Calculus B

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10
Chapter

10 Chapter
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part.2

∗ Divergent Test :- (D.T) مختصر $\sum_{n=}^{\infty} a_n$

$\lim_{n\to\infty} a_n$ → $=0$ → Test faild → تفشل

→ $\neq 0$ → diverge

Note :- If $\sum_{n=}^{\infty} a_n$ is convarget seris then $\lim_{n\to\infty} a_n = 0$

∗ Test for coverget or diverge ??

[1] $\sum_{n=1}^{\infty}\frac{1+n}{5+3n} \xrightarrow{D.T} \lim_{n\to\infty}\frac{1+n}{5+3n} = \frac{\infty}{\infty}$!!

$\xrightarrow{C.R} \lim_{n\to\infty}\frac{\cancel{n}}{3\cancel{n}} = \boxed{\frac{1}{3}} \to \neq 0 \Rightarrow$ Div By D.T

[2] $\sum_{n=5}^{\infty}\frac{1}{2+3^{-n}} \xrightarrow{D.T} \lim_{n\to\infty}\frac{1}{2+3^{-n}} = \frac{1}{2+3^{-\infty}} = \frac{1}{2+0} = \boxed{\frac{1}{2}}$

$\frac{1}{2} \neq 0 \to$ Div By. D.T

$\boxed{3}$ $\sum_{n=3}^{\infty}\left(1-\frac{2}{n}\right)^{n} \xrightarrow{D.T} \lim_{n\to\infty}\left(1-\frac{2}{n}\right)^{n} = \lim_{n\to\infty}\left(1+\frac{-2}{n}\right)^{n}$

$= e^{-2} \to \boxed{\frac{1}{e^{2}}} \to \neq 0 \to$ Div By D.T

$\boxed{4}$ $\sum_{n=1}^{\infty}(-1)^{n}\frac{3n}{n+1} \longrightarrow$ div By D.T

even $\to \lim_{n\to\infty}\frac{+3n}{n+1} = \frac{\infty}{\infty} \xrightarrow{CR} \frac{3}{1} = \boxed{3} \neq 0$ div

$\downarrow \neq$ DNE $\uparrow$

odd $\to \lim_{n\to\infty}\frac{-3n}{n+1} = \frac{\infty}{\infty} \xrightarrow{CR} \frac{-3}{1} = \boxed{-3} \neq 0$ div

$\boxed{5}$ $\sum_{n=1}^{\infty} n^{2}\sin\left(\frac{1}{n}\right) \xrightarrow{DT} \lim_{n\to\infty}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n^{2}}} \xrightarrow{LR}$

$\xrightarrow{LR} \frac{\cos\left(\frac{1}{n}\right) * \frac{-1}{n^{2}}}{\frac{-1}{n^{4}}} = \boxed{\infty} \neq 0$ div By D.T

*Ex. Given $\sum_{n=1}^{\infty} a_n = 2$ $\xrightarrow{\text{conv}}$, then $\lim_{n\to\infty} (4a_n)^2 + 5 = ??$

$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n$

يعني النهاية بتساوي صفر

$= 0 + 5 = \boxed{5}$ ///

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* p. Test :- (P.T)

$\sum_{n=1}^{\infty} \frac{1}{n^p}$

$p > 1 \rightarrow$ converget by P.T

$p \leq 1 \rightarrow$ diverget by P.T

Ex. Test for converget or divergt.

