مراجعة

مؤمن عاشور

Moamen Ashour

1 $\int (2x+5)\, e^{2x+5}\, dx = ??$

By parts

$u = 2x+5 \quad dv = e^{2x+5}$

$du = 2\, dx \quad v = \dfrac{e^{2x+5}}{2}$

$= \dfrac{e^{2x+5}}{2}$

$= (2x+5)\dfrac{e^{2x+5}}{2} - \int \cancel{2} * \dfrac{e^{2x+5}}{\cancel{2}}\, dx$

عامل مشترك

$= (2x+5)\dfrac{e^{2x+5}}{2} - \dfrac{e^{2x+5}}{2} + c$

عامل مشترك بعمل بيك افتعال لـ ②

$= \dfrac{e^{2x+5}}{2}\left(2x+\overset{+4}{5-1}\right) + c$

$= \dfrac{e^{2x+5}}{\cancel{2}}\left(\cancel{2}(x+2)\right) + c$

$e^{2x+5}(x+2) + c$ ← ابسط صورة

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[illegible]

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2 $\int \tan^3(x) \cdot \sec^3(x)\, dx = ??$

$y = \sec(x)$

$dx = \dfrac{dy}{\sec(x)\tan(x)}$

$= \int \tan^{\cancel{3}\,2}(x) \cdot y^{\cancel{3}\,2} \cdot \dfrac{dy}{\sec(x)\tan(x)}$

$= \int \tan^2(x) \cdot y^2\, dy$ متطابقة

$= \int (\sec^2(x) - 1)\, y^2\, dy$

$= \int (y^2 - 1)\, y^2\, dy$

$= \int y^4 - y^2\, dy$

$= \dfrac{y^5}{5} - \dfrac{y^3}{3} + C$

$= \dfrac{\sec^5(x)}{5} - \dfrac{\sec^3(x)}{3} + C$

---

3 $\int_0^\infty e^{-Px}\, dx$ is converge قيمة P = اللي بتخليه

$\lim_{a \to \infty} \int_0^a e^{-Px}\, dx$

$\lim_{a \to \infty} \left. \dfrac{e^{-Px}}{-P} \right]_0^a$

$\lim_{a \to \infty} \dfrac{e^{-Pa}}{-P} \overset{+}{--} + \dfrac{e^0}{P}$ → الاجابة conv لازم

$= \dfrac{e^{-P\infty}}{-P} + \dfrac{1}{P} =$

$e^{\infty} = \infty$

$\boxed{e^{-\infty} = 0} \rightarrow$ conv

خليها كدا

لازم ∞ تكون سالبة

بالتالي لازم قيمة $0 < P$

لو بده Div كان $0 > P$

$\boxed{P > 0}$ #

مسؤوليه
محدود

مراجعة

Modme
Ashour

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$\boxed{3}$ The suitable trigometric sub dhat slove the integral → بدو العوض بس

$$\int \sqrt{5+4x-x^2}\,dx \longrightarrow$$

إكمال
مربع
لازم معامل
x^2 موجب
نكود السالب
ناخد عشترك

$$5+4x-x^2$$

$$-(x^2-4x-5) \qquad \left(\frac{x \text{ معامل}}{2}\right)^2$$

$$-(x^2-4x-5+4-4) \qquad 4=\left(\frac{4}{2}\right)^2$$

$$-(x^2-4x+4\quad -9)$$

$$-((x-2)^2-9)$$

$$-(x-2)^2+9$$

$$9-(x-2)^2$$

x^2 - عدد

a^2-x^2 → sin → $x=a\sin(x)$

a^2+x^2 → tan → $x=\tan(x)$

x^2-a^2 → sec(x) → $x=\sec(x)$

$$x-2=3\sin(\theta)$$

$$\boxed{x=3\sin(\theta)+2}\ \#\#$$

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$\boxed{4}$ The points of intersection between the curves $r=-3\sin(\theta)$ & $r=2+\sin(\theta)$ are??

$r=r$

$r=r$

$-3\sin(\theta)=2+\sin(\theta)$

$\frac{-2}{2}=\frac{4}{2}\sin(\theta)$

$-\frac{1}{2}=\sin(\theta)$

(ربع) ثالث — $\frac{\pi}{6}$ سادس — (ربع) رابع

ربع ثالث	ربع رابع
$\pi+\theta$	$2\pi-\theta$
$\frac{6\pi}{6}+\frac{\pi}{6}$	$\frac{6\times 2\pi}{6}-\frac{\pi}{6}$
$\boxed{\frac{7\pi}{6}}$	$\boxed{=\frac{11\pi}{6}}$
	$\boxed{\frac{-\pi}{6}}$

$r=-3\sin(\theta)$

$r=-3\left(-\frac{1}{2}\right)$

$\boxed{=\frac{3}{2}}$

$\left(\frac{3}{2},\frac{7\pi}{6}\right)$

$\left(\frac{3}{2},\frac{-\pi}{6}\right)$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	1	2	3
cos	3	2	1

$\frac{}{2}$

$\boxed{\frac{\pi}{6}}$

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5 One of the following represents the point $(1, +\sqrt{3})$ in terms of the polar coordinates

هادي بادي

A $(-2, \frac{-\pi}{3})$ B $(-2, \frac{\pi}{3})$ C $(2, \frac{\pi}{3})$ D $(-2, \frac{-\pi}{3})$ E $(2, \frac{4\pi}{3})$

$$\boxed{\begin{aligned}\theta &= \tan^{-1}\left(\frac{y}{x}\right)\\ r^2 &= x^2+y^2\end{aligned}}$$

$$r^2 = x^2+y^2$$
$$= (1)^2 + (-\sqrt{3})^2$$
$$= 1+3$$
$$\sqrt{r^2} = \sqrt{4}$$
$$\boxed{r_1 = 2}$$
$$\boxed{r_2 = -2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$$

