

Rotational Mechanical system T.F:-

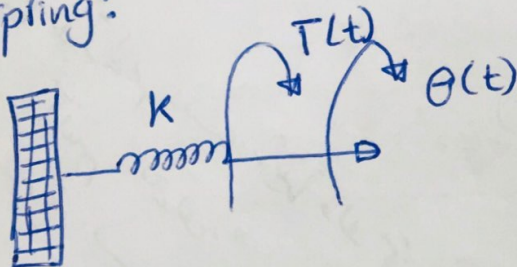
The motion about a fixed axis

$$\sum \text{torques} = J \alpha \quad \text{angular of acceleration}$$

↓
Inertia

$$J = \frac{1}{2} M r^2$$

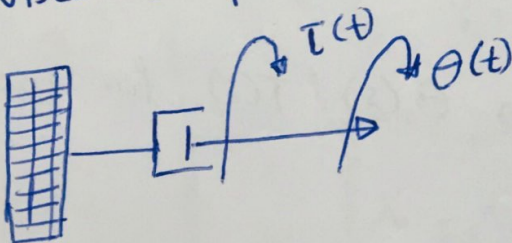
1 spring:



$$T(t) = K \theta(t)$$

« انظر الى هذا الشكل بعناية »
« سرعة زاوية على كرتة الدورانية »

2 viscous damper:

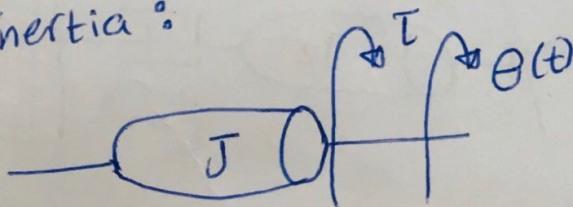


$$T(t) = \beta \dot{\theta}(t) = \beta \omega(t)$$

$$\dot{\theta}(t) = \omega(t) \Rightarrow \text{angular velocity}$$

↓
سرعة زاوية على كرتة الدورانية

3 Inertia:



$$T(t) = J \ddot{\theta}(t)$$

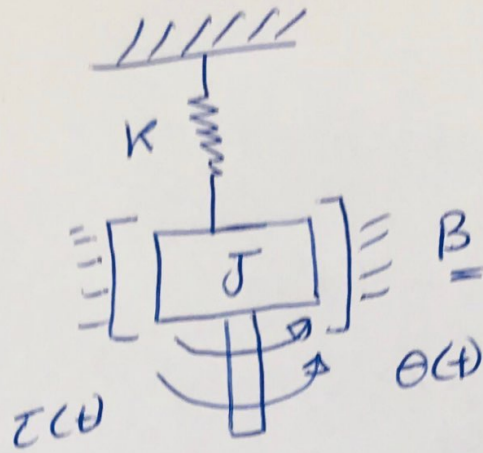
$$= J \alpha(t)$$

$$\alpha(t) = \ddot{\theta}(t)$$

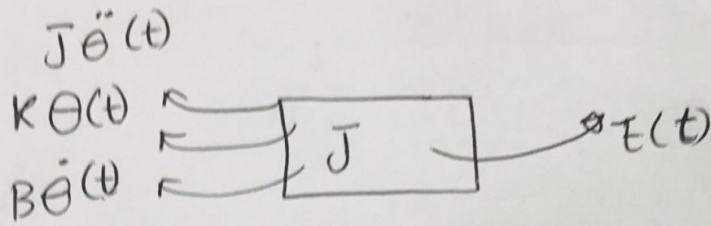
* Inertia: resistance of any physical object to change in its state of motion or rest

« مقاومة أي جسم فيزيائي للتغير في سرعته »