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \xrightarrow{D.T} \lim_{n\to\infty} \frac{1}{\sqrt{n}} = \frac{1}{\infty} = \boxed{0} \rightarrow$ Test faild

جرب Test أخر

$\xrightarrow{P.T} \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$

$\frac{1}{2} < 1 \Longrightarrow$ div By P.T

$\boxed{2}\ \sum_{n=1}^{\infty} 3n^{-5} \xrightarrow{P.T} \sum_{n=1}^{\infty} \frac{3}{n^5} \to 5>1 \to$ conv by PT

$\boxed{3}\ \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{2}{3}}} \to \frac{2}{3}<1 \to$ div by PT

$\boxed{4}\ \sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3} \xrightarrow{D.T} \lim_{n=\infty} \frac{5-2\sqrt{n}}{n^3} = \frac{\infty}{\infty}$!! —

$\xrightarrow{CK} \frac{-2\sqrt{n}}{n^3} \to \frac{W}{\infty} = \boxed{0} \to$ فشل Test fail

$\xrightarrow{P.T} \sum_{n=1}^{\infty} \frac{5}{n^3} - \frac{2\sqrt{n}}{n^3} \to \frac{2n^{\frac{1}{2}}}{n^3} = \frac{2}{n^{\frac{5}{2}}} \to \frac{5}{2}>1$

$3>1$

conv by P.T

conv by P.T

$\boxed{5}\ \sum_{5^n}^{\infty} \frac{1}{5^n} - \frac{1}{n^5} = \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n - \sum_{n=1}^{\infty} \frac{1}{n^5} \xrightarrow{P.T}$

جيودل

$5>1$

$r = \left|\frac{1}{5}\right| < 1$

conv by PT

conv

conv #

* Integral Test (I.T) $\sum_{n=1}^{\infty} a_n$

$$\sum_{n=1}^{\infty} a_n = \int_1^{\infty} a_n \, dn = \lim_{b\to\infty} \int_1^{b} a_n \, dn \; \ldots \overset{conv}{\boxed{\text{عدد}}} \; \ldots \overset{div}{\boxed{\infty}}$$

شروط تطبيقها

1) contiuous متصلة
2) Postive موجبة
3) decrsing متناقصة

كيف نثبت انها Decrsing متناقصة

$a_2 > a_1$

مشتقة $(a_n)' < 0$

*Ex. The series $\underbrace{\sum_{n=1}^{\infty} n e^{-n}}$ is conv or div ??

DT $\rightarrow$ $\lim_{n\to\infty} n e^{-n} = \frac{\infty}{\infty}$!! $\rightarrow \frac{N}{\infty} = \boxed{0} \rightarrow$ فشل

IT $\rightarrow$ $n e^{-n}$ is contiuous, dicrsing ~~and~~ | is postive

$$\sum_{n=1}^{\infty} n e^{-n} = \int_1^{\infty} n e^{-n} \, dn$$

$$= \lim_{b\to\infty} \int_1^{\infty} n e^{-n} \, dn \xrightarrow{} \text{by parts}$$

$a_1 = (1) e^{-(1)} = \boxed{\frac{1}{e}}$

$a_2 = (2) e^{-2} = \boxed{\frac{2}{e^2}}$

$a_1 < a_2$

$u = n \quad dv = e^{-n}$

$du \; dn \quad v = \frac{e^{-n}}{-1}$

$$= \frac{-n}{e^n} + \int e^{-n} \, dn = \left[\frac{-n}{e^n} - \frac{1}{e^n} \right]_1^b$$

*Ex The series $\sum_{n=1}^{\infty} ne^{-n}$ is conv or div

D.T → $\lim_{n\to\infty} \frac{n}{e^n} = \frac{\infty}{\infty}$ $\frac{N}{\infty} = \boxed{0} \rightarrow$ فشل

I.T → is contouns $[1,\infty)$

is postive and dicrsing

$a_1 = (1)e^{-(1)} = \frac{1}{e}$

$a_2 < a_1$

$a_2 = (2)e^{-(2)} = \frac{2}{e^2}$

By Part

$$\sum_{n=1}^{\infty} ne^{-n} = \int_1^{\infty} ne^{-n}\,dn = \lim_{b\to\infty} \int_1^{b} ne^{-n}\,dn$$

