

Calculus B

Moamen B Ashour

x	y	$\frac{dy}{dx}$	R
$\int_a^b$	$\lim_{x \rightarrow \infty}$	$\sum$	$f'(x)$
$\int$	$f'(x)$	$x \rightarrow \infty$	$x^2 x$
$\lim_{x \rightarrow \infty}$	$\lim_{x \rightarrow \infty}$	$f'(x)$	$\int$
$\sum$	$f'(x)$	$x \div$	x^{-1}
$\sqrt{x}$	$\lim_{x \rightarrow \infty}$	$\sum$	$\int$



10 Chapter

10 Chapter
120 Sheets

A5 148 X 210 MM

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* Alternating series :- (A.S.T)

$$\sum_{n=1}^{\infty} (-1)^n a_n \longleftrightarrow \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

شيل $(-1)^n$

* خطوات الحل

① $\lim_{n \rightarrow \infty} |a_n| \neq 0 \rightarrow \text{div by D.T}$
 $\quad \quad \quad = 0$

② a_n is decreasing $\left\{ \begin{array}{l} a_1 > a_2 \\ a_n > a_{n+1} \\ (a_n)' < 0 \end{array} \right.$

③ $\rightarrow \text{A.S.T} \leftarrow \underline{\text{CONV}}$ 3 خطوات في الامتحان

Note $(-1)^n = \cos(n\pi) = \cos^n(\pi)$

لنفس الشكل ان شاء الله

الفرس: التاريخ: اليوم:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \rightarrow \text{A.S.T}$$

$$\lim_{n \rightarrow \infty} |a_n| \rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0 \xrightarrow{\text{D.T}} \text{فصل}$$

$\hookrightarrow a_n$ is decreasing $a_1 = \boxed{\frac{1}{1}} \textcircled{>} a_2 = \boxed{\frac{1}{2}}$

$\Rightarrow$ conv by A.S.T

$$\textcircled{2} \sum_{n=1}^{\infty} \frac{\cos(n)}{n!} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \xrightarrow{\text{A.S.T}}$$

$$\lim_{n \rightarrow \infty} |a_n| \rightarrow \lim_{n \rightarrow \infty} \frac{1}{n!} = \frac{1}{\infty} = 0 \xrightarrow{\text{D.T}} \text{فصل}$$

$\hookrightarrow a_n$ is decreasing $a_1 = \frac{1}{1!} = 1 \quad a_2 = \frac{1}{2!}$

a_n conv by A.S.T

$1 \textcircled{>} \frac{1}{2}$
 $a_1 > a_2$

Moamen B Ashur

3 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n-1}{2n+1} \xrightarrow{A.S.T}$

div
by
D.T

$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{3n-1}{2n+1} = \frac{\infty}{\infty} \xrightarrow{CR} \frac{3n}{2n} = \boxed{\frac{3}{2}}$

4 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5^n} \xrightarrow{A.S.T}$

$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{1}{5^n} = \frac{1}{\infty} = \boxed{0} \xrightarrow{D.T} \text{div}$

$\hookrightarrow a_n$ is decreasing $a_1 = \frac{1}{5}$ $a_2 = \frac{1}{5^2}$

by AST $\leftarrow$ conv $\leftarrow \frac{1}{5} > \frac{1}{25}$

5 $\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{2^n} \xrightarrow{A.S.T} = \sum_{n=1}^{\infty} \frac{n(-1)^n}{2^n}$

$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{n}{2^n} = \frac{\infty}{\infty} = \frac{n}{\infty} = \boxed{0}$

$\lim$ is decreasing $a_1 = \frac{1}{2}$ $a_2 = \frac{2}{2^2}$

$\frac{1}{2} > \frac{2}{4} = \frac{3}{2^3} = a_3$ $\leftarrow \frac{1}{2} = \frac{2}{4}$

conv by AST

$\boxed{5} \sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1} \right)$ ~~As it is~~ $\rightarrow$ ~~div by 0~~