* Example :-



friction between the bob and air



$$\tau(t) = J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t)$$

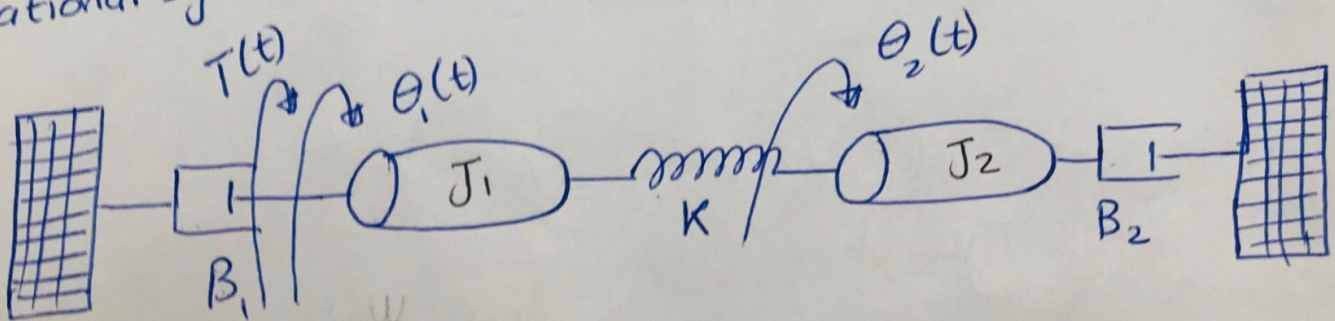
$$\tau(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s)$$

$$\tau(s) = \theta(s)(Js^2 + Bs + K)$$

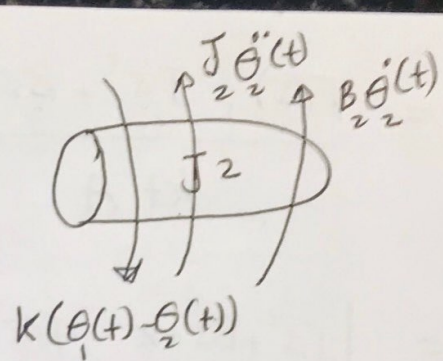
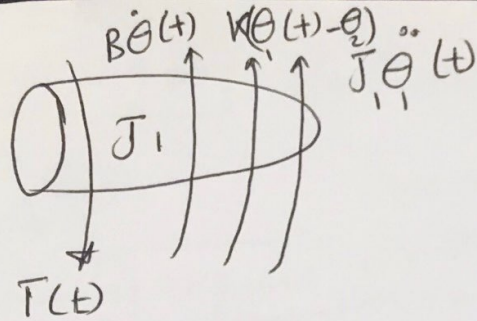
$$\left[\frac{\theta(s)}{\tau(s)} = \frac{1}{Js^2 + Bs + K} \right] \#$$

فاليوم لا نعلم نفه شيئاً ولا نعرفه
إلا ما نعلمه نعلمه
سوف يكون

Example: Find the transfer function $\theta_2(s) / T(s)$, for the rotational system shown below.



Sol :-



$$T(t) = B\dot{\theta}_1(t) + k(\theta_1(t) - \theta_2(t)) + J_1\ddot{\theta}_1(t)$$

$$T(s) = Bs\theta_1(s) + k\theta_1(s) - k\theta_2(s) + J_1s^2\theta_1(s)$$

$$T(s) = (J_1s^2 + Bs + k)\theta_1(s) - k\theta_2(s) \quad \dots \textcircled{1}$$

$$k(\theta_1(t) - \theta_2(t)) = J_2\ddot{\theta}_2(t) + B_2\dot{\theta}_2(t)$$

$$k\theta_1(s) - k\theta_2(s) = J_2s^2\theta_2(s) + B_2s\theta_2(s)$$

$$0 = -k\theta_1(s) + (J_2s^2 + B_2s + k)\theta_2(s) \quad \dots \textcircled{2}$$

$$\begin{bmatrix} T(s) \\ 0 \end{bmatrix} = \begin{bmatrix} J_1s^2 + Bs + k & -k \\ -k & J_2s^2 + B_2s + k \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix}$$

