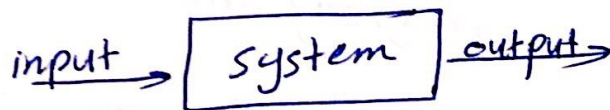


* في البداية هذا الساتر يحتاج في مراجعة موضوع Laplace transform الذي تم تناوله بالتفصيل في مادة الـ signal.
* أي شيء يتعلق بالـ Laplace transform موجود في كتابي لساتر 4 في signal.

السطر 2 //
← أول خطوة قبل البدء بعملية النمذجة للـ system هي تحويل الـ system من physical model إلى mathematical model حتى نتمكن من التعامل مع الـ system.



ولكن علينا تحويل الـ system من physical model إلى mathematical model
بـ "Modeling"

⇒ If you need to find the Mathematical model from physical Model, we need "physical Laws" such as:

- 1) Ohm's Law
 - 2) Kirchhoff's Law → node voltage method.
→ mesh current method.
 - 3) Newton's Laws ⇒ For mechanical system.
- } for electrical system

← من خلال هاتين الطائفتين نتمكن من إيجاد علاقة بين input, output

* There are several ways to find the mathematical model. Lap all

[1] differential equation.

[2] Transfer function. ~ in classical control.

[3] state-space model. ~ in modern control

طريقة // بنية كوك ~ TF , SS , القاب .

بنية بنية كراوية لا Laplace transform

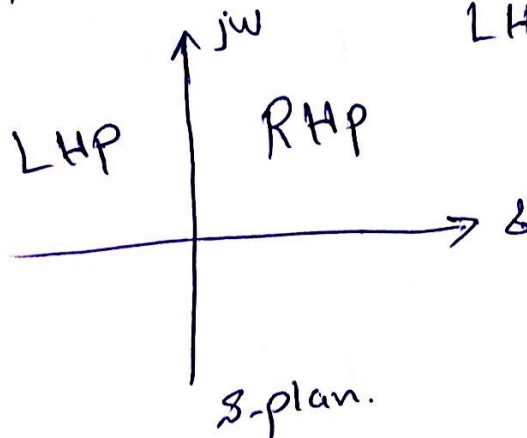
$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt.$$

$$s = \sigma + j\omega$$

~ Complex number ~

Real part imaginary part

~ poles ~ stable ~ system ~ LHP ~ RHP



frequency domain ~ Laplace ~ time domain ~

* If we have two or more poles at zeros, the system is unstable



Use Laplace transform to solve the differential equation
 ~ all the initial condition equal Zero ~

$$\frac{dy^2}{dt^2} + 12 \frac{dy}{dt} + 32y = 32u(t)$$

Solⁿ -

$$s^2 y(s) + 12s y(s) + 32y(s) = 32 \cdot \frac{1}{s}$$

$$y(s) (s^2 + 12s + 32) = \frac{32}{s}$$

$$y(s) = \frac{32}{s(s^2 + 12s + 32)}$$

$$y(s) = \frac{32}{s(s+4)(s+8)} = \frac{K_1}{s} + \frac{K_2}{(s+4)} + \frac{K_3}{(s+8)}$$

$$K_1 = \frac{32 \cancel{s}}{\cancel{s}(s+4)(s+8)} \Big|_{s=0} = \boxed{1}$$

$$K_2 = \frac{32(s+4) \cancel{(s+4)}}{\cancel{(s+4)}(s+8)} \Big|_{s=-4} = \boxed{-2}$$

$$K_3 = \frac{32(s+8) \cancel{(s+8)}}{\cancel{(s+8)}(s+4)} \Big|_{s=-8} = \boxed{1}$$

$$y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$\boxed{y(t) = (1 - 2e^{-4t} + e^{-8t})u(t)} \quad \#$$

أفكر في
 صورة العنصر
 السالبة والنصف العنصر

$$x(t) = 3e^{-2t}u(t) + 4e^t u(-t) \quad , \text{ find } x(s)$$

$$x(s) \Rightarrow ??$$

Sol °

$$x(s) = \frac{3}{s+2} - \frac{4}{s-1} = \frac{3(s-1) - 4(s+2)}{(s+2)(s-1)}$$

$$x(s) = \frac{-s-11}{s^2+s-2}$$

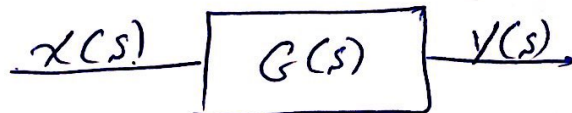
$$ROC \Rightarrow -2 < \operatorname{Re}\{s\} < 1$$

$$x(s) = \frac{3}{s+2} - \frac{4}{s-1} \quad \leftarrow \text{توضيح *}$$

$\swarrow \quad \searrow$
 $ROC: \operatorname{Re}\{s\} > -2 \quad \quad ROC: \operatorname{Re}\{s\} < 1$

Transfer function °—

frequency domain . العلاقة تربط بين Input, output في الـ



$$G(s) = \frac{Y(s)}{X(s)} = \frac{\text{output (freq. domain)}}{\text{input (freq. domain)}} = \text{T.F}$$

$$Y(s) = X(s) G(s) \quad \text{معنى آخر}$$

$$Y(t) \neq X(t) G(t)$$

الشرط لازم الـ Initial condition $\Rightarrow$ Zero

poles $\Rightarrow$ أصفاء المقام
Zeros $\Rightarrow$ أصفاء البسط

$$H(s) = \frac{(s+1)(s-1)}{(s+2)(s+3)} \rightarrow \text{Zeros} \quad \sim$$

$$\quad \quad \quad \rightarrow \text{poles} \quad \sim$$

° رمزها ~
X رمزها ~

4

:-

$$2y''(t) + 3y'(t) + 5y(t) = x'(t) - 2x(t)$$

Find the T.F.

Sol :-

All initial condition equal Zero.

$$2s^2 Y(s) + 3sY(s) + 5Y(s) = sX(s) - 2X(s)$$

$$Y(s) (2s^2 + 3s + 5) = X(s) (s - 2)$$

$$\left| \frac{Y(s)}{X(s)} = \frac{s-2}{2s^2+3s+5} \right| \Rightarrow \text{Transfer function.}$$

Inverse Laplace Transform :-

هناك طريقتان كذا الانقوس ← استخدام بقايفه
Partial fraction ← استخدام

الاجابة بنتم P.F. لانهم نراي بعض القالات :-

* $\text{order den} \gg \text{order num}$ "درجه المقام اكبر من درجه البسط"
لو كانت درجه البسط اكبر من او تساوي درجه المقام بنتم بصيغه اعطونه.

* General forms of solving by partial fraction :-

- [1] Real and distinct.
- [2] Real and repeated.
- [3] Complex and distinct.
- [4] Complex and repeated.

ادعوني اسكن لكم
ليكنكونه عن عبارتي
وهم واضرني

Example # : Find the inverse Laplace transform:-

$$X(s) = \frac{2s^3 + 2s - 10}{s^3 + 3s^2 - s - 3}$$

Sol: -

هذا دالة بولينية ذات درجة 3 مقام 3. نحلها بطريقة

$$\begin{array}{r} s^3 + 3s^2 - s - 3 \overline{) 2s^3 + 2s - 10} \\ \underline{2s^3 + 6s^2 - 2s - 6} \\ -6s^2 + 4s - 4 \end{array} \quad \#$$

$$X(s) = 2 + \frac{-6s^2 + 4s - 4}{s^3 + 3s^2 - s - 3}$$

$$s^3 + 3s^2 - s - 3 = 0 \Rightarrow (s-1)(s+1)(s+3) = 0$$

Poles $\Rightarrow \boxed{s=1}, \boxed{s=-1}, \boxed{s=-3}$ ~ Real and distinct ~

$$X(s) = \frac{-6s^2 + 4s - 4}{s^3 + 3s^2 - s - 3} = \frac{K_1}{(s-1)} + \frac{K_2}{(s+1)} + \frac{K_3}{(s+3)}$$

$$K_1 = \frac{(s-1)(-6s^2 + 4s - 4)}{(s-1)(s+1)(s+3)} \Big|_{s=1} = \boxed{-\frac{3}{4}}$$

$$K_2 = \frac{(s+1)(-6s^2 + 4s - 4)}{(s+1)(s-1)(s+3)} \Big|_{s=-1} = \boxed{\frac{7}{2}}$$

$$K_3 = \frac{(s+3)(-6s^2 + 4s - 4)}{(s+1)(s-1)(s+3)} \Big|_{s=-3} = \boxed{-\frac{35}{4}}$$

6

$$C(s) = 2 - \frac{3/4}{s-1} + \frac{7/2}{s+1} + \frac{-35/4}{s+3}$$

$$x(t) = 2\delta(t) + \left[\frac{7}{2}e^{-t} - \frac{3}{4}e^t - \frac{35}{4}e^{-3t} \right] u(t)$$

Ex #: $x(s) = \frac{2s^2 + 3s - 1}{(s+1)(s+2)^3}$

find $x(t)$?

$$x(s) = \frac{2s^2 + 3s - 1}{(s+1)(s+2)^3} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)^3} + \frac{K_3}{(s+2)^2} + \frac{K_4}{(s+2)}$$

$$K_1 = \frac{(2s^2 + 3s - 1)(s+1)}{(s+1)(s+2)^3} \Big|_{s=-1} = \boxed{-2}$$

$$K_2 = \frac{(2s^2 + 3s - 1)(s+2)^3}{(s+1)(s+2)^3} \Big|_{s=-2} = \boxed{-1}$$

$$K_3 = \frac{1}{1!} \frac{d}{ds} \left(\frac{2s^2 + 3s - 1}{s+1} \right) \Big|_{s=-2}$$

$$K_3 = \frac{2s^2 + 4s + 4}{(s+1)^2} \Big|_{s=-2} = \boxed{4}$$

$$K_4 = \frac{1}{2!} \frac{d}{ds} \left(\frac{2s^2 + 4s + 4}{(s+1)^2} \right) \Big|_{s=-2}$$

$$= \frac{1}{2} \left(\frac{-4}{(s+1)^3} \right) \Big|_{s=-2} = \boxed{2}$$

$$x(s) = \frac{-2}{(s+1)} - \frac{1}{(s+2)^3} + \frac{4}{(s+2)^2} + \frac{2}{(s+2)}$$

$$x(t) = \left(-2e^{-t} - t^2 e^{-2t} + 4t e^{-2t} + 2e^{-2t} \right) u(t)$$

#

7

"وطني السحاب الزفير والنبوءة * منور"
السحاب والأرض إنك كنه ملامح أنتم
تنطقون "سورة الطور"

$$* X(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$$

find $x(t)$

$$X(s) = \frac{100(s+3)}{(s+6)(s+3-j4)(s+3+j4)} = \frac{K_1}{(s+6)} + \frac{K_2}{(s+3-j4)} + \frac{K_2^*}{s+3+j4}$$

