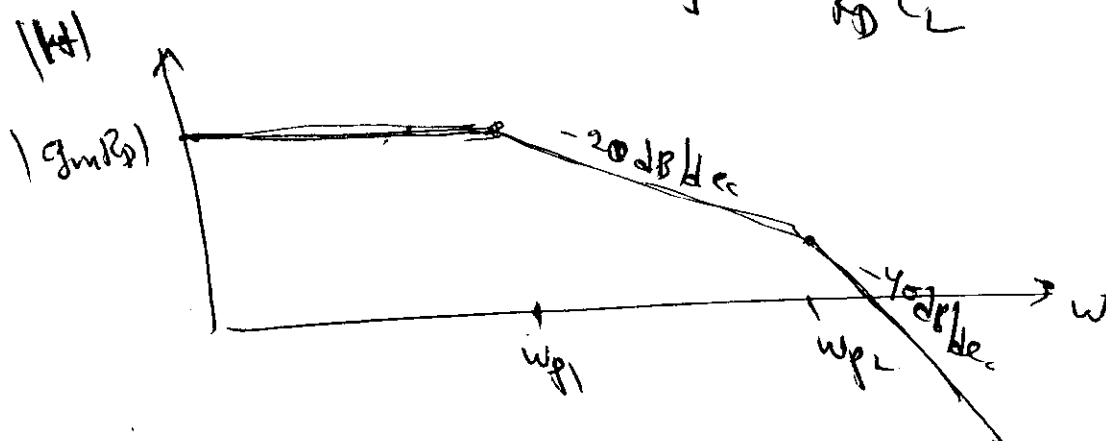
$$H(s) = \frac{-g_m R_D}{1 + \frac{s C_g R_g}{1}}$$



$$H(s) = \frac{-g_m R_D}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

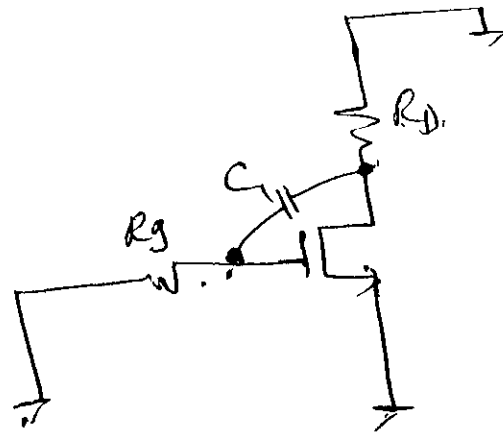
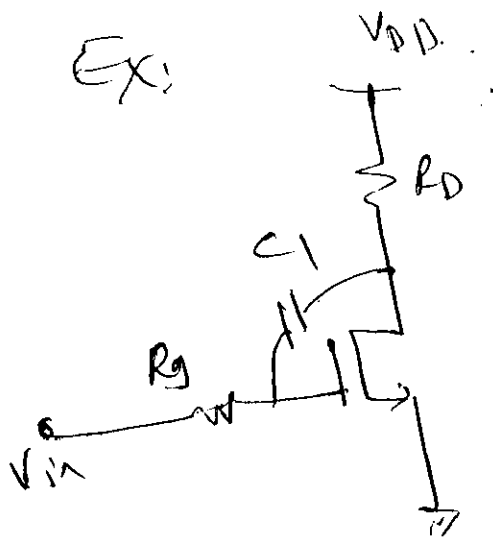
$$\omega_{p1} = -\frac{1}{R_g C_g}$$

$$\omega_{p2} = \frac{1}{R_D C_L}$$



(11)

Find the pole



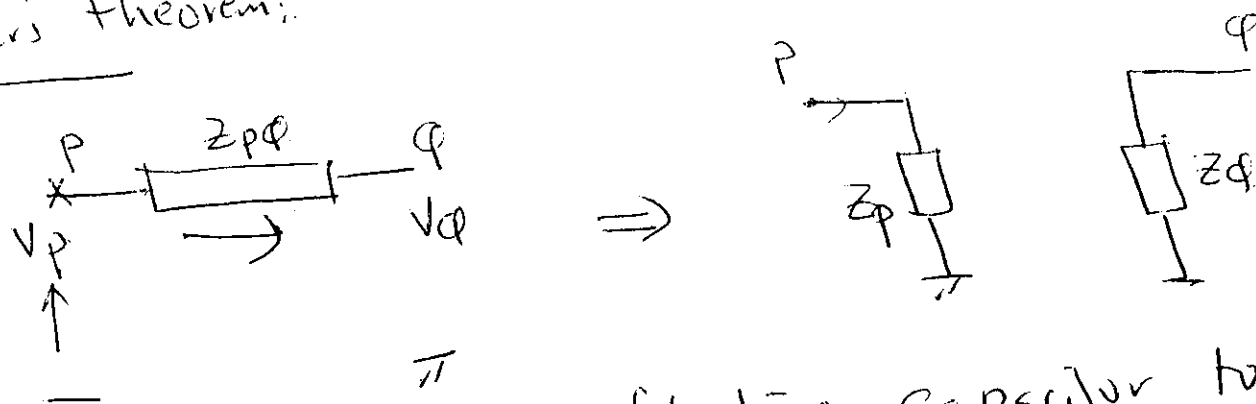
Here is it is hard to find
the pole using inspection method
because we have C_1 that
connected between two node.
(floating capacitor)

Today's lect.

(2)

- The Miller Effect
- high frequency Model of Bipolar Transistor

Miller's theorem:



We can decompose the floating capacitor to two grounded capacitor.

Z_{PQ} : floating capacitor

Z_P, Z_Q : grounded capacitor

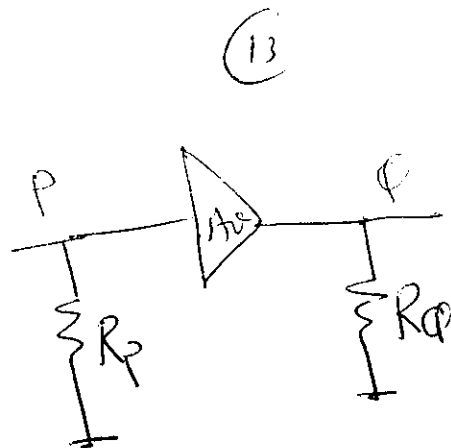
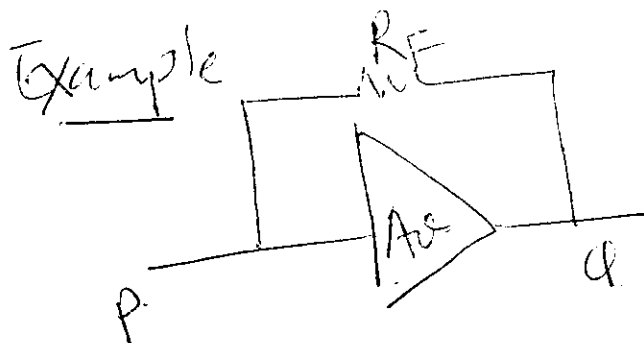
- i) Current drawn from P must be the same for both circuit

$$\frac{V_P - V_Q}{Z_{PQ}} = \frac{V_P}{Z_P} \Rightarrow Z_P = \frac{Z_{PQ}}{1 - V_Q/V_P}$$

The same for

$$Z_Q = \frac{Z_{PQ}}{1 - V_P/V_Q}$$

$\Rightarrow$ This is called Miller Theorem



$$R_P = \frac{R_F}{1 - \frac{V_Q}{V_P}} = \frac{R_F}{1 - A_v}$$

$$R_Q = \frac{R_F}{1 - \frac{V_P}{V_Q}} = \frac{R_F}{1 - \frac{1}{A_v}}$$

ex: $A_v = -10$

$$R_P = \frac{R_F}{11}$$

$$R_Q = \frac{R_F}{1 + \frac{1}{10}} = \frac{R_F}{1.1}$$

Miller effect:

Z_{PQ} is divided by $(1 - A_v)$ as it appears at the input. (Reducing input resistance)

Ques 2 : if $A_v = 10$

$$R_P = \frac{R_F}{1 - 10} = \frac{R_F}{-9} \Rightarrow \text{negative resistance}$$

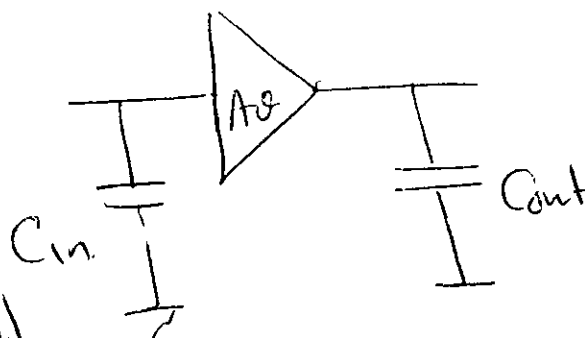
Ex:



$$Z_{CF} = \frac{1}{sC_F}$$

(14)

$$C_{in} = \frac{C_F}{1 - A_v} = \frac{1}{sC_F(1 - A_v)}$$



$$C_{out} = \frac{C_F}{1 - 1/A_v} = \frac{1}{sC_F(1 - 1/A_v)}$$

Miller effect:

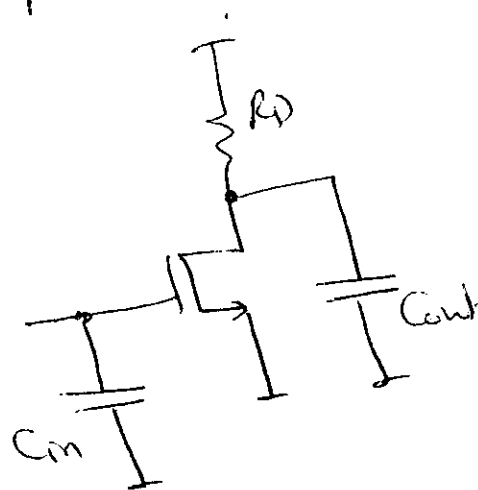
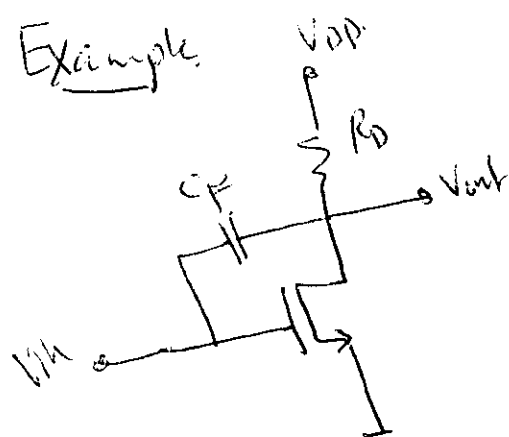
if $A_v = -10$

input capacitance: $C_F(1 - A_v) \Rightarrow 11 C_F$

Output Cap $C_F(1 - 1/A_v) \Rightarrow 1.1 C_F$

$\Rightarrow$ the input cap becomes greater ~~at~~ the input and output cap " smaller at the output "

Example



$$A_v = -g_m R_D$$

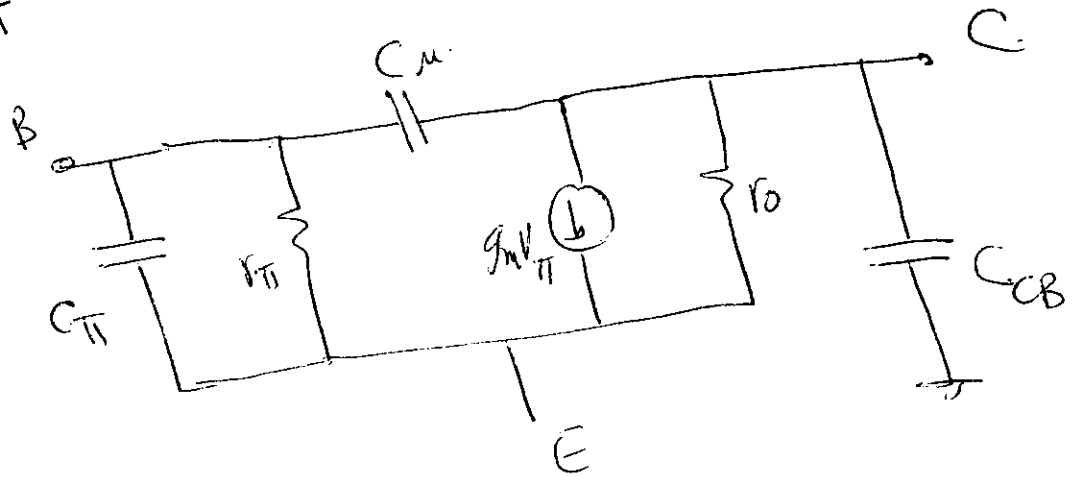
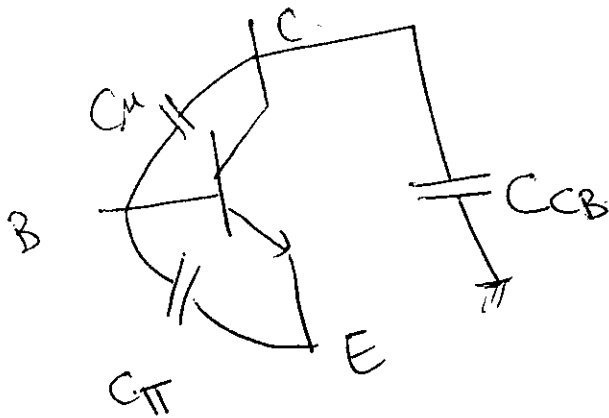
$$C_{in} = C_F(1 - A_v) = C_F(1 + g_m R_D)$$

$$C_{out} = C_F(1 - 1/A_v) = C_F(1 + 1/g_m R_D)$$

Remarks

- 1) Miller Approximation has introduced a new pole.
- 2) Miller " has eliminated the zero.

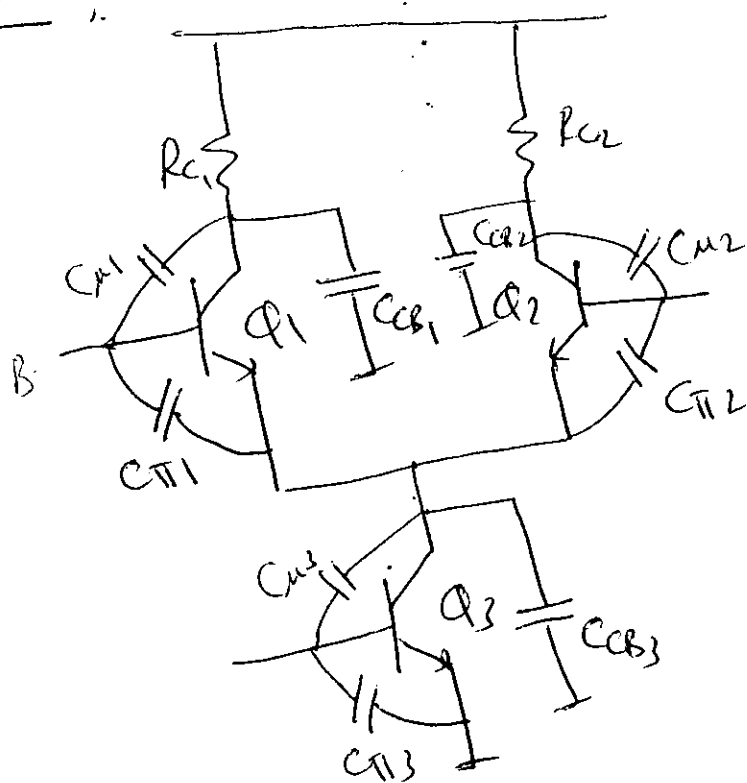
High Frequency Model of BJT:



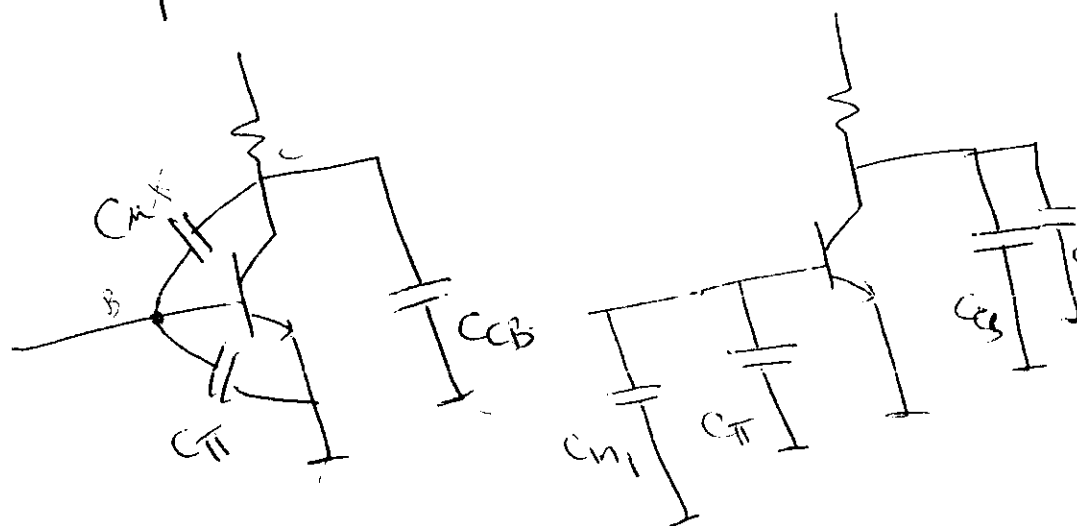
Example:

Put the capacitor

(16)



Quiz: which Cap experiences the Miller-effect



$$C_{in1} = C_{\pi}(1 - A_{v1})$$

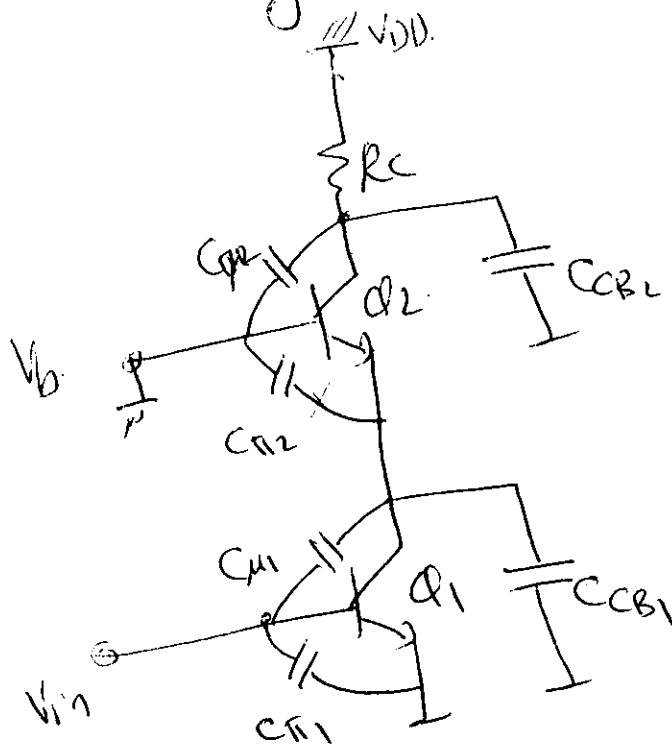
$$C_{out} = C_{\pi} \left(1 - \frac{1}{A_{v1}} \right)$$



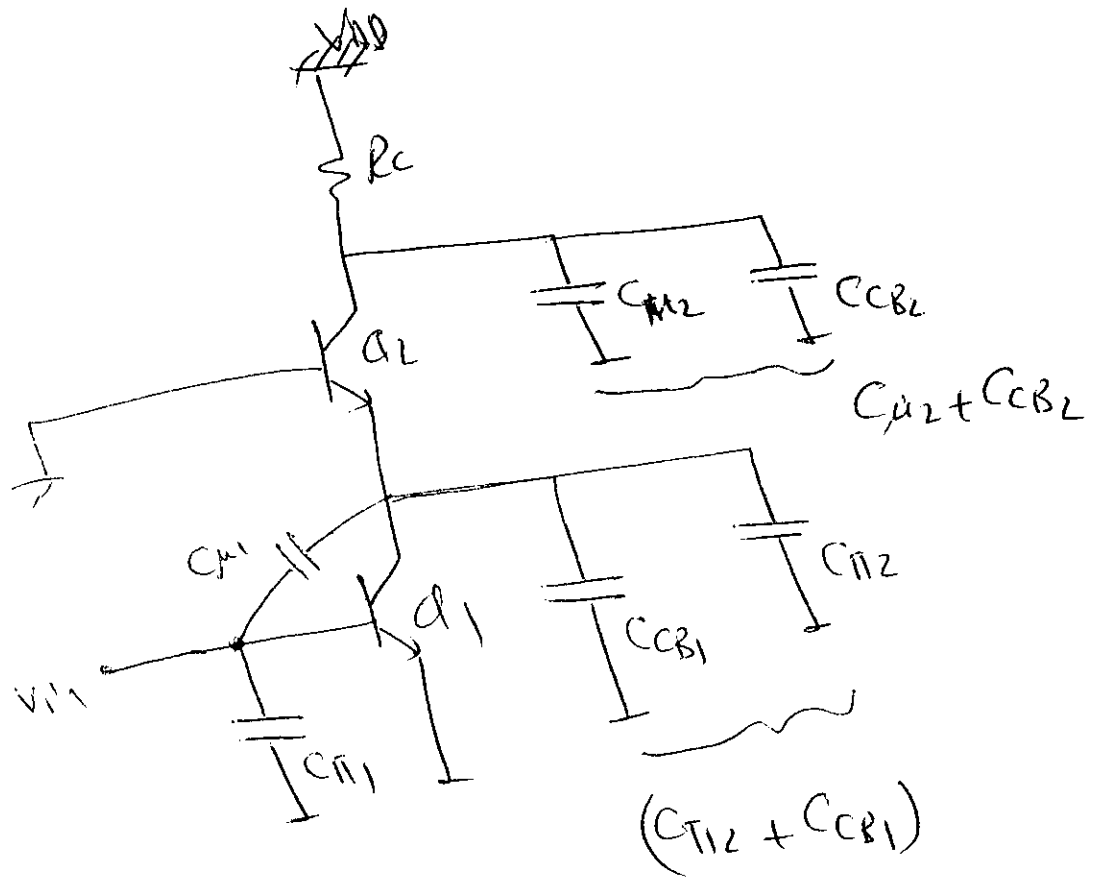
Miller Capacitance.

Ex: Identify all Cap.

(17)



V_b is shorted



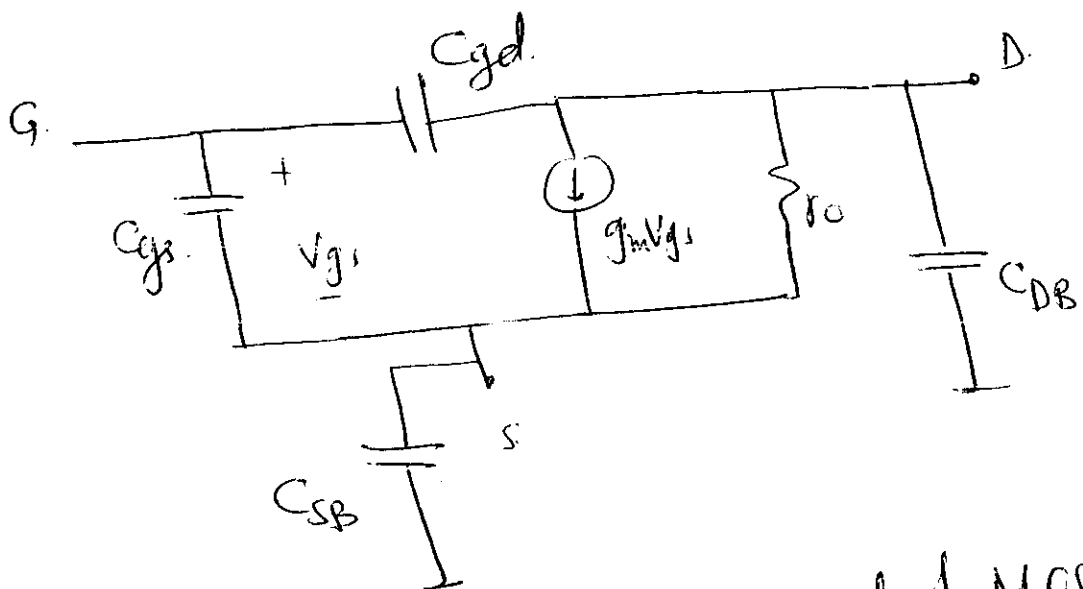
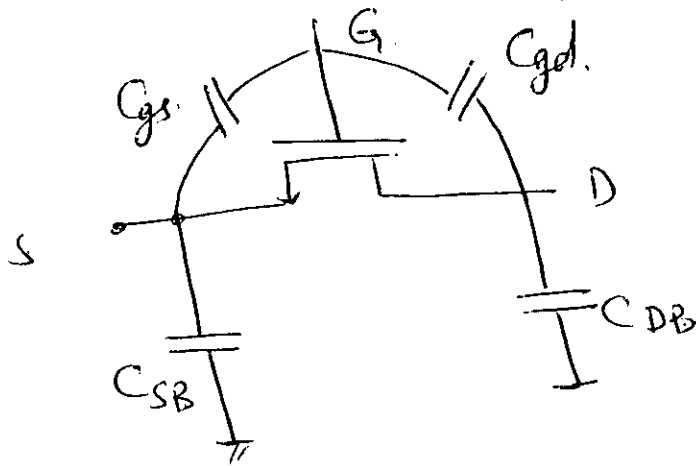
C_{M1} ; expensive, Miller effect

Today's lect

(18)

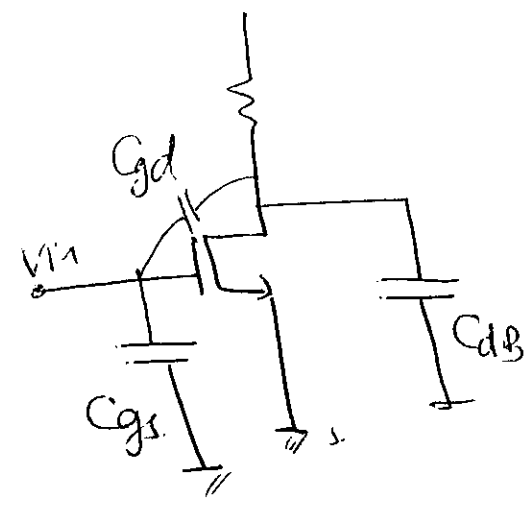
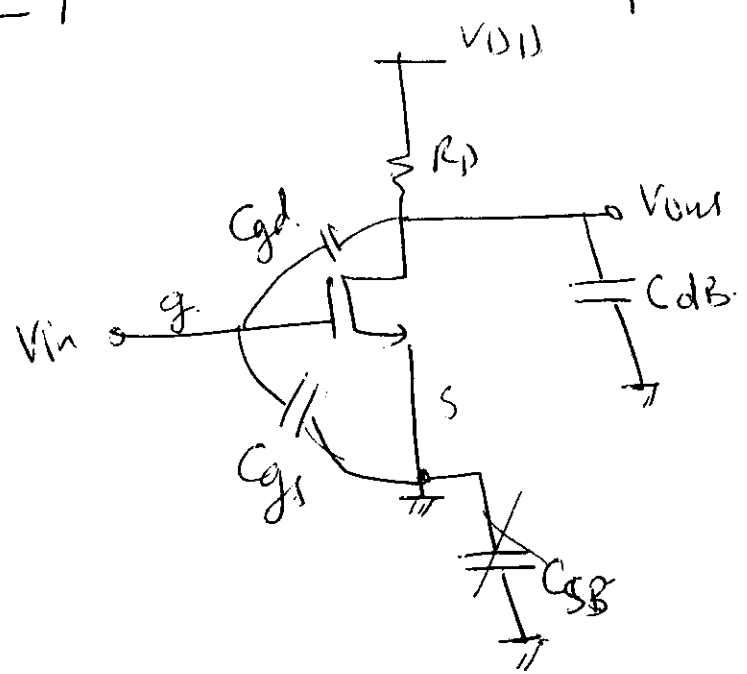
- High frequency Model of MOS
- General Procedure for calculation frequency Resp.
- Frequency Response of CE/CS stage.

* High frequency Model of MOS:



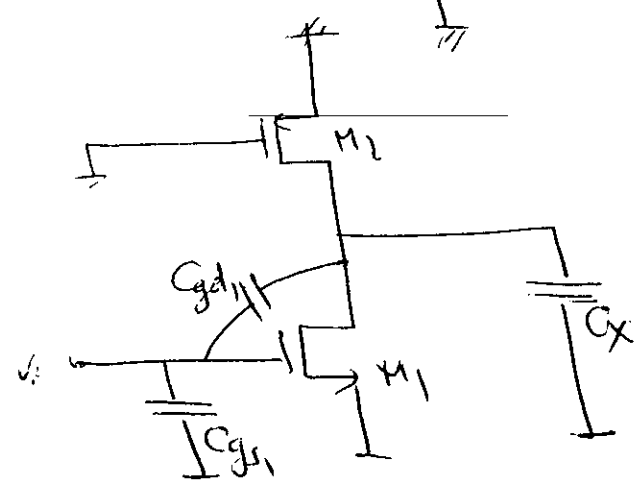
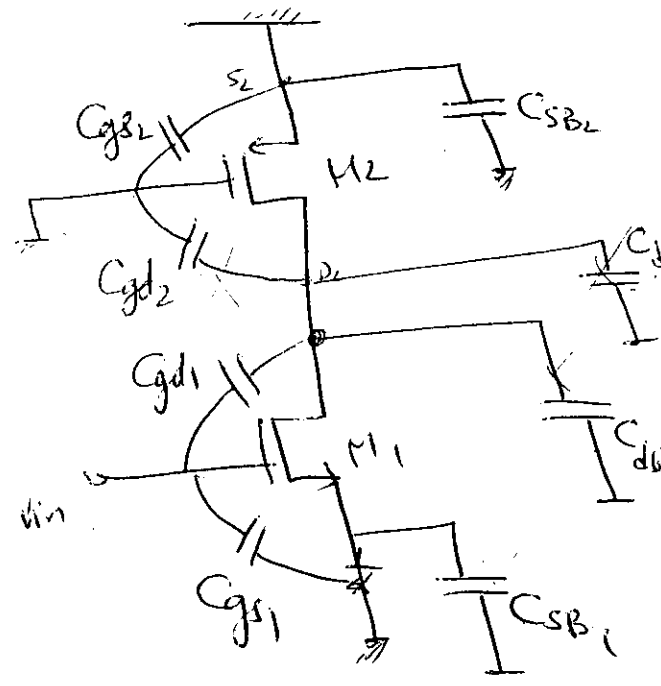
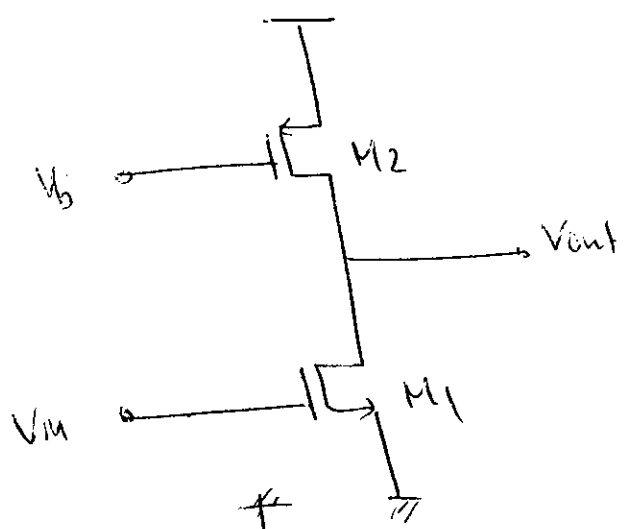
high frequency Model of MOS

Example: Draw all Cap:



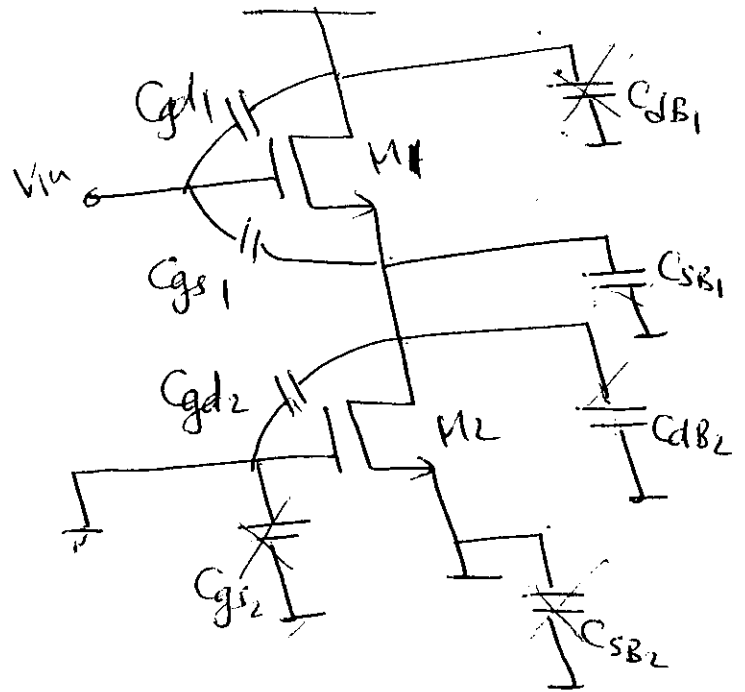
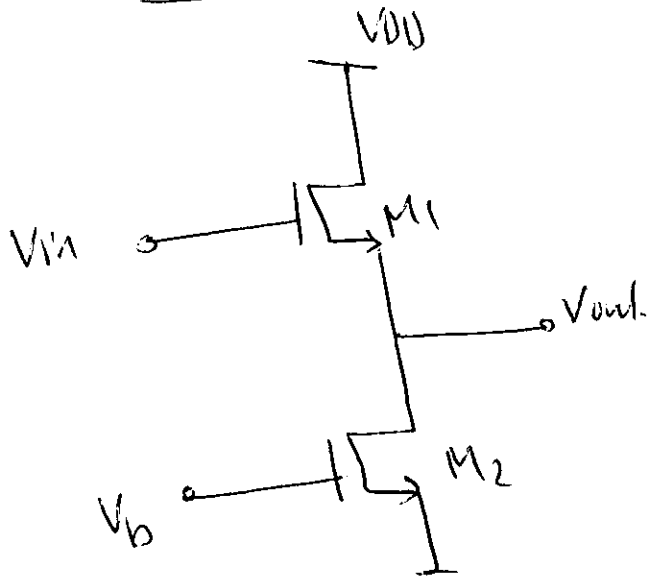
C_{SB} is shorted
 C_{gd} experiences Miller effect.