$u = n$ $dv = e^{-n}$ مؤمنى عالشور

$du = dn$ $v = \frac{e^{-n}}{-1}$

conv By I.T

$$= \frac{-n}{e^n} + \int e^{-n}\,dn = \frac{-n}{e^n} - \frac{1}{e^n}\Big]_1^b$$

$$\lim_{b\to\infty}\left(\frac{-b}{e^b} - \frac{1}{e^b}\right) - \left(\frac{-1}{e} - \frac{1}{e}\right)$$

$\frac{\infty}{\infty} \xrightarrow{CR} \frac{N}{\infty}$

$$\left(\frac{N}{\infty} - \frac{1}{e^{\infty}}\right) + \frac{1}{e} + \frac{1}{e} = 0 - 0 + \frac{2}{e}$$

$\boxed{\frac{2}{e}}$

Ex. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is conv or div

dT $\rightarrow$ $\lim_{n \to \infty} \frac{1}{n^2+1} = \frac{1}{\infty} = \boxed{0} \rightarrow$ فشل

IT $\rightarrow$ is contusses and postive and dicresing

$a_1 = \frac{1}{2}$

$a_2 = \frac{1}{5}$ $a_1 > a_2$

$\Rightarrow \int_1^{\infty} \frac{1}{n^2+1} \rightarrow \lim_{b \to \infty} \int_1^{b} \frac{1}{n^2+1}\, dn$

$\Rightarrow \lim_{n \to \infty} \tan^{-1}(n) \Big]_1^{b}$

$\lim_{n \to \infty} \left(\tan^{-1}(b) - \tan^{-1}(1)\right) \Rightarrow$

$\Rightarrow \tan^{-1}(\infty) - \tan^{-1}(1)$

$\frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$ conv by I.T

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$* \sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^2}$ is conv or div

D.T $\rightarrow \lim_{n\to\infty} \frac{1}{n(\ln(n))^2} = \frac{1}{\infty(\ln(\infty))^2} = \frac{1}{\infty} = \boxed{0}$ فشل

II $\Rightarrow$ is contsue and posbile and icvsiag

$\Rightarrow \int_3^{\infty} \frac{1}{n(\ln(n))^2}\,dn$

$\left\{ \frac{1}{n(\ln(n))^2} < 0 \right.$ تحقق

$\Rightarrow \lim_{b\to\infty} \int_3^{b} \frac{1}{n(\ln(n))^2}\,dn \rightarrow$ فرضي

$\Rightarrow \int \frac{1}{\cancel{n}\, y^2}\,\cancel{n}\,dy$

$y = L(n)$

$dn = \frac{dy}{\frac{1}{n}}$

$\boxed{dn = n\,dy}$

$\Rightarrow \int y^{-2}\,dy \Rightarrow \frac{y^{-1}}{-1} = \frac{-1}{y}$

$\Rightarrow \left.\frac{-1}{Ln(n)}\right]_3^{b} \rightarrow$

$\left(\frac{1}{n(\ln(n))^{p}} \right.$ → $p>1$ conv → $p<1$ div

$\lim_{n\to\infty} \left(\frac{-1}{\ln(b)} + \frac{1}{\ln(3)} \right)$

$\frac{-1}{\infty} + \frac{1}{\ln(3)}$

$0 + \frac{1}{Ln(3)} \longrightarrow \boxed{\frac{1}{Ln(3)}} \rightarrow$ conv by II

* comparison Test (C.T)

مقارنة series

* نقوم بجلب الاختبار الموجودة بالسؤال مع series ثانية (من راسي)
بعدها نقارنهما (هل هي div conv) ونكتب العلاقة بينهم
ونطبق التالي

مش أكبر او اصغر

$$0 \leq a_n \leq b_n$$

If $\sum_{n=}^{\infty} b_n$ is conv $\xrightarrow{then}$ a_n is conv

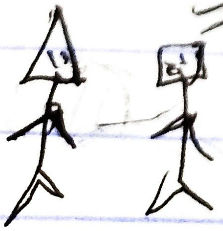
If $\sum_{n=}^{\infty} a_n$ is div $\xrightarrow{then}$ b_n is div