$\theta = \frac{\pi}{3}$ سالب

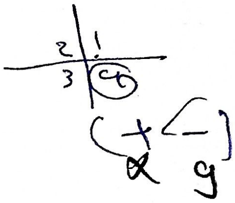
رابع

$$2\pi - \theta$$
$$3\times\frac{2\pi}{3} - \frac{\pi}{3}$$
$$\boxed{= \frac{5\pi}{3}}$$
$$\downarrow$$
$$\boxed{\frac{-\pi}{3}} \#$$

ثاني

$$\pi - \theta$$
$$3\times\frac{\pi}{3} - \frac{\pi}{3}$$
$$\boxed{\frac{2\pi}{3}} \#$$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\sqrt{1}$	$\sqrt{2}$	$\sqrt{3}$
cos	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{1}$
	2		



$(2, -\frac{\pi}{3})$

or

$(-2, \frac{2\pi}{3})$

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لازم تعرف النقطة الاصلية بأي ربع

في الحالة هاي لنقطة $(1, -\sqrt{3})$ ← الرابع هو الاصلي

سالب موجب

بالثاني او الرابع اراح اي (2 ياخد اموجب لـ r

Seq $\begin{cases} \to L \text{ (عدد)} \to \underline{\text{conv}} \\ \pm\infty \to \underline{\text{div}} \end{cases}$

$\lim_{n\to\infty}$ DNE

مراجعة

Series $\boxed{6}$

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Ex. ff $S_n = \frac{2n-1}{n+1}$ find a_n

$S_n = S_{n-1} + a_n$

$\boxed{a_n = S_n - S_{n-1}}$

$S_{n-1} = \frac{2(n-1)-1}{(n-1)+1}$

$S_{n-1} = \frac{2n-3}{n}$

$\to S_n - S_{n-1} = a_n$

$= \frac{2n-1}{n+1} - \frac{3n-3}{n}$

$= \frac{2n^2-n-2n^2-2n+3n+3}{n(n+1)} = \frac{3}{n(n+1)} = a_n$

$\boxed{7}$ find the sum of the series Geometric (اسس)

$$\sum_{n=1}^{\infty} \frac{3^{n+1}-2^{n-2}}{4^n}$$

$$= \sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n} - \frac{2^{n-2}}{4^n}$$

وزع البسط على المقام

$$= \sum_{n=1}^{\infty} \frac{3^n * 3^1}{4^n} - \frac{2^n * 2^{-2}}{4^n}$$

فك الاسس

$\to |r| \Rightarrow \frac{2}{4} = \frac{1}{2} < 1 \to \underline{\text{conv}}$

$$= \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n * 3 - \left(\frac{2}{4}\right)^n * \frac{1}{4}$$

$\to |r| = \frac{3}{4} < 1 \to \underline{\text{conv}}$

$S_n = \frac{a_1 = 3 * \left(\frac{3}{4}\right)}{1 - \frac{3}{4}} - S_n = \frac{a_1 = \frac{1}{4} * \left(\frac{1}{2}\right)}{1 - \frac{1}{2}}$

$= 9 - \frac{1}{4} = \boxed{\frac{35}{4}}$ #

or

$$\sum_{n=1} \frac{3^{n+1}-2^{n-2}}{4^n}$$

وزع السيجما
[illegible]

[redacted]

مراجعة

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8 The sum of the series → Taylor

$$e^x = \sum_{n=0}^{\infty} \frac{(x)^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{7^n\, n!} \text{ is ??}$$

$$= \sum_{n=1}^{\infty} \frac{\left(\frac{2}{7}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{7}{2}\right)^n}{n!} - a_0$$

$\left(\frac{7}{2}\right)^n \to e^{\frac{7}{2}}$

a_0 → تعويض الصفر

$$0! = 1$$

[redacted]

$$\boxed{e^{\frac{2}{7}} - 1}$$

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9 Assume $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sums given by $S_n = 4 + \frac{2}{n-1}$ what is a_6??

$$S_n = S_{n-1} + a_n$$

$$S_6 = S_5 + a_6$$

$$a_6 = S_6 - S_5$$

$$a_6 = \left(\cancel{4} + \frac{2}{6-1}\right) - \left(\cancel{4} + \frac{2}{5-1}\right)$$

$$a_6 = \frac{2}{5} - \frac{2}{4}$$

$$\boxed{a_6 = \frac{-1}{10}} \ \#$$

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$L\left(2\left(-\frac{\pi}{3}\right)\right)$

or

$\boxed{7}$ with partial sums given

$S_n = 4 + \frac{2}{N-1}$ what is

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مراجعة

$\boxed{23}$

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$\boxed{No}$ To detrmine whether the series $\boxed{}$ is conv or div using the Limit comparen Test what should we use

السؤال بحكي شو الغرض لتجد ~~[illegible]~~ بستخدم بس انا بدي اكمل حل عشان كيفه

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+2}}{n^3} \longrightarrow b_n = \frac{\text{اكبر قوة}}{\text{اكبر قوة}} = \frac{\sqrt{n^4}}{n^3} = \frac{n^2}{n^3} =$