Common Source



$C_{SB2} / C_{SB1} \rightarrow$ shorted.
 $C_X = C_{dB1} + C_{dB2} + C_{gd2}$
 C_{gs2} is shorted
 $C_{gd1} \rightarrow$ Miller, of 6.1

Ex: Source follower



$C_{db1} \rightarrow$ shorted

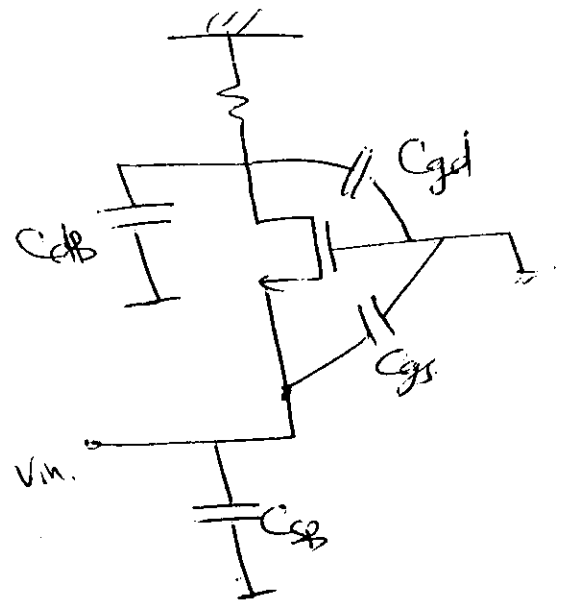
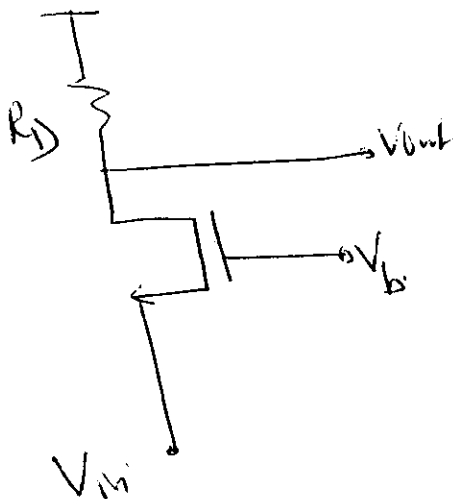
$C_{gs2} \rightarrow$ shorted

$C_{SB2} \rightarrow$ shorted

$$C_{SB1} \parallel C_{db2} \parallel C_{gd2} = C_{SB1} + C_{db2} + C_{gd2}$$

$C_{gs1} \rightarrow$ experiences Miller effect.

Ex: Common Gate:

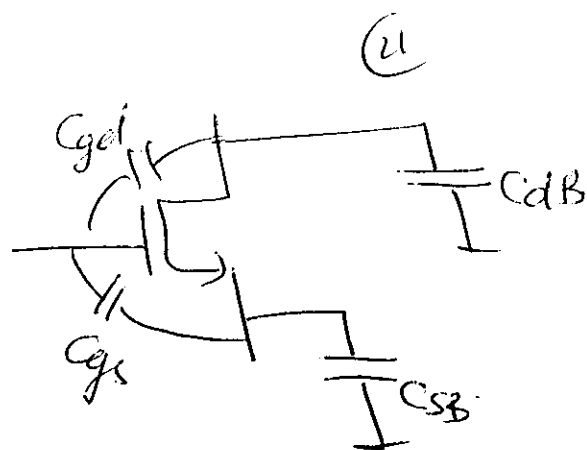
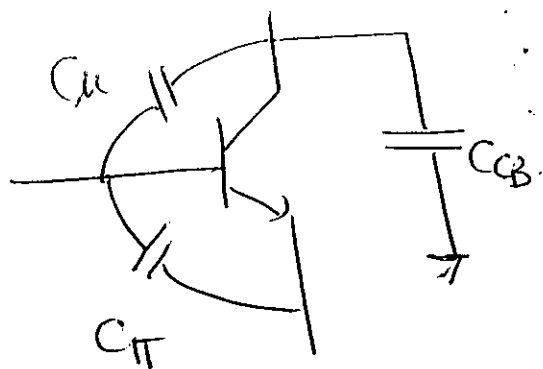


$$C_{SB} \parallel C_{gs} = C_{SB} + C_{gs}$$

$$C_{gd} \parallel C_{db} = C_{gd} + C_{db}$$

$\Rightarrow$ No Miller effect

Review



General Procedure for Calculation Frequency Response:

1) Exact Solution

1- Draw the circuit.

2- Include all device capacitance.

3- Remove or merge capacitance.

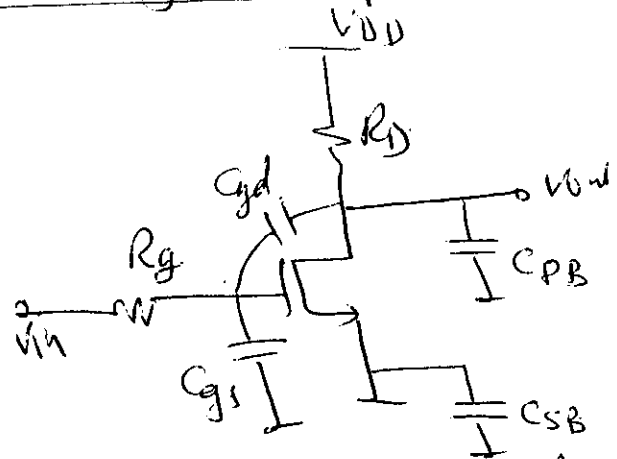
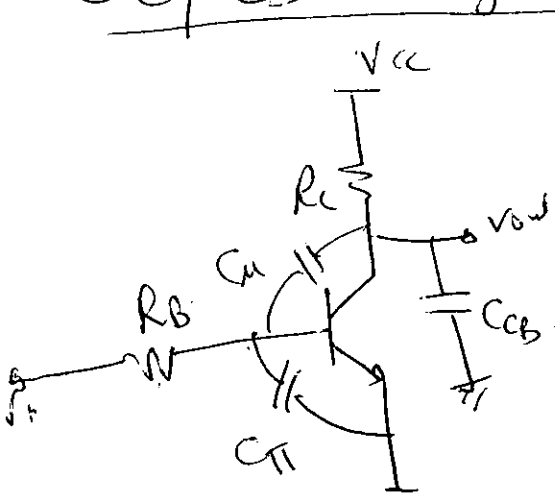
4- Compute the transfer function $H(s)$.

5- Calculate $|H(j\omega)|$ then plot.

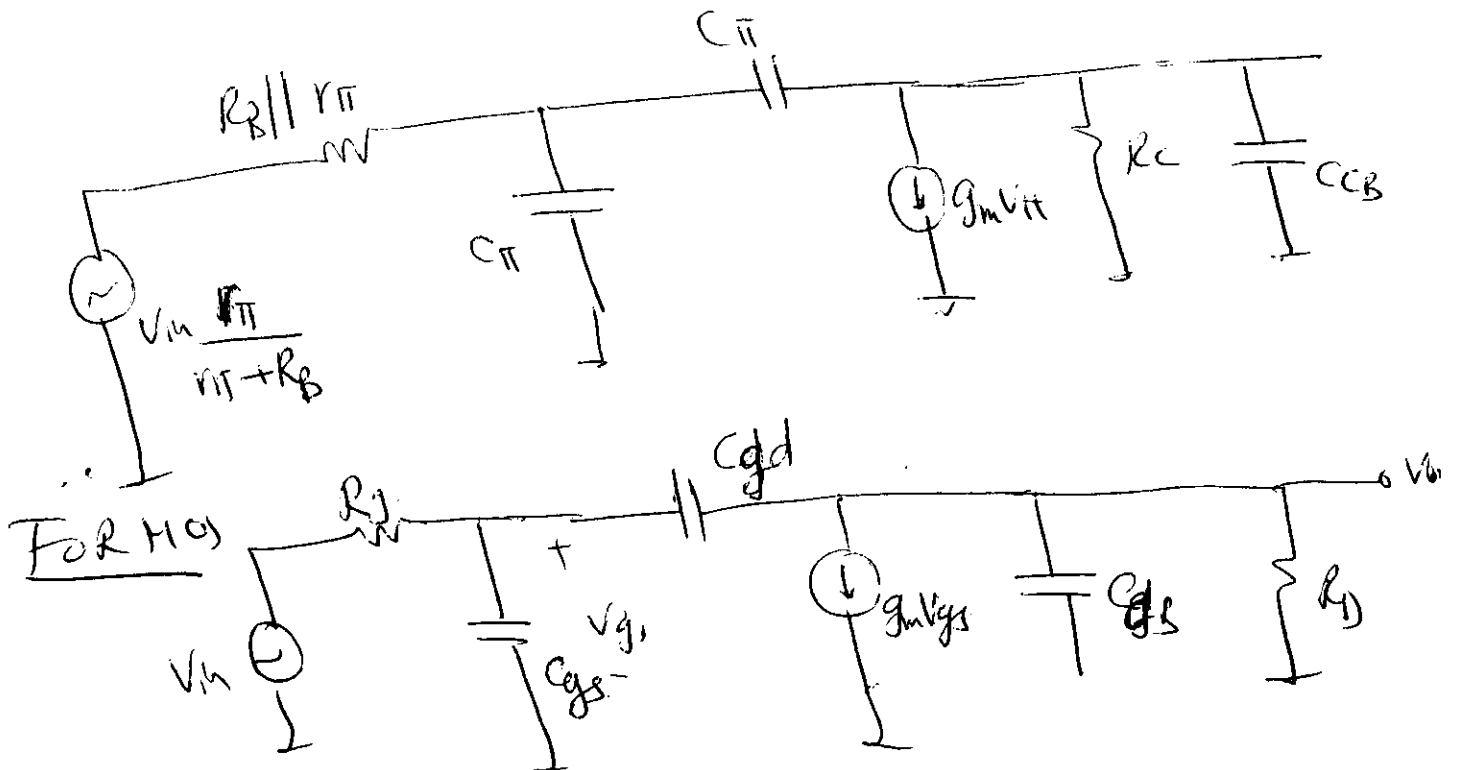
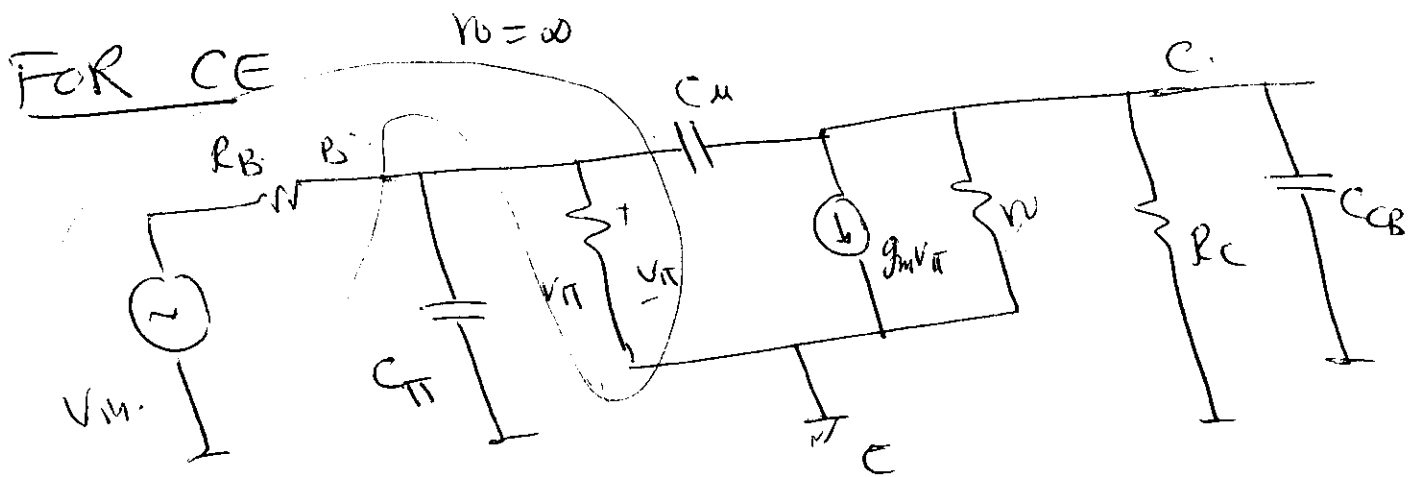
2) Approximation

Instead of (4.5), find the poles by inspection.

CE/CS stage Frequency Response (22)



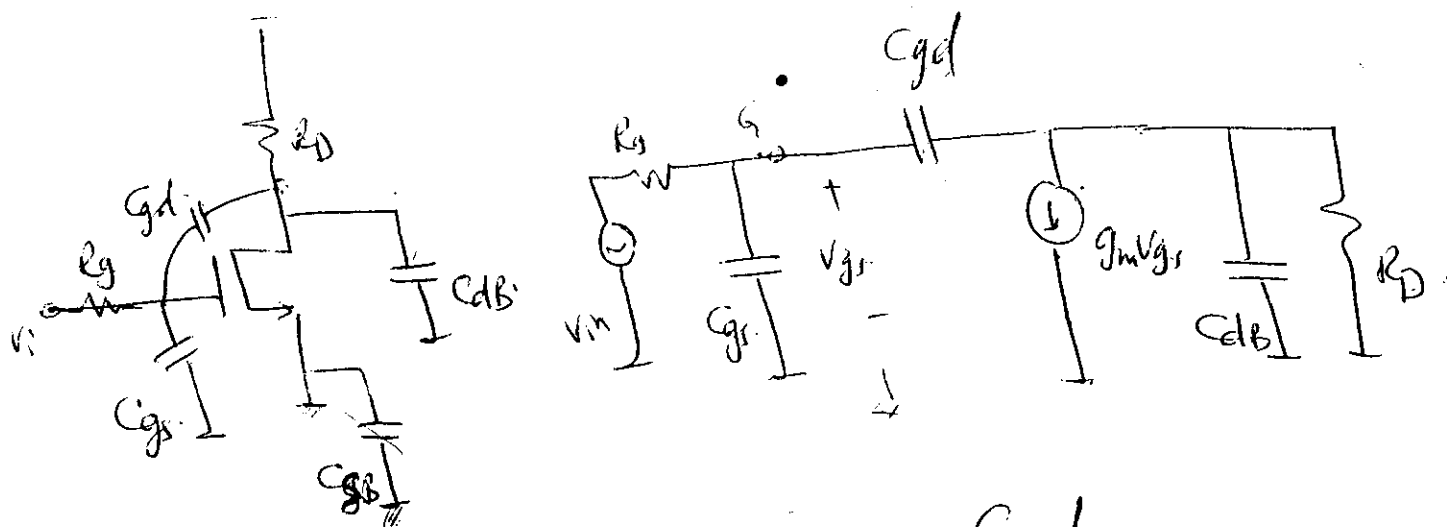
Both R_B, R_G represent the output resistance of the previous stage.



The approximation method:

(23)

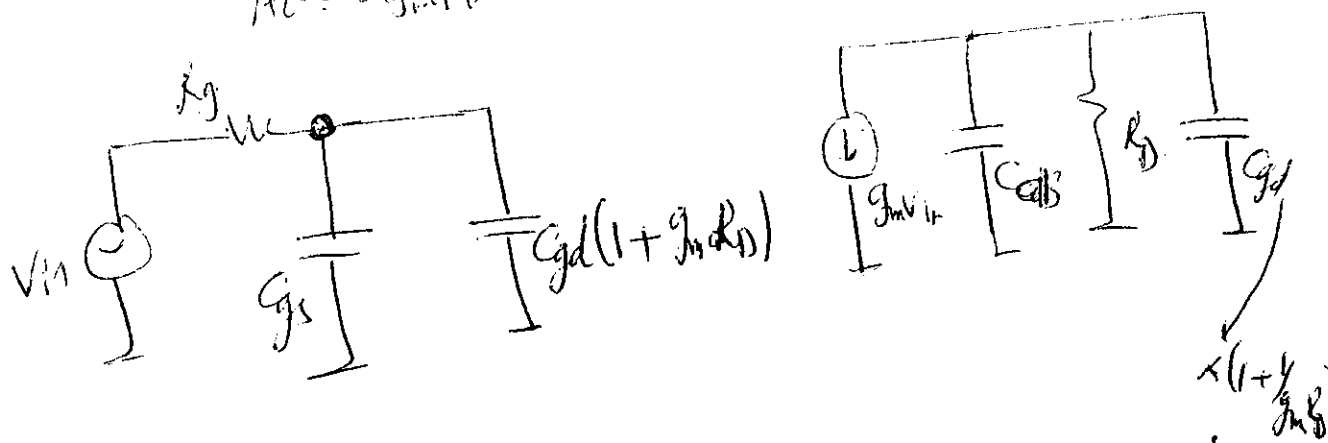
- Finding the pole using inspection.