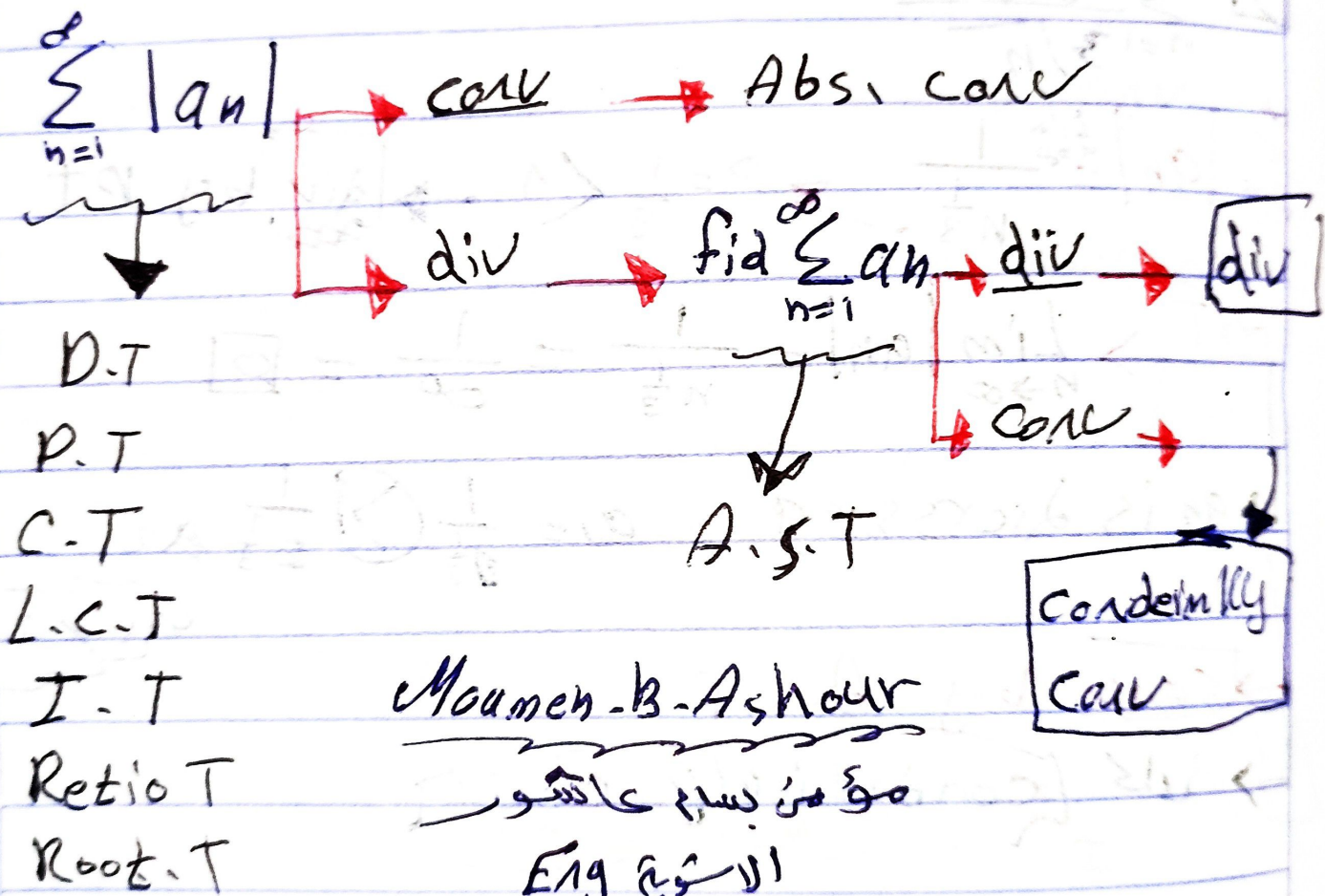
$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{\infty}{\infty} \xrightarrow{\text{CR}} \frac{n}{n} = 1$$

$$\boxed{7} \sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{L(n)} \xrightarrow{\text{AST}} \text{div by } b.f$$

$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{n}{\ln(n)} = \frac{\infty}{\infty} \xrightarrow{\text{LR}} \frac{1}{\frac{1}{n}} = n = \boxed{\infty}$$



* Absolute convergence Conditional convergence Or divergence



Moumen B. Ashour

مؤمن بن بشير عاشور

الاسوية Eng

2007

Ex. Test the following series for
Abs. conv, conditionally conv or div

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$$

$$\sum_{n=1}^{\infty} |a_n| = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{1}{3}}} \rightarrow p = \frac{1}{3} < 1 \rightarrow \boxed{\text{div}} \text{ by p.t}$$

$$\text{A.S.T} \rightarrow \lim_{n \rightarrow \infty} |a_n| = \frac{1}{n^{\frac{1}{3}}} = \frac{1}{\infty} = \boxed{0}$$

$$\rightarrow a_n \text{ is decreasing} \quad a_1 = \frac{1}{\sqrt[3]{1}} > \frac{1}{\sqrt[3]{2}} = a_2$$

$$a_1 > a_2$$

$\rightarrow \boxed{\text{conv}}$ by A.S.T.

