



RCM Engineering Team “ Branch of the Islamic University ”

Chapter 3

for : Control Systems – EELE 3360



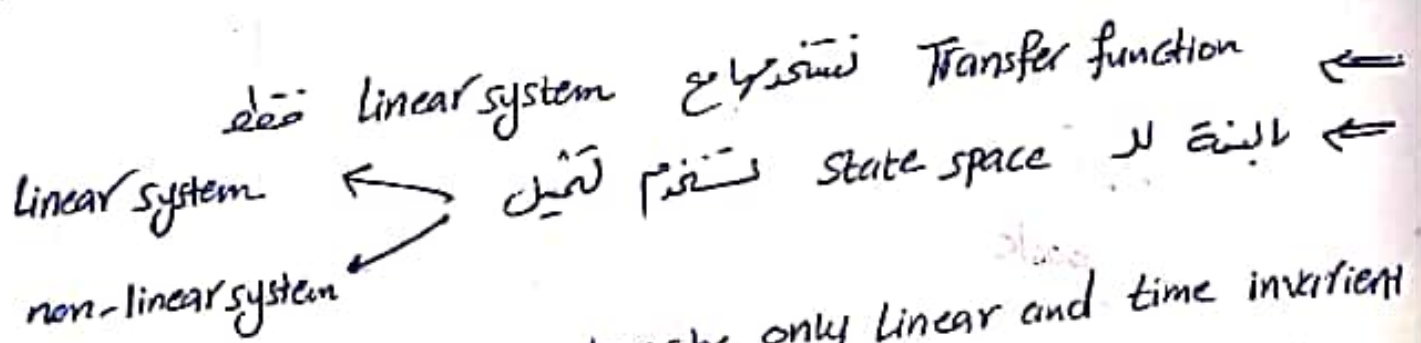
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The second semester of the academic year 2020 - 2021

* Objectives of this chapter :-

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- 1) Find mathematical model "state space model" for linear, time invariant system.
- 2) electrical and mechanical system in state space.
- 3) Convert transfer function to state space.
- 4) Convert state space to transfer function.
- 5) linearization state space representation.



non-linear system

- * Transfer function can describe only linear and time invariant system and that system must be single input, single output.
- * State space Model describes linear and non-linear, time invariant and time variant and any type of system.
Multi-input, Multi-output or SISO "single input, single output"

S.S // $\frac{1}{s}$
time domain $\rightarrow$ $\frac{1}{s}$
Modeling in time domain
T.F $\rightarrow$ s-domain

« مثل الزبد ينقصون أرواحهم في جحيمٍ إلى كل حين
لم ينبت سبع سنابل في كل حيناً منه حبة
والله راضٍ عنهم في النار ، والله واسعٌ عليمٌ »
سورة البقرة -

* In state space Model if we have n -th order differential Equation we have to find n -Equation with first order only.

* State space Model consists of two matrix Equations:-

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

"state equation"

"output equation"

$x \Rightarrow$ state vector.

$\dot{x} \Rightarrow$ derivative of the state vector.

$y \Rightarrow$ output vector.

$u \Rightarrow$ input or control vector.

$A \Rightarrow$ system matrix.

$B \Rightarrow$ input matrix.

$C \Rightarrow$ output matrix.

$D \Rightarrow$ feedforward matrix.

Applying the state space representation

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

* number of state variables = number of order system (n th order system)

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{np} \end{bmatrix}$$

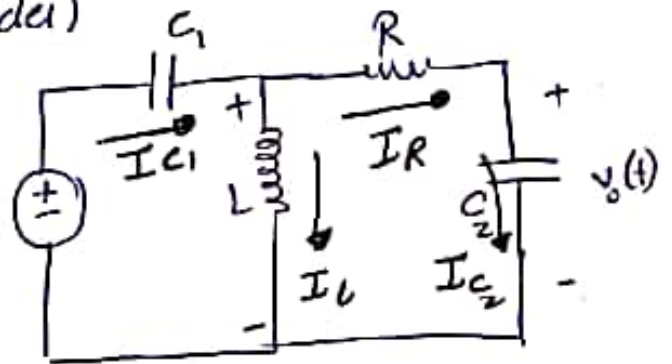
* The input and output system are given in the system

