

Calculus B

Moamen B Ashour

x	y	$\frac{\partial c}{\partial x}$	$ R $
$\int_1^x$	$\lim_{x \rightarrow \infty}$	$\sum$	$f'(x)$
$\int$	$f(x)$	∞	$x^2 x$
$ i = \infty$	$\lim_{x \rightarrow \infty}$	$f(x)$	$\int$
$\sum$	$f(x)$	$x \div$	x^{-1}
$\sqrt[n]{x}$	$\lim_{x \rightarrow \infty}$	$\sum$	$\int_1^x \frac{1}{t} dt$



11

Chapter

11 Chapter
120 Sheets

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Polar Coordinates

Chapter 11

مؤمن
عاشور
Moamen-B. Ashour

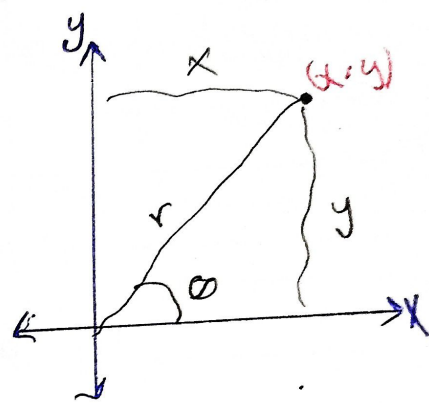
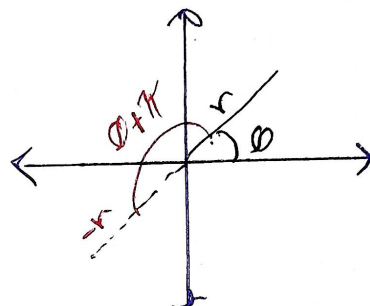
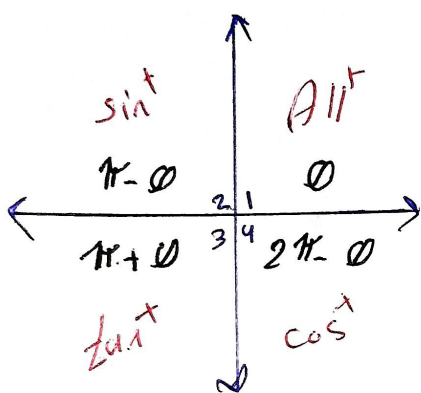
* Coordinates

→ Cartesian → (x, y)

→ Polar → (r, θ)

طول الخط
المستقيم
لنقطة الأصل

الزاوية
x محور



$$(x, y) \leftrightarrow (r, \theta)$$

$$x = r \cos(\theta)$$

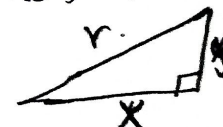
$$y = r \sin(\theta)$$

$$r^2 = x^2 + y^2$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_2 = \theta_1 + \pi$$

مسد كلهم اعدادات اجب من
احسنت جانب الزاوية



عن طريقه فينا عوارى
وبافيه السلة لتجيب اعدادات

Ex. The cartesian coordinates of $\left(3, \frac{\pi}{2}\right)$

مطلوب به (r, θ) حول (x, y) عن طريقه
المتغيرات لتستطيع اعداد

$$x = r \cos(\theta) \rightarrow 3 * \cos\left(\frac{\pi}{2}\right) = 3 * 0 = 0 = x$$

$$y = r \sin(\theta) \rightarrow 3 * \sin\left(\frac{\pi}{2}\right) = 3 * 1 = 3 = y$$

$$\Rightarrow (0, 3)$$

* The Polar coordinates of $(-\sqrt{3}, 1)$ is (r, θ)

$$r^2 = x^2 + y^2$$

$$r^2 = (-\sqrt{3})^2 + (1)^2$$

$$r^2 = 3 + 1$$

$$\sqrt{r^2} = \sqrt{4}$$

$$r = 2$$

$$-r = -2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$$

Diagram showing the angle θ in the second quadrant. The reference angle is $\frac{\pi}{6}$. The angle θ is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

$$\theta = \frac{5\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

على التوازيات عادية كوني

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	1	2	3
cos	3	2	1
	2		

$$\tan = \frac{\sin}{\cos}$$

$$\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

Moumen B. Ashour

نوجد θ_1 يكون θ_2 من θ_1 $\Rightarrow (r_1, \theta_1)$ or (r, θ_2)