$\rightarrow$ It's conditionally conv

مثال P.T
مثال P.T

ای اختصار به لا وینیت

$$\boxed{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^3+1} \xrightarrow{\text{C.T.}}$$

$$a_n = \frac{1}{n^3+1} < \frac{1}{n^3} \rightarrow p=3 > 1 \rightarrow \text{conv}$$

Ab اگر conv یعنی conv و طالت دکری یعنی conv

$$\boxed{3} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{n}{2^n}$$

→ L.C.T
→ Root Test
→ C.T

Root Test

$$\lim_{n \rightarrow \infty} a_n^{\frac{1}{n}} = \lim_{n \rightarrow \infty} \left(\frac{n}{2^n} \right)^{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{n}}}{2} = \frac{1}{2}$$

$$\boxed{\frac{1}{2} < 1} \text{ conv by Root Test}$$

⇒ Ab.s conv #

$$[4] \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1} \xrightarrow{\text{L.O.T.}}$$

div by P.T

$$b_n = \frac{\text{أكبر هو}}{\text{أصغر هو}} = \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}} \rightarrow \boxed{\frac{1}{2} < 1}$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\sqrt{n}}{n+1} \times \frac{\sqrt{n}}{1} = \frac{n}{n+1}$$

$$= \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{\infty}{\infty} \xrightarrow{\text{CR}} \frac{n}{n} = \boxed{1} < 1 \rightarrow \text{نقطة التوقف}$$

$\sum_{n=1}^{\infty} |a_n|$ is div by L.C.T

$$\xrightarrow{\text{A.S.T.}} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}$$

$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+1} = \frac{\infty}{\infty} = \boxed{0}$$

$\rightarrow a_n$ is decreasing

cond. conv

$$a_1 = \frac{1}{2} \quad \boxed{>} \quad a_4 = \frac{\sqrt{4}}{5} \rightarrow \text{easy by A.S.T.}$$

$$\boxed{5} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^n \cdot n^3}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{3^n}{2^n \cdot n^3}$$

على n قوة

Root Test

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \left(\frac{3^n}{2^n \cdot n^3} \right)^{\frac{1}{n}} \rightarrow \text{قوة القوة}$$

$$\lim_{n \rightarrow \infty} \frac{3}{2 + n^{\frac{3}{n}}} \rightarrow \lim_{n \rightarrow \infty} \frac{3}{2 + 1} = \frac{3}{2} > 1 \quad \text{div by Root}$$

AST $\rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^n \cdot n^3}$

$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{3^n}{2^n \cdot n^3} \text{ أكبر } = \boxed{\infty} \neq 0$$

is div by AST

an = div $\nparallel$

مؤقتاً

* Power Series

نقطة على Roots و Rat

$$\sum_{n=0}^{\infty} a_n (x-c)^n = \text{نقطة}$$

$\Rightarrow$ Interval of convergence

I. c

فترة التقارب

$\Rightarrow$ Radius of convergence

مركز الفترة الحرجية

$$r = \frac{b-a}{2}$$

$\Rightarrow$ Roots و Rat Ratio و Root Ratio

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\lim_{n \rightarrow \infty} \left| (a_n)^{\frac{1}{n}} \right|$$

$> 1 \rightarrow \text{div}$

$< 1 \rightarrow \text{conv}$

Yamen-B-Ashar

* find the interval of convergence
and the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{4^n}{n!} (x-1)^n$$

Ratio T $\rightarrow \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{4^{n+1}}{(n+1)!} (x-1)^{n+1} \cdot \frac{n!}{4^n (x-1)^n} \right|$

$$\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{4 \cdot 4^n \cdot (x-1)^{n+1} \cdot n!}{(n+1) \cdot n! \cdot 4^n (x-1)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{4(x-1)}{n+1} \right|$$