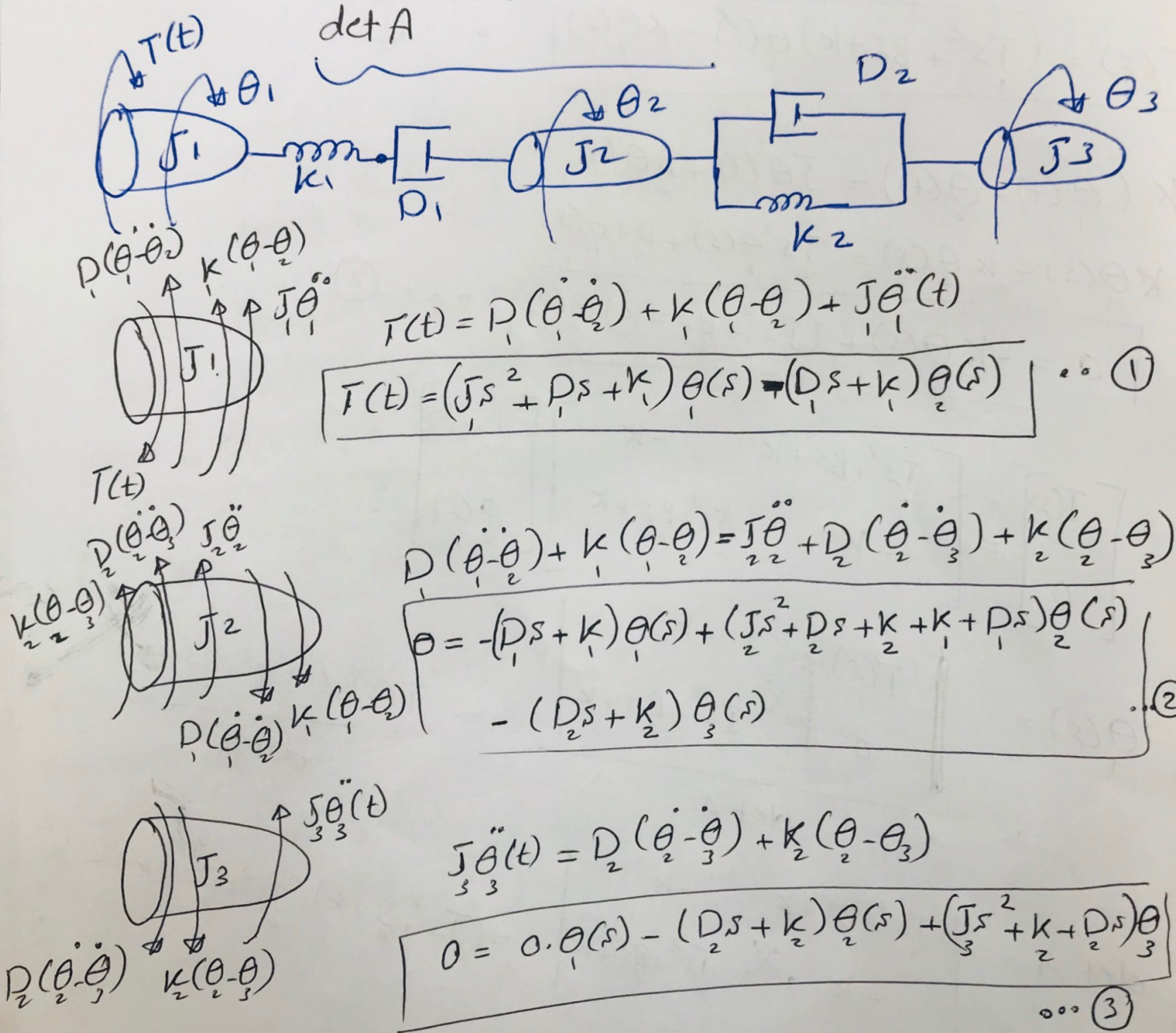
$$\theta_1(s) = \frac{\begin{vmatrix} T(s) & -k \\ 0 & J_2s^2 + B_2s + k \end{vmatrix}}{\det A} = \frac{T(s)(J_2s^2 + B_2s + k)}{\det A}$$

$$\det A = \begin{vmatrix} J_1s^2 + Bs + k & -k \\ -k & J_2s^2 + B_2s + k \end{vmatrix} = (J_1s^2 + Bs + k)(J_2s^2 + B_2s + k) - k^2$$