$$K_1 = \frac{100(s+3)(s+6)}{(s+6)(s^2+6s+25)} \Big|_{s=-6} = \boxed{-12}$$

$$K_2 = \frac{100(s+3)(s+3-j4)}{(s+6)(s+3-j4)(s+3+j4)} \Big|_{s=-3+j4} = 6-j8 = \boxed{10 \angle -53.13^\circ}$$

$$K_2^* = 6+j8 = \boxed{10 \angle 53.13^\circ}$$

$$X(s) = \frac{-12}{s+6} + \frac{10 \angle -53.13^\circ}{s+3-j4} + \frac{10 \angle 53.13^\circ}{s+3+j4}$$

$$x(t) = \left[-12e^{-6t} + 20e^{-3t} \cos(4t - 53.13^\circ) \right] u(t) \quad \# \text{ صواباً}$$

$$\mathcal{L} \left\{ \frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta} \right\} = 2|K|e^{-\alpha t} \cos(\beta t + \theta^\circ) \quad // \text{ ملاحظة}$$

$$K = |K| \angle \theta^\circ$$

$$* X(s) = \frac{768}{(s^2+6s+25)^2} \quad \text{find } x(t) \quad s^2+6s+25 = (s+3-j4)^2(s+3+j4)^2$$

$$X(s) = \frac{768}{(s^2+6s+25)^2} = \frac{K_1}{(s+3-j4)^2} + \frac{K_2}{(s+3-j4)} + \frac{K_1^*}{(s+3+j4)^2} + \frac{K_2^*}{(s+3+j4)}$$

$$K_1 = \frac{768(s+3-j4)^2}{(s+3-j4)^2(s+3+j4)^2} \Big|_{s=-3+j4} = \boxed{-12 \angle 0^\circ}$$

8

$$= \frac{1}{1!} \frac{d}{ds} \left(\frac{768}{(s+3+j4)^2} \right) \bigg|_{s=-3+j4} \Rightarrow$$

$$= \frac{d}{ds} \left(\frac{768}{(s+3+j4)^2} \right) = \frac{-1536}{(s+3+j4)^3} \bigg|_{s=-3+j4} = 3 \angle -90^\circ$$

$$R_1^* = -12$$

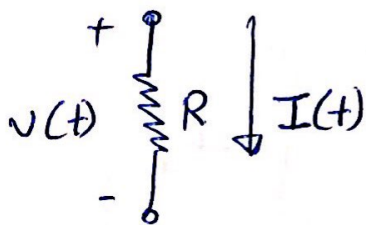
$$R_2^* = 3 \angle 90^\circ$$

$$X(s) = \frac{-12 \angle 0^\circ}{(s+3-j4)^2} + \frac{3 \angle -90^\circ}{(s+3-j4)} + \frac{12 \angle 0^\circ}{(s+3+j4)^2} + \frac{3 \angle 90^\circ}{(s+3+j4)}$$

$$x(t) = \left(+24t e^{-3t} \cos(4t) + 6e^{-3t} \cos(3t - 90^\circ) \right) u(t)$$

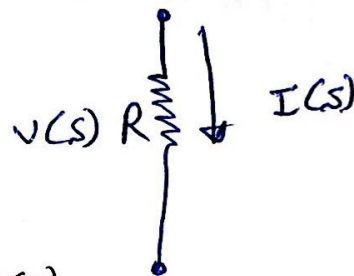
* Electrical network transfer function :-

Time domain

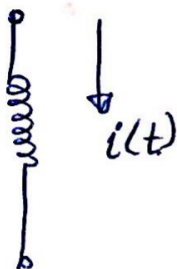


$\Rightarrow$

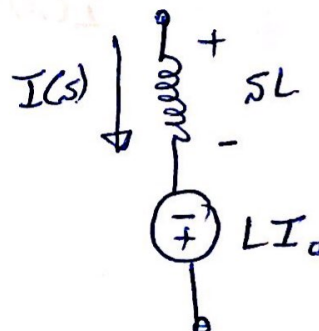
s-domain



$$V(s) = R I(s)$$



$\Rightarrow$



$$v(t) = L \frac{di}{dt}$$

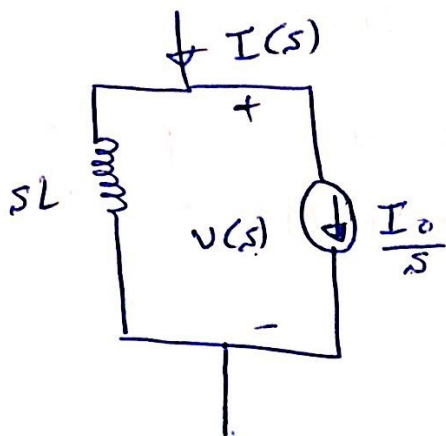
$$i(t) = \frac{1}{L} \int v(t) dt + I_0$$

$$V(s) = L [sI(s) - I_0]$$

$$V(s) = sLI(s) - \frac{I_0 L}{s}$$

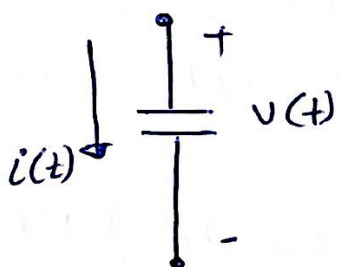
9

OR use current source :-

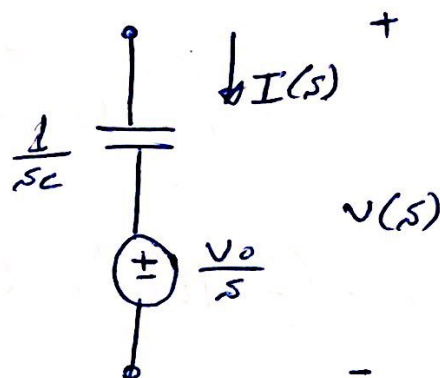


$$I(s) = \frac{v(s)}{sL} + \frac{I_o}{s}$$

$$\frac{d}{dt} \rightarrow s$$



$\Rightarrow$

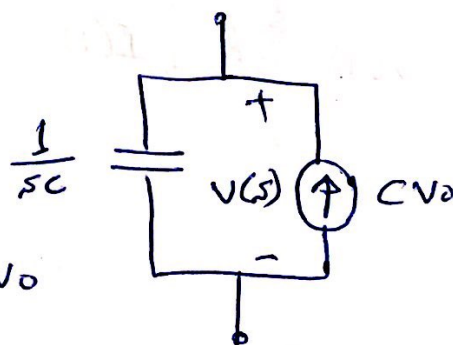


$$i = C \frac{dv}{dt}$$

$$v(t) = \frac{1}{C} \int i(t) dt + v_0$$

$$v(s) = \frac{I(s)}{sC} + \frac{v_0}{s}$$

OR

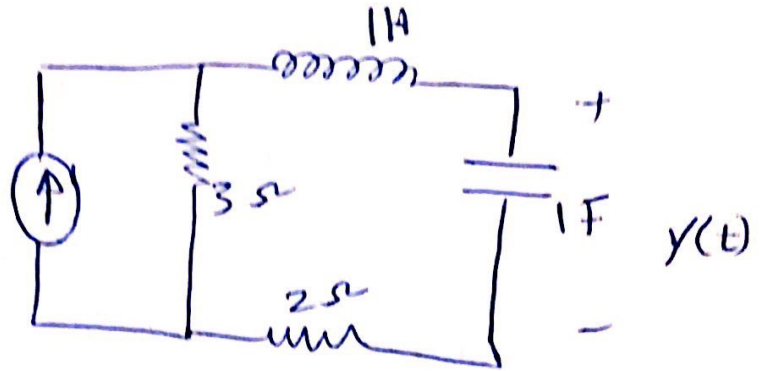


$$I(s) = sCv - CV_0$$

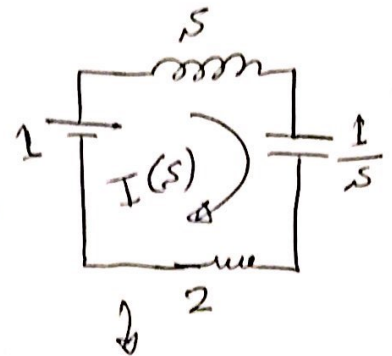
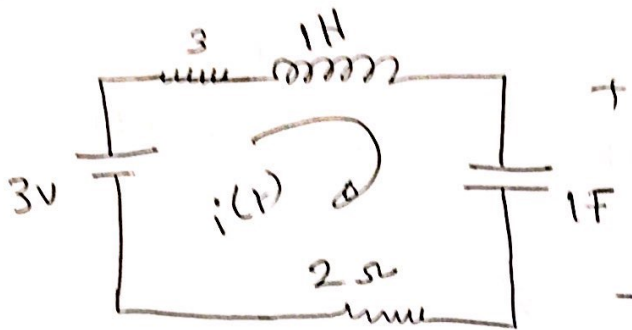
* Find $y(t)$ if $v_c(0) = 1V$, $i_l(0) = 2A$,
 $x(t) = \delta(t)$

$\delta(t) \xrightarrow{L=1}$

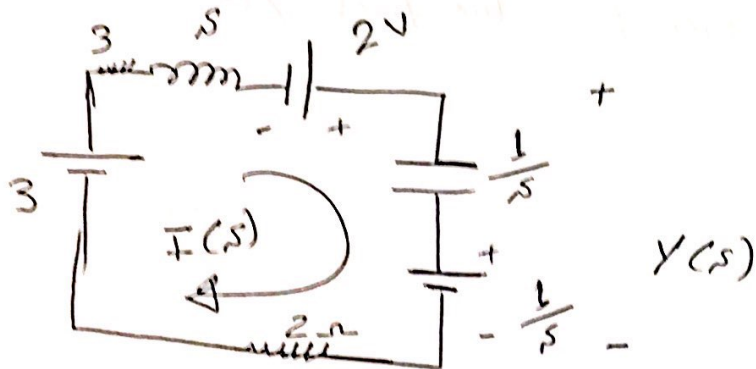
$x(t) = \delta(t)$



Sol :-



Initial condition



$$y(s) = I(s) \cdot \frac{1}{s} + \frac{1}{s}$$

By KVL:

$$3 = 3I(s) + sI(s) - 2 + \frac{I(s)}{s} + \frac{1}{s} + 2I(s)$$

$$3 = I(s) \left(3 + s + \frac{1}{s} + 2 \right) - 2 + \frac{1}{s}$$

$$-\frac{1}{s} + 5 = I(s) \left(5 + s + \frac{1}{s} \right)$$

$$I(s) = \frac{5 - \frac{1}{s}}{s + \frac{1}{s} + 5} \cdot \frac{s}{s}$$

$$\Rightarrow \frac{5s - 1}{s^2 + 5s + 1}$$



$$Y(s) = \frac{1}{s} \cdot I(s) + \frac{1}{s}$$

$$= \frac{1}{s} \cdot \left(\frac{5s-1}{s^2+5s+1} \right) + \frac{1}{s}$$

$$= \frac{5s-1}{s(s^2+5s+1)} + \frac{s^2+5s+1}{s(s^2+5s+1)}$$

$$= \frac{5s-1 + s^2+5s+1}{s(s^2+5s+1)} \Rightarrow \frac{s^2+10s}{s(s^2+5s+1)}$$

$$\boxed{Y(s) = \frac{s+10}{s^2+5s+1}} \quad \#$$

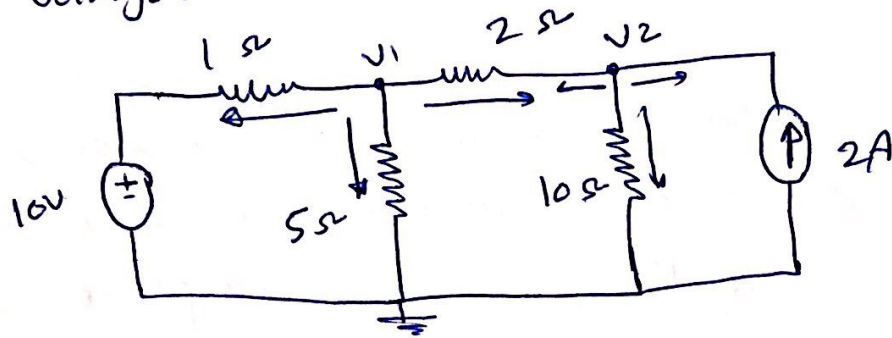
By using inverse Laplace transform we find $y(t)$.