- Applying Miller theorem to C_{gd}

$$C_{gd} \rightarrow \begin{cases} C_{gd}(1 - A_{vc}) \rightarrow \text{input} = C_{gd}(1 + g_m R_D) \\ C_{gd}(1 - 1/A_{vc}) \rightarrow \text{output} = C_{gd}(1 + 1/g_m R_D) \end{cases}$$

$A_{vc} = -g_m R_D$

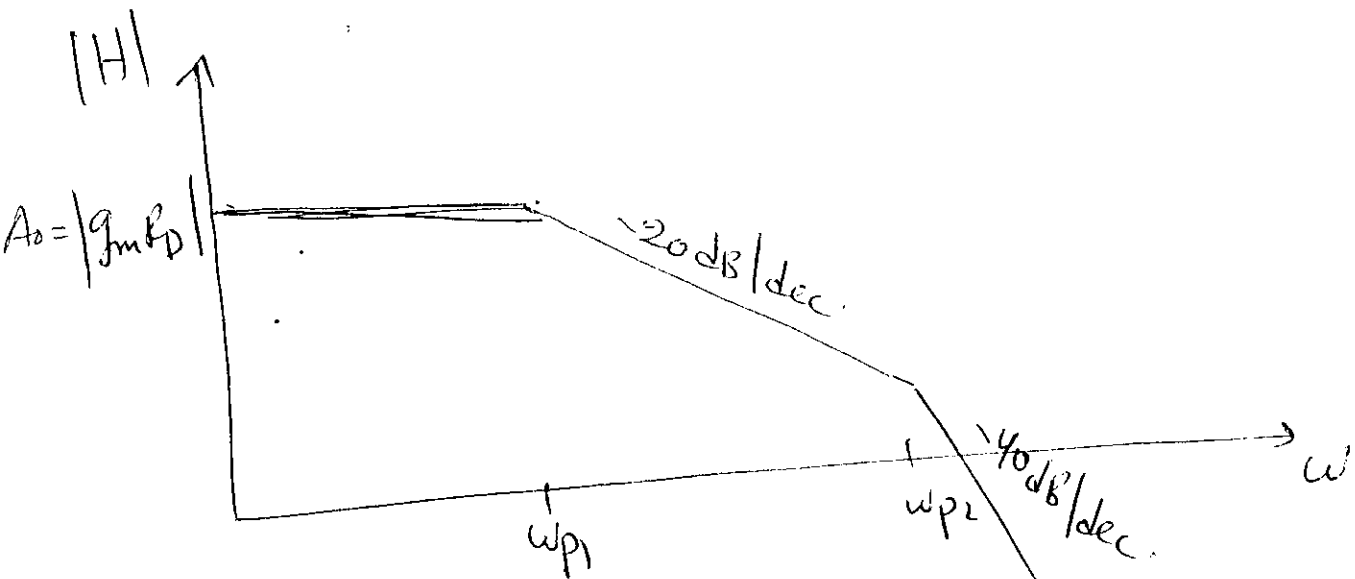


$$\omega_{p, in} = \frac{1}{R_g (C_{gs} + C_{gd}(1 + g_m R_D))}$$

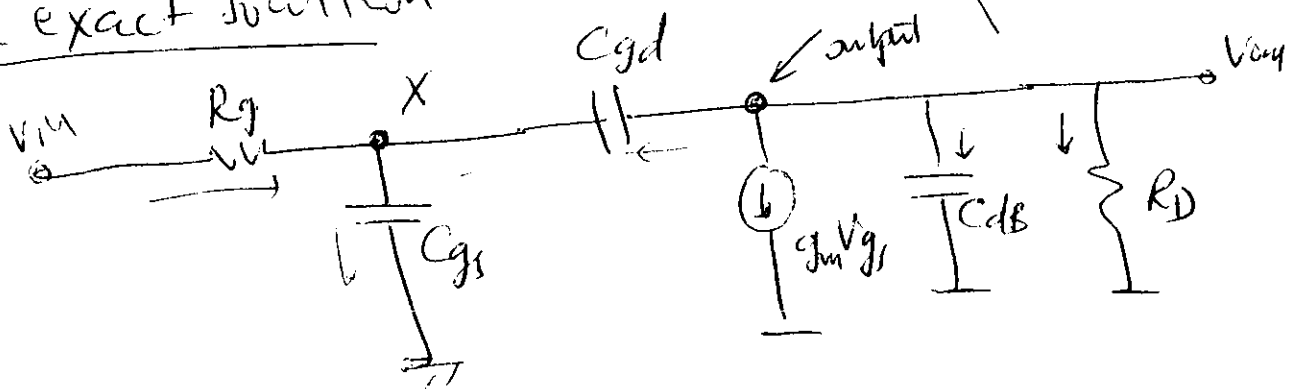
$$\omega_{p, out} = \frac{1}{R_D (C_{LB} + C_{gd}(1 + 1/g_m R_D))}$$

(24)

$$|A(s)| = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$



The exact solution



Using Nodal Analysis

KCL @ output node:

$$\frac{V_{out}}{R_D} + sC_{db}V_{out} + g_m V_{gs} + sC_{gd}(V_{out} - V_{in}) = 0 \quad \rightarrow (1)$$

KCL @ X node ..

$$\frac{V_{in} - V_{gs}}{R_g} + sC_{gs}V_{gs} + sC_{gd}(V_{gs} - V_{out}) = 0 \quad \rightarrow (2)$$

Solve (1, 2)

(25)

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{(sC_{gd} - g_m) R_D}{R_{g,b} (C_{gs} C_{gd} + C_{gs} C_{db} + C_{gd} C_{db}) s^2 + [C_{gd}(A + g_m R_D) + C_{gs}] R_G + R_D (C_{db} + C_{gd}) s + 1}$$

We can see that the system has zero and two poles

* $\omega_z = \frac{g_m}{C_{gd}} \rightarrow$ very high frequency not important.

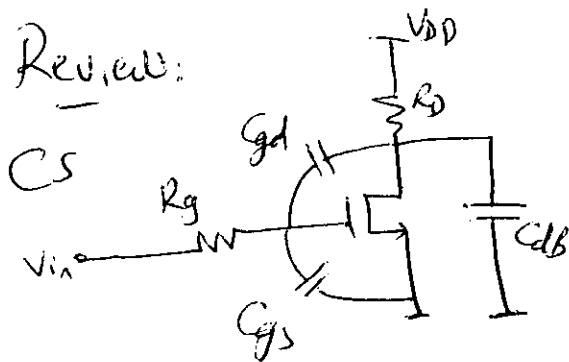
* we have two pole (Dominant poles)
($\omega_{p1} \ll \omega_{p2}$)

W

Today's lect.

- Dominant pole Approximation
- Insight into CE/CS Frequency Response
- Examples
- Frequency Response of CB/CG stage

Review:



$$\frac{V_{out}}{V_{in}} = \frac{(sC_{gd} - g_m)R_D}{(C_{gs}C_{gd} + C_{gd}C_{db} + C_{gs}C_{db})R_gR_Ds^2 + s[(1 + g_mR_D)C_{gd}]R_g + (C_{gd} + C_{db})R_D} + 1$$

Dominant pole Approximation:-

$$D(s) = (1 + s/\omega_{p1})(1 + s/\omega_{p2})$$

$$= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

Assume: $\omega_{p1} \ll \omega_{p2} \Rightarrow \frac{1}{\omega_{p1}} \gg \frac{1}{\omega_{p2}}$

$$D(s) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{1}{\omega_{p1}}s + 1$$

$$\omega_{p1} = \frac{1}{[(1 + g_mR_D)C_{gd} + C_{gs}]R_g + (C_{gd} + C_{db})R_D}$$

To find ω_{p2}

$$\omega_{p1} \omega_{p2} = \frac{1}{(C_{gs} C_{gd} + C_{gd} C_{ds} + C_{gs} C_{dB}) R_g R_D}$$

$$\omega_{p2} = \frac{\omega_{p1} \omega_{p2}}{\omega_{p1}}$$

From Miller Theorem

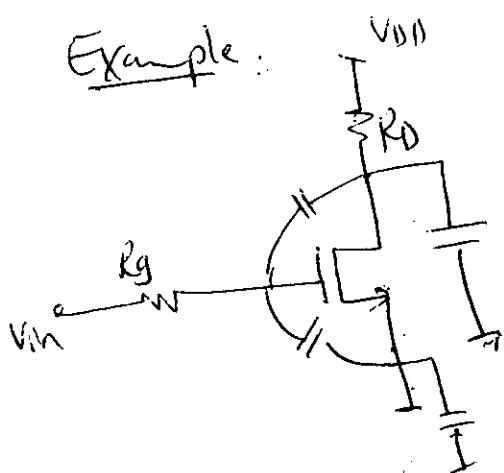
$$\omega_{p1} = \frac{1}{[(1 + g_m R_D) C_{gd} + C_{gs}] R_g}$$

Comparing the dominant pole ω_{p1} with $(\omega_{p1}) \Rightarrow$ we see that they are approximately the same.

A few points: check our expression of $H(s)$

- 1) In $\frac{V_o}{V_{in}}$, if $s=0 \Rightarrow$ (low frequency) $A_v = -g_m R_D$
- 2) At high frequency $s \rightarrow \infty$, $\frac{V_{out}}{V_{in}} = 0$

Example:



* what happens if $\omega \rightarrow 2\omega$ but I_D is constant.

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

when g_m becomes double.

$$g_m' = \sqrt{2} g_m$$

(28)

$$\omega_{p1} = \frac{1}{[(1 + g_m R_D) C_{gd} + C_{gs}] R_G + (C_{gd} + C_{db}) R_D}$$

doubling of width $\Rightarrow$ doubling of capacitance

$$\omega_{p1}' = \frac{1}{((1 + \sqrt{2} g_m R_D) 2 C_{gd} + 2 C_{gs}) R_G + (2 C_{gd} + 2 C_{db}) R_D}$$

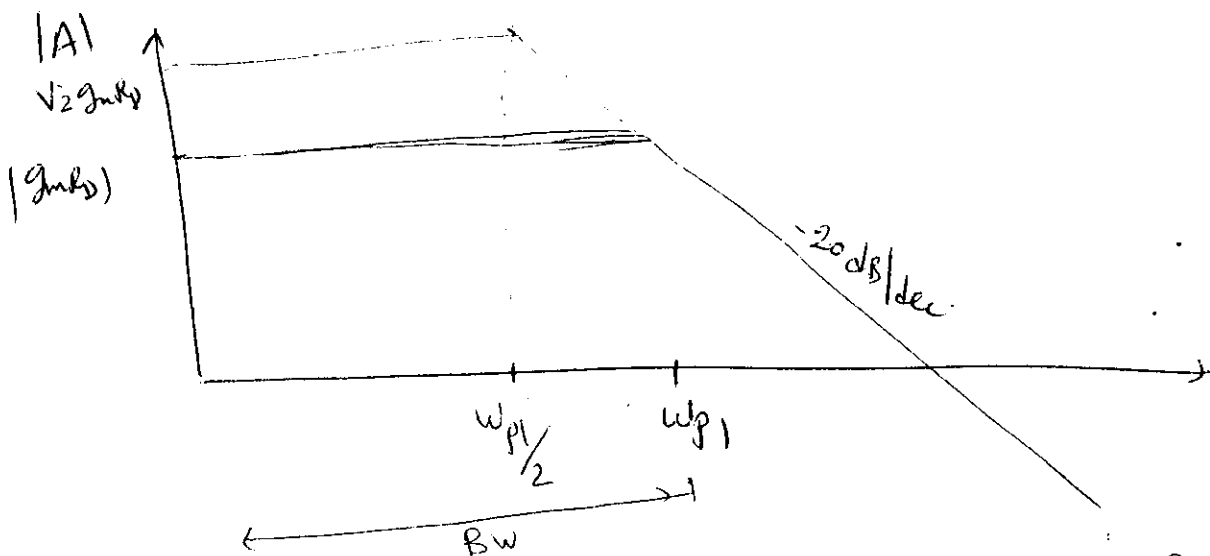
$\Rightarrow$ that means ω_{p1} goes down by at least a factor of 2 (BW reduces by 2)

At low frequency

$$A_v = -g_m R_D$$

$\xrightarrow[\omega]{\text{doubling}}$

$$A_v' = -\sqrt{2} g_m R_D = \sqrt{2} A_v$$

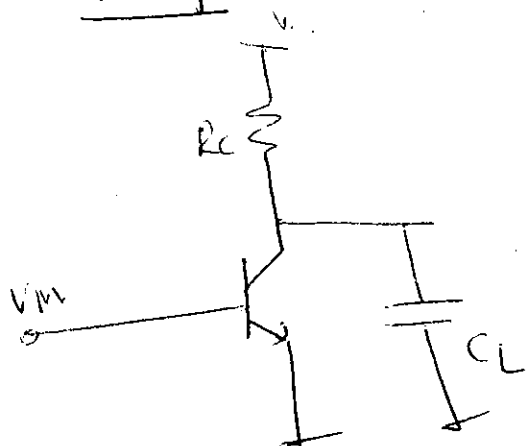


We see trade off between A_v and BW

Example

Ignore the parasitic C_{eq}

(29)



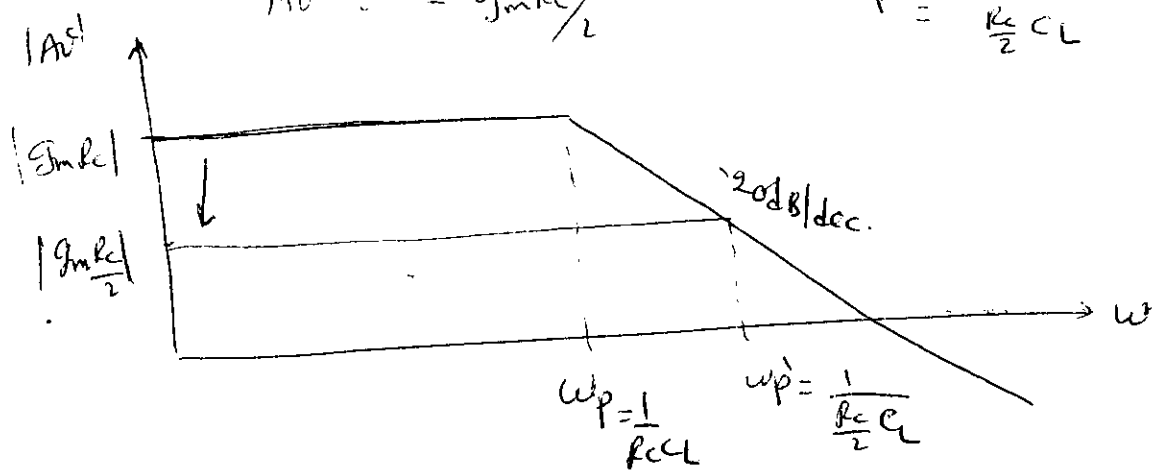
A1) If $R_c \rightarrow \frac{R_c}{2}$, what happens ($I_C = \text{cts}$)

$$A_v = -g_m R_c$$

$$A_v' = -g_m \frac{R_c}{2}$$

$$\omega_p = \frac{1}{R_c C_L}$$

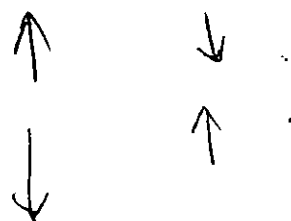
$$\omega_p' = \frac{1}{\frac{R_c}{2} C_L}$$



We see that the gain is reduced by a factor of 2, and BW is increased by " = 2.

The Gain Bandwidth $\therefore = \text{gain} \times \text{BW} = \text{cts constant}$

$$= g_m R_c \times \frac{1}{R_c C_L} = \frac{g_m}{C_L}$$



If $R_c' \rightarrow \frac{R_c}{2}$, and $I_c' = 2I_c$ what happens

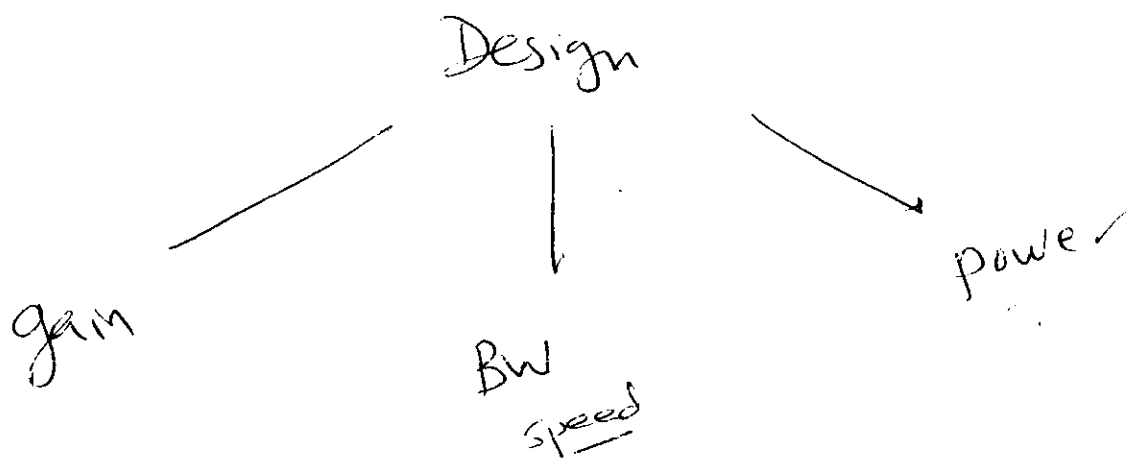
$$g_m = \frac{I_c}{V_T} \Rightarrow g_m' = \frac{2I_c}{V_T} \text{ doubling}$$

$$GBW = g_m R_c \times \frac{2}{R_c C_L} = 2 \frac{g_m}{C_L}$$

$\Rightarrow$ By doubling $I_c \Rightarrow$ BW is improved

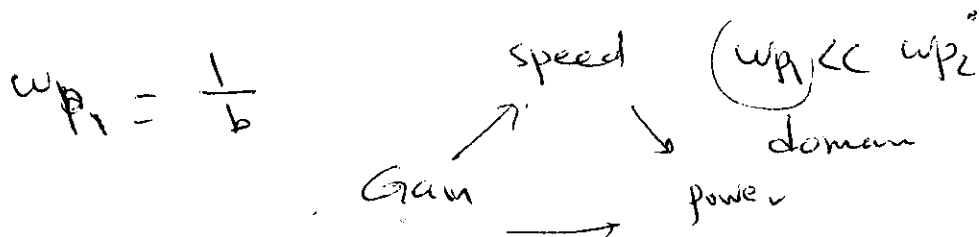
$$\text{Power} = V_{CC} (2I_{CC})$$

So the power increases.