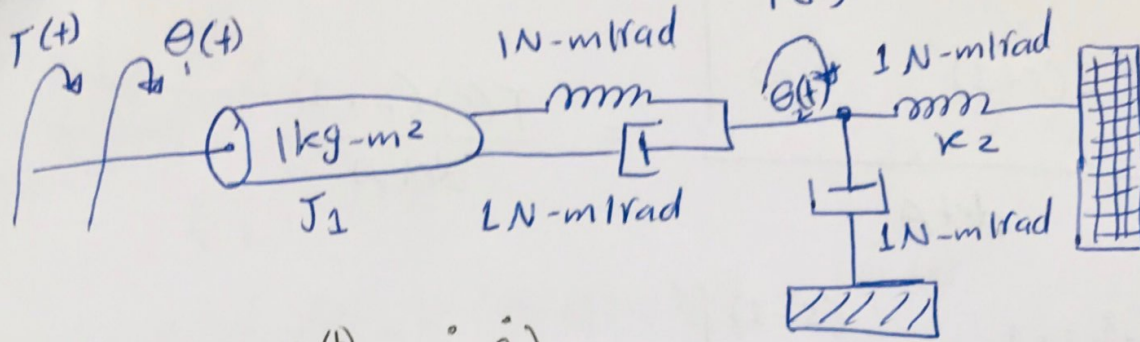
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$$\theta_1(s) = \frac{T(s) (J_2 s^2 + B_2 s + k)}{\det A}$$

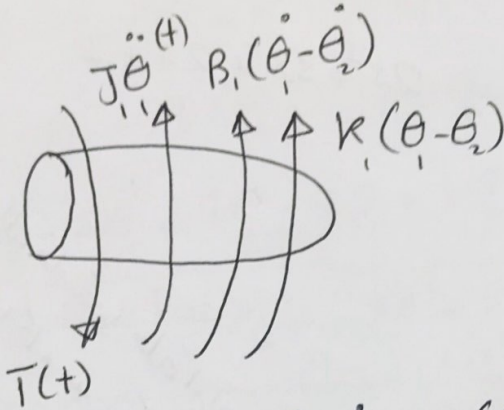
$$\theta_2(s) = \frac{\begin{vmatrix} J_1 s^2 + B_1 s + k & T(s) \\ -k & 0 \end{vmatrix}}{\det A} = \frac{k T(s)}{\det A}$$



Exercise 2.9 # Find the $G(s) = \frac{\Theta_2(s)}{T(s)}$



So:-



$$T(t) = J_1 \ddot{\theta}_1(t) + B_1 (\dot{\theta}_1 - \dot{\theta}_2) + K_1 (\theta_1 - \theta_2)$$

$$T(s) = J_1 s^2 \Theta_1(s) + B_1 s \Theta_1(s) - B_1 s \Theta_2(s) + K_1 \Theta_1(s) - K_1 \Theta_2(s)$$

$$T(s) = (J_1 s^2 + B_1 s + K_1) \Theta_1(s) + (-B_1 s - K_1) \Theta_2(s)$$

$$T(s) = (s^2 + s + 1) \Theta_1(s) + (-s - 1) \Theta_2(s) \quad \dots \quad (1)$$

$$B_1 (\dot{\theta}_1 - \dot{\theta}_2) + K_1 (\theta_1 - \theta_2) = B_2 \dot{\theta}_2 + K_2 \theta_2$$

$$B_1 s \Theta_1(s) - B_1 s \Theta_2(s) + K_1 \Theta_1(s) - K_1 \Theta_2(s) = B_2 s \Theta_2(s) + K_2 \Theta_2(s)$$

$$s \Theta_1(s) - s \Theta_2(s) + \Theta_1(s) - \Theta_2(s) = s \Theta_2(s) + \Theta_2(s)$$

$$-(s+1) \Theta_1(s) + (2s+2) \Theta_2(s) = 0 \quad \dots \quad (2)$$

أفترض به الله سبحانه وتعالى
السمو، الأيمن طوعاً وكراهة
آل عمران.

$$\Theta_2(s) = \frac{\begin{vmatrix} s^2 + s + 1 & T(s) \\ -(s+1) & 0 \end{vmatrix}}{\det A} = \frac{T(s)(s+1)}{\det A}$$

