

Calculus B

Moamen B Ashour

10 Chapter

10 Chapter

120 Sheets

A5 148 X 210 MM

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* series → متسلسلة

$$a_1 + a_2 + a_3 + a_4 + \dots + a_n = \sum_{i=1}^n a_i = S_n$$

$$\Rightarrow S_1 = a_1$$

$$\Rightarrow S_2 = a_1 + a_2$$

$$\Rightarrow S_3 = a_1 + a_2 + a_3$$

$$S_n = S_{n-1} + a_n$$

$$\Rightarrow \text{Ex. If } S_n = \frac{2n-1}{n+1}, \text{ then } a_n = ??$$

$$S_n = S_{n-1} + a_n$$

n-1 → n بدل

$$S_{n-1} = \frac{2(n-1)-1}{(n-1)+1} = \frac{2n-2-1}{n-1+1} = \frac{2n-3}{n}$$

$$a_n = S_n - S_{n-1} \Rightarrow \frac{2n-1}{n+1} - \frac{2n-3}{n}$$

$$a_n = \frac{(2n-1)n - (2n-3)(n+1)}{(n+1)n} \Rightarrow \frac{2n^2 - n - (2n^2 + 2n - 3n - 3)}{n^2 + n}$$

$$a_n = \frac{2n^2 - n - 2n^2 - 2n + 3n + 3}{n^2 + n}$$

$$\Rightarrow \frac{3}{n+1} = a_n$$

Two questions on series

Test for conv or div

find the sum

- Divergent
- p -Test
- Integral Test
- Comparison Test
- Limit comparison Test
- Ratio Test
- Root Test

- Telescoping series
Partial sum
- Geometric series

Moamen B. Alkhour

* Telescoping Series (partial sum)

$$\sum_{n=1}^{\infty} f(n) - g(n) \quad \text{اقتران - اقتران}$$

مؤمن عاشور

$$\Rightarrow \text{Direct} \rightarrow \text{اقتران - اقتران}$$

$$\Rightarrow \text{خواص } \ln \rightarrow \sum \ln\left(\frac{A}{B}\right) = \sum \ln(A) - \sum \ln(B)$$

$$\Rightarrow \text{كسور جزئية} \rightarrow \sum \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \rightarrow \text{لأول وحدة صفر لثانية}$$

يعني لو فيه جوان

* find the sum (if exist) for

اول واحد واخر واحد

$$\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+2} = a_1 + a_2 + a_3 + a_4 + \dots + a_{n-1} + a_n$$

$$S_n = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S_n = \left(\frac{1}{2} - \frac{1}{n+2}\right)$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} S_n$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{2} - \frac{1}{n+2}\right) = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

Ex. find the sum (if exist) for

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

$$S_n = \left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \dots$$
$$+ \left(\frac{1}{n-2} - \frac{1}{n} \right) + \left(\frac{1}{n-1} - \frac{1}{n+1} \right) + \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

$$S_n = \left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$S_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$= 1 + \frac{1}{2} - \frac{1}{\infty} - \frac{1}{\infty}$$

$$\Rightarrow 1 + \frac{1}{2} - 0 - 0$$

$$\Rightarrow 1 + \frac{1}{2} - 1 - 1$$

$$\Rightarrow 1 + \frac{1}{2} - 1 - 1$$

Moumen B. Ashok

2) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n} \rightarrow$ كسر جزئي

$$\frac{1}{n^2 + n} = \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$\rightarrow 1 = A(n+1) + B(n) \rightarrow \text{بالمقارنة}$$

$$n=0 \rightarrow 1 = A + 0 \Rightarrow A = 1$$

$$n=-1 \rightarrow 1 = 0 + -B \Rightarrow B = -1$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

$$S_n = \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n} \right) + \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$S_n = 1 - \frac{1}{n+1}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = \left(1 - \frac{1}{\infty} \right) = (1 - 0) = 1$$

Momen B. Ashour

ممنون
عبد الطاهر

3] $\sum_{n=1}^{\infty} \ln\left(\frac{n+3}{n+1}\right)? \rightarrow$ نلاحظ النمط

$\sum_{n=1}^{\infty} \ln(n+3) - \ln(n+1)$

$S_n = \cancel{\ln(1)} - \ln(2) + \cancel{\ln(3)} - \ln(3) + \cancel{\ln(4)} - \ln(4) + \dots + \ln(n+1) - \ln(n-1) + \ln(n+2) + \ln(n) + \ln(n+3) - \ln(n+1)$

$S_n = -\ln(2) - \ln(3) + \ln(n+2) + \ln(n+3)$

$\lim_{n \rightarrow \infty} -\ln(2) - \ln(3) + \ln(n+2) + \ln(n+3)$ لا حل

$\infty + \infty = \infty$

الشيخ أبي يعقوب بالاسئلة عنداء ادلتي لازم تذببه انوكم العارقي
بين الحديث اذا كان (1) تاخذ الحد الاول والاخير فقط
اذا كان العارقه جهدار (2) تاخذ اول وثاني حد من ابداءه والنهاية
ويمكن ان يكون الاول ~~من~~ دائما فيه السؤال هذا الحد الثاني
كيسه حدت انية رح تلاحظي بنظر من يكر مع الحل