Here, we see that the design should trade off among (power, BW, gain)

Review: $D(s) = as^2 + bs + 1$ two pole (ω_{p1}, ω_{p2})



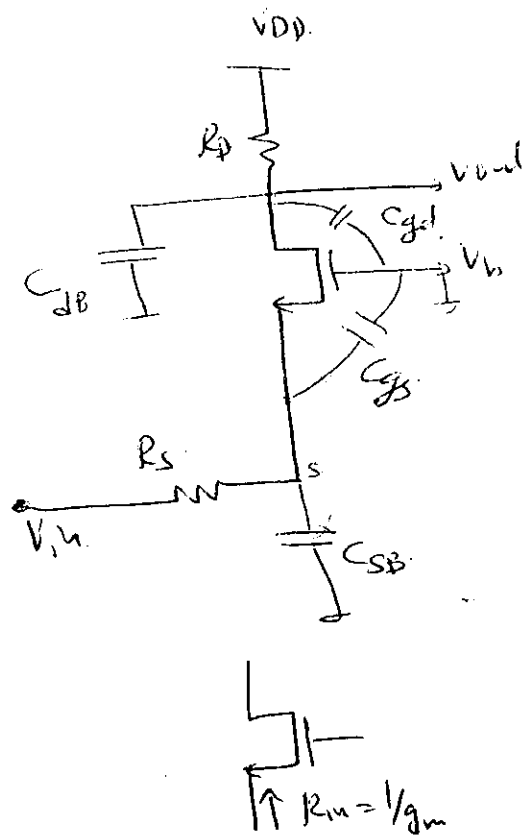
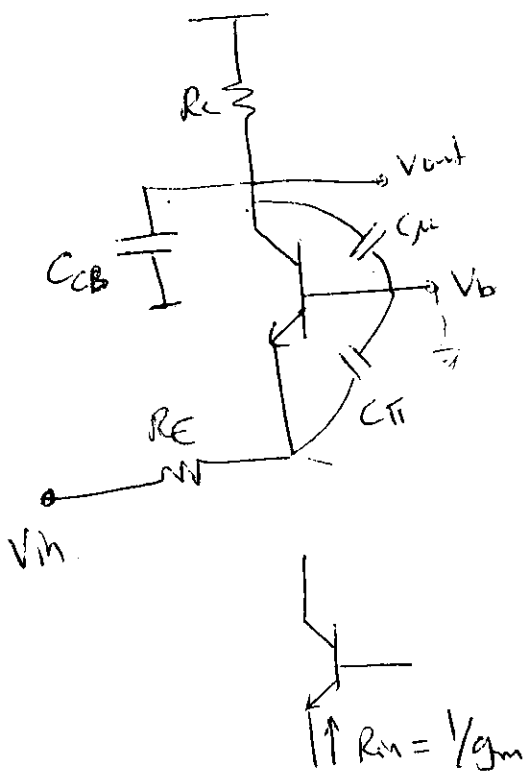
Today's lect

(31)

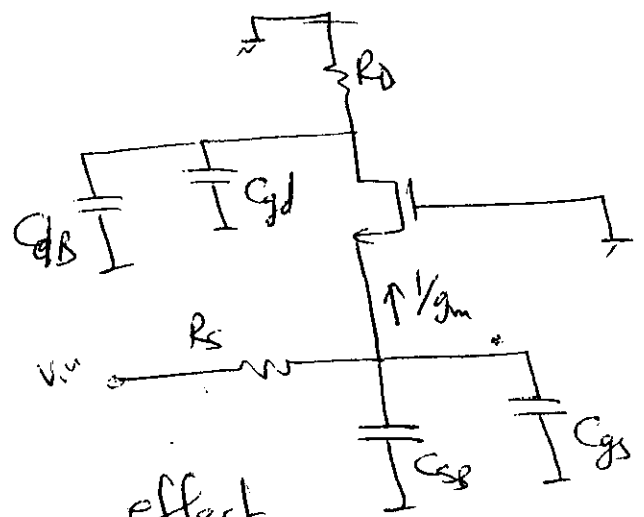
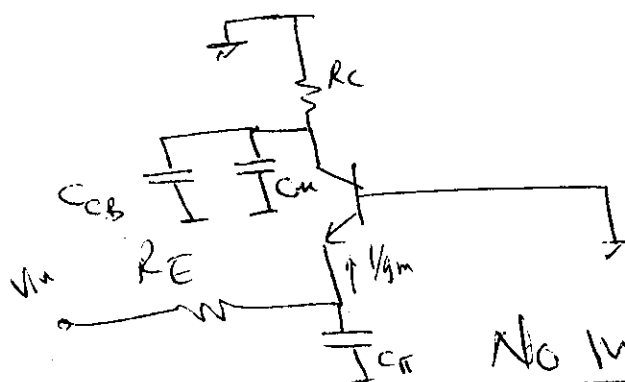
- Frequency Response of CB/CG stage

- Example 1:

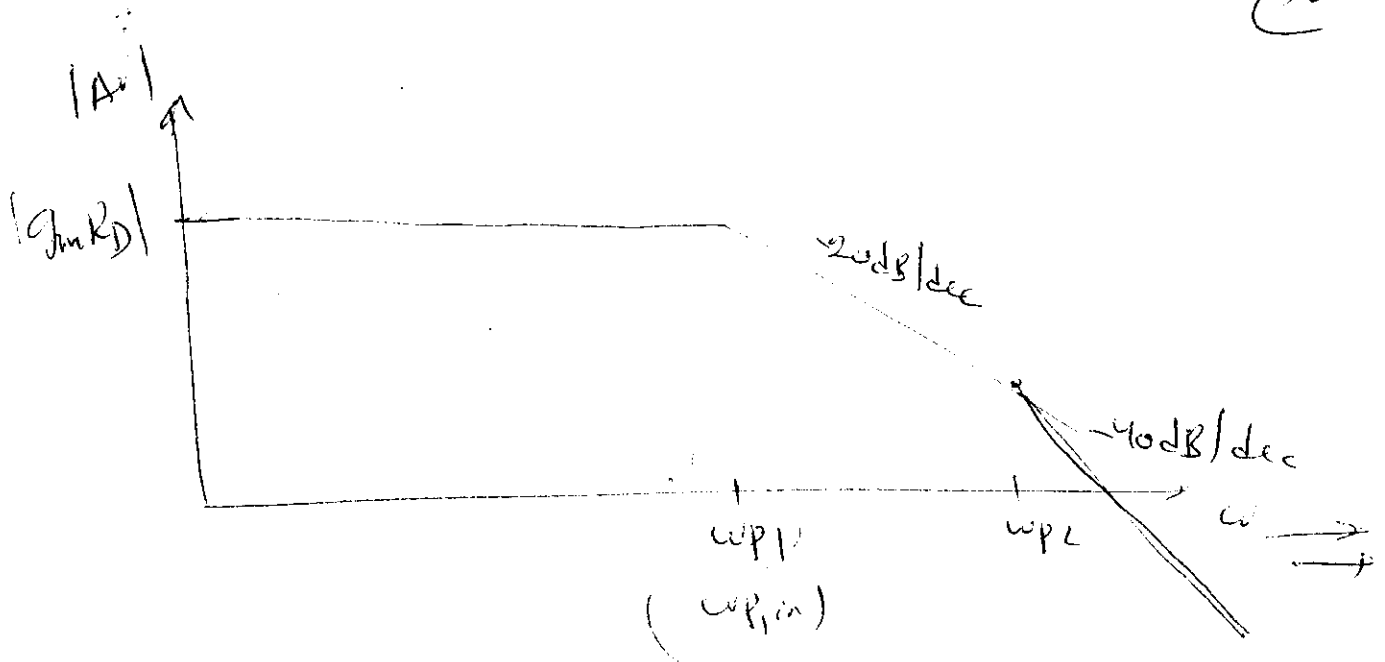
Frequency Response of CB/CG stage



- 1) Identify all Capacitors
- 2) Merge Cap



No Miller effect



Using Inspection method to find the poles

* Input pole;

CE: Capacitance to GND $\rightarrow C_{\pi}$
Resistance to ac gnd $\rightarrow (R_E \parallel 1/g_m)$

$$\omega_{pin} = \frac{1}{C_{\pi} (R_E \parallel 1/g_m)}$$

MO: Capacitance to GND = $(C_{gs} + C_{sb})$
Resistance to ac gnd = $R_s \parallel 1/g_m$

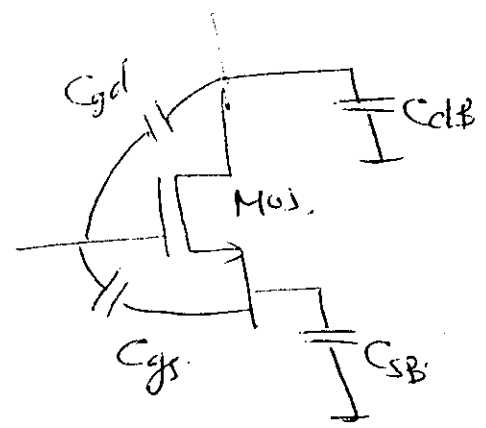
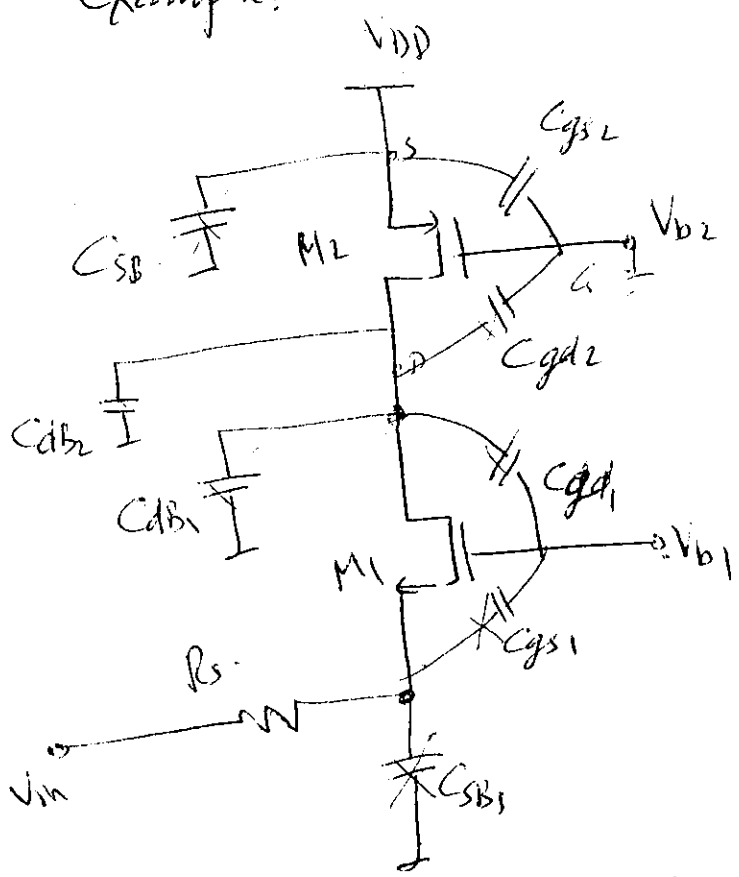
$$\omega_{pin} = \frac{1}{(C_{gs} + C_{sb}) (R_s \parallel 1/g_m)}$$

Output pole

CE: $\omega_{pout} = \frac{1}{C_{cb} + C_u} R_c$ / MO: $\omega_{pout} = \frac{1}{R_D (C_{gd} + C_{db})}$

Example:

- 1) Find the poles.
- 2) draw all Cap.
- 3) simplify.



$$(C_{gs1} \parallel C_{SB1}) \rightarrow (C_{gs1} + C_{SB1})$$

$$(C_{db1} \parallel C_{gd1}) \parallel C_{db2} \parallel C_{gd2} = C_{gd1} + C_{db1} + C_{db2} + C_{gd2}$$

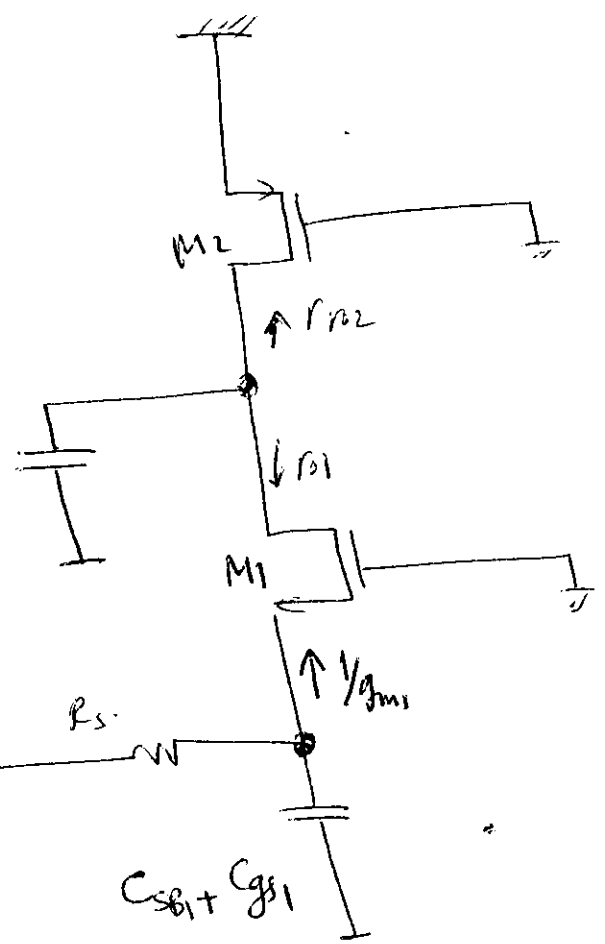
$C_{SB2} \rightarrow$ shunted
 $C_{gs2} \rightarrow$ shunted

$$C_{gd1} + C_{db1} + C_{db2} + C_{gd2}$$

$$\omega_{p, in} = \frac{1}{(R_s \parallel \frac{1}{g_{m1}})(C_{SB1} + C_{gs1})}$$

$\omega_{p, out}$:

Case 1: $\lambda = 0$ / $r_o = \infty$
 $\omega_{p, out} = \frac{1}{\infty} = 0$



when $\lambda \neq 0$

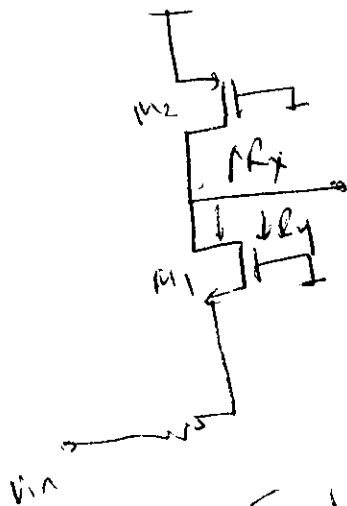
(34)

$$R_{out} = R_x \parallel R_y$$

$$R_x = r_{o2}$$

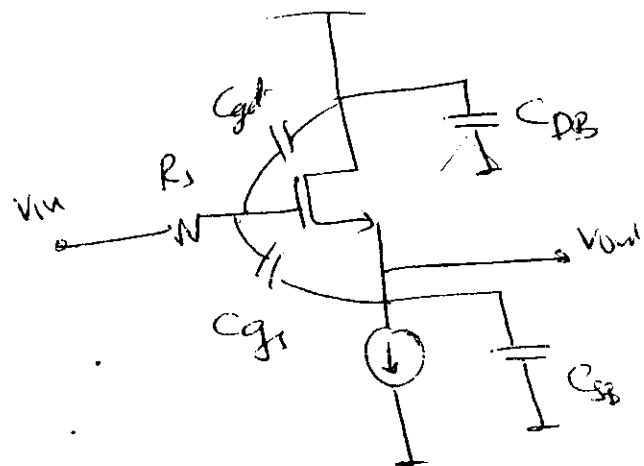
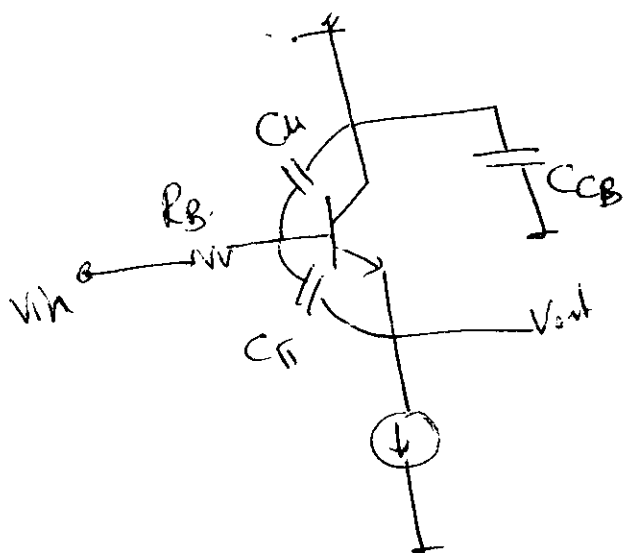
$$R_y = (1 + g_{m1} r_{o1}) R_s + r_{o1}$$

$$\omega_{p,out} = \frac{1}{R_{out} C_{out}}$$



$$C_{out} = C_{dB1} + C_{gd1} + C_{dB2} + C_{gd2}$$

Frequency Response of Emitter-follower
Source follower.