$b_n = \frac{1}{n^1} \rightarrow p = 1 \leq 1 \Rightarrow \underline{div}$

$L.CT \rightarrow \lim_{n\to\infty} \frac{a_n}{b_n} \Rightarrow \frac{\sqrt{n^4+2}}{n^{3}\,2} * \cancel{n}$

$\lim_{n\to\infty} \sqrt{\frac{n^4+2}{n^4}} \Rightarrow \lim_{n\to\infty} \sqrt{\frac{n^4}{n^4}+\frac{2}{n^4}} = \lim_{n\to\infty} \sqrt{1+\frac{2}{n^4}} \Rightarrow \sqrt{1+\frac{2}{\infty}} = \sqrt{1+0}$

$\boxed{div}$ لها نفس السلوك يعني ← $0 < \boxed{=1}$

$\boxed{2}$ $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^3}\right)$

$b_n = \frac{1}{n^3}$

$\boxed{3}$ $\frac{\ln(n)}{\sqrt{n}} \Rightarrow \boxed{b_n = \frac{1}{\sqrt{n}}} = \frac{1}{n^{\frac{1}{2}}} \rightarrow p = \frac{1}{2} < 1 \quad \underline{div}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\ln(n)}{\cancel{\sqrt{n}}} * \cancel{\sqrt{n}} = \ln(\infty) = \infty > 0 \Rightarrow$ نفس السلوك يعني $\underline{div}$

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موقف
بام
خشو

مراجعة

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$\boxed{11}$ Using the root test, the series

$$\sum_{n=1}^{\infty} (-1)^n \left(n \sin\left(\tfrac{1}{n}\right)\right)^n$$

$$\text{Root-Test} \rightarrow \lim_{n\to\infty} \left| a_n^{\frac{1}{n}} \right| = \lim_{n\to\infty} \left(\left(n \sin\left(\tfrac{1}{n}\right)\right)^{\cancel{n}}\right)^{\cancel{\frac{1}{n}}}$$

$$= \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = \boxed{1}$$

- $> 1 \rightarrow$ div
- $< 1 \rightarrow$ conv
- $= 1 \rightarrow$ Test fail ##

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

لجيب sin زاوية
على نفس الزاوية
بيساوي واحد

علشان عدد قوة n

$\boxed{12}$ The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)^2}$ is [redacted] conv abs or div
or conv condita or Ge~~[illegible]~~
or Power
$\cancel{\Sigma}(x)^n$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{(n+1)^2} < b_n = \frac{1}{n^2}$$

Conv بتاع الصغير Conv اذا الكبير $P = 2 > 1$

Conv by P-T

$\Rightarrow$ conv abs

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$\boxed{13}$

[illegible]

$\boxed{25}$

fact

find the interval of convergens and radius of conv the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{2n+1}$$

$$2(n+1)+1 \qquad 2n+2+1$$

Ratio Test $\longrightarrow$ $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \Rightarrow \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-2)^{n+1}}{2n+3} * \frac{2n+1}{(-1)^n (x-2)^n} \right|$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{\cancel{(-1)^n} * (-1)^1 * \cancel{(x-2)^n} * (x-2)^1}{2n+3} * \frac{2n+1}{\cancel{(-1)^n} \cancel{(x-2)^n}} \right|$$

$$\Rightarrow |x-2| \lim_{n\to\infty} \frac{2n+1}{2n+3} = \frac{\infty}{\infty} \xrightarrow{CR} \frac{2n}{2n} = \boxed{1}$$

$$\Rightarrow |x-2| * 1 < 1$$

$$-1 < x-2 < 1$$

$$\boxed{1 < x < 3}$$

$x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2n+1}$ $= \boxed{1}$ $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

$\sum \frac{1}{2n+1} \xrightarrow{L.CT} b_n = \frac{1}{2n} \longrightarrow$ div

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{2n+1} * \frac{2n}{1}$$

$$\lim_{n\to\infty} \frac{2n}{2n+1} \frac{\infty}{\infty} \xrightarrow{CR} \frac{2n}{2n} = \boxed{1}$$

نفس السلوك .

$$r = \frac{3-1}{2} = \frac{2}{2} = \boxed{r = 1}$$

$x = 3 = \sum_{n=1} \frac{(-1)^n}{2n+1}$

$\xleftrightarrow{A.T}$

$\lim_{n\to\infty} |a_n|$

اثبت انها متناقصة

$= \lim_{n\to\infty} \frac{1}{2n+1} = \frac{1}{\infty} = \boxed{0}$

Is diersing

$\boxed{\text{Conv}}$ $\boxed{1 < x \leq 3}$

$r_1 = 2$

$r_2 = -2$

Root Test $\lim_{n\to\infty}$

$\lim_{x\to 0}\frac{\sin x}{x}$

$\Rightarrow \lim_{n\to\infty}\left|\frac{(-1)^n * (-1)^1 * (x-2)^n * (x-2)}{2n+3}\right|$

$\frac{2n}{2n} = \boxed{1}$

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مراجعة

$\boxed{26}$

#14 Express the function $f(x) = \frac{1}{(x+1)^2}$ as a power series

لما تشوف المقام علية تربيع اعرف انه ناتج عن مشتقة

$\frac{1}{1-x} = \sum (x)^n$

$$\frac{-1}{x+1} \rightarrow \frac{\overset{+}{\cancel{-}}1}{(x+1)^2}$$

$$\Rightarrow -\frac{1}{x+1}$$

$$\Rightarrow -\frac{1}{1-(-x)}$$

$$\Rightarrow (-)\sum_{n=0}^{\infty}(-x)^n$$

$$\Rightarrow \sum_{n=0}^{\infty}(-1)(-1)^n(x)^n$$

اشتق الطرفين

$$\Rightarrow \sum_{n=0}^{\infty}(-1)^{n+1}(x)^n = \frac{-1}{x+1}$$

$$\boxed{\Rightarrow \sum_{n=1}^{\infty}(-1)^{n+1}\,n(x)^{n-1} = \frac{1}{(x+1)^2}}$$