Control analysis can be done by two methods:-

1. Node voltage Method.

2. Mesh current Method.

Node voltage:-



number of node = n

number of equation = n-1

شرح طريقة الكل :

- 1] نعوام بتجيد essential node ، نعوام بتجيد 3 عناصر.
- 2] بتجيد وحدة من essential node أروحي أي $V=0$ ، بتجيد Reference.
- 3] بتجيد (بجانب السيار) بتجيد من essential node.
- 4] نعوام بتجيد (بجانب السيار) بتجيد من essential node.
- 5] نعوام بتجيد (بجانب السيار) بتجيد من essential node.

Sol:-

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0 \quad \dots ①$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0 \quad \dots ②$$

$$V_1 - 10 + \frac{1}{5}V_1 + \frac{1}{2}V_1 - \frac{1}{2}V_2 = 0$$

$$\Rightarrow 1.7V_1 - \frac{1}{2}V_2 = 10$$

$$\frac{1}{2}V_2 - \frac{1}{2}V_1 + \frac{V_2}{10} = 2$$

$$\Rightarrow -\frac{1}{2}V_1 + 0.6V_2 = 2$$

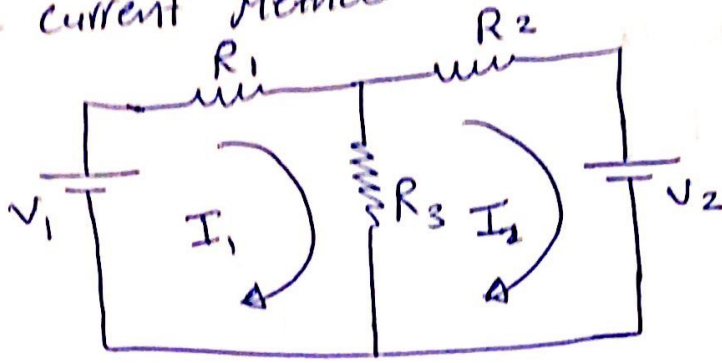
$$V_1 = 9.09V$$

$$V_2 = 10.909V$$

#

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[2] Mesh current Method :-



- سرخ طريقة اكل :
- [1] نقوم بتحديد اتجاه التيار في كل حلقة فصلة ~ نعرض مع عقارب الساعة ~
 - [2] نقوم بكتابة المعادلات بكتابة التيارات ~ نأخذ Loop كاملة ~
 - [3] نقوم باستخدام طريقة Cramer's كل المعادلات ~ إيجاد الجاهيل

$$Ax = B$$

$A \Rightarrow$ square matrix $n \times n$

$x \Rightarrow$ vector of the variable

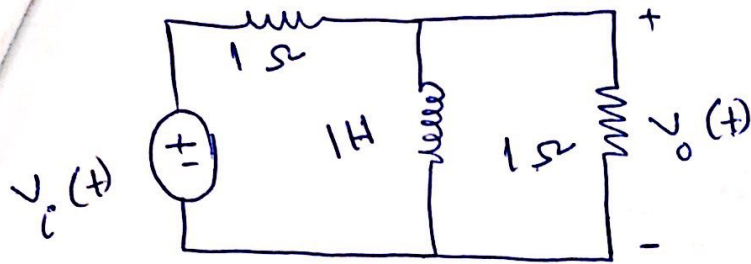
$B \Rightarrow$ Column vector

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

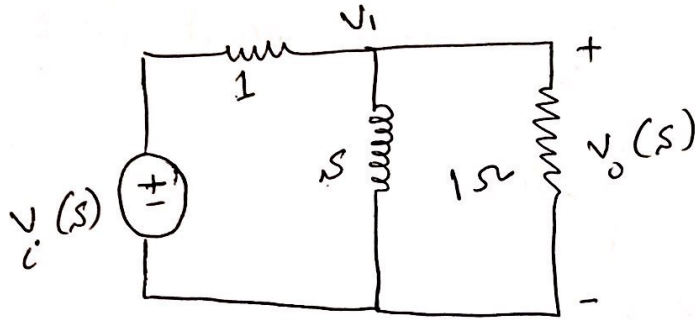
$$x_i = \frac{\det(A_i)}{\det(A)} \quad i = 1, 2, \dots, n$$

A_i : matrix form by replacing the i -th column of A by the column vector B

Find the transfer function, $G(s) = V_o(s) / V_i(s)$ for each network shown in fig.



Sol :-



$$\frac{V_1 - V_i(s)}{1} + \frac{V_1}{s} + \frac{V_1 - V_o(s)}{1} = 0$$

حيث $V_1 = V_o$

$$V_1 - V_i + \frac{V_1}{s} = 0$$

$$\frac{sV_1 - sV_i + V_1}{s} = 0 \Rightarrow sV_1 - sV_i + V_1 = 0$$

$$(s+1)V_1 = sV_i \Rightarrow \frac{V_o}{V_i} = \frac{s}{s+1}$$

$V_o = V_1$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s+1}$$

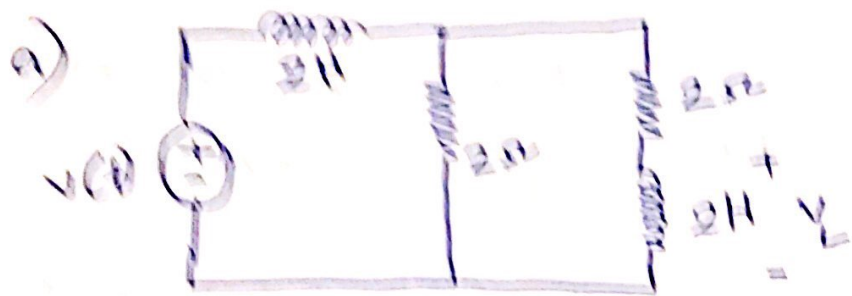
هو المطلوب



"إننا ودينا ما وعدنا على الله"
ولا تخزنا يوم القيامة، إنك
لا تخلف العباد"
سورة آل عمران

[15]

[7] Find the transfer function $G(s) = V(s)/V(s)$ for the network:-



Sol:-



mesh current $I_1(s)$

$$V(s) = (2s + 2)I_1(s) - 2I_2(s)$$

$$0 = -2I_1(s) + (4 + 2s)I_2(s)$$

$$\begin{bmatrix} 2s+2 & -2 \\ -2 & 2s+4 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

$AX = b$ $\Rightarrow X = A^{-1}b$

$$I_1 = \frac{\begin{vmatrix} V(s) & -2 \\ 0 & 2s+4 \end{vmatrix}}{\det A}$$

$$\det A = \begin{vmatrix} 2s+2 & -2 \\ -2 & 2s+4 \end{vmatrix} = (2s+2)(2s+4) - 4$$

$$= 4s^2 + 12s + 4$$

$$I_1(s) = \frac{V(s)(2s+4)}{4s^2 + 12s + 4} = \frac{V(s)(s+2)}{2(s^2 + 3s + 1)}$$

$$I_1(s) = \frac{V(s)(s+2)}{2s^2 + 6s + 2}$$

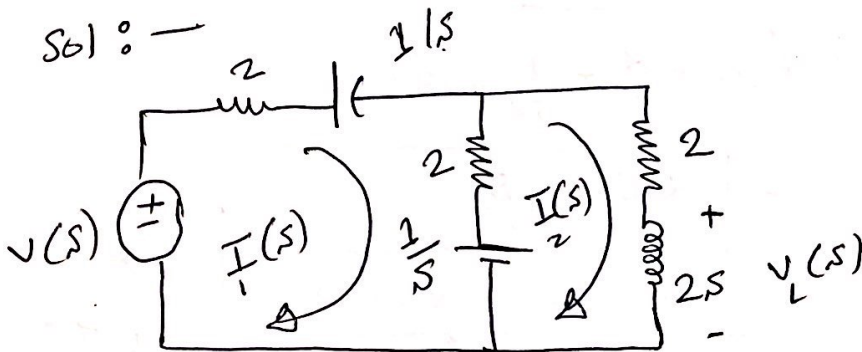
Eng. Hadeel

(16)

[Handwritten signature]

$$V_L(s) = (2s) \frac{(2v(s))}{4s^2 + 12s + 4}$$

انا فتحنا لك فتحاً مبيناً
لنصر لك الله بما تقدم
عليك
وهديك صراطاً مستقيماً
سورة الفتح



$$\begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + \frac{2}{s} & -(2 + \frac{1}{s}) \\ -(2 + \frac{1}{s}) & 4 + \frac{1}{s} + 2s \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