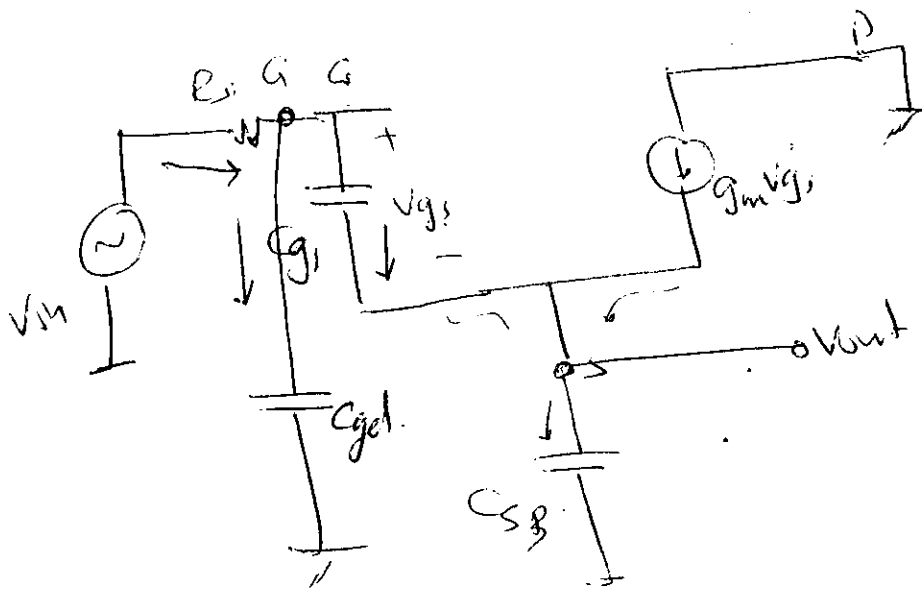


Adding Cap

Using large signal equivalent a

To find $\frac{V_{out}}{V_{in}}$ (transfer function)

(35)



KCL @ gate

$$\frac{V_{in} - V_G}{R_s} = s C_{gd} V_G + s C_{gs} V_{gs}$$

$$V_G = V_{gs} + V_{out}$$

$$\frac{V_{in} - (V_{gs} + V_{out})}{R_s} = s C_{gd} (V_{gs} + V_{out}) + s C_{gs} V_{gs}$$

KCL @ output node

$$s C_{gs} V_{gs} + g_m V_{gs} = s C_{SB} V_{out} \rightarrow (2)$$

from (1, 2)

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{1}{(C_{gd} C_{gs} + C_{gs} C_{SB} + C_{gd} C_{SB}) \frac{R_s}{g_m} s^2 + \dots}$$

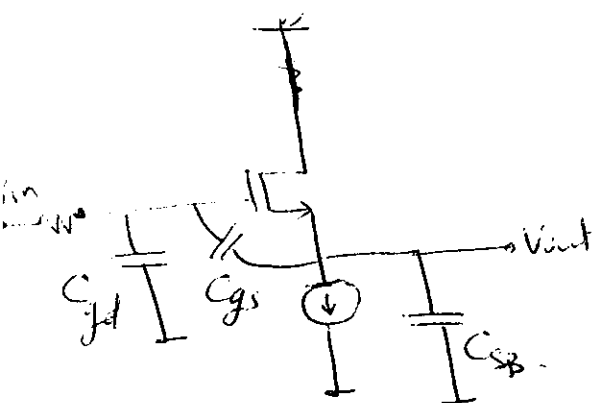
$$\Rightarrow \left[R_s C_{gd} + \frac{C_{gs}}{g_m} + \frac{C_{SB}}{g_m} \right] s + 1$$

Today's lect.

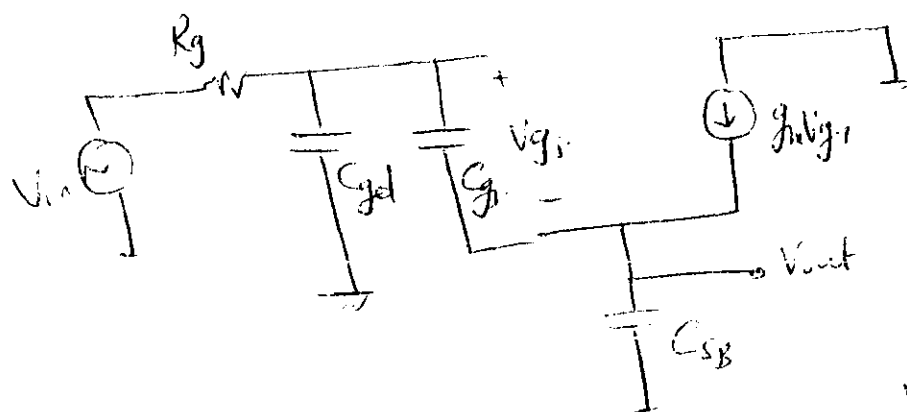
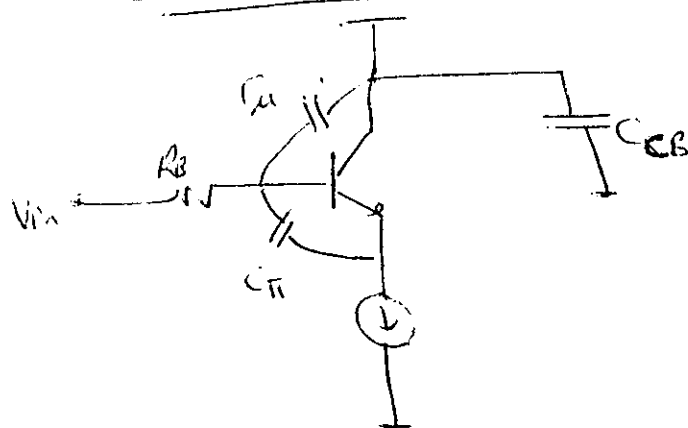
- Input / Output Impedance of Emitter follower
- Frequency Response of Cascode
- Frequency Response of Diff pairs

Review:-

Source follower



Source Emitter

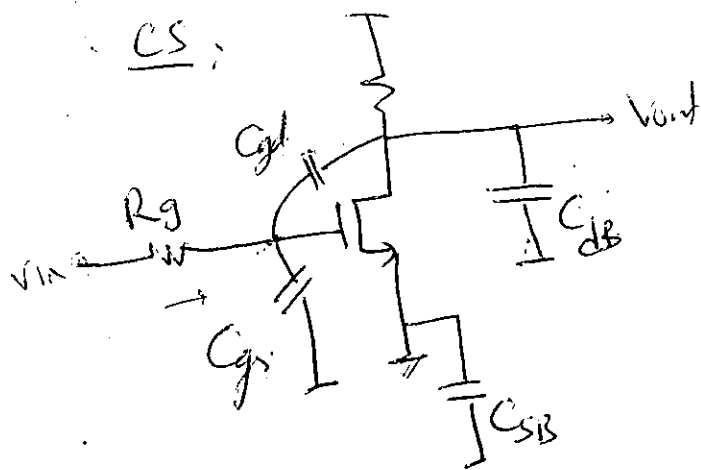


Here, it is difficult to apply inspection method

$$\frac{V_{out}}{v_{in}} = \frac{1}{(C_{gd}C_{gs} + C_{gs}C_{SB} + C_{gd}C_{SB})\frac{R_g}{g_m}s^2 + \left[A_s C_{gd} + \frac{C_{gs}}{g_m} + \frac{C_{SB}}{g_m}\right]s + 1}$$

Input/output Capacitance of follower

(37)



$$C_{gd} \Rightarrow \begin{cases} C_{gd}(1 - A_v) \rightarrow \text{input} \\ C_{gd}(1 - 1/A_v) \rightarrow \text{output} \end{cases}$$

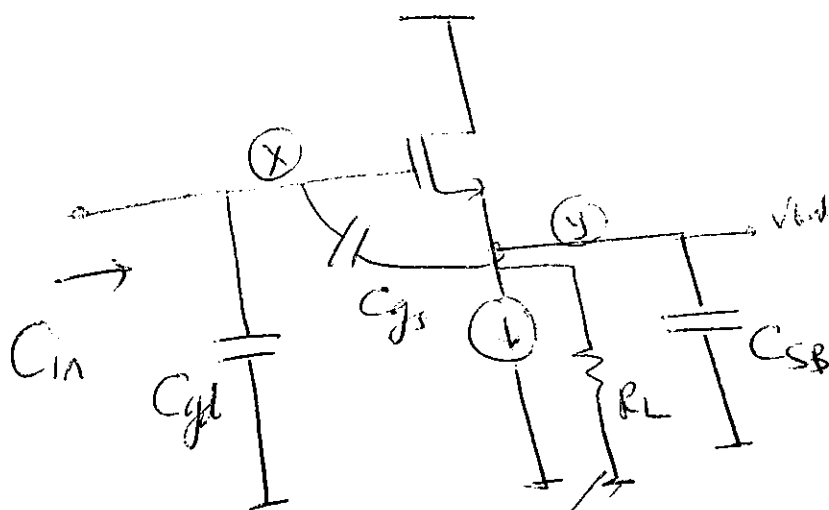
$$A_v = -g_m R_D$$

$$C_{gd} = \begin{cases} C_{gd}(1 + g_m R_D) \rightarrow \text{input} \\ C_{gd}(1 + 1/g_m R_D) \rightarrow \text{output} \end{cases}$$

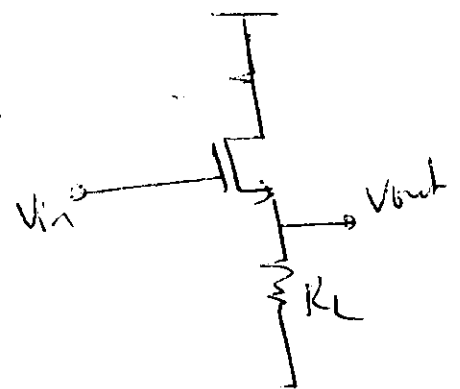
$$C_{in} = C_{gs} + C_{gd}(1 + g_m R_D)$$

$$C_{out} = C_{db} + C_{gd}(1 + 1/g_m R_D)$$

Source follower:



AO:



$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$= \frac{g_m R_L}{1 + g_m R_L}$$

R_L : model of resistance of current source

$$C_{gs} \rightarrow \begin{cases} x: \rightarrow C_{gs}[1 - A_v] = C_{gs}\left[1 - \frac{g_m R_L}{1 + g_m R_L}\right] \\ y: \rightarrow C_{gs}\left[1 - \frac{1}{A_v}\right] \end{cases}$$

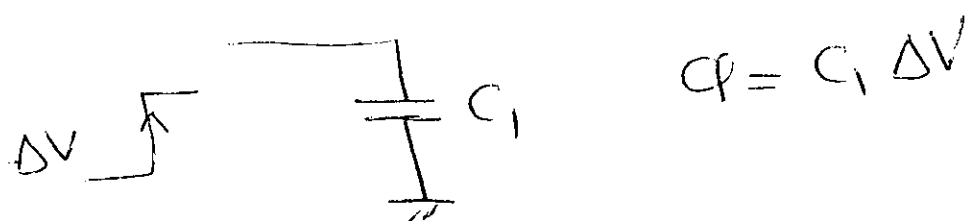
$$C_{in} = C_{gd} + C_{gs} \left[1 - \frac{g_m R_L}{1 + g_m R_L} \right]$$

$$= C_{gd} + \frac{C_{gs}}{1 + g_m R_L} \approx \frac{C_{gs}}{1 + g_m R_L}$$

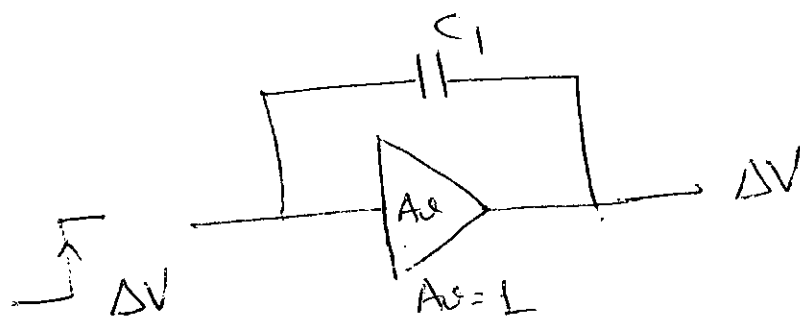
$\Rightarrow$ Input Capacitance is reducing.

Explain: why input cap of source follower is less than C_{gs} .

Case 1:

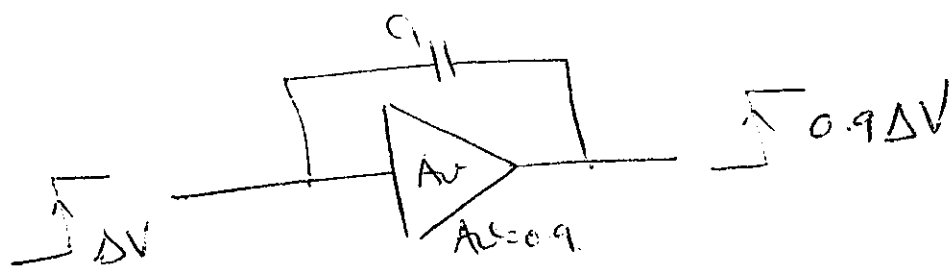


Case 2:



$$Q_{C_1} = C_1 [\Delta V_{out} - \Delta V_{in}] = 0$$

Case 3:



$$\text{Voltage change } C_1 = \Delta V - 0.9\Delta V = 0.1\Delta V$$

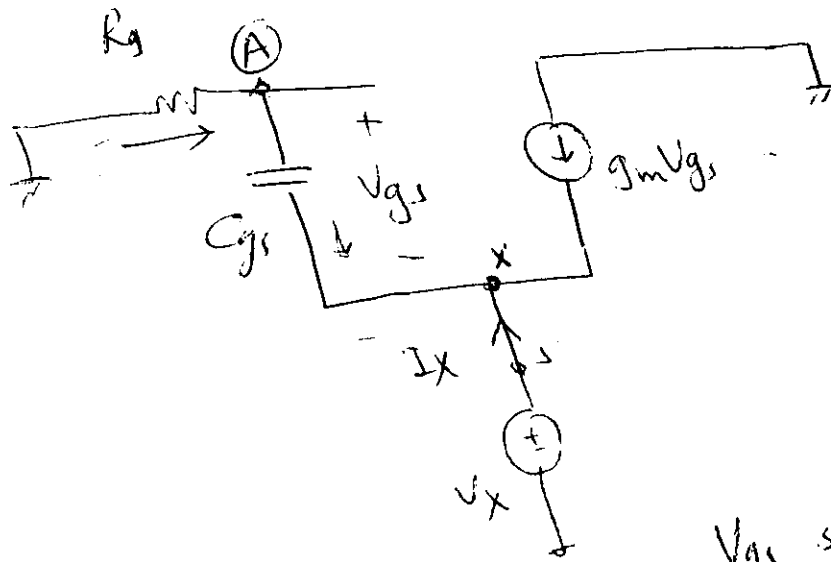
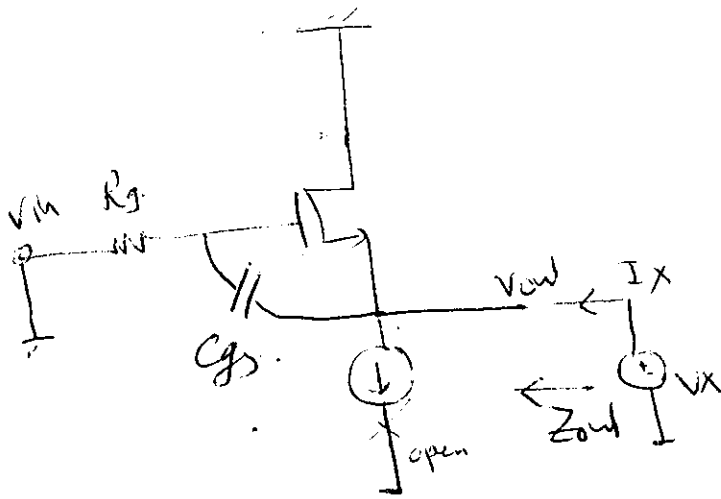
$$Q = C_1 \times 0.1\Delta V = \underbrace{0.1 C_1 \Delta V}_{\text{input cap.}}$$