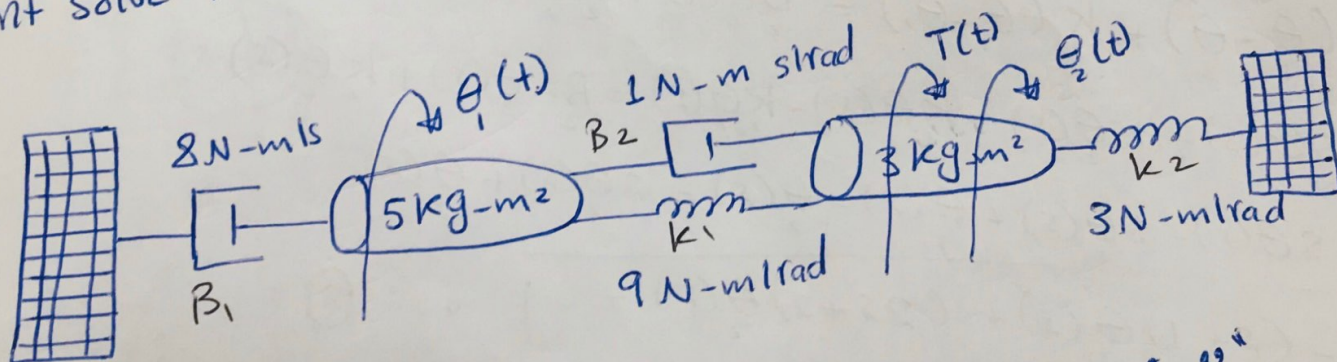
$$\det A = \begin{vmatrix} s^2 + s + 1 & -(s+1) \\ -(s+1) & (2s+2) \end{vmatrix} = 2s^3 + 3s^2 + 2s + 1$$

$$\Theta_2(s) = \frac{T(s)(s+1)}{2s^3 + 3s^2 + 2s + 1}$$

$$\left| \frac{\Theta_2(s)}{T(s)} = \frac{1}{2s^2 + s + 1} \right| \quad \# \text{ مطلوب}$$

«تدعوه الله فابعدوا عنه ابراهيم
حسيناً وما كان من القرية»
آل عمران -

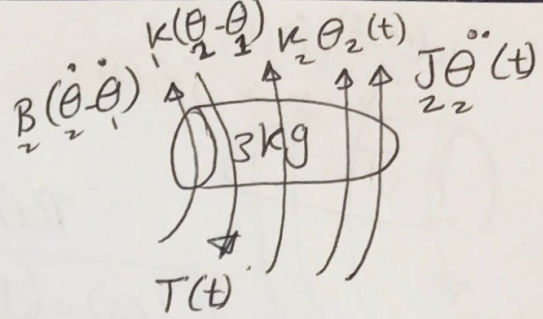
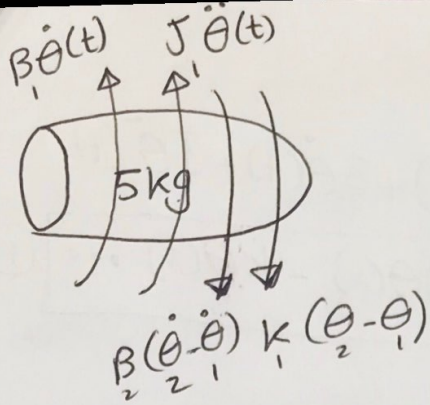
[30] For each of the rotational mechanical system, write but don't solve the equations of motion.



Sol: —

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توابع الأخرى
توابعها
آل عمران -

[6]



$$T(t) = B_2 (\dot{\theta}_2 - \dot{\theta}_1) + K_1 (\theta_2 - \theta_1) + K_2 \theta_2 + J_2 \ddot{\theta}_2(t)$$

$$T(t) = \ddot{\theta}_2 - \dot{\theta}_1 + 9(\theta_2 - \theta_1) + 3\theta_2 + 3\ddot{\theta}_2$$

$$T(s) = s\ddot{\theta}_2 - s\dot{\theta}_1 + 9\theta_2 - 9\theta_1 + 3\theta_2 + 3s^2\theta_2$$

$$T(s) = -(9+s)\theta_1(s) + (3s^2 + s + 12)\theta_2(s) \quad \dots (1)$$

$$B_2 (\dot{\theta}_2 - \dot{\theta}_1) + K_1 (\theta_2 - \theta_1) = B_1 \dot{\theta}_1(t) + J_1 \ddot{\theta}_1(t)$$

$$B_2 s \theta_2(s) - B_2 s \theta_1(s) + K_1 \theta_2(s) - K_1 \theta_1(s) = B_1 s \theta_1(s) + J_1 s^2 \theta_1(s)$$