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$$\det A = \begin{vmatrix} 4 + \frac{2}{s} & -(2 + \frac{1}{s}) \\ -(2 + \frac{1}{s}) & 4 + \frac{1}{s} + 2s \end{vmatrix} = \frac{8s + \frac{8}{s} + \frac{1}{s^2} + 16}{s^2}$$

$$\frac{I_1(s)}{1} = \frac{\begin{vmatrix} v(s) & -(2 + \frac{1}{s}) \\ 0 & 4 + \frac{1}{s} + 2s \end{vmatrix}}{\det A} = \frac{v(s) \left(4 + \frac{1}{s} + 2s \right)}{8s + \frac{8}{s} + \frac{1}{s^2} + 16}$$

$$\frac{I_2(s)}{2} = \frac{\begin{vmatrix} 4 + \frac{2}{s} & v(s) \\ -(2 + \frac{1}{s}) & 0 \end{vmatrix}}{\det A} = \frac{v(s) \left(2 + \frac{1}{s} \right)}{8s + \frac{8}{s} + \frac{1}{s^2} + 16}$$

$$v_L(s) = 2s \frac{I_2(s)}{2} \Rightarrow \frac{2s (v(s) \left(2 + \frac{1}{s} \right))}{8s + \frac{8}{s} + \frac{1}{s^2} + 16}$$

$$G(s) = \frac{v_L(s)}{v(s)} = \frac{2s v(s) \left(2 + \frac{1}{s} \right)}{v(s) \left(8s + \frac{8}{s} + \frac{1}{s^2} + 16 \right)}$$

$$G(s) = \frac{(2s) \left(2 + \frac{1}{s} \right)}{8s + \frac{8}{s} + \frac{1}{s^2} + 16} = \frac{4s + \frac{2s}{s}}{8s + \frac{8}{s} + \frac{1}{s^2} + 16} \quad \#$$

$$G(s) = \frac{4s + 2}{8s + \frac{8}{s} + \frac{1}{s^2} + 16}$$

#



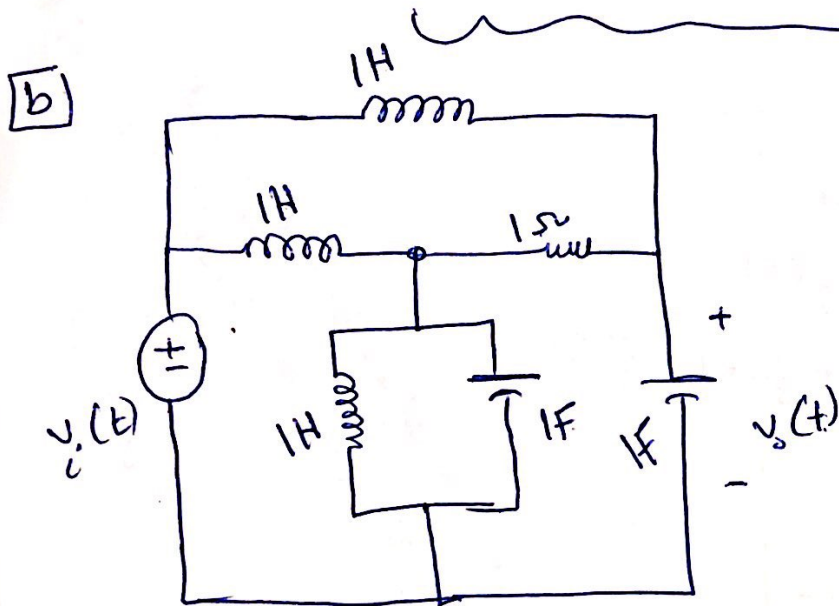
ان شاء الله
بالتوفيق
والهداية
الى
الحل
الصحيح

والله اعلم

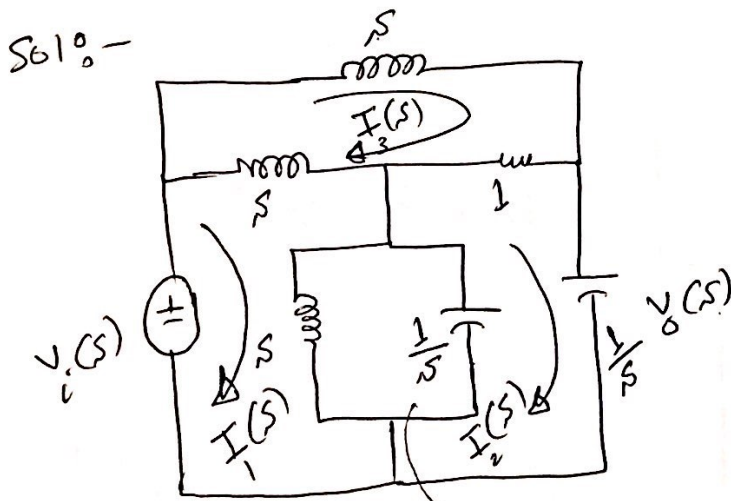
$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{(3s)}{6s^2 + 5s + \frac{2}{s} + 4}$$

"انا و جهنم جهنم" نفق
الجبر، انه "أواب"
سورة من -

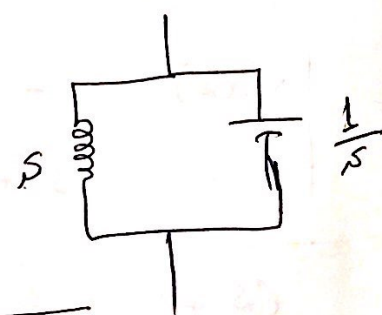
$$G(s) = \frac{3s}{6s^2 + 5s + \frac{2}{s} + 4} \quad \# \text{ وهو المطلوب}$$



"اقترب للناس جانبهم ومع في
نظرة" مرحبهم "
سورة الانبياء -

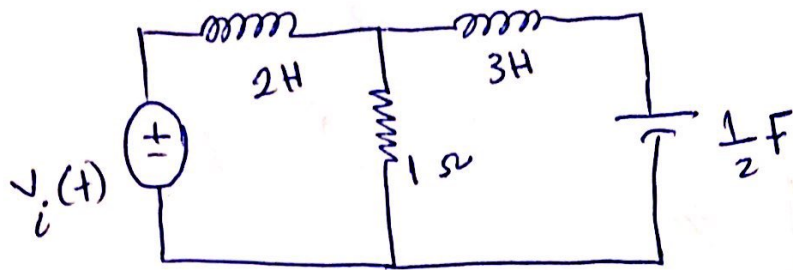


تتمتع بايجاد ال equivalent impedance
للنظر الى بعض عناصره كمثل

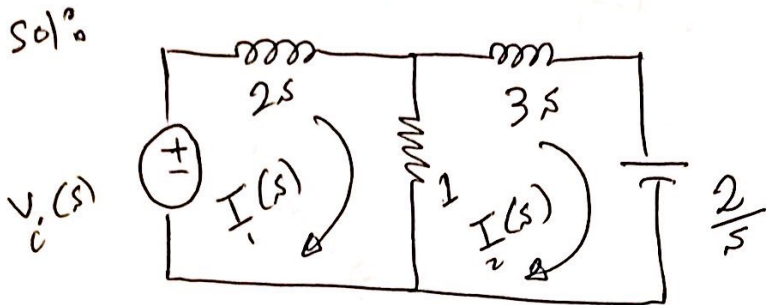


$$Z_{eq} = \frac{\frac{1}{s} \times s}{\frac{1}{s} + s} \Rightarrow \frac{1}{s^2 + 1} = \frac{s}{s^2 + 1}$$

[18] Find the transfer function $G(s) = V_o(s) / V_i(s)$, For each network show in figure. solve the problem using mesh analysis.



by using mesh current



$$V_i(s) = (2s + 1)I_1(s) - I_2(s)$$

$$0 = -I_1(s) + (3s + \frac{2}{s} + 1)I_2(s)$$

$$I_1(s) = \frac{\begin{vmatrix} V_i(s) & -1 \\ 0 & 3s + \frac{2}{s} + 1 \end{vmatrix}}{\det A}$$

$$\det A = \begin{vmatrix} 2s + 1 & -1 \\ -1 & 3s + 1 + \frac{2}{s} \end{vmatrix} = 6s^2 + 5s + \frac{2}{s} + 4$$

$$I_1(s) = \frac{\begin{vmatrix} V_i(s) & -1 \\ 0 & 3s + \frac{2}{s} + 1 \end{vmatrix}}{6s^2 + 5s + \frac{2}{s} + 4}$$

$$= \frac{V_i(s) (3s + \frac{2}{s} + 1)}{6s^2 + 5s + \frac{2}{s} + 4} \#$$

$$I_2(s) = \frac{\begin{vmatrix} 2s + 1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\det A}$$

$$= \frac{V_i(s)}{6s^2 + 5s + \frac{2}{s} + 4} \#$$

Eng. Hadil

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$$I_1(s) = \left(\frac{s}{s^2+1} + s \right) I_1(s) - \left(\frac{s}{s^2+1} \right) I_2(s) - s I_3(s)$$

$$0 = - \left(\frac{s}{s^2+1} \right) I_1(s) + \left(\frac{s}{s^2+1} + \frac{1}{s} + 1 \right) I_2(s) - \frac{I_3(s)}{3}$$

$$0 = - s I_1(s) - \frac{I_2(s)}{2} + (2s+1) \frac{I_3(s)}{3}$$

$$\det A = \begin{vmatrix} \frac{s}{s^2+1} + s & -\frac{s}{s^2+1} & -s \\ -\left(\frac{s}{s^2+1}\right) & \frac{s}{s^2+1} + \frac{1}{s} + 1 & -1 \\ -s & -1 & 2s+1 \end{vmatrix}$$

$$\det A = \frac{1}{s^2+1} + \frac{s^3}{s^2+1} + \frac{s^2}{s^2+1} + \frac{s^2}{s^2+1} + s^2 + \frac{2s}{s^2+1} + s + 1$$

$$\frac{I_2(s)}{2} = \begin{vmatrix} \frac{s}{s^2+1} + s & \psi(s) & -s \\ -\left(\frac{s}{s^2+1}\right) & 0 & -1 \\ -s & 0 & 2s+1 \end{vmatrix}$$

$\det A$

$$\frac{I_2(s)}{2} = \frac{-\psi(s) \cdot -\left(\frac{s}{s^2+1}\right) (2s+1) - s}{\det A} = \frac{\left(\frac{2s^2+s}{s^2+1}\right) + \frac{s}{1} \psi(s)}{\det A}$$

$$\psi_0(s) = \frac{I_2(s)}{2} \cdot \frac{1}{s}$$

$$\frac{I_2(s)}{2} = \frac{(s^2+2s+2) \cdot s}{s^4+2s^3+3s^2+3s+2} \psi_0(s)$$

$$\psi_0(s) = \frac{(s^2+2s+2) \psi(s)}{s^4+2s^3+3s^2+3s+2}$$

$$\Rightarrow \frac{\psi_0}{\psi_0} = \frac{s^2+2s+2}{s^4+2s^3+3s^2+3s+2}$$

Translation mechanical system :-

The motion of Mechanical system can be described in various dimensions as translational ^①, rotational ^②, or combinations ^③ of both.

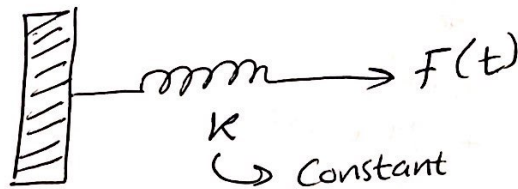
* Mechanical systems like electrical system have three passive linear components.

* The spring and mass are energy storage elements, but viscous damper dissipate energy.