Output Impedance of source follower

(39)

Assume only C_{gs} exists:

$$Z_{out} = \frac{V_{out}}{I_{out}} \Big|_{v_{in}=0} = \frac{V_X}{I_X} \Big|_{v_{in}=0}$$



KCL at node x:

$$V_{gs} sC_{gs} + g_m V_{gs} + I_X = 0$$

$$V_{gs} = \frac{-I_X}{sC_{gs} + g_m} \rightarrow (1)$$

KCL at input loop

$$-V_{gs} sC_{gs} R_g = V_{gs} + V_X$$

$$V_{gs} = -\frac{V_X}{1 + sC_{gs} R_g} \rightarrow (2)$$

$$\text{from (1, 2)} \quad \frac{I_X}{sC_{gs} + g_m} = \frac{V_X}{1 + sC_{gs} R_g} \Rightarrow \frac{V_X}{I_X} = \frac{1 + sC_{gs} R_g}{sC_{gs} + g_m} = Z_{out}$$

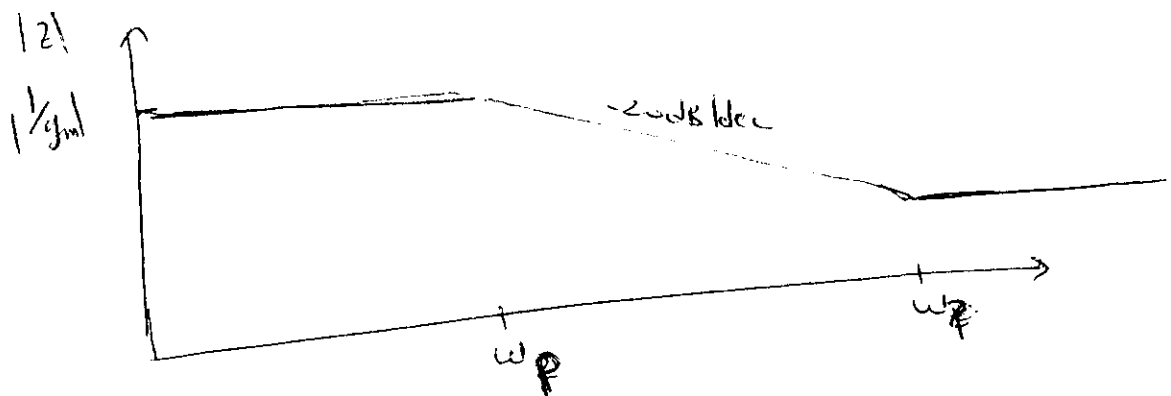
$$\text{If } s=0 \Rightarrow \left(\frac{V_X}{I_X} = \frac{1}{g_m} \right)$$

$$Z_{out} = \frac{1 + s C_{gs} R_g}{g_m + s C_{gs}} \quad \begin{matrix} \leftarrow \text{zero} \\ \uparrow \text{pole} \end{matrix}$$

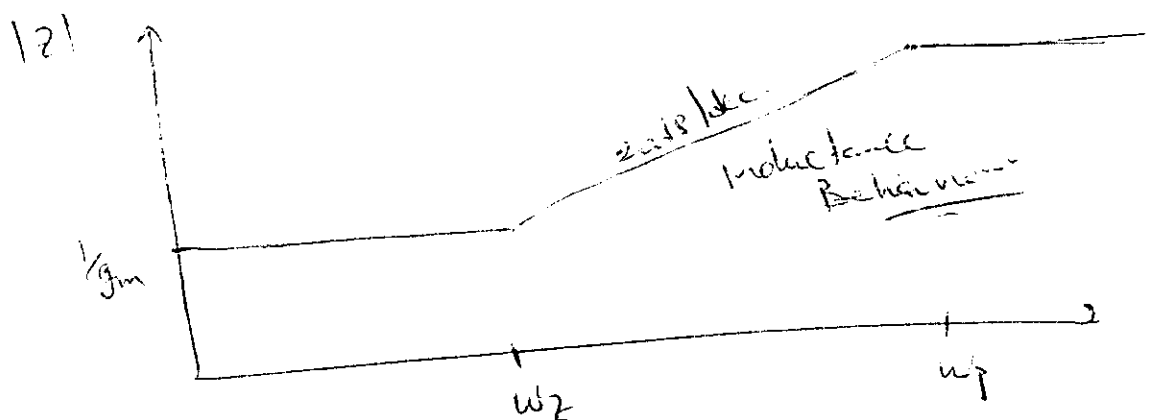
$$\omega_z = -\frac{1}{C_{gs} R_g} \quad \omega_p = -\frac{g_m}{C_{gs}}$$

let plot $|Z_{out}|$

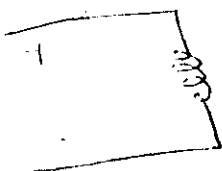
Assume $\omega_p < \omega_z \Rightarrow g_m < 1/R_g$



Case 2. $\omega_z < \omega_p$ $g_m > 1/R_g$



We see in Case 2, the gain increases by frequency, It is the same as inductor

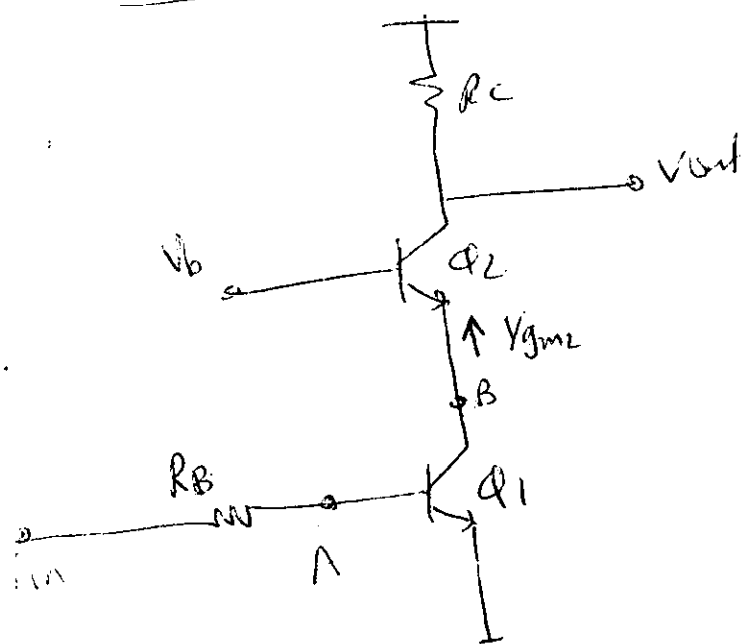


$$V = \underbrace{j\omega L}_Z I \Rightarrow \text{active inductor}$$

Frequency Response of Cascode: (less Miller effect)

(45)

Bipolar Cascode:



To find

$$\frac{V_B}{V_A} \quad ?$$

Q_1 acts as CE

$$A_{vE} = -g_{m1} \left(\frac{1}{g_{m2}} \right)$$

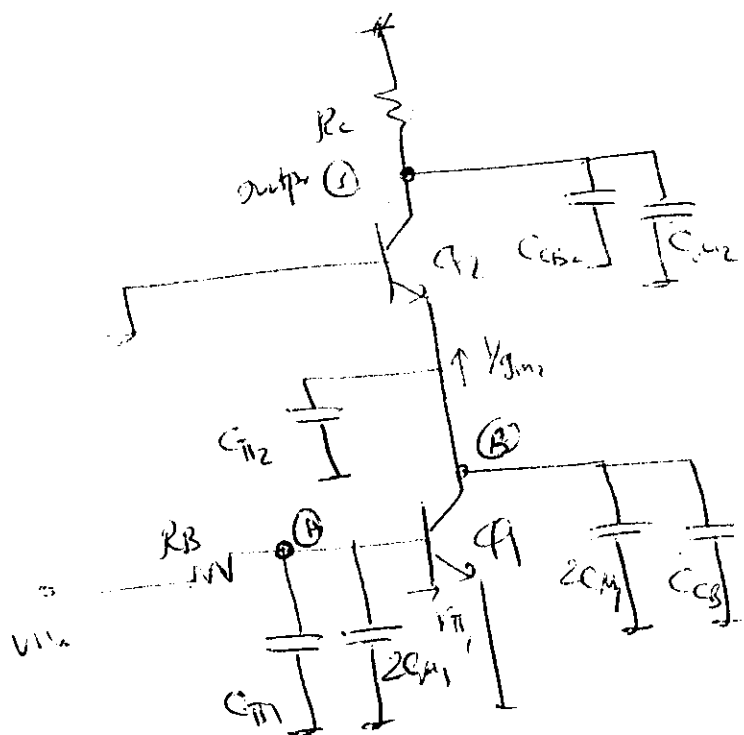
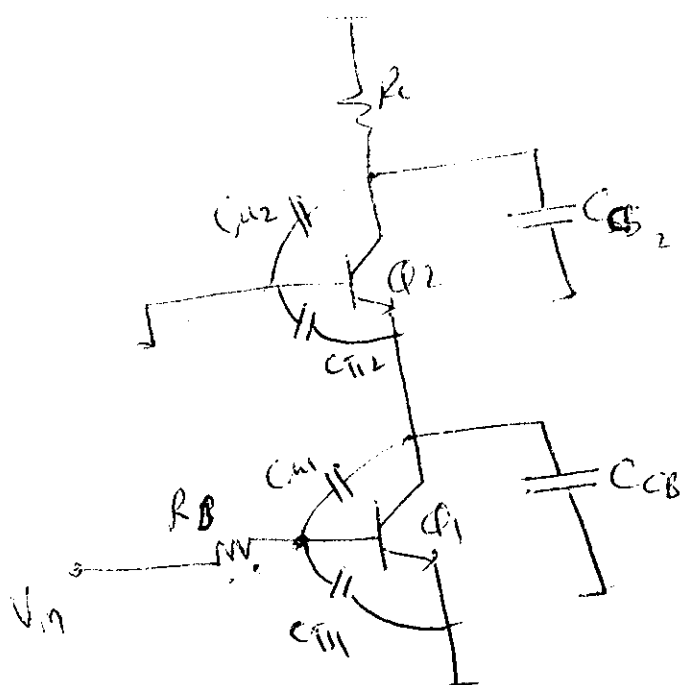
$$= -\frac{g_{m1}}{g_{m2}}$$

ignore (V_A)

$$I_{C1} \approx I_{C2}$$

$$A_{vE} = -\frac{g_{m1}}{g_{m2}} = \frac{-\frac{I_{C1}}{V_T}}{-\frac{I_{C2}}{V_T}} = -1$$

Adding all Cap



$$C_{M1} \Rightarrow \begin{cases} C_{M1}(1 - A_{vE}) \rightarrow \text{input} \rightarrow C_{M1}(1 + 1) = 2C_{M1} \\ C_{M1}(1 - 1/A_{vE}) \rightarrow \text{output} \rightarrow C_{M1}(1 + 1) = 2C_{M1} \end{cases}$$

3 poles

$$\omega_{p1A} = \frac{1}{R_A C_A}$$

$$R_A = r_{\pi 1} \parallel R_B$$

$$C_A = C_{\pi 1} + 2C_{\mu 1}$$

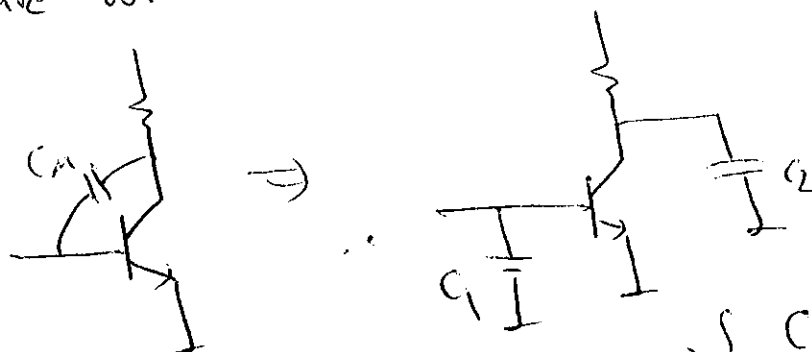
$$\omega_{p1A}^{\text{impul}} = \frac{1}{(r_{\pi 1} \parallel R_B)(C_{\pi 1} + 2C_{\mu 1})}$$

$$\omega_{p1B} = \frac{1}{\frac{1}{g_{m2}} [C_{\pi 2} + 2C_{\mu 1} + C_{CB1}]}$$

$$\omega_{p1out} = \frac{1}{R_C \times [C_{CB2} + C_{\mu 2}]}$$

Typically ω_{p1B} is very high ω_{p1A} , ω_{p1out} : limit the BW

Compare with simple CE



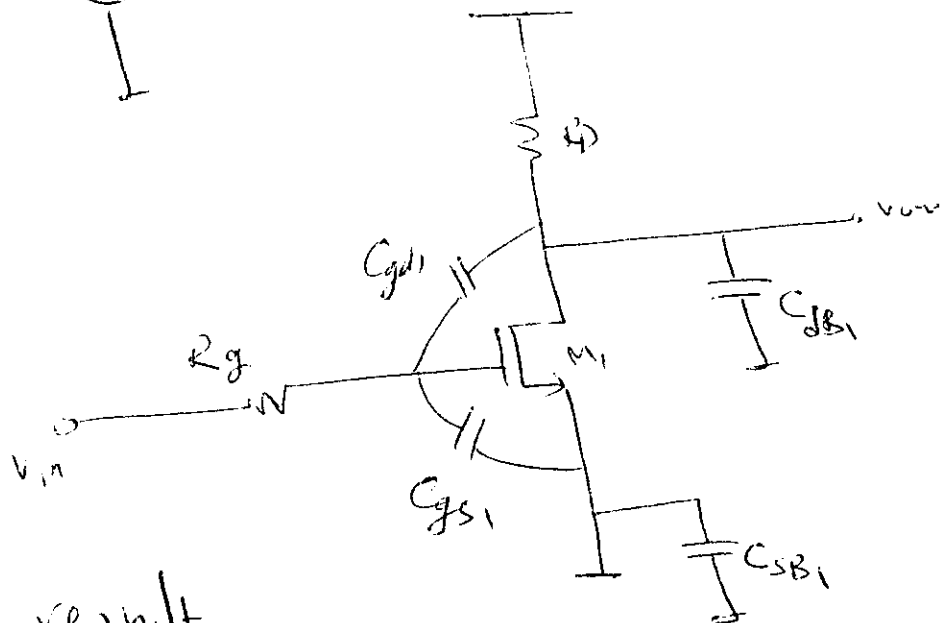
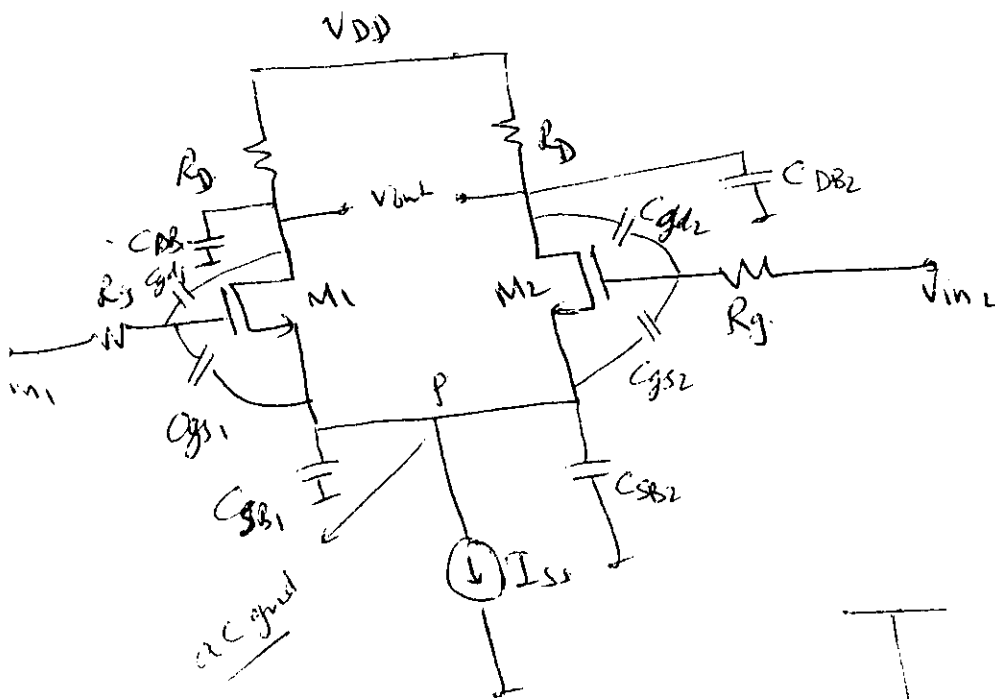
$$C = C_{\mu} (1 + g_m R_C)$$

 $\Rightarrow \left\{ \begin{array}{l} C_1 \text{ appears as} \\ \text{greater in CE than} \\ \text{Cascode} \end{array} \right.$

Today's lect

- Frequency Response of Diff pair
- The transit frequency f_T
- MOS Cascode
- CS stage with current source load.