وَمَا تَوْفِيقِي إِلَّا بِاللَّهِ عَلَيْهِ تَوَكَّلْتُ وَإِلَيْهِ أُنِيبُ
وَلَا يَغْنِبُهَا اللَّهُ يَوْمَ الْقِيَامَةِ
سَوَاءٌ الْبَرَّةُ

There are two methods to find the state variables:

- 1] A minimum number of state variable must be selected as component of the state vector
- 2] The component of the state vector must be linearly independent.
(Can't be written as linear combination of each other)

Example #1 Find the state space of the electrical network if the output is volt. (Electrical Model)



Sol: اول خطوة بنا كذا عدد ال state variable - حيت في عدد ال state variable يساوي عدد ال component اجزاء الدائرة مثل العلاقات والمكونات.

3 State variable يساوي ← يوجد لنا مكونين في
state variable voltage $v(t)$ in capacitor
current $i(t)$ in inductor.

عنا جنب مقاومة ال state variable بنا جنب الجهد على الحث
والتيار في الحث (بنا كذا نستفيد منهم على جنب الحث)
مع مقاومة ان:

$$V_L = L \frac{di}{dt}$$

$$I_C = C \frac{dv}{dt}$$

* احنا بنا مشتقة التيار، مشتقة الجهد
state variable S

$$\underline{V_L} = L \frac{di}{dt}, \quad \underline{I_{C1}} = C_1 \frac{dv_1}{dt}, \quad \underline{I_{C2}} = C_2 \frac{dv_2}{dt}$$

(i_L, v_{C1}, v_{C2}) state variable الى نه عله ←

$$* I_{C1} = I_L + I_{C2}$$

$$I_{C1} = I_L + \frac{V_L - v_{C2}}{R} \Rightarrow \boxed{I_{C1} = I_L + \frac{V_L}{R} - \frac{v_{C2}}{R}} \quad \dots (*)$$

$$* I_{C2} = I_R = \frac{V_L - v_{C2}}{R} \Rightarrow \boxed{I_{C2} = \frac{V_L}{R} - \frac{v_{C2}}{R}} \quad \dots (**)$$

$$* V_L = v_i - v_{C1}$$

$$L \frac{di}{dt} = v_i - v_{C1} \Rightarrow \boxed{\frac{di}{dt} = \frac{v_i}{L} - \frac{v_{C1}}{L}} \quad \dots (***)$$

$$I_{C1} = I_L + \frac{V_L}{R} - \frac{v_{C2}}{R}$$

← (*) متعلقة به

$$C_1 \frac{dv_1}{dt} = I_L + \frac{V_L}{R} - \frac{v_{C2}}{R}$$

كل واحد في صيغة متعلقة بالآخر

State variable الى نه

$$\boxed{\frac{dv_1}{dt} = \frac{I_L}{C_1} + \frac{v_i - v_{C1}}{RC_1} - \frac{v_{C2}}{RC_1}} \quad \dots (1)$$

$$V_L = v_i - v_{C1}$$

$$I_{C2} = \frac{V_L}{R} - \frac{v_{C2}}{R}$$

(**) متعلقة به

$$C_2 \frac{dv_2}{dt} = \frac{v_i}{R} - \frac{v_{C1}}{R} - \frac{v_{C2}}{R} \Rightarrow \boxed{\frac{dv_2}{dt} = \frac{v_i}{RC_2} - \frac{v_{C1}}{RC_2} - \frac{v_{C2}}{RC_2}} \quad \dots (2)$$