$(2, \frac{5\pi}{6})$ or $(-2, \frac{11\pi}{6})$

[2] The Polar coordinates of $(-1, -\sqrt{3})$ is (r, θ)

$$\Rightarrow r^2 = x^2 + y^2$$

$$r^2 = (-1)^2 + (-\sqrt{3})^2$$

$$r^2 = 1 + 3$$

$$r^2 = 4$$

$$r = 2$$

$$-r = -2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right)$$

$$\theta = \tan^{-1}(\sqrt{3})$$

Diagram showing the angle θ in the third quadrant. The reference angle is $\frac{\pi}{3}$. The angle θ is $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

$$\theta = \frac{4\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	1	2	3
cos	3	2	1
	2		

$$\tan = \frac{\sin}{\cos}$$

"

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$\Rightarrow (r_1, \theta_1)$ or $(-r, \theta_2)$

$\Rightarrow (2, \frac{4\pi}{3})$ or $(-2, \frac{\pi}{3})$

Identify and sketch

قبل كل شيء لازم نذكر
بمعاداة الدائرة

$$(x-a)^2 + (y-b)^2 = r^2$$

center (a,b)
r → نصف القطر

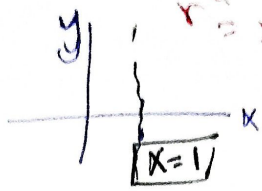
Ex. sketch

$$r = \sec(\theta)$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$r^2 = x^2 + y^2$$



$$r = \sec(\theta)$$

$$\frac{r}{1} = \frac{1}{\cos(\theta)}$$

$$r \cos(\theta) = 1$$

$$x = 1$$

Ex. Identify the equation

$$r = 2 \sin(\theta)$$

أولنا اننا نريد
بـ r

$$r = 2 \sin(\theta) \rightarrow * r$$

$$r^2 = 2r \sin(\theta)$$

$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y = 0$$

$$x^2 + y^2 - 2y + 1 = 1$$

$$x^2 + (y-1)^2 = 1 \rightarrow r = \sqrt{1} = 1$$

$$\text{center} = (0,1)$$

$$\pm \left(\frac{r \sin \theta}{2} \right)^2$$

$$1 = \left(\frac{-2}{2} \right)^2$$

$$r^2 = x^2 + y^2$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

Ex. the equation $r = 2 \cos(\theta) - 4 \sin(\theta)$ represents?

$$r = 2 \cos(\theta) - 4 \sin(\theta) \rightarrow * r$$

$$r^2 = 2r \cos(\theta) - 4r \sin(\theta)$$

$$x^2 + y^2 = 2x - 4y$$

$$x^2 - 2x + y^2 + 4y = 0$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 0 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 5$$

$$r = \sqrt{5}$$

دائرة
المركز (1, -2)

Amr B-Ashkar

Muhammed B. Ashour Eng

Limacons

$r = a \pm b \sin(\theta)$
سويك

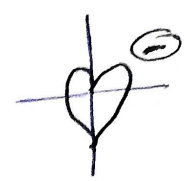
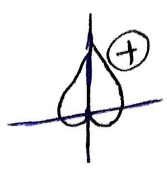
or $r = a \pm b \cos(\theta)$
أفقي

الاتجاه
كل موقع
اليد

1) ci
→
→

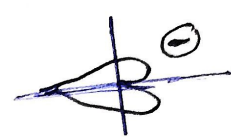
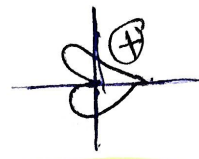
1) $a = b \rightarrow$ Cardioid

$r = a \pm b \sin(\theta)$ →



2) f

$r = a \pm b \cos(\theta)$ →

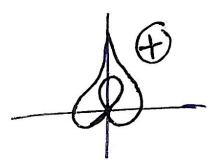


→

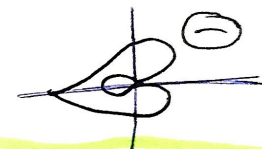
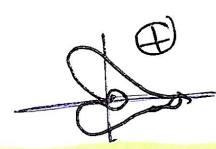
2) $a < b \rightarrow$ Inner Loop Limacons

3)

