

Calculus B

Moamen B Ashour

10

Chapter

10 Chapter
120 Sheets

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شركة الحضارصات الرقمية البرجم



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chapter 10

⇒ Infinite sequences and series

⇒ sequences

$$1 + 18 = 19$$

مجموعة من القيم (الأعداد) يسبقها فواصل
ونكتب ما بعدها من قبل القاعده
العامه a_n

$$\Rightarrow \left\{ \begin{matrix} 1 & 4 & 9 & 16 & 25 & \dots \\ 1^2 & 2^2 & 3^2 & 4^2 & 5^2 & \end{matrix} \right\}$$

$$\rightarrow a_n = n^2 \quad n=1$$

⇒ series

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$$1 + 4 + 9 + 16 + 25 \dots$$

$$\rightarrow \sum_{n=1}^{\infty} n^2$$

☒ write the first 3 terms of the sequence

$$\boxed{1} \quad a_n = \frac{2n}{n^2+1} \quad n=1$$

$$a_1 = \frac{2(1)}{(1)^2+1} = \boxed{1}$$

$$\boxed{2} \quad a_1 = 2 \quad a_{n+1} = \frac{a_n}{1+a_n}$$

$$a_2 = \frac{2(2)}{(2)^2+1} = \boxed{\frac{4}{5}}$$

$$a_2 = \frac{a_1}{1+a_1} = \frac{2}{1+2} = \boxed{\frac{2}{3}}$$

$$a_3 = \frac{2(3)}{(3)^2+1} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

$$a_3 = \frac{a_2}{1+a_2} = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \boxed{\frac{2}{5}}$$

* find a formula for the general term a_n of the sequence

[A] $\left[5, 8, 11, 14, 17, \dots \right] a_n = [3n + 2]_{n=1}$

هذه آلية معينة لحل مثل هذه المسألة
وشرط الأساس الذي يتعلق بجميع كل القيم
ولما يكون في جيب وسط وطاقم خارجي افعلهم جميع كل
مادة لحال وطاقم وطاقم

[B] $\left[0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \dots \right] a_n = \left[\frac{n^2 - 1}{n + 1} \right]_{n=1}$

[C] $\left[4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \dots \right] a_n = \left[\frac{(-1)^{n+1}}{(4)^{n-2}} \right]_{n=1}$

* لما استوفى الاحداد بتغير البنية
مع اختيار ما حرة $(-1)^n$

كما تكون بتغير المعجب للمعجب للمعجب

$(-1)^{n+1} \Rightarrow (-1)^{n-1}$

* Monotonicity: الترتيب

Sequence is always

Increasing Decreasing

$$a_{n+1} > a_n \quad a_{n+1} < a_n \quad \text{Monoton-B. Askhar}$$

$$a_{n+1} - a_n > 0 \quad a_{n+1} - a_n < 0$$

$$\frac{a_{n+1}}{a_n} > 1$$

$$\frac{a_{n+1}}{a_n} < 1$$

$$(a_n)' > 0$$

الزيادة

$$(a_n)' < 0$$

النقصان

$$\square a_n \{1 - 2^n\}_{n=1} \Rightarrow a_1 = 1 - 2^1 = -1$$

$$a_2 = 2 - 2^2 = -2$$

$$a_1 > a_2$$

monotonic
Decreasing

* The Limit of infinite sequences

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \begin{cases} L \text{ اى عدد} \rightarrow \text{Converge} \\ +\infty, -\infty, DNE \rightarrow \text{Diverge} \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \begin{cases} CR \text{ ما في لى} \rightarrow \frac{\text{أكبر معامل لى}}{\text{أكبر معامل لى}} \\ LR \text{ لى} \rightarrow \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \dots \checkmark \end{cases}$$

$$\Rightarrow \infty - \infty = !! \begin{cases} \text{توحيد مقامات} \\ \text{على مشتقة} \rightarrow \text{poly} \\ \text{ما يكون جبر} \rightarrow \text{عزب باعلاقة} \end{cases}$$

* Determine the following sequences Converge و Diverge

$$[1] a_n = \frac{3 + 5n^2}{n + n^2} \rightarrow \lim_{n \rightarrow \infty} \frac{3 + 5n^2}{n + n^2} \xrightarrow{CR} \lim_{n \rightarrow \infty} \frac{5n^2}{n^2} = [5]$$

$$[2] a_n = \frac{3n}{4 + \sqrt{n}} \rightarrow \lim_{n \rightarrow \infty} \frac{3n}{4 + \sqrt{n}} \xrightarrow{CR} \lim_{n \rightarrow \infty} \frac{3n}{\sqrt{n}} = 3\sqrt{n} \rightarrow \infty [D]$$

$$[3] a_n = \frac{4 + 3n^2}{1 - n^3} \rightarrow \lim_{n \rightarrow \infty} \frac{4 + 3n^2}{1 - n^3} \xrightarrow{CR} \lim_{n \rightarrow \infty} \frac{3n^2}{-n^3} = \frac{-3}{\infty} = 0$$