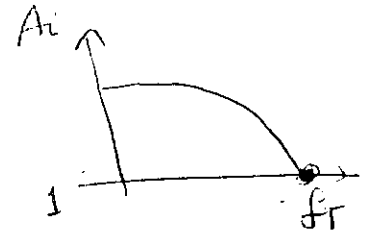
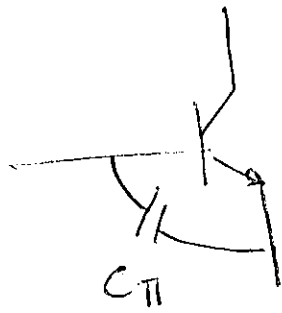
Frequency Response of Diff pair:



The same result
as CS stage.

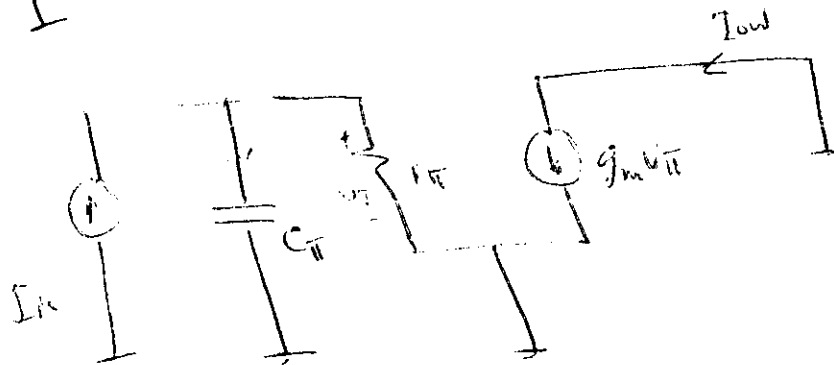
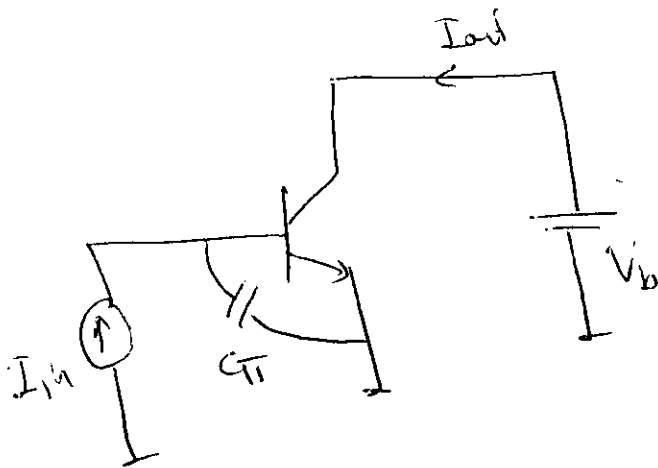
The transit frequency

(44)



f_T is the frequency which the current gain of the device drop to one.

Find $\frac{I_{out}(s)}{I_{in}} \quad s = j\omega$



$$V_{\pi} = r_{\pi} \left[\frac{I_{in}}{1 + sC_{\pi}r_{\pi}} \right]$$

$$\begin{aligned} I_o &= g_m V_{\pi} \\ &= \frac{g_m r_{\pi} I_{in}}{1 + sC_{\pi}r_{\pi}} \end{aligned}$$

$$\frac{I_o}{I_{in}} = \frac{g_m r_{\pi}}{1 + sC_{\pi}r_{\pi}}$$

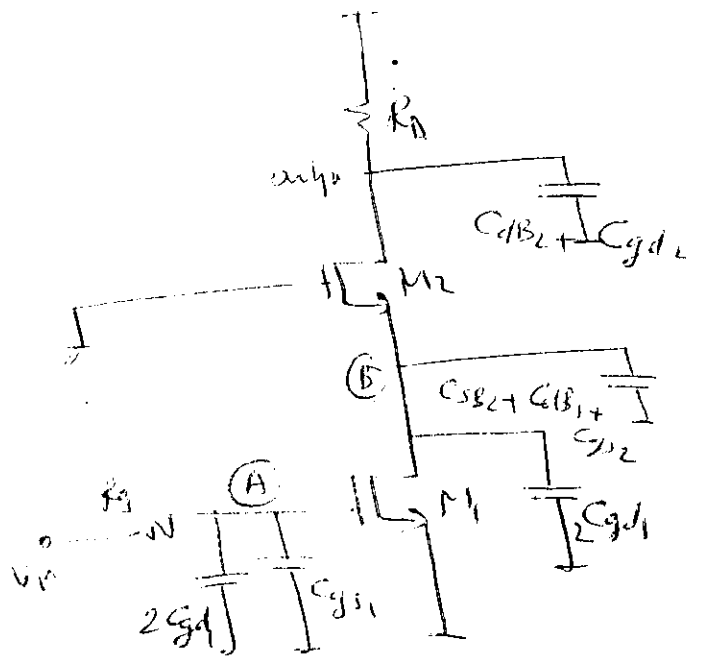
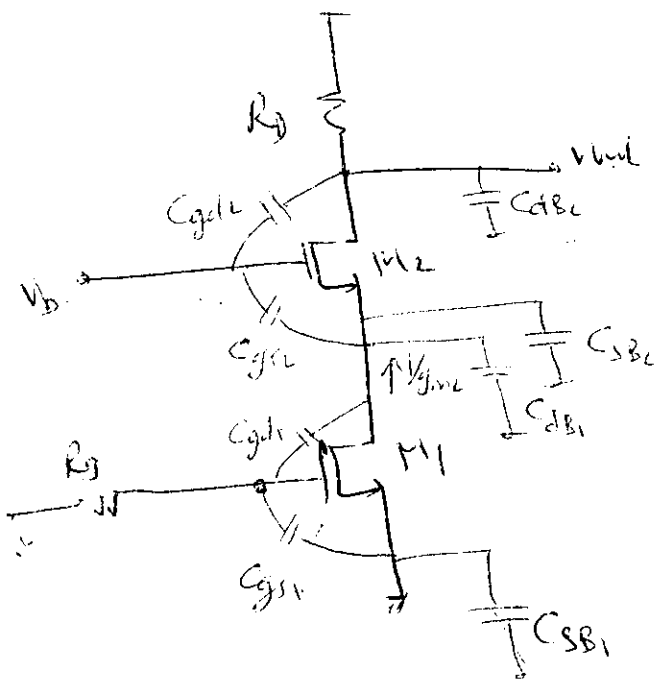
$s = j\omega$

$$\left| \frac{I_o}{I_{in}} \right| = \frac{g_m r_{\pi}}{\sqrt{1 + (C_{\pi}r_{\pi}\omega)^2}} = 1$$

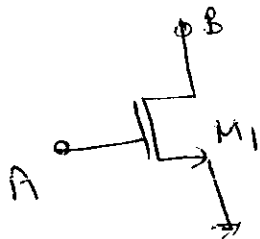
- [illegible]

MOS Cascode

46



$$C_{gd1} \rightarrow \begin{cases} \text{input section} & C_{gd1}(1 - A_{v1}) = 2C_{gd1} \\ \text{output section} & C_{gd1}(1 - 1/A_{v1}) = 2C_{gd1} \end{cases}$$



$$A_V = \frac{V_B}{V_A}$$

M1: acts as common source

$$A_{v1} = \frac{V_B}{V_A} = -g_{m1} \left(\frac{1}{g_{m2}} \right) \approx -1$$

The poles

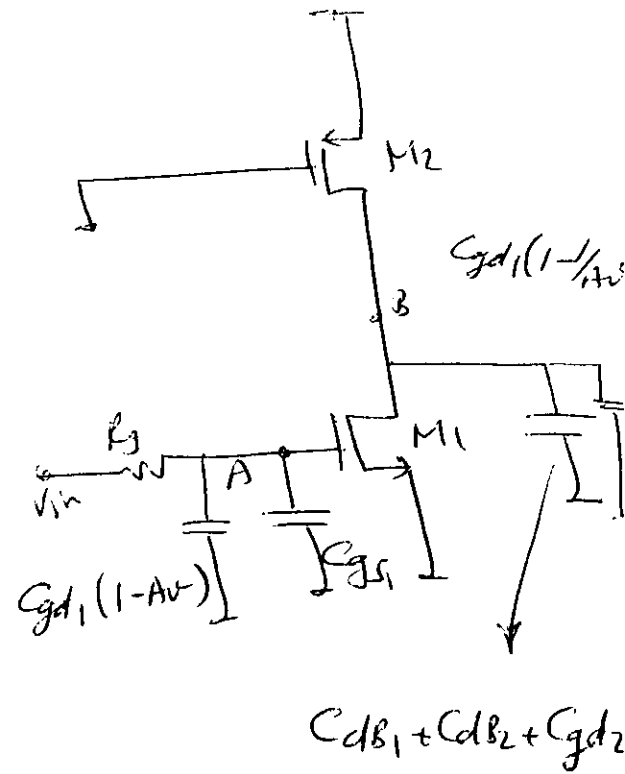
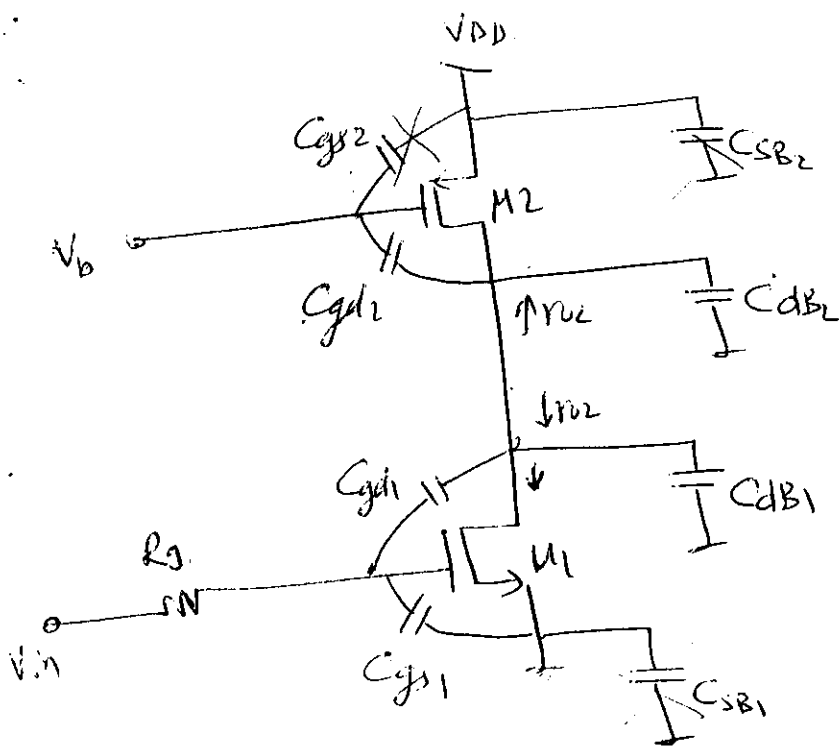
$$\omega_{p1A} = \frac{1}{R_G \times (2C_{gd1} + C_{gs1})}$$

$$\omega_{p1B} = \frac{1}{\frac{1}{g_{m2}} (2C_{gd1} + C_{SB2} + C_{DB1} + C_{gs2})}$$

$$\omega_{p1out} = \frac{1}{R_D (C_{DB2} + C_{gd2})}$$

CS stage with current source load:-

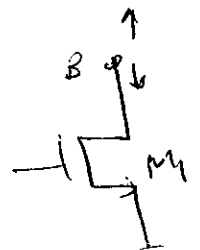
(48)



$C_{gs2}, C_{SB1}, C_{SB2} \rightarrow$ shorted

$$C_{gd1} \rightarrow \begin{cases} C_{gd1}(1 - A_v) \\ C_{gd1}(1 - 1/A_v) \end{cases}$$

$$A_v = \frac{V_B}{V_A} = -g_{m1}(r_{o1} \parallel r_{o2})$$

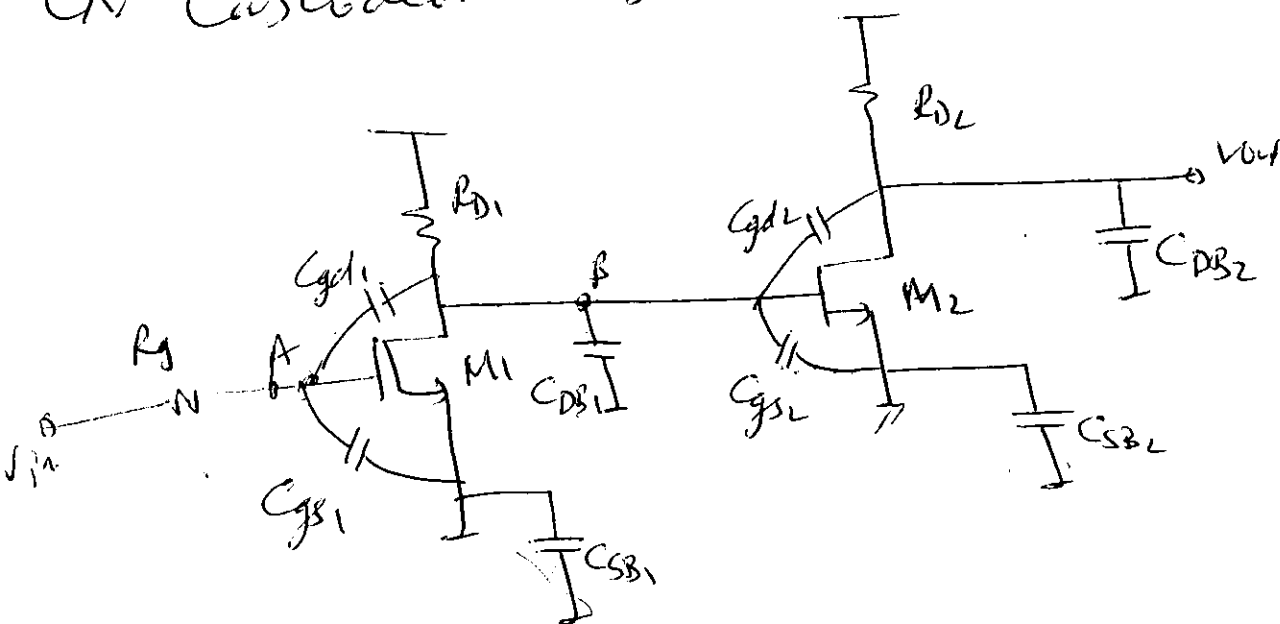


$$C_{gd1} \rightarrow \begin{cases} C_{gd1}(1 + g_{m1}(r_{o1} \parallel r_{o2})) \rightarrow \text{input side} \\ C_{gd1}\left(1 + \frac{1}{g_{m1}(r_{o1} \parallel r_{o2})}\right) \rightarrow \text{output side} \end{cases}$$

$$\omega_{p, \text{in}} = \frac{1}{R_s (C_{gs1} + C_{gd1}(1 + g_{m1}(r_{o1} \parallel r_{o2})))}$$

$$\omega_{p, \text{out}} = \frac{1}{(r_{o1} \parallel r_{o2})(C_{db1} + C_{db2} + C_{gd2} + C_{gd1}(1 + \frac{1}{g_{m1}(r_{o1} \parallel r_{o2})}))}$$

Ex: Cascoded stage



$$\otimes C_{DB1} \parallel C_{gs2} = C_{DB1} + C_{gs2}$$

$C_{SB1}, C_{SB2} \rightarrow \text{shorted}$

$$C_{gd1} \rightarrow \begin{cases} C_{gd1} (1 + g_{m1} R_{D1}) \\ C_{gd1} (1 + \frac{1}{g_{m1} R_D}) \end{cases}$$

$$C_{gd2} \rightarrow \begin{cases} C_{gd2} (1 + g_{m2} R_D) \\ C_{gd2} (1 + \frac{1}{g_{m2} R_D}) \end{cases}$$

$$\omega_{P1A} = \frac{1}{R_g (C_{gs1} + C_{gd1} (1 + g_{m1} R_{D1}))}$$

$$\omega_{P1B} = \frac{1}{R_D \left[(C_{DB1} + C_{gs2} + C_{gd1} (1 + \frac{1}{g_{m1} R_{D1}})) + C_{gd2} (1 + g_{m2} R_D) \right]}$$

$$\omega_{P1out} = \frac{1}{R_{D2} \parallel C_{DB2} + C_{gd2} (1 + \frac{1}{g_{m2} R_{D2}})}$$

Transit frequency

$$f_T = \frac{g_m}{2\pi C_{gs}}$$

what happens if W/L is double
 where 1) I_D is constant
 1) I_D is doubled.

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}{2\pi C_{gs}}$$

when (W/L) is doubled

$$\begin{aligned} (W/L)' &\longrightarrow 2(W/L) \\ C_{gs}' &\longrightarrow 2C_{gs} \end{aligned}$$

$$f_T' = \frac{f_T}{\sqrt{2}}$$

when I_D is double

$$\begin{aligned} g_m &\longrightarrow 2g_m \\ C_{gs}' &\longrightarrow 2C_{gs} \end{aligned}$$

$f_T \longrightarrow$ remain the same.