رح تلاحظي 100%
Moamer B. Ashall

* Geometric series $\sum_{n=0}^{\infty} (r)^n$

$$\sum_{n=0}^{\infty} a(r)^n = \begin{cases} |r| \geq 1 & \rightarrow \text{diverge} \quad \text{no sum} \\ |r| < 1 & \rightarrow \text{converge} \quad \text{yes sum} \end{cases}$$

$-1 < r < 1$

$$S_n = \frac{a_1}{1-r}$$

Ex * find the sum (if ext) for the following series

$$\textcircled{1} \sum_{n=0}^{\infty} \frac{2}{3^n} = \sum_{n=0}^{\infty} 2 * \frac{1}{3^n} = \sum_{n=0}^{\infty} 2 * \left(\frac{1}{3}\right)^n$$

$$S_n = \frac{a_1}{1-r} \Rightarrow \frac{2 * 1}{1 - \frac{1}{3}} = \boxed{3} \# \quad r = \frac{1}{3} \quad \left|\frac{1}{3}\right| < 1$$

$$\textcircled{2} \sum_{n=1}^{\infty} 2^{2n+1} 8^{-n} = \frac{2^{2n} * 2^1}{8^n} = \frac{2(2^2)^n}{8^n} \quad \underline{\text{conv}}$$

$$= \sum_{n=1}^{\infty} 2 * \left(\frac{4}{8}\right)^n = \sum_{n=1}^{\infty} 2 * \left(\frac{1}{2}\right)^n \rightarrow \left|\frac{1}{2}\right| < 1 \quad \underline{\text{conv}}$$

$$S_n = \frac{a_1}{1-r} = \frac{1}{1 - \frac{1}{2}} = \boxed{2} \#$$

$$\boxed{3} \sum_{n=1}^{\infty} \frac{3^n + 5^n}{5^{n+1}} = \sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}} + \frac{5^n}{5^{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}} + \frac{5^n}{5^{n+1}} = \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n \cdot \frac{1}{5} + \sum_{n=1}^{\infty} \left(\frac{5}{5}\right)^n \cdot \frac{1}{5}$$

$$r = \left|\frac{3}{5}\right| < 1$$

$$\left|\frac{5}{5}\right| > 1$$

div

conv + conv = conv

conv + div = div

div + div = div

div - div = !

conv

$$S_n = \frac{a_1}{1-r}$$

$$S_n = \frac{3}{2 \cdot 5} + \frac{5}{1 - \frac{3}{5}}$$

$$+ \text{div} = \boxed{\text{div}}$$

No sum

$$\boxed{4} \sum_{n=1}^{\infty} \frac{3^{n-1} + 5^n}{4^{2n}} = \sum_{n=1}^{\infty} \frac{3^{n-1}}{4^{2n}} + \frac{5^n}{4^{2n}}$$

$$= \sum_{n=1}^{\infty} \frac{3^n}{3 \cdot 4^{2n}} + \sum_{n=1}^{\infty} \frac{5^n}{(4^2)^n}$$

$$\frac{78}{143}$$

$$S_n = \frac{a_1}{1-r} = \frac{\frac{1}{3}}{1 - \frac{5}{16}}$$

$$= \sum_{n=1}^{\infty} \frac{1}{3} \left(\frac{3}{16}\right)^n + \sum_{n=1}^{\infty} \left(\frac{5}{16}\right)^n$$

$$\left|\frac{3}{16}\right| < 1$$

$$\left|\frac{5}{16}\right| < 1$$

conv + conv = conv

Ex write the number $0.\overline{4}$ as a ratio of integers

$$0.\overline{4} = 0.44444444 \dots$$

$$= 0.4 + 0.04 + 0.004 + 0.0004 \dots$$

$$= \frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \frac{4}{10000} \dots$$

$$= \frac{4}{(10)^1} + \frac{4}{(10)^2} + \frac{4}{(10)^3} + \frac{4}{(10)^4} \dots$$

$$\sum_{n=1}^{\infty} \frac{4}{(10)^n} = \sum_{n=1}^{\infty} 4 \times \left(\frac{1}{10}\right)^n$$

مؤسس على 10

في اول سؤال بالمشي

$$r < 1 \rightarrow \text{conv}$$

$$S_n = \frac{a_1}{1-r} = \frac{\frac{4}{10}}{1-\frac{1}{10}} = \boxed{\frac{4}{9}}$$

2) ... $4.\overline{12}$

$$4.\overline{12} = 4 + 0.\overline{12} \rightarrow 4.121212121212 \dots$$

$$= 4 + 0.12 + 0.0012 + 0.000012 + \dots$$

$$= 4 + \frac{12}{100} + \frac{12}{(100)^2} + \frac{12}{(100)^3} + \dots$$

$$\boxed{\frac{136}{33}}$$

$$\sum_{n=0}^{\infty} 4 + \frac{12}{(100)^n} = 4 \sum_{n=0}^{\infty} 12 \times \left(\frac{1}{100}\right)^n \rightarrow S_n = 4 + \frac{\frac{12}{100}}{1-\frac{1}{100}}$$

$$|r| < 1 \rightarrow \text{conv}$$

مؤمن بسام عاشور
Moamen B Ashour

وَتَسَاءُ أَنْتَ مِنَ الْأَمَانِي نَجْمَةٌ
وَيَسَاءُ رَبُّكَ أَنْ يُنَاكِ الْقَمَرُ



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