$r = a \pm b \sin(\theta)$ →



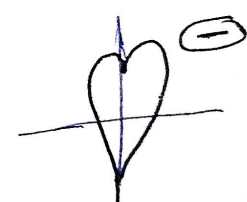
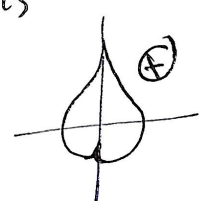
$r = a \pm b \cos(\theta)$ →



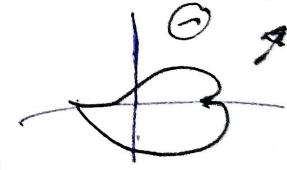
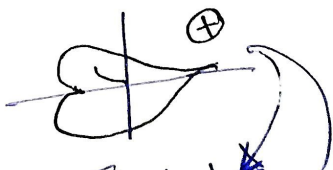
4

3) $a > b \rightarrow$ Dimpled Limacons

$r = a \pm b \sin(\theta)$ →



$r = a \pm b \cos(\theta)$ →



أفقي

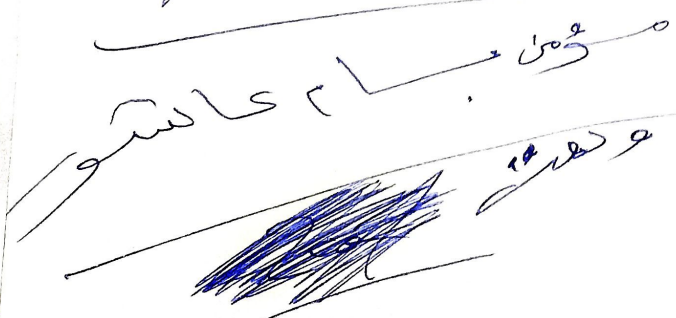
Muhammed B. Ashour

$a+b$ أفقي
 $(a-b)$ أفقي

$r = a \sin(n\theta)$
 $r = a \cos(n\theta)$

الوردة
Rose

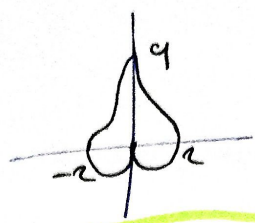
$n \rightarrow \text{odd} \rightarrow = n$
 $n \rightarrow \text{even} \rightarrow = 2n$



Moamen B. Ashour
مؤمن بسم عاشور
220297363

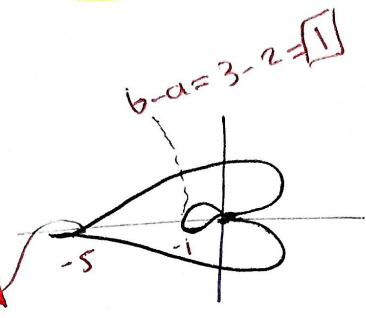
Ex. sketch the following:

1) $r = 2 + 2 \sin(\theta)$
 $a = b$
 Cordoid



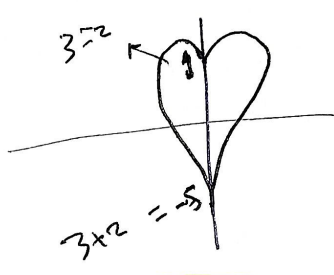
كيف حيت 2/2
كل طريقه تقوى
[theta = 0] او [theta = pi]

2) $r = 2 - 3 \cos(\theta)$
 $a < b$
 inner loop



$a + b = 5 = 3 + 2$

3) $r = 3 - 2 \sin(\theta)$
 $a > b$
 Dimpled



Ex. The curve $r = 2 + 2 \sin(\theta)$ cuts the x-axis in ...??

$\Rightarrow r = 2 + 2 \sin(\theta)$
 $r = 2 + 2 \neq 0$
 $r_1 = 2$
 $r_2 = -2$

Point Polar $\Rightarrow (r_1, \theta_1)$ or (r_2, θ_2)
 $(2, 0)$ or $(-2, \pi)$

هنا كله باربع قوتها
بسته في طريقة عمل
انك تقوى
 $r_1 \leftarrow \theta = 0$
او $r_2 \leftarrow \theta = \pi$
هاد لو كان به x-axis
لو كان y-axis 2 غير اقصوى
 $r \leftarrow \theta = \frac{\pi}{2}$
او $r_2 \leftarrow \theta = \frac{3\pi}{2}$

مؤمن

* Ex. The point of intersection between

Her cores $r = 3 \sin(\theta)$ & $r = 2 - \sin(\theta)$ is where??