الاصغر تتبع الأكبر وليس العكس ليس كدا ا تقريبا

Maumen B Ashoor

مؤمن بسام عاشور

2007



الدرس :

من راسي

نفس السؤال بطريقة II

Ex Test for conv or div

$\boxed{1}\ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \xrightarrow{DT} \lim_{n\to\infty} \frac{1}{n^2+1} = \frac{1}{\infty} = \boxed{0} \rightarrow$ فشل

$\xrightarrow{CT} \frac{1}{n^2+1} < \frac{1}{n^2} \xrightarrow{PT} P = 2 > 1 \Rightarrow$ conv

⇐ اذا الكبيرة conv يعني الصغيرة conv مسهل الحياة الله

$\boxed{2}\ \sum_{n=3}^{\infty} \frac{1}{n-1} \xrightarrow{DT} \lim_{n\to\infty} \frac{1}{n-1} = \frac{1}{\infty} = \boxed{0} \rightarrow$ فشل

$\xrightarrow{C.T} \frac{1}{n-1} > \frac{1}{n^1} \rightarrow P = 1 \leq 1 \rightarrow$ div P.T

الصغيرة Div يعني الكبيرة Div ← By C,T

$\boxed{3}\ \sum_{n=1}^{\infty} \frac{4}{n^3+5n} \xrightarrow{DT} \lim_{n\to\infty} \frac{4}{n^3+5n} = \frac{4}{\infty} = \boxed{0} \rightarrow$ فشل

$\xrightarrow{CT} \frac{4}{n^3+5n} < \frac{4}{n^3} \xrightarrow{PT} P = 3 > 1 \rightarrow$ conv

اذا الكبير conv اذن الصغيرة conv

$\boxed{4}$ $\sum_{n=1}^{\infty}\frac{1}{n+3^n}$ $\xrightarrow{D.T}$ $\lim_{n\to\infty}\frac{1}{n+3^n}=\frac{1}{\infty}=\boxed{0}\to$ فشل

$\xrightarrow{C.T}$ $\frac{1}{n+3^n} < \frac{1}{n}$ PT $1 \leq 1 \to$ div

لان الكبير Div إذن الاختبار فشل شو نعمل ولاشي غير الفرق

$\xrightarrow{C.T}$ $\frac{1}{n+3^n} < \frac{1}{3^n}$ جيومتري $\left(\frac{1}{3}\right)^n \to |r| < 1 \to$ Conv

* الكبير Conv اذن الاصغر Conv وخلص السؤال يا باشا!!!!

$\boxed{5}$ $\sum_{n=2}^{\infty}\frac{\ln(n)}{n^5}$ $\xrightarrow{D.T}$ $\lim_{n\to\infty}\frac{\ln(n)}{n^5}=\frac{\infty}{\infty}$!!

$\xrightarrow{LR}$ $\frac{\frac{1}{n}}{5n^4}=\frac{1}{5n^4}=\frac{1}{\infty}=\boxed{0}\to$ ☺ اووها!!! فشل

$\xrightarrow{C.T}$ $\frac{\ln(n)}{n^5} < \frac{n}{n^5} \to \frac{\ln(n)}{n^5} < \frac{1}{n^4}$ PT $4 > 1 \to$ Conv

!!الكبير Conv فالصغير Conv | إلي بالاصغر هاي مقارنة لازم يكون ما فيها حد ln(n) مع n
وبعدها المقسم اقرب وهيك

$-1 < \sin < 1$

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{\sin(n)+3}{n^3}$ DT → $\lim_{n\to\infty} \frac{\sin(n)+3}{n^3} = \frac{4}{\infty} = 0$ ← مثل

CT → $\sin(n) < 1$ → $\frac{\sin(n)+3}{n^3} < \frac{4}{n^3}$

$3 > 1$ PT → conv وهي الكبيرة يعني بردو conv

$\boxed{2}$ $\sum_{n=1}^{\infty} \frac{5^{n+1}}{2^{n-1}}$ → Div by تجميع

Geometric → $\sum_{n=1}^{\infty} \frac{5^n * 5^1}{2^n * 2^{-1}} = \sum_{n=1}^{\infty} 10\left(\frac{5}{2}\right)^n$ → $\frac{5}{2} > 1$