حالا انو ذيل النهاية n ليس x فنسب x

ثابتة ممكن يصلح ج x $\Rightarrow$ $\lim_{n \rightarrow \infty} \frac{4}{n+1} = 0$

$$\lim_{n \rightarrow \infty} \frac{4}{n+1} = 0$$

$$|x-1| \cdot 0 < 1$$

$$< 1$$

$$I_c \Rightarrow (-\infty, \infty)$$

$$r = \frac{b-a}{2} = \frac{\infty - (-\infty)}{2} = \frac{2\infty}{2} = \infty$$

$$\boxed{2} \sum_{n=1}^{\infty} \frac{n}{3^{n+1}} \cdot (x)^n \xrightarrow{\text{Ratio}}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{n+1}{3^{n+2}} \cdot (x)^{n+1} \cdot \frac{3^{n+1}}{n(x)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{n+1 \cdot (x)^n \cdot (x)}{3^n \cdot 3^2} \cdot \frac{3^n \cdot 3}{n \cdot (x)^n} \right|$$

$$\left| \frac{x}{3} \right| \lim_{n \rightarrow \infty} \frac{n+1}{n} = \frac{\infty}{\infty} \xrightarrow{\text{CR}} \frac{n}{n} = 1 \cdot \left| \frac{x}{3} \right|$$

$$\left| \frac{x}{3} \right| < 1$$

لازم أقل من واحد

$$|x| < 3$$

$$\boxed{-3 < x < 3}$$

$$\sum_{n=1}^{\infty} \frac{n}{3^{n+1}} \cdot (3)^n = \sum_{n=1}^{\infty} \frac{n}{3} = \frac{\infty}{3} = \infty$$

div by

Note $\Rightarrow |x| < 5$



$$|x| > 5$$



لازم نفس الاطراف
وكونها بالاحكام

استدق يعني

لو Converge حتما متساوية

$$\boxed{3} \sum_{n=1}^{\infty} \frac{(x)^n}{n \cdot 5^n} \xrightarrow{\text{Ratio T}}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x)^{n+1}}{(n+1) \cdot 5^{n+1}} \cdot \frac{n \cdot 5^n}{(x)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{\cancel{(x)^n} \cdot (x)}{(n+1) \cdot \cancel{5^n} \cdot 5} \cdot \frac{n \cdot \cancel{5^n}}{\cancel{(x)^n}} \right|$$

$$\left| \frac{x}{5} \right| \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{\infty}{\infty} \xrightarrow{\text{CR}} \frac{n}{n} = \boxed{1} \cdot \left| \frac{x}{5} \right|$$

← is less than 1

$$\left| \frac{x}{5} \right| < 1$$

$$|x| < 5$$

$$\boxed{-5 < x < 5}$$

$$r = \frac{b-a}{2} = \frac{5 - (-5)}{2} = \frac{10}{2} = \boxed{5}$$

$$x=5 \rightarrow \sum_{n=1}^{\infty} \frac{(5)^n}{n \cdot 5^n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{div by } \rightarrow p=1 \leq 1$$

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{n \cdot 5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (5)^n}{n \cdot 5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \xrightarrow{\text{A.S.T}}$$

$$\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = \boxed{0} \rightarrow \text{is diverging}$$

$(-5)^n$ is $\rightarrow$ Case $a_1 > a_2$

$$\boxed{4} \sum_{n=1}^{\infty} n! (x-3)^n \rightarrow \text{Ratot}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)! (x-3)^{n+1}}{n! (x-3)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1) \cancel{n!} \cdot \cancel{(x-3)^n} \cdot (x-3)}{\cancel{n!} \cdot \cancel{(x-3)^n}} \right|$$

$$\lim_{n \rightarrow \infty} |(n+1)(x-3)| = |x-3| \lim_{n \rightarrow \infty} (n+1) = \boxed{\infty} \cdot |x-3|$$

$$|x-3| \cdot \infty < 1 \rightarrow \text{لا يوجد نقطة تقاطع} \rightarrow \underline{x=3}$$

$$\boxed{x=3} \rightarrow \text{نقطة تقاطع}$$

$$R.C = \boxed{x = \text{Zero}} \neq$$

~~$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n^2 3^n}$$~~

$$\boxed{5} \sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n^2 3^n} \quad \text{Ratio}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (x+2)^{n+1}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 3^n}{(-1)^n (x+2)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (x+2)^{n+1}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 3^n}{(-1)^n (x+2)^n} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (x+2)^{n+1} (n)^2}{3 (n+1)^2} \right|$$