Intersection $\rightarrow r = r$ $\Rightarrow$ solve

$$r = r$$

$$3 \sin \theta = 2 - \sin \theta$$

$$4 \sin \theta = \frac{2}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

cos 90

$$\theta = \pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}$$

$$\theta_1 = \frac{5\pi}{6}$$

عوض ای زاویه
یا معادله

$$\theta = 0$$

$$\theta_2 = \frac{\pi}{6}$$

$$r = 3 \sin \theta$$

$$r = 3 \sin \left(\frac{\pi}{6} \right)$$

$$r = 3 \times \frac{1}{2}$$

$$r = \frac{3}{2}$$

$$\text{Point} \rightarrow \left(\frac{3}{2}, \frac{\pi}{6} \right) \text{ and } \left(\frac{3}{2}, \frac{5\pi}{6} \right)$$

Momen-B-Ashour

* Area in polar coordinates :-

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r)^2 d\theta$$

مساحة المنطقة المحيطة بالقطب

عدد الشكالات θ_1 و θ_2

Intersection

$$r_1 = r_2$$



Given

معطى بالسؤال

Ex. find the area that enclosed by the curve $r = 2 \sin(\theta)$

حل آخر باستخدام الشكالات

$$r = 2 \sin(\theta) \rightarrow *r$$

$$r^2 = 2r \sin(\theta) \rightarrow y$$

$$x^2 + y^2 = 2y \rightarrow \text{معادلة دائرة}$$

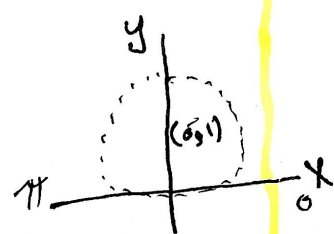
$$x^2 + y^2 - 2y = 0$$

$$x^2 + y^2 - 2y + 1 = 1$$

$$x^2 + (y-1)^2 = 1 \rightarrow \text{دائرة نصف قطرها 1 ومركزها (0,1)}$$

$$\Rightarrow \text{مركز (0,1)}$$

$$r = 1$$



مساحة المنطقة المحيطة بالقطب

$$A = \pi r^2$$

$$= \pi (1)^2$$

$$= \pi$$

هذا حل بديل باستخدام الشكالات

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r^2) d\theta$$

$$= \frac{1}{2} \int_0^{\pi} (2 \sin(\theta))^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi} (2)^2 (\sin^2(\theta)) d\theta$$

$$= \int_0^{\pi} (1 - \cos(2\theta)) d\theta$$

$$\int_0^{\pi} (1 - \cos(2\theta)) d\theta$$

$$= \theta - \frac{\sin(2\theta)}{2} \Big|_0^{\pi}$$

$$= \pi - \frac{\sin(2\pi)}{2} = \pi - 0 = \pi$$

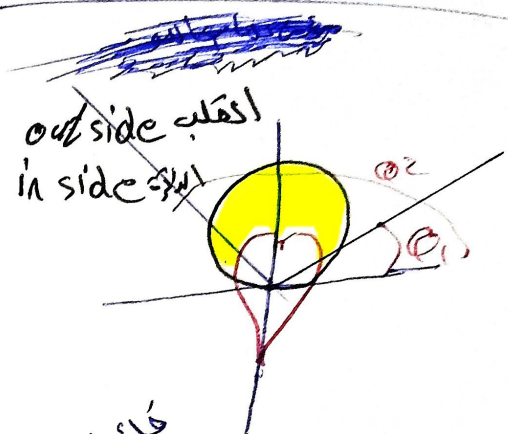
$$= 0 - \frac{\sin(2 \cdot 0)}{2} = 0$$

$$\pi - 0 = \pi$$

* مساحة حلقة خفيفة في حال وجود أكثر من معادلة يمين (أكثر من r)
 يا هندي
 ويكون المطلوب احساحة المنطقة بينهم لتستخدم
 الفاصول الناتجة بالـ θ

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r_{\text{inside}})^2 - (r_{\text{outside}})^2 d\theta$$

Noonan-B-Ashour



* Find the area inside $r=3\cos(\theta)$
 outside $r=1+\cos(\theta)$??