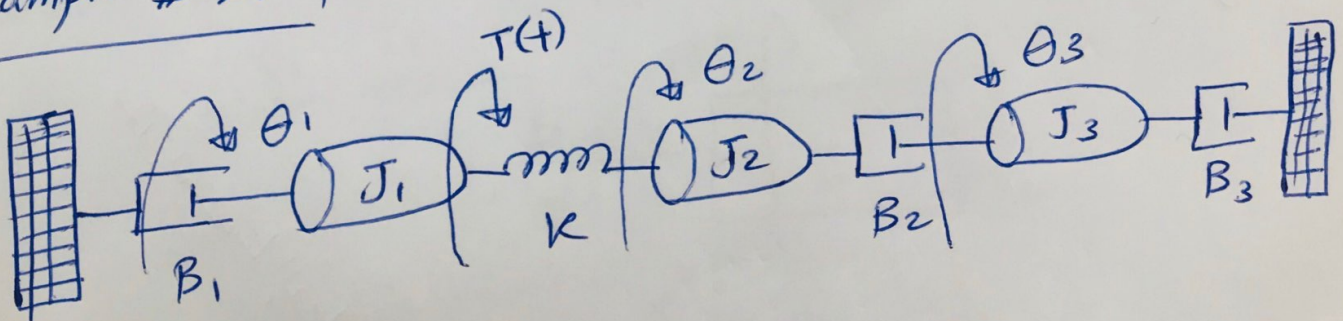
$$s \theta_2(s) - s \theta_1(s) + 9 \theta_2(s) - 9 \theta_1(s) = 8s \theta_1(s) + 5s^2 \theta_1(s)$$

$$(5s^2 + 8s + s + 9) \theta_1(s) - (s + 9) \theta_2(s) = 0$$

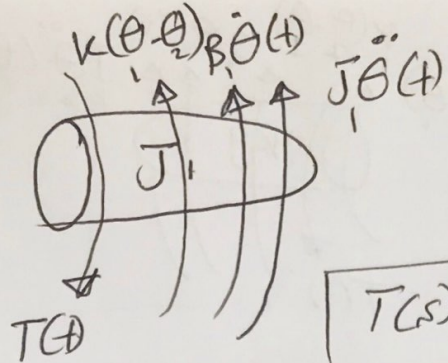
$$(5s^2 + 9s + 9) \theta_1(s) - (s + 9) \theta_2(s) = 0 \quad \dots (2)$$

Example # 2.201

Find the equations, without solve :-

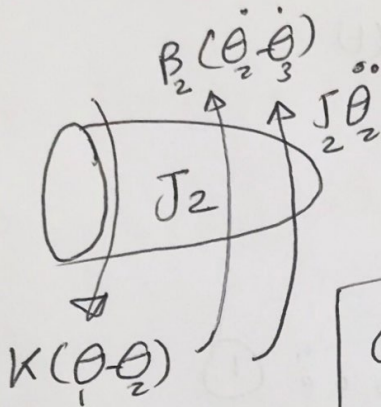


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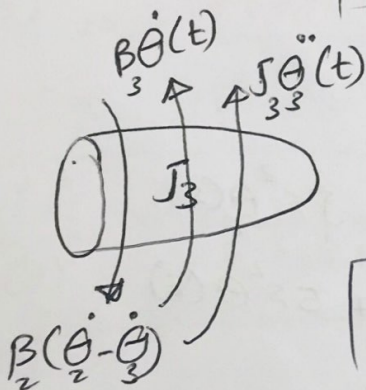
$$T(t) = k(\theta_1 - \theta_2) + B_1 \dot{\theta}_1 + J_1 \ddot{\theta}_1$$

$$T(s) = (J_1 s^2 + B_1 s + k) \theta_1(s) - k \theta_2(s) \dots \textcircled{1}$$