نفس صيغة السؤال بس

$$f(x) = \frac{x^3}{x-4}$$

حاول دايما تجيب على الشكل العام ليقدم

$$= x^3 \cdot \frac{1}{x-4}$$

$$= \frac{x^3}{-4}\cdot\frac{1}{\left(1-\frac{x}{4}\right)}$$

$$\frac{x^3}{-4}\cdot\sum_{n=0}^{\infty}\left(\frac{x}{4}\right)^n$$

$$\sum_{n=0}^{\infty}\frac{(x)^n * (x)^3}{(4)^n * (-4)}$$

$$\boxed{\sum_{n=0}^{\infty}\frac{(x)^{n+3}}{(-1)(4)^{n+1}}}\ \#$$

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مراجعة

15 $\int x \sin^{-1}(5x)\,dx$ → by parts

$u = \sin^{-1}(5x)$ $dv = x$

$du = \frac{5}{\sqrt{1-(5x)^2}}$ $v = \frac{x^2}{2}$

$$\boxed{= \frac{x^2}{2}\sin^{-1}(5x) - \int \frac{5x^2}{2\sqrt{1-25x^2}}\,dx}$$

16 The substitution $\boxed{x = \sec(\theta)}$ transforms the

$dx = \sec(\theta)\tan(\theta)$

$\int x^5 (x^2-1)^{\frac{3}{2}}\,dx$ into

$$\int (\sec\theta)^5 (\underbrace{\sec^2(\theta)-1}_{})^{\frac{3}{2}} \sec(\theta)\tan(\theta)\,d\theta$$

$\tan^2$

$$\int (\sec(\theta))^6 ((\tan^2(\theta))^{\frac{3}{2}} \tan(\theta)\,d\theta$$

$$\boxed{\int \sec^6(\theta)\tan^4(\theta)\,d\theta} \#$$

17 The partial decomposition of

$$\frac{x}{(x+3)(x^3+27)} \text{ is}$$

$$\rightarrow \frac{x}{(x+3)(x+3)(x^2-3x-9)}$$

$$\frac{x}{(x+3)^2(x^2-3x+9)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{Cx+D}{x^2-3x+9}$$

$x^3+y^3 =$ ↑ لا تحل

$(x+y)(x^2-xy+y^2)$

$x^3-y^3 =$

$(x-y)(x^2+xy+y^2)$

Note

$du \quad \frac{5}{\sqrt{1-(5x)^2}} \quad u = \frac{x^2}{2}$

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18 The substiation $x=u^5$ transforms the $\int \frac{\sqrt[3]{x}}{\sqrt{x}+4}\,dx$

$x = u^5$

$\sqrt[3]{x} = \sqrt[3]{u^5} = u^2 \quad \rightarrow \sqrt[3]{x} = ?? \rightarrow \sqrt{x} \rightarrow ??$

$\sqrt{x} = \sqrt[2]{u^5} = u^3 \qquad dx = 5u^5\,du$

$$\rightarrow \int \frac{u^2}{u^3+4} 5u^5\,du$$

$$\int \frac{5u^7}{u^3+4}\,du$$

19 $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) =$

$S_n = \left(\cos(\pi) - \cos\left(\frac{\pi}{2}\right)\right) + \ldots$

$+\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right)\right) + \left(\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right)\right)$

$S_n = \cos(\pi) - \cos\left(\frac{\pi}{n+1}\right)$

$\lim_{n\to\infty} S_n \rightarrow \cos(\pi) - \cos\left(\frac{\pi}{n+1}\right)$

$\rightarrow -1 - \cos\left(\frac{\pi}{\infty}\right)$

$\rightarrow -1 - \cos(0)$

$\rightarrow -1 - 1 = \boxed{-2}$ #

20 If $\sum_{n=1}^{\infty} 4a_n = 12$ then $\lim_{n\to\infty} a_n =$

بما انه طلع جواب
يعني هو conv
بالتالي ديفر
D.T هيفشل
ومش نشغل
لحد النهاية
وشادي صفر
ديفيرجس الجواب
مطبوع

صفر

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21 The Maclaurin series for $x^7 \sin(x^3)$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

$x^7 \sin(x^3)$

$$x^7 * \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n+1}}{(2n+1)!}$$

$$x^7 * \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{6n+3}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)^7 (x)^{6n+3}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{6n+10}}{(2n+1)!}$$

✓

18 The substitution $x = 4^5$ transforms

19 $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right)$

$\left|\cos(\pi) - \cos\left(\frac{\pi}{2}\right)\right| + \dots$

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مراجعة

22 The Taylor series for $f(x) = x^3 + x + 7$ at $x = 3$ is $\sum_{n=0}^{\infty} b_n (x-3)^n$. Then $b_2 = ??$

$$\text{Taylor} = f(a) + \underbrace{f'(a)(x-a)}_{b_1} + \underbrace{\frac{f''(a)}{2!}}_{b_2}(x-a)^2 + \underbrace{\frac{f'''(a)}{3!}}_{b_3}(x-a)^3 + \dots$$

$$= \sum_{n=0}^{\infty} b_n (x-3)^n$$

$$b_2 = \frac{f''(3)}{2!} \rightarrow \frac{18}{2!} = \frac{18}{2*1} = \boxed{9}$$