⇒ باستخدام C.T → حل اخر

$\frac{5^{n+1}}{2^{n-1}} > \frac{5^n}{2^n}$ → $\left(\frac{5}{2}\right)^n$ → $\frac{5}{2} > 1$ → div

الصغير Div اذن الكبير Div

كفاية يا دنيا !!!!

* Limit comparison Test :- (L.C.T)

خطوات الحل:

1) نفرض series (من راسي) تكون سهلة وبتقرب للسؤال ونسميها bn

2) نحسب قيمة $\lim_{n\to\infty} \frac{a_n}{b_n}$ ونطبق كالتالي:

من السؤال

$\lim_{n\to\infty} \frac{a_n}{b_n} =$

من راسي

- > 0 → an & bn → الاتنين نفس السلوك
- $= 0$ → and bn is conv → an conv
- $= \infty$ → and bn is div → an div
  - $= 0$ → bn is div → Test فشل
  - $= \infty$ → ~~bn~~ an is conv → Test فشل

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كسري $\frac{\text{عمليات جمع وطرح}}{\text{عمليات ضرب جمع}}$ → L.C.T

مؤمن بسام عاشور

* حالات فرض الـ bn

① اذا كانت $a_n = \frac{Poly}{Poly}$ ← $b_n = \frac{\text{أكبر قوة بالبسط}}{\text{أكبر قوة بالمقام}}$

② اذا كانت $a_n = \sin(\;)$ / $\cos(\;)$ ← $b_n = $ (الزاوية) ر (الزاوية)$^{\text{قوة}}$

③ اذا كان مقام الـ a_n اقتران وحيد ← $b_n = \frac{1}{\text{المقام}}$

④ الاقترانات الجذرية بتعاملها زي رقم $\boxed{1}$ $\sqrt[b]{x^a} = x^{\frac{a}{b}}$

* Are the following series conv or div

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{n+4}{n^2+1} \xrightarrow{D.T} \lim_{n\to\infty} \frac{n+4}{n^2+1} = \frac{\infty}{\infty} \xrightarrow{CR} \frac{n}{n^2} = \frac{1}{n} = \frac{1}{\infty} = \boxed{0}$ فشل

$\xrightarrow{L.C.T} b_n = \frac{n}{n^2} = \frac{1}{n} \longrightarrow P = 1 \leq 1 \rightarrow$ Div.

الأكبر

الأكبر

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+4}{n^2+1}}{\frac{1}{n}} \times \frac{n}{1} = \lim_{n\to\infty} \frac{n^2+4n}{n^2+1} = \frac{\infty}{\infty} \xrightarrow{CR}$

$\xrightarrow{CR} \frac{n^2}{n^2} = \boxed{1} > 0$

an and bn is div

دقق [illegible]

أعلى من السا خاوة D.T يفشل

$\boxed{2}\ \sum_{n=1}^{\infty} \frac{n^{2}+n-1}{2n^{5}+5n^{4}-2n} \xrightarrow{D.T} \lim_{n\to\infty}$ → بدون D.T ؛ إذا الحكام

$\xrightarrow{L.C.T}\ a_n = \frac{n^{2}+n-1}{2n^{5}+5n^{4}-2n}\quad b_n = \frac{\cancel{n^{2}}\ 1}{2n^{5}\,3} \rightarrow P = 3 > 1 \rightarrow$ conv

$\lim_{n\to\infty} \frac{a_n}{b_n}\ \frac{n^{2}+n-1}{2n^{5}+5n^{4}-2n} * \frac{2n^{3}}{1}$ $\boxed{\text{Conv}}$ لهم نفس السلوك يعني