$$k(\theta_1 - \theta_2) = B_2(\dot{\theta}_1 - \dot{\theta}_2) + J_2 \ddot{\theta}_2$$

$$0 = -k \theta_1(s) + (J_2 s^2 + B_2 s + k) \theta_2(s) - B_2 s \theta_1(s) \dots \textcircled{2}$$



$$B_2(\dot{\theta}_2 - \dot{\theta}_3) = B_3 \dot{\theta}_3 + J_3 \ddot{\theta}_3$$

$$0 = -B_2 s \theta_2(s) + (J_3 s^2 + B_2 s + B_3 s) \theta_3(s) \dots \textcircled{3}$$

$$T(s) = (J_1 s^2 + B_1 s + k) \theta_1(s) - k \theta_2(s) + 0 \theta_3(s)$$

$$0 = -k \theta_1(s) + (J_2 s^2 + B_2 s + k) \theta_2(s) - B_2 s \theta_3(s)$$

$$0 = 0 \cdot \theta_1(s) - B_2 s \theta_2(s) + (J_3 s^2 + B_2 s + B_3 s) \theta_3(s)$$

المعادلات الثلاثة

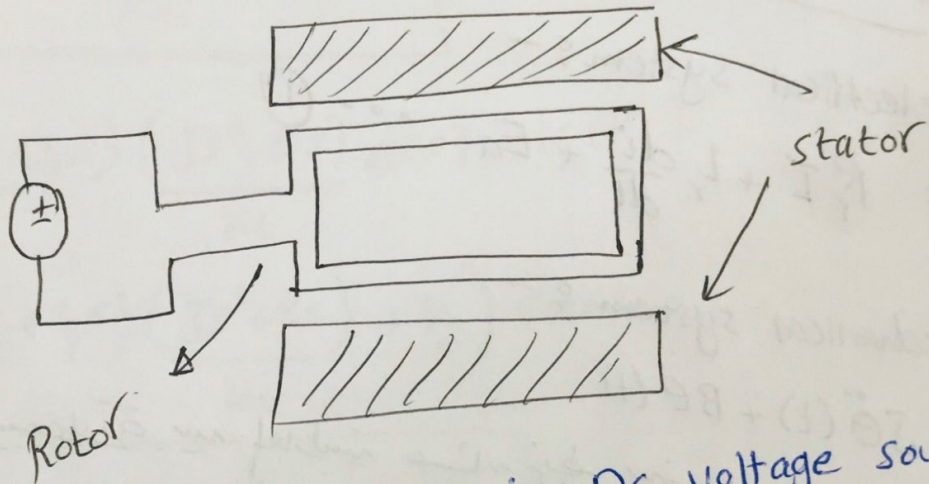
من درجتها الأولى بالله ناه

* Modeling for electromechanical system: —

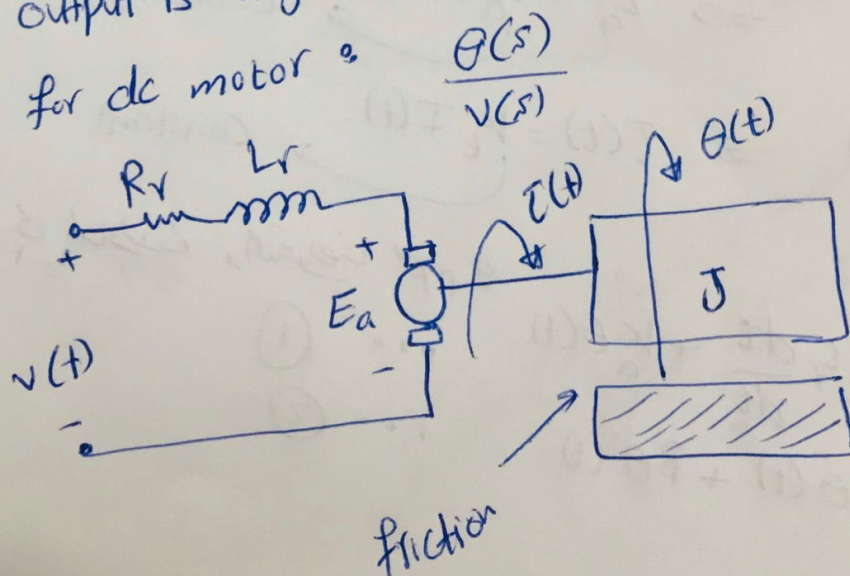
Example # Dc-motor:

Dc motor consists of two parts

- stator: stationary part of the motor
" The outer shell " which contain the magnetic source
- rotor: which lies inside the stator and consists of a metal core and covered by wires, and the dc source connected to the end of the wire.