$$= \left| \frac{x+2}{3} \right| \lim_{n \rightarrow \infty} \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2} = \boxed{1} \cdot \left| \frac{x+2}{3} \right|$$

$$\left| \frac{x+2}{3} \right| < 1$$

$$|x+2| < 3$$

$$-3 < x+2 < 3$$

$$\boxed{-5 < x < 1}$$

$$\boxed{r=3}$$

$$x=1 \rightarrow$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad \text{AST} \rightarrow \text{converges}$$

$$x=-5$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-3)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n^2} \quad \text{converges}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2} = \frac{1}{n^2} \quad p=2 > 1$$

* Representation of functions as power series

$$\sum_{n=0}^{\infty} (r)^n = \frac{1}{1-r} \rightarrow |r| < 1$$

$$\Rightarrow \sum_{n=0}^{\infty} (x)^n = \frac{1}{1-x}$$

$$\Rightarrow \frac{1}{1-(x)} = \sum_{n=0}^{\infty} (x)^n$$

نقطة
لازم كذا

مفاتيح

Ex. Express the given functions as power series

$$\text{I} f(x) = \frac{1}{1+x} \Rightarrow \sum_{n=0}^{\infty} (-1)^n (x)^n$$

$$\Rightarrow f(x) = \frac{1}{1-(-x)}$$

$$\Rightarrow \sum_{n=0}^{\infty} (-x)^n$$

$$I.C \Rightarrow |x| < 1$$

$$-1 < x < 1$$

$$(-1, 1)$$

لازم نقطة لكل اشارة

$$\frac{1}{1-(x)}$$

$$[2] f(x) = \frac{1}{1+x^2}$$

$$\Rightarrow f(x) = \frac{1}{1-(-x)^2}$$

$$\Rightarrow \sum_{n=0}^{\infty} (-x)^{2n}$$

مجموعه
از توانها

$$\sum_{n=0}^{\infty} (-1)^n (x)^{2n}$$

$$|x^2| < 1$$

$$x^2 < 1 = |x| < 1$$

$$[3] f(x) = \frac{2x}{1-x^3}$$

$$\Rightarrow f(x) = 2x * \frac{1}{1-x^3}$$

$$\Rightarrow 2x * \sum_{n=0}^{\infty} (x^3)^n$$

$$\Rightarrow \sum_{n=0}^{\infty} 2x * x^{3n}$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(x)^{3n+1}$$

جمع توانها

$$[4] f(x) = \frac{x^3}{2+x}$$

عبارت مبداء (2)

$$\Rightarrow f(x) = x^3 * \frac{1}{2(1+\frac{x}{2})}$$

$$\Rightarrow f(x) = \frac{x^3}{2} * \frac{1}{1-(-\frac{x}{2})}$$

$$\Rightarrow \frac{x^3}{2} \sum_{n=0}^{\infty} \left(-\frac{x}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{x^3}{2} * \frac{(-1)^n}{2^n} * x^n$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} * (x)^{n+3}$$

مجموعه توانها

* Differentiation and integration of power series

التميز
والحد

$$\frac{d}{dx} \left[\sum_{n=0}^{\infty} a_n (x-c)^n \right] = \sum_{n=1}^{\infty} a_n \cdot n (x-c)^{n-1}$$

$$\int \left[\sum_{n=0}^{\infty} a_n (x-c)^n \right] = \sum_{n=0}^{\infty} a_n \frac{(x-c)^{n+1}}{n+1} + C$$

* Ex: Express the following functions as a power series

1 $f(x) = \tan^{-1}(x)$

$$f'(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n (x)^{2n}$$

فعلية لازم نعرف كل كمال اسم استقامة بالحد في حدان زوج

$$\int \frac{1}{1+x^2} dx = \int \sum_{n=0}^{\infty} (-1)^n (x)^{2n} dx$$

د $\tan^{-1}$ لازم كمال

علشان نحسب عوض

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} + C$$

بالاخر $\boxed{0} = \underline{x}$

$$\tan^{-1}(0) = 0 + C$$

$$\boxed{0 = C}$$

مؤمن بسام عاشور
Moamen B Ashour

وَتَسَاءُ أَنْتَ مِنَ الْأَمَانِي نَجْمَةٌ
وَيَسَاءُ رَبُّكَ أَنْ يُنَاكِ الْقَمَرُ



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