Zero $\leftarrow$

$$9 \quad a_n = \left(1 - \frac{3}{n}\right)^{2n} \rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{\frac{n}{2}}\right)^{2n}$$

$$\rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{-6}{2n}\right)^{2n} \rightarrow e^{-6} \rightarrow \underline{\underline{\text{conv}}}$$

$$10 \quad a_n = \left(\frac{n}{n+1}\right)^n \rightarrow \lim_{n \rightarrow \infty} \left(\frac{n}{n(1 + \frac{1}{n})}\right)^n$$

عائد متسلسلة

$$\Rightarrow \frac{1}{\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n} = \frac{1}{e^1} = \frac{1}{e} = e^{-1} \rightarrow \underline{\underline{\text{conv}}}$$

$$11 \quad a_n = \sqrt{n^2 + n} - n \rightarrow \lim_{n \rightarrow \infty} \sqrt{n^2 + n} \rightarrow \infty \rightarrow \infty$$

مربع متسلسلة

$$\lim_{n \rightarrow \infty} \sqrt{n^2 + n} - n = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} = \lim_{n \rightarrow \infty} \frac{n^2 + n - n^2}{\sqrt{n^2 + n} + n} = \frac{0}{\infty}$$

$$\in \mathbb{R} \rightarrow \frac{n}{\sqrt{n^2 + n}} \rightarrow \lim_{n \rightarrow \infty} \frac{n}{2n} \rightarrow \frac{1}{2} \rightarrow \underline{\underline{\text{conv}}}$$

1/n + n

$$12 \quad a_n = \ln(n+5) - \ln(n+2)$$

$$\lim_{n \rightarrow \infty} \ln\left(\frac{n+5}{n+2}\right) = \frac{\infty}{\infty} \xrightarrow{\text{CR}} \lim_{n \rightarrow \infty} \ln\left(\frac{1}{1}\right) = \ln(1) = 0$$

$$13 \quad e^n - n \rightarrow \lim_{n \rightarrow \infty} e^n - n \quad \infty - \infty !!$$

$$\lim_{n \rightarrow \infty} n\left(\frac{e^n}{n} - 1\right) \rightarrow \infty(\infty) \quad \boxed{\text{div}}$$

Moumen B. Ashour

*Note

$$\lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \\ \infty & \text{otherwise} \end{cases}$$

$r > 1$ & $r < -1$
converge

① $a_n = 3 + \left(\frac{-1}{2}\right)^n \rightarrow \lim_{n \rightarrow \infty} 3 + \left(\frac{-1}{2}\right)^n \rightarrow 3 + 0 = (3)$
 $\checkmark -1 < \frac{-1}{2} < 1$

② $a_n = \frac{e^n}{e^n} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{e}{2}\right)^n \rightarrow \boxed{\infty} \rightarrow \text{div}$
 $\checkmark \frac{e}{2} > 1$

③ $a_n = \frac{3^{n+1}}{4^n} \xrightarrow{n \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{3^n \times 3}{4^n} \rightarrow \lim_{n \rightarrow \infty} 3 + \left|\frac{3}{4}\right|^n \rightarrow 3 + 0 = (3)$
 $\checkmark \left(\frac{3}{4}\right) < 1$ converge

④ $a_n = \frac{e^n - 7^n}{8^n} \xrightarrow{n \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{e^n}{8^n} - \frac{7^n}{8^n} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{e}{8}\right)^n - \left(\frac{7}{8}\right)^n$
 $\Rightarrow 0 - 0 \rightarrow \boxed{0} \rightarrow \text{div}$ $-1 < \frac{e}{8} < 1$ $\frac{7}{8} < 1$

⑤ $a_n = \frac{5^{n+1}}{4^{2n-1}} \xrightarrow{n \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{5^n \times 5}{(4^2)^n} = \lim_{n \rightarrow \infty} 20 \times \frac{5^n}{16^n}$
 $\Rightarrow \left(\frac{5}{16}\right)^n \neq 20 \Rightarrow 0 \neq 20 = (0) \text{ conver}$
 $\checkmark -1 < \frac{5}{16} < 1$