$f(x) = x^3 + x + 7$

$f'(x) = 3x^2 + 1$

$f''(x) = 6x$

$f''(3) = 6*3 = \boxed{18}$

23 The Taylor series $f(x) = \frac{1}{x}$ at $x = 3$

لبدنا نحول التايلور لماكلارين من طريقة طرح وجمع $\underline{X}$ ن ≥

$$f(x) = \frac{1}{(x-3)+3}$$

عامل $\textcircled{3}$

$$f(x) = \frac{1}{3 + (x-3)}$$

$$f(x) = \frac{1}{3\left(1 + \frac{(x-3)}{3}\right)}$$

$$f(x) = \frac{1}{3\left(1 - \left(\frac{-(x-3)}{3}\right)\right)}$$

$$f(x) = \frac{1}{3} * \sum_{n=0}^{\infty} \left(\frac{-(x-3)}{3}\right)^n$$

$$f(x) = \frac{1}{3} * \sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{(3)^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{(3)^{n+1}}$$

Done

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24 $\frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} + \dots$

اكتشف العلاقة بعدين حلها ورجعه أصلي

$$\sum_{n=2}^{\infty} \frac{4^n}{n!} - a_0 - a_1 \qquad e^{\square} \sum_{n=0}^{\infty} \frac{(\square)^n}{n!}$$

له عشان يرجع العدا للصفر

$$e^4 - 1 - 4 \rightarrow \boxed{e^4 - 5}$$

$a_0 = \frac{4^0}{0!} = \boxed{1} \qquad a_1 = \frac{4^1}{1!} = \boxed{4}$

مراجعة

25 The Interval of conv of $\sum_{n\to\infty}^{\infty} \frac{(x-3)^n}{n}$

هاي Power [illegible]
على Ribko

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$= \lim_{n\to\infty} \left|\frac{(x-3)^{n+1}}{n+1} * \frac{n}{(x-3)^n}\right|$

$\lim_{n\to\infty} \left|\frac{\cancel{(x-3)^n} * (x-3) * n}{(n+1)\cancel{(x-3)^n}}\right|$

اعتقد بروح مع الأكسات
والمابقي مع $\underline{n}$

$|x-3| \lim_{n\to\infty} \frac{n}{n+1}$ $\frac{\infty}{\infty}$ ok

$\frac{n}{n} = \boxed{1}$ عوض

$|x-3| * 1 < 1$

$|x-3| < 1$

$-1 < x-3 < 1$

$\boxed{2 < x < 4}$

لازم اعرف انها
مفتوحة او مسكرة
كيف عرفت

$\boxed{x=2}$ [illegible]

$\boxed{x=2}$

$\sum_{n\to\infty}^{\infty} \frac{(-1)^n}{n}$ — AST

$\lim_{n\to\infty} |a_n| = \frac{1}{n}$

$= \frac{1}{\infty} = \boxed{0}$

dicring

$a_1 > a_2$

$\boxed{\text{is conv}}$

مغلقة

$x=4$

$\sum_{n=\infty}^{\infty} \frac{(1)^n}{n}$

$\sum_{n=\infty}^{\infty} \frac{1}{n^1}$

$P = 1 \leq 1$

div by P.T

مفتوحة

26 The Polar equation $r = 2\cos(\theta) + 4\sin(\theta)$ represents

$r^2 = 2r\cos(\theta) + 4r\sin(\theta)$ ×r

$x^2 + y^2 = 2x + 4y$

$x^2 + y^2 - 2x - 4y + 1 + 4 = 0 + 1 + 4$ → اكمال مربع $\left(\frac{\text{معامل}}{2}\right)^2$

[redacted] $x^2 - 2x + 1 + y^2 - 4y + 4 = 5$

$(x-1)^2 + (y-2)^2 = 5$

center (1, 2)
$r = \sqrt{5}$

هي طريقة اسرع واسهل
لبدك كل هاد اعرف انه
$\boxed{r \text{ دجذره}}$ وخليهم دخطرند و $\boxed{\sin \text{ مع } y}$ $\boxed{\cos \text{ مع } x}$
لما بعدها اقسم على $\boxed{2}$ المعامل
بيطلع center
بدل $\boxed{r}$ خدها ترتيب
لحتى الحل

$r = \frac{2\cos(\theta)}{2} + \frac{4\sin(\theta)}{2}$

$\boxed{(1, 2)}$

$r = \sqrt{1^2 + 2^2} = \boxed{\sqrt{5}}$

Moamen_B.Asher

31

مراجعة

Ashour

Momen

27 one of the following repersents the point with cartsin Coordinates $(-2, 2\sqrt{3})$ in polar??