$\lim_{n\to\infty} \frac{2n^{5}+2n^{4}-2n^{3}}{2n^{5}+5n^{4}-2n} = \frac{\infty}{\infty}\ !!\ \xrightarrow{C.R} \frac{2\cancel{n^{5}}}{2\cancel{n^{5}}} = \boxed{1} > 0$

$\boxed{3}\ \sum_{n=2}^{\infty} \frac{n^{3}}{\sqrt{n^{7}-2n^{2}+1}} \xrightarrow{D.T} \frac{\cancel{\infty}}{\infty} = \boxed{0} \rightarrow$ فشل

$\xrightarrow{L.C.T}\ b_n = \frac{n^{3}}{\sqrt{n^{7}}} = \frac{n^{3}}{n^{\frac{7}{2}}} = \frac{1}{n^{\frac{1}{2}}} = \frac{1}{\sqrt{n}}\quad P = \frac{1}{2} < 1$ div

$\lim_{n\to\infty} \frac{a_n}{b_n}\ \frac{n^{3}}{\sqrt{n^{7}-2n^{2}+1}} * n^{\frac{1}{2}} = \frac{n^{\frac{7}{2}}}{n^{\frac{7}{2}}} = \boxed{1} > 0$

نفس السلوك يعني حرفي Div

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div

$\boxed{4}\ \sum_{n=1}^{\infty} \sin\left(\frac{\pi}{n}\right) \xrightarrow{L.C.T} b_n = \frac{\pi}{n} \rightarrow P = 1 \leq 1$

نفس السلوك

$\lim_{n\to\infty} \frac{a_n}{b_n}\ \frac{\sin\left(\frac{\pi}{n}\right)}{\frac{\pi}{n}} \rightarrow \lim_{x\to 0} \frac{\sin(x)}{x} = (1) > 0$

$x = \frac{\pi}{2}$ فرض

$\Rightarrow$ an's div by L.C.T

$\boxed{5}\ \sum_{n=1}^{\infty} \sin^2\left(\frac{\pi}{n}\right) \xrightarrow{L.C.T} b_n = \left(\frac{\pi}{n}\right)^2 \xrightarrow{p.t} P = 1 \leq 1 \rightarrow$ div

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^2} = \lim_{x\to 0} \left(\frac{\sin(x)}{x}\right)^2 = (1)^2 = \boxed{1} > 0$

نفس السلوك

يعني div

$x = \frac{\pi}{2}$ فرض

$P = \frac{1}{2} < 1$ div

مثل

$\boxed{6}\ \sum_{n=3}^{\infty} \frac{\ln(n)}{\sqrt{n}} \xrightarrow{C.T} \frac{\ln(n)}{\sqrt{n}} < \frac{n}{\sqrt{n}} = \frac{\ln(1)}{\sqrt{n}} \leq \frac{1}{n^{\frac{1}{2}}}$

$\xrightarrow{L.C.T} b_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}} \rightarrow P = \frac{1}{2} < 1 \rightarrow$ div

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\ln(n)}{\sqrt{n}} * \frac{\sqrt{n}}{1} = \ln(n) \rightarrow \ln(\infty) = \boxed{\infty}$

bn و هي div

$\boxed{7}$ $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2} \xrightarrow[L.C.T]{D.T \to \text{فشل}} b_n = \frac{1}{n^2} \to P = 2 > 1 \to \underline{\text{conv}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{e^{\frac{1}{n}}}{\cancel{n^2}} * \frac{\cancel{n^2}}{1} = e^{\frac{1}{n}} = e^{\frac{1}{\infty}} = e^0 = \boxed{1} > 0$

نفس السلوك يعني Conv

$\boxed{8}$ Ex. suppose $a_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{\frac{1}{n^2}} = 5$ → b_n

a_n = is conv or div ??