* test for converges

$$\boxed{8} a_n = \frac{\sin(2n)}{1+\sqrt{n}} \rightarrow \lim_{n \rightarrow \infty} \frac{\sin(2n)}{1+\sqrt{n}} \rightarrow \frac{0}{\infty} = \boxed{0} \xrightarrow{\text{conv}}$$

هنا نحلها على الساندوتش ~~اللي~~

Note → $n^a < a^n < n! < n^n$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\text{صغير}}{\text{كبير}} &= \frac{N}{\infty} = 0 \rightarrow \text{conv} \\ \lim_{n \rightarrow \infty} \frac{\text{كبير}}{\text{صغير}} &= \frac{\infty}{N} = \infty \rightarrow \text{div} \end{aligned}$$

$$5! = 5 * 4 * 3 * 2 * 1 = \boxed{120}$$

$$n! = n(n-1)(n-2)(n-3) \dots$$

$$(n+3)! = (n+3)(n+2)(n+1)n!$$

في جيبه وكرتبه
ولمسة

* Test for Converges $n^a < a^n < n! < n^n$

$$[1] a_n = \frac{n!}{3+4n} \rightarrow \lim_{n \rightarrow \infty} \frac{n!}{3+4n} \rightarrow \frac{\text{كبير}}{\text{صغير}} = \frac{\infty}{\infty} = \boxed{\infty} \text{ div}$$

$$[2] a_n = \frac{4-n^2}{5^n} \rightarrow \lim_{n \rightarrow \infty} \frac{4-n^2}{5^n} \rightarrow \frac{\text{صغير}}{\text{كبير}} = \frac{\infty}{\infty} = \boxed{0} \text{ conv}$$

$$[3] a_n = \frac{2n^3+n!}{1+n^n} \rightarrow \lim_{n \rightarrow \infty} \frac{2n^3+n!}{1+n^n} \rightarrow \frac{\text{صغير}}{\text{كبير}} \rightarrow \frac{\infty}{\infty} = \boxed{0} \text{ conv}$$

$$[4] a_n = \frac{(n+1)!}{(n-1)!} \rightarrow \lim_{n \rightarrow \infty} \frac{(n+1)(n)(\cancel{n-1})!}{(\cancel{n-1})!}$$

$$\lim_{n \rightarrow \infty} (n+1)(n) = \boxed{\infty} \text{ div}$$

$$[5] a_n = \left(1 + \frac{4}{n}\right)^n \rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{4}{n}\right)^n \rightarrow \boxed{e^4} \rightarrow \text{conv}$$

$$[N] (1+N)(2+N)(3+N) \dots (N+N)$$

$$6) a_n = \frac{2^n}{n^2 + 4} \rightarrow \lim_{n \rightarrow \infty} \frac{2^n}{n^2 + 4} = \frac{\infty}{\infty} \text{ LR } \rightarrow \frac{2^n \ln(2)}{2n} \rightarrow \infty$$

$$\text{LR } \lim_{n \rightarrow \infty} \frac{2^n (\ln(2))^2}{2} \rightarrow \boxed{\infty} \rightarrow \underline{\text{diverges}}$$

$$7) a_n = \frac{2 \ln(n^2)}{n} \rightarrow \lim_{n \rightarrow \infty} \frac{2 \ln(n^2)}{n} = \frac{\infty}{\infty} !!$$

$$\text{LR } \frac{2 + \frac{1}{n}}{1} = \frac{0}{1} = \boxed{0} \text{ CON}$$

$$-1 < \sin < 1 \rightarrow \boxed{0}$$

$$8) a_n = \frac{3 + \sin^2(n)}{n^3} \rightarrow \lim_{n \rightarrow \infty} \frac{3 + \sin^2(n)}{n^3} = \frac{0}{\infty} = \boxed{0} \text{ CON}$$

~~9) a_n = \frac{4}{1 + 9^n}~~

$$9) a_n = \frac{4}{1 + 9^n} \rightarrow \lim_{n \rightarrow \infty} \frac{4}{1 + 9^n} = \frac{4}{\infty} = \boxed{0} \text{ CON}$$

$$10) a_n = \ln(4k^3 + 1) - \ln(k^3 + 4) \rightarrow \lim_{k \rightarrow \infty} \ln \left(\frac{4k^3 + 1}{k^3 + 4} \right)$$

$$\text{CR } \ln \left(\lim_{k \rightarrow \infty} \frac{4k^3}{k^3} \right) = \ln(4) \rightarrow \underline{\text{CON}}$$

Moumen B. Ashour

مؤمن بسام عاشور
Moamen B Ashour

وَتَسَاءُ أَنْتَ مِنَ الْأَمَانِي نَجْمَةٌ
وَيَسَاءُ رَبُّكَ أَنْ يُنَاكِ الْقَمَرُ



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