* The motion of translation is defined as a motion that takes place along a straight or curved path. The variables that are used to describe translational motion are acceleration, velocity and displacement.

$$\sum \text{force} = Ma$$

* Spring :



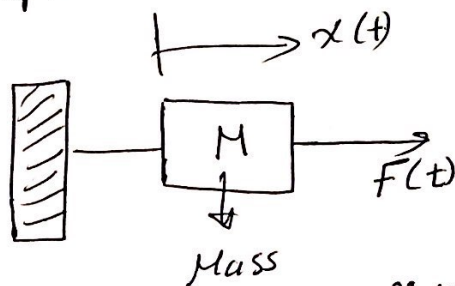
$$F(t) = kx(t)$$

$\downarrow$ input $\downarrow$ output

$$\Rightarrow \boxed{F(s) = kx(s)}$$

" Laplace "

* Mass :

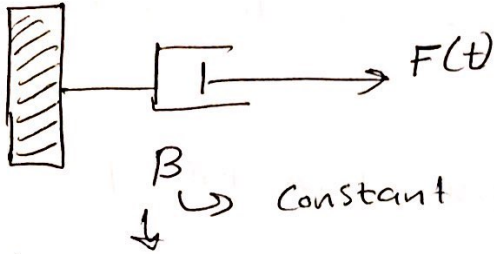


$$F(t) = ma = m\ddot{x}(t)$$

$$\boxed{F(s) = ms^2x(s)}$$

"دروس كتاب فضاء جاذبة"
الرجاء، حتى لا أكون
سوء الفهم -

* viscous damper :-

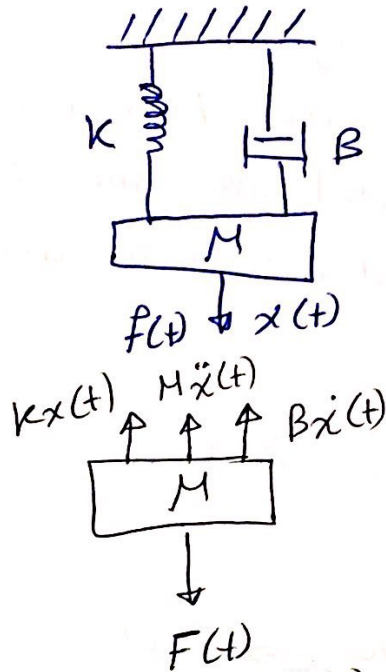


$$F(t) = B\dot{x}(t)$$

$$\boxed{F(s) = Bs x(s)}$$

علاقة بين القوة والحركة

Example :-



Sol :-

By using Newton's Law

$$F = M\ddot{x}$$

$$F_K(t) = Kx(t)$$

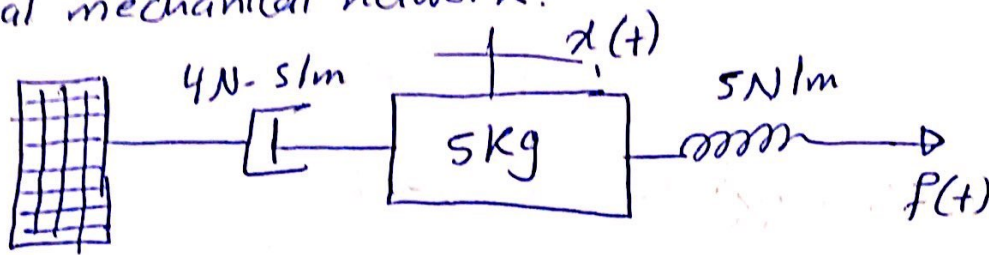
$$F_B(t) = B\dot{x}(t)$$

$$F(t) = B\dot{x}(t) + Kx(t) + M\ddot{x}(t)$$

$$F(s) = sBx(s) + Kx(s) + Ms^2x(s)$$

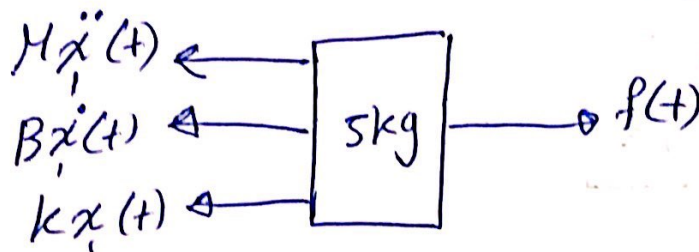
« وجعلنا من بينه أديبهم مرداً ومنه خلغهم مرداً فأغنىناهم فلم لا يسرركم »
- سورة يس -

Find the transfer function, $G(s) = x(s) / F(s)$, for the translational mechanical network.



في المثال الثاني

Sol:



$$F(t) = M\ddot{x}(t) + B\dot{x}(t) + kx(t)$$

$$F(s) = Ms^2x(s) + Bs\dot{x}(s) + kx(s)$$

$$F(s) = x(s) [Ms^2 + Bs + k]$$

$$\left| \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k} = G(s) \right|$$

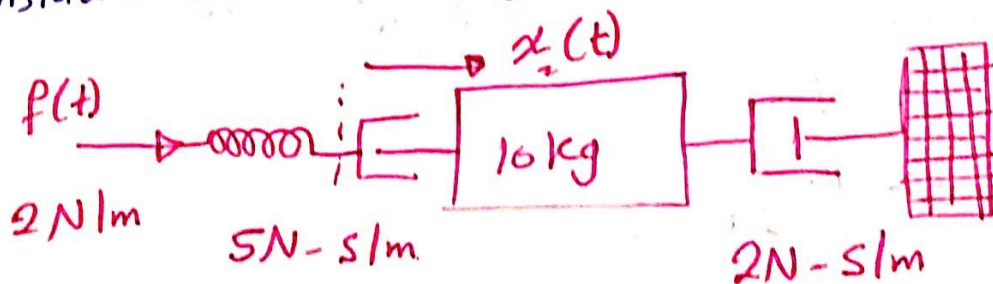
$$\left| G(s) = \frac{1}{5s^2 + 4s + 5} \right|$$

#

"وهو آخر مثال عن دوائر الكتلة والزنبرك والاحتكاك"
 سنرى في اليوم التالي
 - بعض الأمثلة -

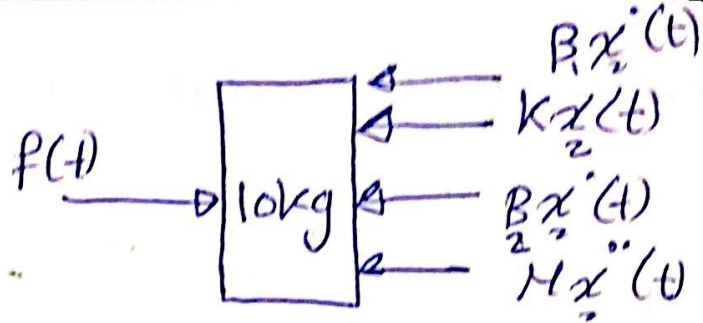
* Find the transfer function $G(s) = x(s) / F(s)$, for the translational mechanical system.

في المثال الثاني



hint: "place a zero mass at $x(t)$ "

Sol :-



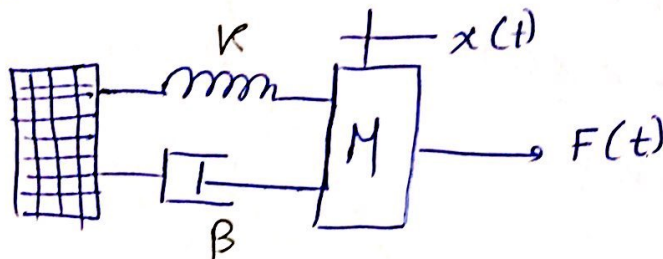
$$f(t) = kx_2(t) + B_1x_2'(t) + Mx_2''(t) + B_2x_2'(t)$$

$$f(s) = 2x(s) + 5sx(s) + 10s^2x(s) + 2sx(s)$$

$$f(s) = x(s) [2 + 5s + 10s^2 + 2s]$$

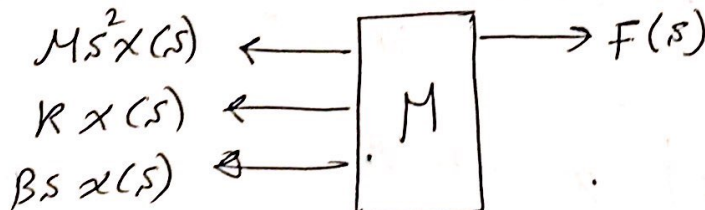
$$\boxed{\frac{x(s)}{F(s)} = \frac{1}{10s^2 + 7s + 2}} \quad \#$$

* Find the transfer function $x(s)/F(s)$



منه سلاسل الدوائر

Sol :-



$$F(s) = Ms^2x(s) + Kx(s) + Bs x(s)$$

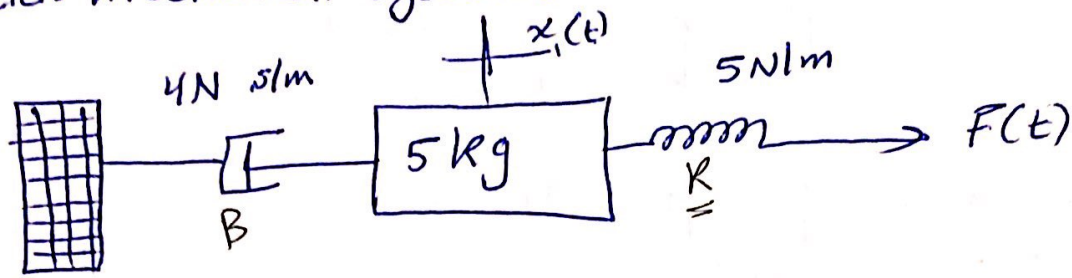
$$F(s) = x(s) (Ms^2 + K + Bs)$$

$$\boxed{\frac{x(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}} \quad \#$$

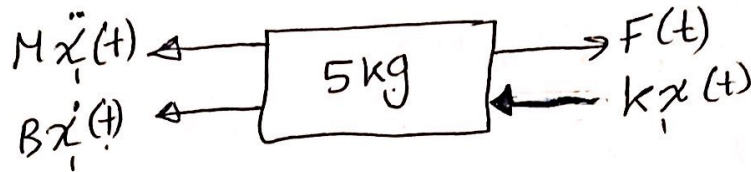
و هو المطلوب

"و تری اجمال حبیبها طارده" هي لحن من السحاب
 "طبع" الله الذي أتقنه كل شيء
 - صورة الضل -
 (ش)

Find the transfer function $G(s) = X_1(s) / F(s)$ for the translation mechanical system:-