سالب موجب

[illegible] ©

$r^2 = x^2 + y^2$

$r^2 = (-2)^2 + (2\sqrt{3})^2$

$r^2 = 4 + 12$

$r^2 = 16$

$\boxed{r_1 = 4}$

$\boxed{r_2 = -4}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$

$\theta = \tan^{-1}(-\sqrt{3})$

$\boxed{\theta = \frac{\pi}{3}}$

سالبة

$\frac{\pi}{3}$

الرابع

$2\pi - \theta$

$3 \times \frac{2\pi}{3} - \frac{\pi}{3}$

$\boxed{\frac{5\pi}{3}}$

الثاني

$\pi - \theta$

$\frac{3\pi}{3} - \frac{\pi}{3}$

$\boxed{\frac{2\pi}{3}}$

$\left(4, \frac{2\pi}{3}\right)$

$\left(-4, \frac{5\pi}{3}\right)$

---

28 set up the integral that finds the erea

inside $r = 9\cos(\theta)$

outside $r = 3 + \cos(\theta)$

بدو المسألة التكامل بس

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} (r)^2_{in} - (r)^2_{out}$$

$$= \frac{1}{2}\int (9\cos(\theta))^2 - (3 + \cos(\theta))^2 \, d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 81\cos^2(\theta) - (3 + \cos(\theta))^2$$

لما نجيب الحدود بنساوي $r = r$

$9\cos(\theta) = 3 + \cos(\theta)$

$8\cos(\theta) = 3$

$\cos(\theta) = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}}$

cos

موجب

$\frac{\pi}{3}$

الربع الرابع

$2\pi - \theta$

$3 \times \frac{2\pi}{3} - \frac{\pi}{3}$

$\boxed{\theta = \frac{5\pi}{3}}$

$\boxed{\frac{-\pi}{3}}$

الربع الاول

$\theta = \theta$

$\boxed{\frac{\pi}{3}}$

مؤمن

عاشور

((n+1|+))

مراجعة

32

29

$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 3^n}{n!}$ is $\overset{abs}{\text{conv}}$ or condsly conv or div

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{(n+1)3^n}{n!}$

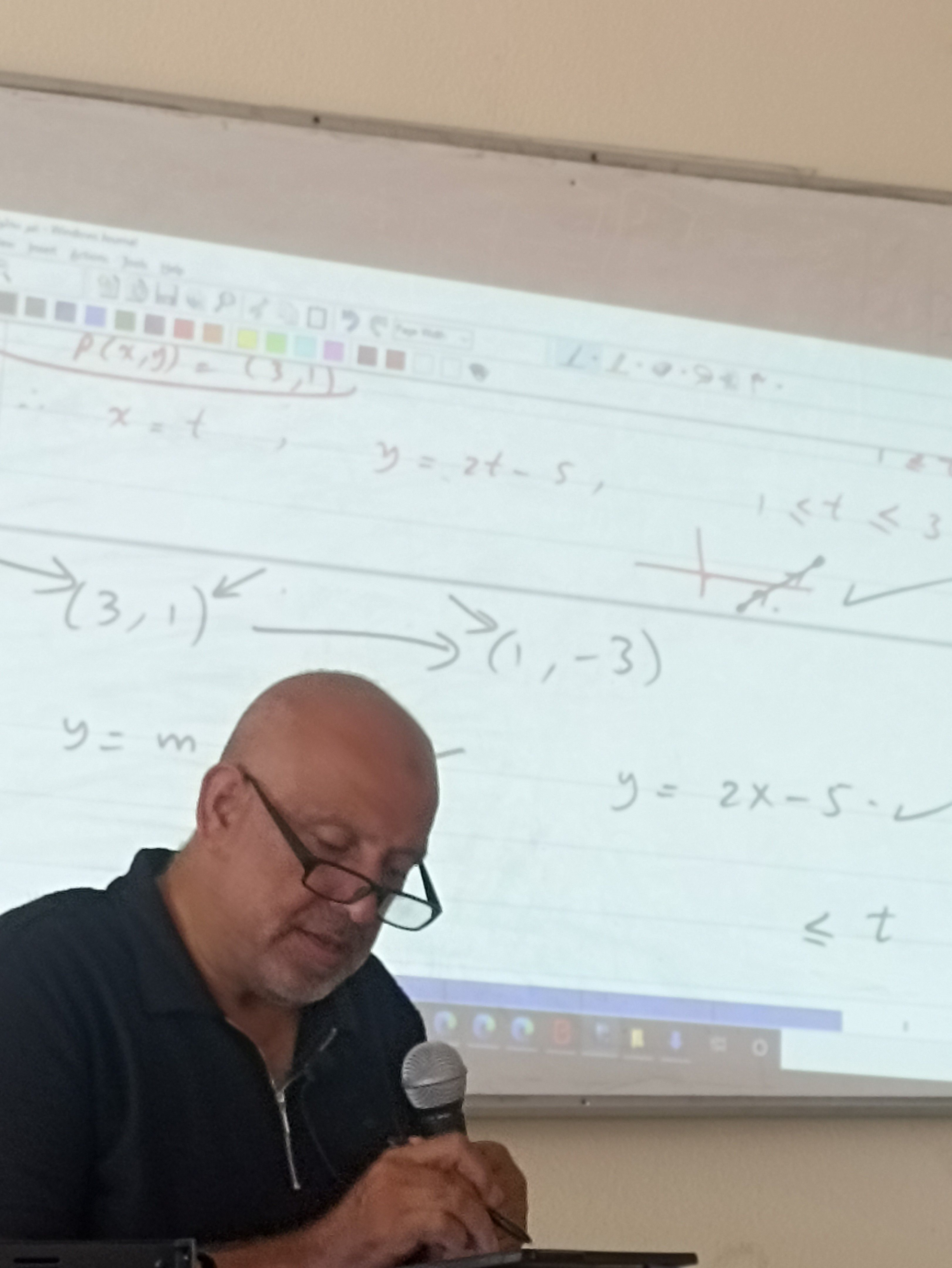
Moamen-B.Ashour

Ritoh Test $\longrightarrow$ $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+2)3^{n+1}}{(n+1)!} * \frac{n!}{(n+1)\,3^n}$