* for Dc-motor, the input is Dc voltage source, and the output is angular displacement $\theta(t)$, so the transfer function for dc motor is $\frac{\theta(s)}{V(s)}$

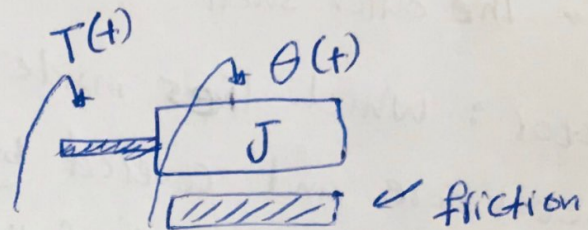
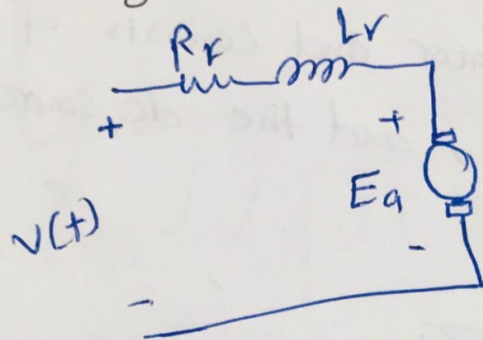


R_r : resistance rotor

L_r : inductance rotor

E_a : the applied voltage on the rotor.

* To find the transfer function, we will separate it into electrical system and mechanical system.



for electrical system :-

$$V(t) = R_r I + L_r \frac{di}{dt} + E_a \dots \textcircled{1}$$

for mechanical system :-

$$T(t) = J \ddot{\theta}(t) + B \dot{\theta}(t)$$

نحاول في هذه العلاقة مع المعادلات على أن نربط بين

for dc motor :

$$E_a \propto \theta(t)$$

$$\Rightarrow E_a = K_a \theta(t) \quad \text{Constant}$$

$$T(t) \propto I(t)$$

$$\Rightarrow T(t) = K_t I(t) \quad \text{Constant}$$

الربط بين المعادلات، المعادلات هي

$$\rightarrow V(t) = R_r I + L_r \frac{dI}{dt} + K_a \theta(t) \dots \textcircled{1}$$

$$\rightarrow K_t I(t) = J \ddot{\theta}(t) + B \dot{\theta}(t) \dots \textcircled{2}$$

$$L(t) = \frac{J\ddot{\theta}(t) + B\dot{\theta}(t)}{k_t}$$

② معادلة

$$V(s) = R_f I(s) + L_f s I(s) + k_a \theta(s)$$

$$V(s) = (R_f + L_f s) I(s) + k_a \theta(s) \quad \dots \textcircled{A}$$

$$I(s) = \frac{Js^2 \theta(s) + Bs \theta(s)}{k_t}$$

$$I(s) = \frac{(Js^2 + Bs) \theta(s)}{k_t} \quad \dots B$$

$$V(s) = (R_f + L_f s) \frac{(Js^2 + Bs) \theta(s)}{k_t} + k_a \theta(s)$$

$$V(s) = \left((R_f + L_f s) \frac{(Js^2 + Bs)}{k_t} + k_a \right) \theta(s)$$

$$\frac{\theta(s)}{V(s)} = \frac{1}{(R_f + L_f s) \frac{(Js^2 + Bs)}{k_t} + k_a}$$

وهو المطلوب

ما أظنك في محبة في الأرض ولا في أفق
إلا في كتاب في جبل أم بين أها
سورة الحديد

linear system $\rightarrow$ linear $\rightarrow$ non-linear system
 Linearization $\rightarrow$ system $\rightarrow$ linear $\rightarrow$ non-linear system

* Linear system possesses two properties :

$\Rightarrow$ superposition, homogenous.