Sol:



$$F(t) = M\ddot{x}_1(t) + B\dot{x}_1(t) + Kx_1(t)$$

$$F(s) = Ms^2X_1(s) + BsX_1(s) + KX_1(s)$$

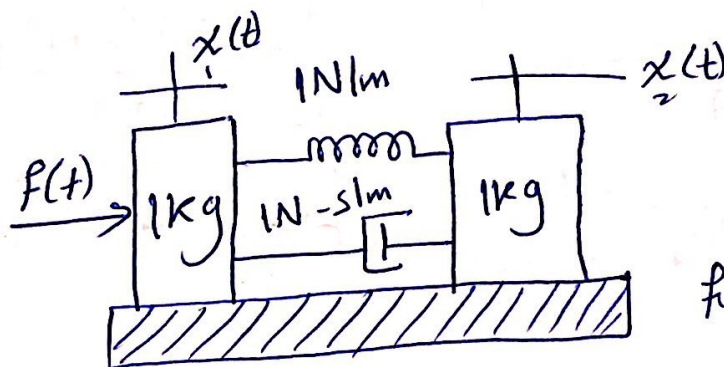
$$F(s) = X_1(s) [Ms^2 + Bs + K]$$

$$F(s) = X_1(s) [5s^2 + 4s + 5]$$

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{1}{5s^2 + 4s + 5}} \quad \#$$

لدينا له اسطوانة البعير والاربعين وبعدها
رأيه من سكة الاربع فجده لكنه لا
تفهمون
سواء لا سراء

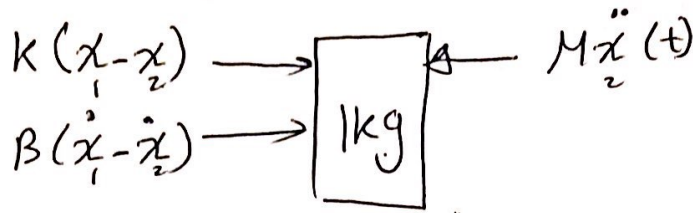
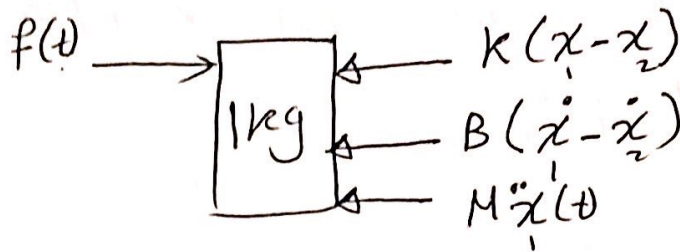
24 Find the transfer function: $X_2(s) / F(s)$



لا في الصلابة
functionless

الذي

Sol :-



$$f(t) = K(x_1 - x_2) + B(\dot{x}_1 - \dot{x}_2) + M\ddot{x}_1$$

$$f(s) = x_1(s) - x_2(s) + s x_1(s) - s x_2(s) + s^2 x_1(s)$$

$$f(s) = (1 + s + s^2) x_1(s) + (-1 - s) x_2(s) \quad \dots (1)$$

$$K(x_1 - x_2) + B(\dot{x}_1 - \dot{x}_2) = M\ddot{x}_2(t)$$

$$K x_1(s) - K x_2(s) + B s x_1(s) - B s x_2(s) = M s^2 x_2(s)$$

$$x_1 - x_2 + s x_1 - s x_2 = s^2 x_2$$

$$(1 + s) x_1(s) + (-1 - s - s^2) x_2(s) = 0 \quad \dots (2)$$

$$x_2(s) = \frac{\begin{vmatrix} s^2 + s + 1 & f(s) \\ s + 1 & 0 \end{vmatrix}}{\det A}$$

$$= \frac{-f(s)(s+1)}{\det A}$$

$$\det A = \begin{vmatrix} s^2 + s + 1 & -1 - s \\ s + 1 & -s^2 - s - 1 \end{vmatrix}$$

$$= \boxed{-s^4 + 2s^3 + 2s^2}$$

$$x_2(s) = \frac{f(s)(s+1)}{(-s^4 + 2s^3 + 2s^2)}$$

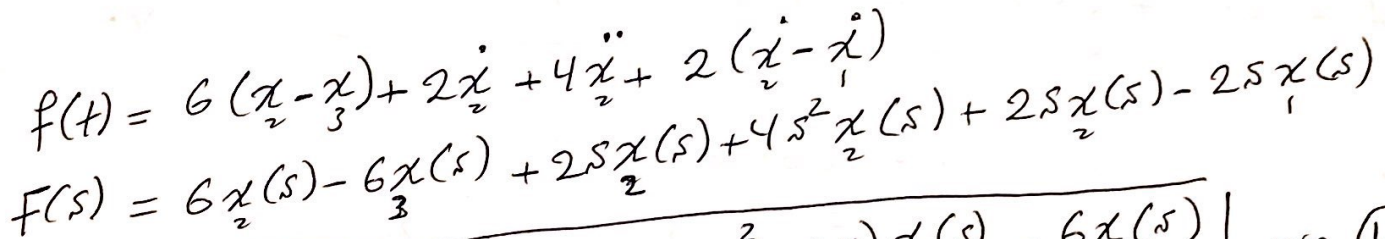
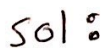
$$G(s) = \frac{x_2(s)}{f(s)} = \frac{f(s)(s+1)}{f(s)(-s^4 + 2s^3 + 2s^2)}$$

$$= \frac{(s+1)}{(-s^4 + 2s^3 + 2s^2)}$$

المطلوب

هذا بيان للنائب هادي صويطة
للجمعية فلا تكتبوا ولا تقرأوا
وأنتم الأعلمون إنه لكم
حقوقه
صوفي آل عمران

Find the transfer function $G(s) = X(s)/F(s)$, for the translational mechanical system shown in fig.



$$6(x_2 - x_3) = 4x_3 + 2x_3$$

$$6(x_2 - x_3) = 4x_3 + 2x_3$$

$$6x_2 - 6x_3 = 4s^2x_3(s) + 2sx_3(s)$$

$$6x_2(s) - 6x_3(s) - 4s^2x_3(s) - 2sx_3(s) = 0$$

$$6x_2(s) + (-6 - 4s^2 - 2s)x_3(s) = 0$$

$$\boxed{0x_1(s) - 6x_2(s) + (6 + 2s + 4s^2)x_3(s) = 0} \dots (2)$$

Eng. Hadil

$$2(\ddot{x}_2 - \ddot{x}_1) = 4\ddot{x}_1 + 6x_1$$

$$2s^2x_2(s) - 2sx_1(s) = 4s^2x_1(s) + 6x_1(s)$$

$$\boxed{(6 + 2s + 4s^2)x_1(s) - 2sx_2(s) + 0x_3(s) = 0} \quad \dots (3)$$

$$x_3(s) = \begin{vmatrix} -2s & 4s^2 + 4s + 6 & f(s) \\ 0 & -6 & 0 \\ 6 + 2s + 4s^2 & -2s & 0 \end{vmatrix}$$

بستخدام كرامر.

$$\Delta = \begin{vmatrix} -2s & 4s^2 + 4s + 6 & -6 \\ 0 & -6 & 4s^2 + 2s + 6 \\ 6 + 2s + 4s^2 & -2s & 0 \end{vmatrix}$$

$$\boxed{\Delta = 64s^6 + 128s^5 + 352s^4 + 392s^3 + 384s^2 + 216s}$$

بستخدام الكوساكن

$$x_3(s) = \frac{f(s) (0 - (-6)(6 + 2s + 4s^2))}{64s^6 + 128s^5 + 352s^4 + 392s^3 + 384s^2 + 216s}$$

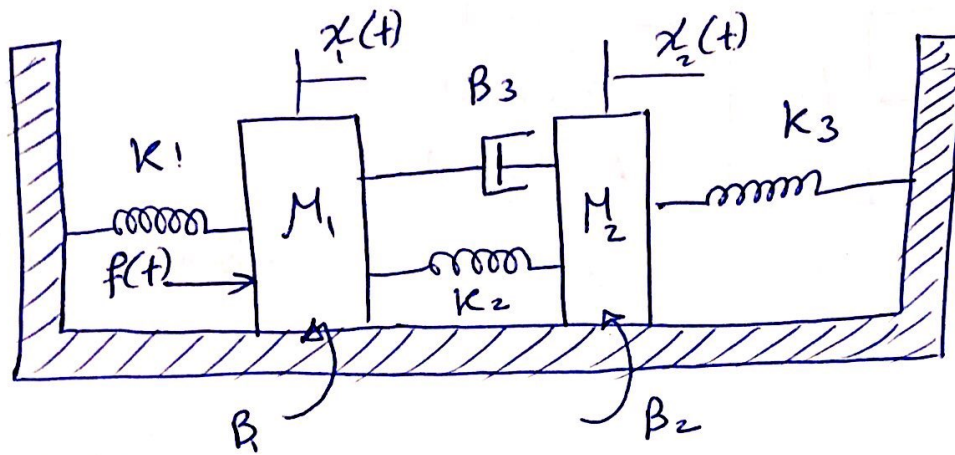
$$x_3(s) = \frac{f(s) (24s^2 + 12s + 36)}{64s^6 + 128s^5 + 352s^4 + 392s^3 + 384s^2 + 216s}$$

$$G(s) = \frac{x_3(s)}{f(s)} = \frac{24s^2 + 12s + 36}{64s^6 + 128s^5 + 352s^4 + 392s^3 + 384s^2 + 216s}$$

$$\boxed{G(s) = \frac{3}{8s^4 + 12s^3 + 26s^2 - 18s}} \quad \#$$

29

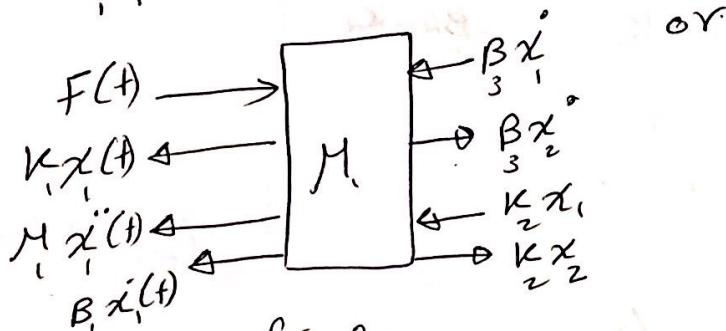
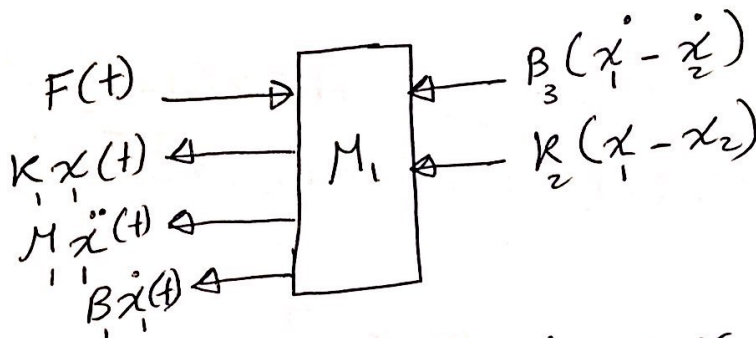
and the transfer function $x_2(s)/F(s)$