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r_{\text{in}})^2 - (r_{\text{out}})^2 d\theta$$

$$= \frac{1}{2} \int (3\cos(\theta))^2 - (1+\cos(\theta))^2 d\theta$$

$$= \frac{1}{2} \int 9\cos^2(\theta) - (1 + 2\cos(\theta) + \cos^2(\theta)) d\theta$$

$$= \frac{1}{2} \int 9\cos^2(\theta) - 1 - 2\cos(\theta) - \cos^2(\theta) d\theta$$

$$\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 8\cos^2(\theta) - 2\cos(\theta) - 1 d\theta$$

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

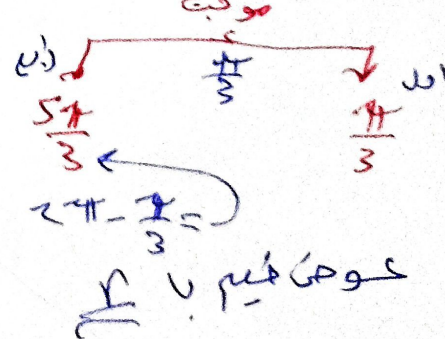
$$\frac{1}{2} [5\pi - \pi] = 2\pi$$

حدد الزوايا كيف
 $r_1 = r_2$

$$3\cos(\theta) = 1 + \cos(\theta)$$

$$\frac{2\cos(\theta)}{2} = \frac{1}{2}$$

$$\cos(\theta) = \frac{1}{2}$$



$$\frac{1}{2} \int 8 \left(\frac{1+\cos(2\theta)}{2} \right) - 2\cos(\theta) - 1 d\theta$$

$$\frac{1}{2} \int 4 + 4\cos(2\theta) - 2\cos(\theta) - 1 d\theta$$

$$\frac{1}{2} \int 3 + 4\cos(2\theta) - 2\cos(\theta) \Rightarrow = \frac{1}{2} [3\theta + 2\sin(2\theta) - 2\sin(\theta)]$$

$$\left(\frac{3}{2} \right) \left(\frac{5\pi}{3} \right) - \left(\frac{3}{2} \right) \left(\frac{\pi}{3} \right)$$

Conic Sections

Chapter 11

القطع المخروطية

معادلاتهم

1) circle

2) Ellipse

3) parabola

4) Hyperbola



1) circle

$$\rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

2) Ellipse

$$\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

القطع الإهليلجي
محاوره الرئيسية
حسب الأعداد

3) Parabola

$$\rightarrow y = ax^2$$

$$\rightarrow x = by^2$$

واحد منهم رئيسي
والآخر
خارج

4) Hyperbola

$$\rightarrow \pm \frac{x^2}{a^2} \mp \frac{y^2}{b^2} = 1$$

القطع
ذو الفروع
خارج

Ex. The following equation represents

$$[1] \frac{x^2}{4} + \frac{y^2}{4} = 1 \rightarrow \text{circle}$$

لبن دائرة

$$(0,0) \quad r = \sqrt{4} = [2]$$

عشاق صوب
وترسيم والأرقام
تقسى

$$[2] \frac{x^2}{36} + \frac{y^2}{16} = 1 \rightarrow \text{ellipse}$$



with major axis is [x]

كشاك
a > b

$$[3] \frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$\frac{y^2}{9} + \frac{x^2}{4} = 1 \rightarrow \text{ellipse}$$



with major axis is [y]

a < b

$$[4] \frac{x^2}{9} - \frac{y^2}{16} = 1 \rightarrow \text{Hyperbola}$$

right
and
left

أوجه بيتر
مفاتيح يكون
ديك الاتجاه
x^2

$$[5] \frac{y^2}{4} = \frac{x^2}{25} + 1$$

$$- \frac{x^2}{25} + \frac{y^2}{4} = 1$$

Hyperbola

أوجه
هو

up and down



فوقه دنت

Moamen-B-Ashgar

مؤمن عاشور

مؤمن بسام عاشور
Moamen B Ashour

وَتَسَاءُ أَنْتَ مِنَ الْأَمَانِي نَجْمَةٌ
وَيَسَاءُ رَبُّكَ أَنْ يُنَاكِ الْقَمَرُ



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