$\lim_{n\to\infty} \frac{a_n}{b_n}$
- → > 0 → نفس السلوك
- → = 0 → & b_n is conv → a_n conv
- → = ∞ → & b_n is div → a_n div

$b_n = \frac{1}{n^2} \xrightarrow{P.T} P = 2 > 1 \to b_n \to \underline{\text{conv}}$

وهي نفس السلوك لانه اجابة Lim بتساوي ⑤

يعني $a_n \to$ conv

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* Ratio Test

⇚ يستخدم هذا الاختبار عند وجود مضروب factorial !

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \begin{cases} <1 \rightarrow \text{conv by Ratio test} \\ >1 \rightarrow \text{div by Ratio test} \\ =1 \rightarrow \text{Test fail} \end{cases}$$

فشل

$5! = 5*4*3*3*2*1 = 120$

$5! = 5*4!$

$n! = n(n-1)(n-2)!$

$$\boxed{n^a < a^n < n! < n^n}$$

[1] $\sum_{n=1}^{\infty} \frac{2^n}{n!} \xrightarrow{R.T}$ $a_{n+1} = \frac{2^{n+1}}{(n+1)!}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2^{n+1}}{(n+1)!} * \frac{n!}{2^n}$

التعديل والاختصار حيكون على a_{n+1}

$\lim_{n\to\infty} \frac{\cancel{2^n} * 2^1}{(n+1)\cancel{n!}} * \frac{\cancel{n!}}{\cancel{2^n}} =$

$= \lim_{n\to\infty} \frac{2}{n+1} = \frac{2}{\infty} = \boxed{0 < 1} \rightarrow \text{conv R.T}$

$\boxed{2}\ \sum_{n=1}^{\infty}\frac{n^n}{n!}$ $\frac{\text{كبير}}{\text{صغير}} = \frac{\infty}{N}$ $\boxed{\infty} > 1 \to$ div by R.T

R.T $\rightarrow$ $a_{n+1} = \frac{(n+1)^{(n+1)}}{(n+1)!}$ ~~*~~ ملاحظة

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{(n+1)^{(n+1)}}{(n+1)!} * \frac{n!}{n^n}$ عامل مشترك

$\Rightarrow \frac{(n+1)^n\cancel{(n+1)}}{\cancel{(n+1)}\cancel{n!}} * \frac{\cancel{n!}}{n^n} = \frac{(n+1)^n}{n^n} = \left(\frac{n+1}{n}\right)^n$

$\Rightarrow \left(\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}}\right)^n \Rightarrow \left(1+\frac{1}{n}\right)^n = e^1 = e > 1 \rightarrow$ div

$\boxed{3}\ \sum_{n=1}^{\infty}\frac{n!}{(3n+2)!}$ R.T $\rightarrow$ $a_{n+1} = \frac{(n+1)!}{(3(n+1)+2)!}$

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(3n+5)!} * \frac{(3n+2)!}{n!}$

$\Rightarrow \frac{(n+1)\cancel{n!}}{(3n+5)(3n+4)(3n+3)\cancel{(3n+2)!}} * \frac{\cancel{(3n+2)!}}{\cancel{n!}}$

$\lim_{n\to\infty}\frac{n+1}{(3n+5)(3n+4)(3n+3)} = \frac{n}{27n^3} = \frac{1}{\infty} = \boxed{0} < 1$

$\frac{\infty}{\infty} \xrightarrow{CR} \frac{n}{27\,3n}$ $\frac{1}{27n^2}$

Conv

$\boxed{4}\ \sum_{n=1}^{\infty} \frac{(2n+1)!}{(n+3)!\ 5^{n}}$ $\quad a_{n+1} = \frac{(2(n+1)+1)!}{((n+1)+3)!\ 5^{n+1}}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(2n+3)!}{(n+4)! * 5^{n+1}} * \frac{(2n+1)!}{(n+3)! * 5^{n}}$$

$$= \frac{(2n+3)(2n+2)\cancel{(2n+1)!}}{(n+4)\cancel{(n+3)!} * \cancel{5^{n}} * 5^{1}} * \frac{\cancel{(n+3)!} * \cancel{5^{n}}}{\cancel{(2n+1)!}}$$