مسألة

طريقة

Sol:-

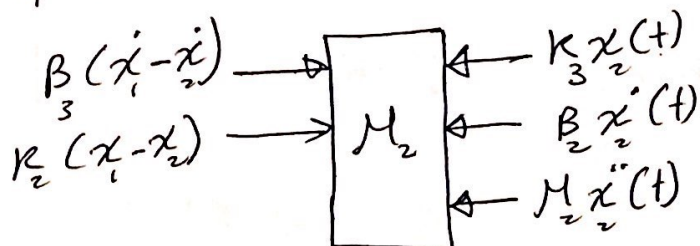
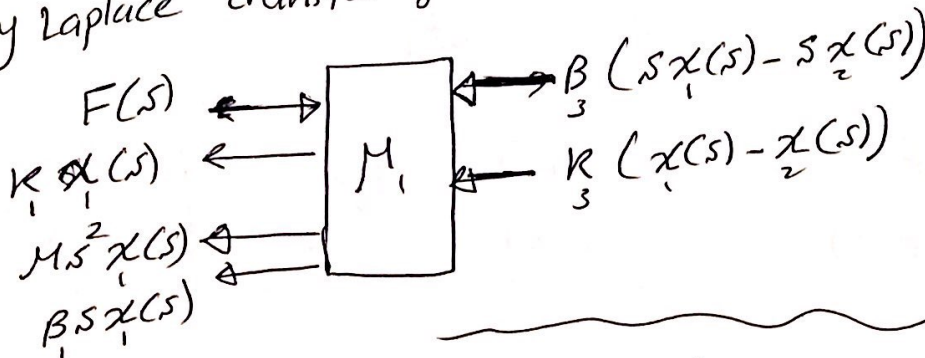


or

* تكون $F(t)$ باجهه القوى الاخرى جميعها
في على رجا $F(t)$
* $F(t)$ تكون باجهه $x(t)$

* $F(t)$ نغيرها input
* $x(t)$ هي المخرجة نغيرها output

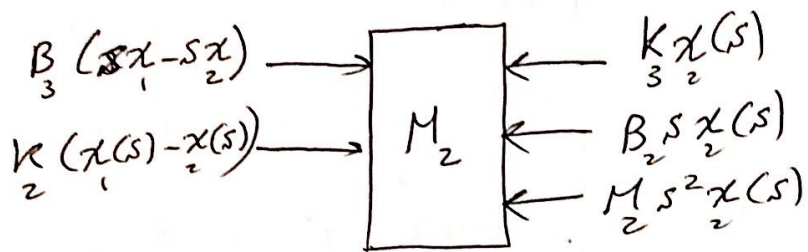
by Laplace transform:-



⇒

الجسم الاول
 M_1

الجسم الثاني
 M_2



$$F(s) = B_3 s x_1(s) - B_3 s x_2(s) + K_3 x_1(s) - K_3 x_2(s) + K_1 x_1(s) + M_1 s^2 x_1(s) + B_1 s x_1(s)$$

$$F(s) = (M_1 s^2 + (B_1 + B_3) s + K_1 + K_3) x_1(s) + (-B_3 s - K_3) x_2(s) \quad \dots (1)$$

$$B_3 s x_1(s) - B_3 s x_2(s) + K_2 x_1(s) - K_2 x_2(s) = K_3 x_2(s) + B_2 s x_2(s) + M_2 s^2 x_2(s)$$

$$[(B_3 s + K_2) x_1(s) + (-M_2 s^2 + (-B_2 - B_3) s + (-K_2 - K_3)) x_2(s) = 0] \dots (2)$$

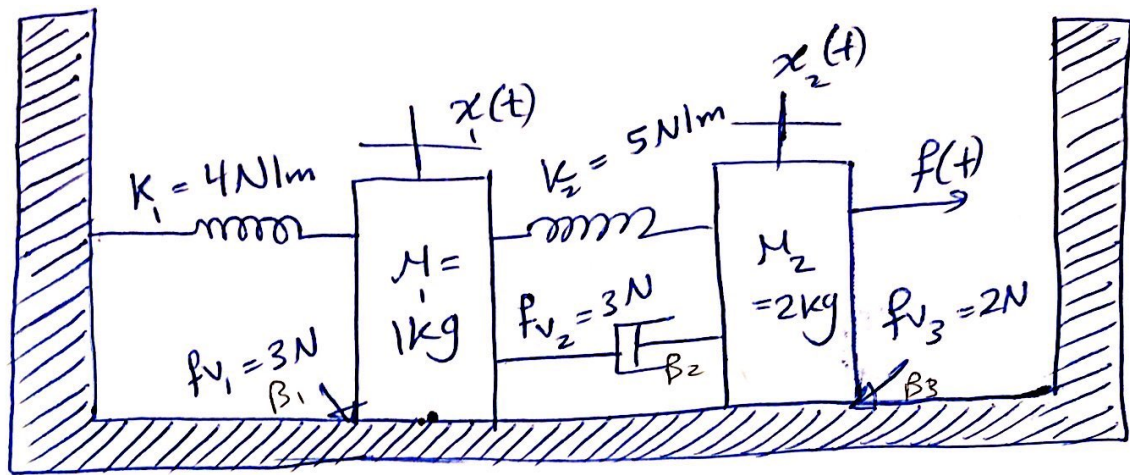
$$x_2(s) = \frac{\begin{bmatrix} M_1 s^2 + (B_1 + B_2) s + K_1 + K_3 & 0 \\ B_3 s + K_2 & (-K_2 - K_3) - (B_2 + B_3) s - M_2 s^2 \end{bmatrix}}{\Delta} = \frac{(B_3 s + K_2) F(s)}{\Delta}$$

$$\frac{x_2(s)}{F(s)} = \frac{(B_3 s + K_2) F(s)}{\Delta F(s)} \Rightarrow \left| \frac{x_2(s)}{F(s)} = \frac{B_3 s + K_2}{\Delta} \right| \quad \# \text{ copy}$$

$$\Delta = \begin{vmatrix} M_1 s^2 + (B_1 + B_2) s + K_1 + K_3 & -B_3 s - K_3 \\ B_3 s + K_2 & (-K_2 - K_3) - (B_2 + B_3) s - M_2 s^2 \end{vmatrix}$$

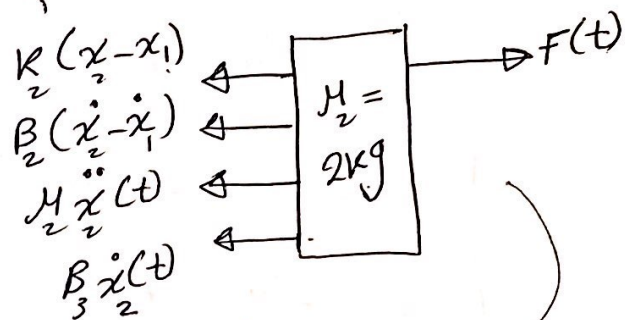
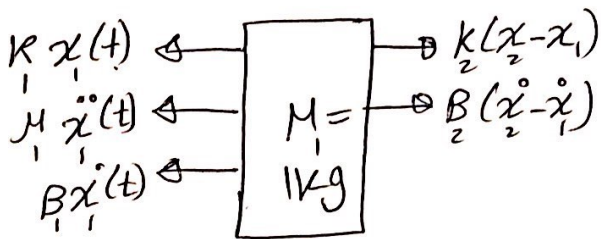
(31)

and the transfer function $G(s) = x_2(s)/F(s)$.



Sol:

free body



$$F(t) = K_2(x_2 - x_1) + B_2(\dot{x}_2 - \dot{x}_1) + M_2 \ddot{x}_2(t) + B_3 \dot{x}_2(t)$$

$$F(s) = K_2 x_2(s) - K_2 x_1(s) + B_2 s x_2(s) - B_2 s x_1(s) + M_2 s^2 x_2(s) + B_3 s x_2(s)$$

$$F(s) = 5x_2(s) - 5x_1(s) + 3s x_2(s) - 3s x_1(s) + 2s^2 x_2(s) + 2s x_2(s)$$

$$F(s) = (-5 - 3s)x_1(s) + (5 + 3s + 2s^2 + 2s)x_2(s)$$

$$F(s) = (-3s - 5)x_1(s) + (2s^2 + 5s + 5)x_2(s) \quad \dots \textcircled{1}$$

$$K_2(x_2 - x_1) + B_2(\dot{x}_2 - \dot{x}_1) = K_1 x_1(t) + M_1 \ddot{x}_1(t) + B_1 \dot{x}_1(t)$$

$$K_2 x_2(s) - K_2 x_1(s) + B_2 s x_2(s) - B_2 s x_1(s) = K_1 x_1(s) + M_1 s^2 x_1(s) + B_1 s x_1(s)$$

$$5x_2(s) - 5x_1(s) + 3s x_2(s) - 3s x_1(s) = 4x_1(s) + s^2 x_1(s) + 3s x_1(s)$$

$$(-5 - 3s - 4 - s^2 - 3s)x_1(s) + (5 + 3s)x_2(s) = 0$$

$$(-s^2 - 6s - 9)x_1(s) + (5 + 3s)x_2(s) = 0 \quad \dots \textcircled{2}$$

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$$F(s) = (-3s-5)x_1(s) + (2s^2+5s+5)x_2(s)$$

$$0 = (-s^2-6s-9)x_1(s) + (5+3s)x_2(s)$$

$$x_1(s) = \frac{\begin{vmatrix} F(s) & 2s^2+5s+5 \\ 0 & 5+3s \end{vmatrix}}{\Delta}$$

$$\Delta = \det A = \begin{vmatrix} -3s-5 & 2s^2+5s+5 \\ -s^2-6s-9 & 5+3s \end{vmatrix} = \boxed{2s^4+17s^3+44s^2+45s+20}$$

$$x_1(s) = \frac{\begin{vmatrix} F(s) & 2s^2+5s+5 \\ 0 & \cancel{5+3s} \end{vmatrix}}{2s^4+17s^3+44s^2+45s+20} = \frac{\boxed{+ F(s)(5+3s)}}{2s^4+17s^3+44s^2+45s+20}$$