$\lim_{n\to\infty} \frac{(n+2) * 3^n * 3}{(n+1)\cancel{n!}} * \frac{\cancel{n!}}{(n+1)\cancel{3^n}}$

$\lim_{n\to\infty} \frac{(n+2)3}{(n+1)^2} = \frac{\infty}{\infty} \xrightarrow{ck} \frac{3n}{n^2} = \frac{3}{n} = \frac{3}{\infty} = \boxed{0} < 1$ conv by Ratio Test

$\boxed{\text{Abs conv}}$



$P(x,y) = (3,1)$

$x = t \,, \qquad y = 2t - 5 \,, \qquad 1 \le t \le 3$

$(3,1) \longrightarrow (1,-3)$

$y = m$

$y = 2x - 5 .$ ✓

$\le t$

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مراجعة

33

30 Find the sum (if exist)

$(\text{عدد})^n$ → Geometric

→ telescopy

$$\sum_{n=1}^{\infty} \frac{2^{n+1}+5^n}{10^n}$$

$$= \sum_{n=1}^{\infty} \frac{2^n * 2^1 + 5^n}{10^n}$$

اقتران - اقتران

$\times$

$$\sum_{n=1}^{\infty} 2\left(\frac{2+5}{10}\right)^n$$

$$\sum_{n=1}^{\infty} 2 * \left(\frac{7}{10}\right)^n \rightarrow |r| < 1$$

conv

$$\text{sum} = \frac{a_1}{1-r} = \frac{1.4}{1-\frac{7}{10}} = \boxed{4.6}$$

32 Test for convergence

$$\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2+1} \rightarrow \text{I.T},\ \text{C.T}$$

ال

$$\boxed{\frac{\tan^{-1}(n)}{n^2+1} < \frac{\frac{\pi}{2}}{n^2+1}}$$

→ اللي بالاسود هادتشي لازم تكون علاقة

=> الي انا فرضتها برجوبها C.T حتعرض عماز مرة وحشوفه

$$\boxed{\frac{\frac{\pi}{2}}{n^2+1} < \frac{\frac{\pi}{2}}{n^2}}$$

→ $P = 2 < 1$

conv by P.T

بالتالي اللي فرضتها قبل conv واللي جيم برضو conv

conv

$\boxed{\text{conv}}$

مؤمن

31 find the sum (if exist)

تلسكوب

$$\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6} \rightarrow$$

كسور جزئية حل

$$\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+2)} = \frac{A}{n+3} + \frac{B}{n+2}$$

$$1 = A(n+2) + B(n+3)$$

→ معادلة ابعا

$$x=-2 \rightarrow 1 = 0 + B \rightarrow \boxed{B=1}$$

$$x=-3 \rightarrow 1 = -A + 0 \rightarrow \boxed{A=-1}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3} \rightarrow$$

دا التلسكوب

$$\Rightarrow S_n = \left(\frac{1}{3} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{n+2}} - \frac{1}{n+3}\right)$$

$$\lim_{n\to\infty}\left(\frac{1}{3} - \frac{1}{n+3}\right) = \frac{1}{3} - \frac{1}{\infty} = \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$$

33

$$\sum_{n=1}^{\infty} n^2 \sin^2\left(\frac{2}{n}\right) \xrightarrow{\text{D.T}}$$

$$\lim_{n\to\infty} n^2 \sin^2\left(\frac{2}{n}\right) \Rightarrow \infty * 0 = \boxed{0}$$

فشل

غير كمل السؤال لبعد وهعام

$$\sum_{n=1}^{\infty} \frac{4\sin^2\left(\frac{2}{n}\right)}{\left(\frac{1}{n}\right)^2} \Rightarrow$$

$$\Rightarrow 4\lim_{n\to\infty} \left(\frac{\sin\left(\frac{2}{n}\right)}{\left(\frac{2}{n}\right)}\right)^2 = 4 * (1)^2$$

$$\boxed{=4}$$

مؤمن
عاشور

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$\boxed{34}$ $\sum_{n=0}^{\infty} \frac{(2n)!}{5n+1}$ $\xrightarrow{DI}$ $\frac{\text{كبير}}{\text{صغير}}$

$\Rightarrow \frac{\infty}{\text{عدد}} = \boxed{\infty \nrightarrow 0} \rightarrow \boxed{\text{div}}$

Ratio Test

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(2n+2)!}{5n+6} * \frac{5n+1}{(2n)!}$

$\Rightarrow \lim_{n\to\infty} \frac{(2n+2)(2n+1)\cancel{(2n)!}}{5n+6} * \frac{5n+1}{\cancel{(2n)!}}$

$\lim_{n\to\infty} \frac{20n^{\cancel{3}2}}{5\cancel{n}} = \frac{20n^2}{5} = \frac{\infty}{5} = \boxed{\infty} > 1$

by Ratio ← div

$\boxed{35}$ Test the sequence

$\sqrt[n]{4^{1+2n}}$ conv or div

$\lim_{n\to\infty} 4 \rightarrow$ عدد $\rightarrow$ conv

$\rightarrow \pm\infty \rightarrow$ div

$\lim_{n\to\infty} (4^{1+2n})^{\frac{1}{n}}$

$\lim_{n\to\infty} (4^{\frac{1+2n}{n}})$ $\boxed{\text{conv}}$

$\lim_{n\to\infty} 4^{\frac{1}{n}+2} = 4^{0+2} = 4^2 = \boxed{16}$

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إدهاقر

$\boxed{36}$ Determine conv Abs or div

or conditionally conv

$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 3^n}{n!}$

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{(n+1)3^n}{n!}$

Rito Test

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

$\lim_{n\to\infty} \frac{(n+2)*3^{n+1}}{(n+1)!} * \frac{n!}{(n+1)3^n}$

$\lim_{n\to\infty} \frac{(n+2)*\cancel{3^n}*3^1}{(n+1)\cancel{n!}} * \frac{\cancel{n!}}{(n+1)\cancel{3^n}}$