$$\lim_{n\to\infty} \frac{(2n+3)(2n+2)}{5n+20} \quad \frac{\infty}{\infty}!! \xrightarrow{cR} \frac{4n^{2}}{5\cancel{n}}$$

$$\lim_{n\to\infty} \frac{4n}{5} = \boxed{\infty} > 1 \rightarrow \text{conv}$$

Moamen-B-Ashour

Eng الإشؤ

* Root Test :-

يستخدم هذا الإختبار عند وجود القوة $\underline{\underline{n}}$ للـ series

$(a_n)^{\frac{1}{n}}$

$\lim\limits_{n\to\infty} \sqrt[n]{a_n} =$
- $< 1 \rightarrow$ conv by Root
- $> 1 \rightarrow$ div by Root
- $= 1 \rightarrow$ test faild فشل

$\boxed{1}$ $\sum\limits_{n=1}^{\infty} \left(\frac{4n-3}{2n+1}\right)^{n} \rightsquigarrow$ Root Test

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \left(\left(\frac{4n-3}{2n+1}\right)^{\cancel{n}}\right)^{\cancel{\frac{1}{n}}} \rightarrow \frac{\infty}{\infty} = \xrightarrow{LR} \frac{4\cancel{n}}{2\cancel{n}} = \boxed{2}$$

$\boxed{2 > 1} \longrightarrow$ div by Root Test

$\boxed{2}$ $\sum\limits_{n=1}^{\infty} \left(1+\frac{2}{n}\right)^{n^2} \rightsquigarrow$ Root Test

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \left(\left(1+\frac{2}{n}\right)^{n^{\cancel{2}}}\right)^{\cancel{\frac{1}{n}}}$$

Moamen B Adel

$$\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{n} = \boxed{e^2 > 1} \longrightarrow \text{div by Root}$$

$\boxed{4}$ $\sum_{n=1}^{\infty}\left(\frac{2n^2+n}{10n^2-3}\right)^n \rightsquigarrow$ Root Test

$\lim_{n\to\infty} a_n^{\frac{1}{n}} = \left(\left(\frac{2n^2+n}{10n^2-3}\right)^{\cancel{n}}\right)^{\cancel{\frac{1}{n}}} = \frac{\infty}{\infty} \xrightarrow{cR} \frac{2\cancel{n^2}}{10\cancel{n^2}} = \boxed{\frac{1}{5}}$

$\boxed{\frac{1}{5} < 1} \rightarrow$ conv by Root Test

$\boxed{5}$ $\sum_{n=1}^{\infty}\left(\frac{n!}{n^4}\right)^n \rightsquigarrow$ Root Test

$\lim_{n\to\infty} a_n^{\frac{1}{n}} = \left(\left(\frac{n!}{n^4}\right)^{\cancel{n}}\right)^{\cancel{\frac{1}{n}}}$

$\lim_{n\to\infty} \frac{n!}{n^4}$ $\frac{\text{كبير}}{\text{صغير}} = \frac{\infty}{N} = \boxed{\infty} > 1 \rightarrow$ div by Root

$\boxed{6}$ $\sum_{n=1}^{\infty} n^{20} e^{-n} \rightsquigarrow$ Root: $\sum_{n=1}^{\infty} \frac{n^{20}}{e^n}$

$\lim_{n\to\infty} a_n^{\frac{1}{n}} = \left(\frac{n^{20}}{e^n}\right)^{\frac{1}{n}} = \frac{n^{\frac{20}{n}}}{e} = \frac{1}{e} < 1 \rightarrow$ conv

Note $\lim_{n\to\infty} n^{\frac{b}{n}} = ①$

وَتَشَاءُ أَنْتَ مِنَ الأَمَانِي نَجْمَةً
وَيَشَاءُ رَبُّكَ أَنْ يُنَالِكَ القَمَرُ

مؤمن بسام عاشور
Moamen B Ashour

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