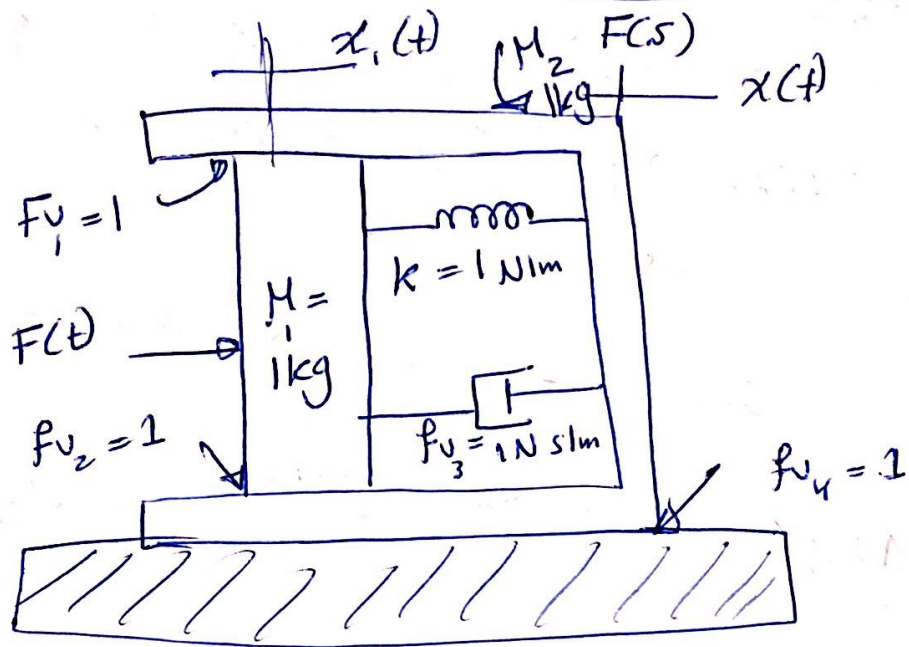
$$\frac{x_1(s)}{F(s)} = \frac{\cancel{F(s)}(5+3s)}{(2s^4+17s^3+44s^2+45s+20) \cancel{F(s)}}$$

$$\boxed{\frac{x_1(s)}{F(s)} = \frac{5+3s}{2s^4+17s^3+44s^2+45s+20}}$$

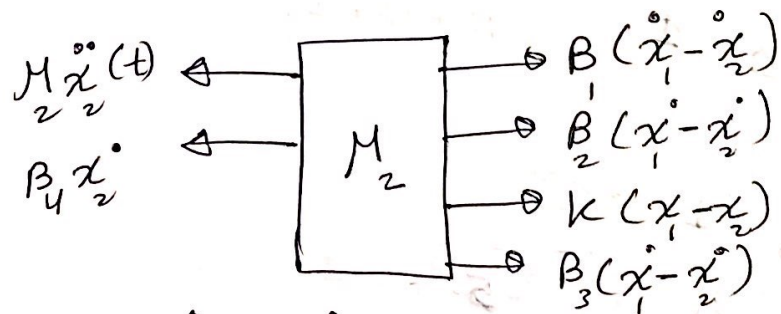
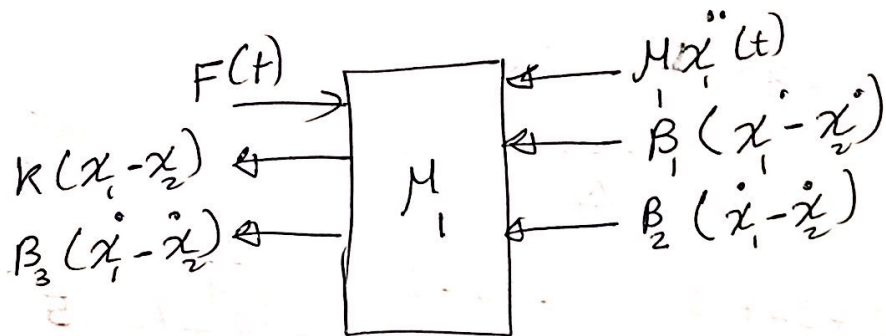
calculation

[Signature]

the transfer function $G(s) = \frac{x_2(s)}{F(s)}$



Solⁿ —



$$F(t) = M_1 \ddot{x}_1(t) + b_1 \dot{x}_1 - b_1 \dot{x}_2 + b_2 \dot{x}_1 - b_2 \dot{x}_2 + k_1 x_1 - k_1 x_2 + b_3 \dot{x}_1 - b_3 \dot{x}_2$$

$$F(t) = \ddot{x}_1(t) + \dot{x}_1(t) - \dot{x}_2(t) + x_1 - x_2 + \dot{x}_1 - \dot{x}_2 + x_1 - x_2 + \dot{x}_1 - \dot{x}_2$$

$$F(s) = s^2 x_1(s) + s x_1(s) - s x_2(s) + s x_1(s) - s x_2(s) + x_1(s) - x_2(s) + s x_1(s) - s x_2(s)$$

$$F(s) = (s^2 + s + s + 1 + s) x_1(s) + (-s - s - 1 - s) x_2(s)$$

$$F(s) = (s^2 + 3s + 1) x_1(s) + (-3s - 1) x_2(s) \quad \dots (1)$$

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$$M_2 \ddot{x}_2(t) + B_4 \dot{x}_2(t) = B_1(\dot{x}_1 - \dot{x}_2) + B_2(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) + B_3(\dot{x}_1 - \dot{x}_2)$$

$$\ddot{x}_2(t) + \dot{x}_2(t) = \dot{x}_1 - \dot{x}_2 + \dot{x}_1 - \dot{x}_2 + x_1 - x_2 + \dot{x}_1 - \dot{x}_2$$

$$s^2 x_2(s) + s x_2(s) = s x_1(s) - s x_2(s) + s x_1(s) - s x_2(s) + x_1(s) - x_2(s)$$

$$x_2(s) + s x_1(s) - s x_2(s)$$

$$-(s + s + 1 + s) x_1(s) + (s^2 + s + s + s + 1 + s) x_2(s) = 0$$

$$\boxed{(-3s-1) x_1(s) + (s^2+4s+1) x_2(s) = 0} \quad \dots (2)$$

$$x_2(s) = \frac{\begin{vmatrix} s^2+3s+1 & F(s) \\ -3s-1 & 0 \end{vmatrix}}{\Delta} = \frac{(2s+2) F(s)}{\Delta}$$

$$\Delta = \begin{vmatrix} s^2+3s+1 & -3s-1 \\ -3s-1 & s^2+4s+1 \end{vmatrix} = \boxed{s^4 + 7s^3 + 5s^2 + s + 1}$$

$$\frac{x_2(s)}{F(s)} = \frac{(2s+2) F(s)}{F(s) (s^4 + 7s^3 + 5s^2 + s + 1)}$$

$$\frac{x_2(s)}{F(s)} = \frac{(2s+2)}{s^4 + 7s^3 + 5s^2 + s + 1}$$

$$\boxed{\frac{x_2(s)}{F(s)} = G(s) = \frac{3s+1}{s^4 + 7s^3 + 5s^2 + s}} \quad \#$$

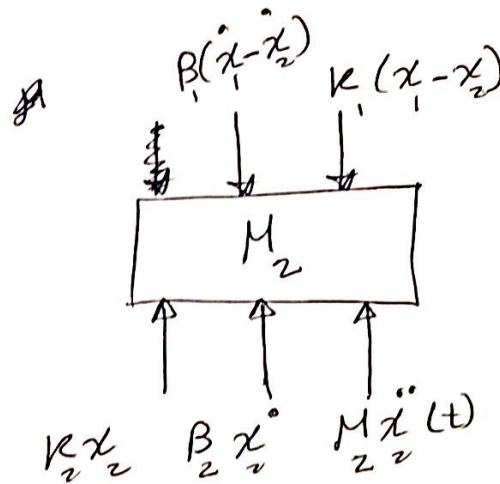
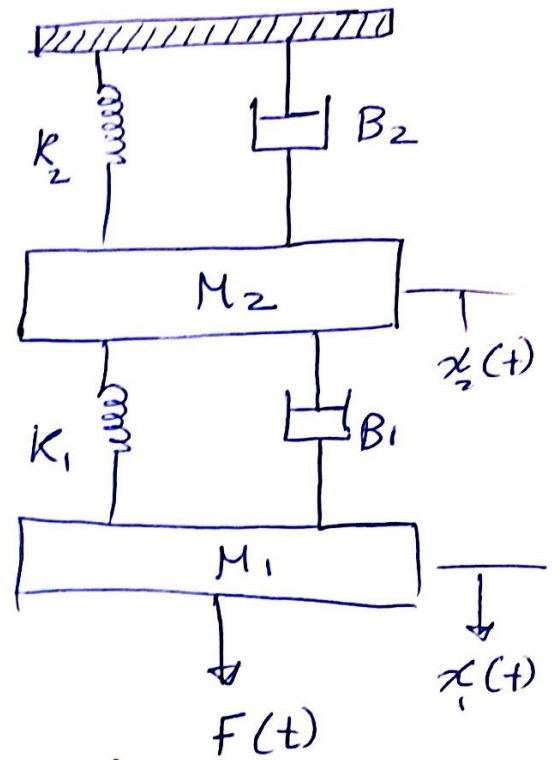
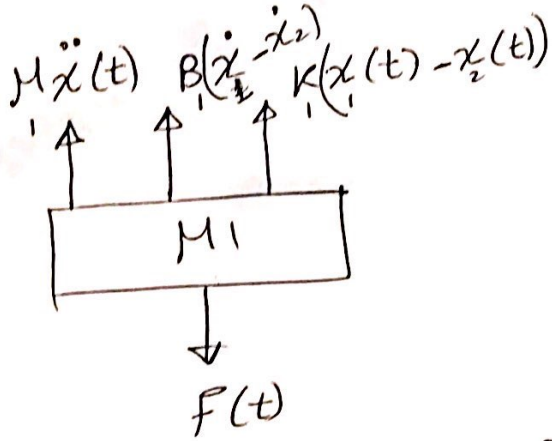
نصف المطلوب

the differential equation that describe the system:-

sol:-

* free body diagram

$F(t)$ and $x_1(t)$ and $x_2(t)$.



$$F(t) = M_1 \ddot{x}_1(t) + B_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2)$$

$$F(s) = M_1 s^2 x_1(s) + B_1 s x_1(s) - B_1 s x_2(s) + K_1 x_1(s) - K_1 x_2(s)$$

$$F(s) = (M_1 s^2 + B_1 s + K_1) x_1(s) + (-B_1 s - K_1) x_2(s) \quad \dots (1)$$

$$B_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2) = K_2 x_2 + B_2 \dot{x}_2 + M_2 \ddot{x}_2$$

$$B_1 s x_1(s) - B_1 s x_2(s) + K_1 x_1(s) - K_1 x_2(s) = K_2 x_2(s) + B_2 s x_2(s) + M_2 s^2 x_2(s)$$

$$(B_1 s + K_1) x_1(s) + (-B_1 s - K_1 - K_2 - B_2 s - M_2 s^2) x_2(s) = 0$$

$$(B_1 s + K_1) x_1(s) + (-M_2 s^2 + (-B_1 - B_2) s + (-K_1 - K_2)) x_2(s) = 0 \quad \dots (2)$$

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$$F(s) = (M_1 s^2 + \beta_1 s + K_1) x_1(s) + (-\beta_1 s - K_1) x_2(s) \quad \dots (1)$$

$$0 = (\beta_1 s + K_1) x_1(s) + (-M_2 s^2 + (-\beta_1 - \beta_2) s + (-K_1 - K_2)) x_2(s) \quad \dots (2)$$

وهو المطلوب