$\lim_{n\to\infty} \frac{3n+6}{(n+1)^2} = \frac{\infty}{\infty} \xrightarrow{CK} \frac{3n}{n^2} = \frac{3}{n} = \frac{3}{\infty}$

conv $\Leftarrow 1 > \boxed{\text{Zero}} =$

by. Rito Test $\rightarrow$ $\boxed{\text{conv A.bs}}$ #

Kamel Ashour

[redacted]

مؤمن عاشور

(35)

34

37 find the sum (if exist)

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} - \frac{1}{10^{n-1}}$$

(عدد)^n جيومترك

تحلها تلسكوبي بجزئية

$$\sum_{n=2}^{\infty} \frac{1}{(n+1)(n-1)} = \frac{A}{(n+1)} + \frac{B}{n+1}$$

$1 = A(n+1) + B(n-1)$

$x = 1 \rightarrow 2A + 0 = 1 \rightarrow \boxed{A = \frac{1}{2}}$

$x = -1 \rightarrow 0 + -2B = 1 \rightarrow \boxed{B = \frac{-1}{2}}$

$$\sum_{n=2}^{\infty} \frac{\frac{1}{2}}{n-1} - \frac{\frac{1}{2}}{n+1}$$

$$S_n = \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \left(\frac{1}{4} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}}\right)$$

$$+ \left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{12}}\right) \dots + \left(\cancel{\frac{\frac{1}{2}}{n-n}} - \frac{\frac{1}{2}}{n-1}\right) + \left(\cancel{\frac{\frac{1}{2}}{n-1}} - \frac{\frac{1}{2}}{n+1}\right)$$

$$\left(\cancel{\frac{\frac{1}{2}}{2-n}} - \cancel{\frac{\frac{1}{2}}{n-2}}\right)$$

$$\lim_{n \to \infty} \left(\frac{1}{2} + \frac{1}{4} - \cancel{\frac{\frac{1}{2}}{n-1}}^{0} - \cancel{\frac{\frac{1}{2}}{n+1}}^{0}\right)$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{8}} \rightarrow \sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

محيبة السؤال

$$\sum_{n=2}^{\infty} \frac{1}{10^{n-1}}$$

$$= \sum_{n=2}^{\infty} \frac{1}{10^n \cdot 10^{-1}}$$

$$\sum_{n=2}^{\infty} 10 \left(\frac{1}{10}\right)^n$$

$r =< 1$

conv

$\text{sum} = \frac{a_1}{1-r}$

$$= \frac{\frac{1}{10}}{1 - \frac{1}{10}} = \boxed{\frac{1}{9}}$$

$$\sum_{n=2}^{\infty} \frac{1}{10^{n-1}} = \frac{1}{9}$$

$$\frac{3}{8} - \frac{1}{9} =$$

$$\frac{27-8}{72} = \boxed{\frac{19}{72}}$$ ##

→ conv A.bs

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مراجعة

$\boxed{38}$ $\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{16}\right)+\cdots\left(\frac{1}{2^n}+\frac{1}{4^n}\right)$

لو جابلك اوجد المجموع
حبيب مش هاد
هاد يا عزيزي بدك
الك تخليه سيريز
يا نعزي يا خد اخر
شي هيه a_n
وبعدها حل

$$\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{4^2}\right)$$

$$\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n+\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$$

$r=\frac{1}{4}<1$
conv

$r=\frac{1}{2}<1 \Rightarrow$ conv

$$S_n=\frac{a_1}{1-r}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=\boxed{1}$$

$$S_n=\frac{a_1}{1-r}=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\boxed{\frac{1}{3}}$$

$$\boxed{\frac{4}{3}}=1+\frac{1}{3}$$

$\boxed{39}$ $\sum_{n=2}^{\infty}\frac{1}{n\sqrt[3]{\ln(n)}} \rightarrow \frac{1}{n(\ln(n))^p} \Rightarrow$ I.T

مستمر

I.T → Positive
continuous
decreasing
$a_1 > a_2$

$$\int_2^{\infty}\frac{1}{n(\ln(n))^{\frac{1}{3}}}\,dn$$

$$\lim_{a\to\infty}\int_2^{a}\frac{1}{n(\ln(n))^{\frac{1}{3}}}\,dn$$

$y=\ln(n)$

$$\boxed{dn=\frac{dy}{\frac{1}{n}}}$$

$$=\int_2^{a}\frac{1}{\cancel{n}\,y^{\frac{1}{3}}}\;\frac{dy}{\frac{1}{\cancel{n}}}$$

$$\int_2^{a} y^{-\frac{1}{3}}\,dy$$

$$=\frac{3}{2}y^{\frac{2}{3}}$$

$$=\frac{3}{2}\left[\ln(n)\right]^{\frac{3}{3}}\Big]_2^{a}$$

$$\lim_{a\to\infty}\frac{3}{2}\left[(\ln(a))^{\frac{3}{2}}-(\ln(2))^{\frac{3}{2}}\right]$$

$\frac{3}{2}*\infty$ - عدد

$\boxed{\text{div. by I.T}}$