$$\boxed{\frac{di}{dt} = \frac{v_i}{L} - \frac{v_{C1}}{L}} \quad \dots (3)$$

$$\boxed{v_o = v_{C2}}$$

→ output الى نه

(4)

$$\frac{di_L}{dt} = \frac{I_L}{C_1} + \frac{V_i}{RC_1} - \frac{V_{C1}}{RC_1} - \frac{V_{C2}}{RC_1} \dots (1)$$

$$\frac{dV_2}{dt} = \frac{V_i}{RC_2} - \frac{V_{C1}}{RC_2} - \frac{V_{C2}}{RC_2} \dots (2)$$

$$\frac{di_L}{dt} = \frac{V_i}{L} - \frac{V_{C1}}{L} \dots (3)$$

$$V_o = V_{C2}$$

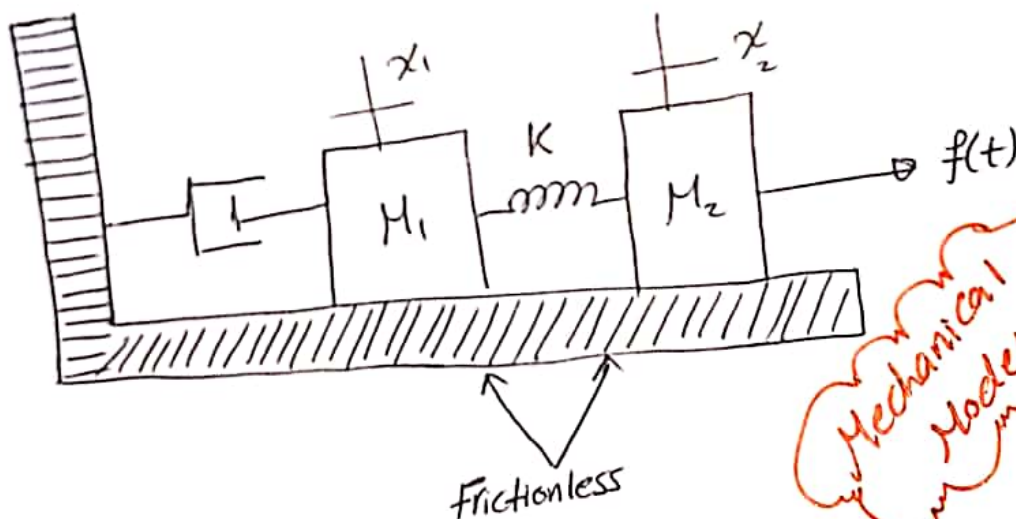
$$\begin{bmatrix} \dot{i}_L \\ \dot{V}_{C1} \\ \dot{V}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C_1} & -\frac{1}{RC_1} & -\frac{1}{RC_1} \\ 0 & -\frac{1}{RC_2} & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{RC_1} \\ \frac{1}{RC_2} \end{bmatrix} V_i$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_{C1} \\ V_{C2} \end{bmatrix} + 0$$

Example #

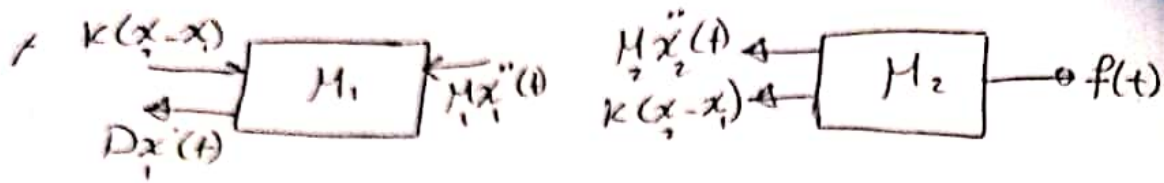
نموذج الكهر

Find the state equations for the translational mechanical system shown in figure?



Mechanical Model

5



B

$$f(t) = M_2 \ddot{x}_2(t) + kx_2 - kx_1$$

$$kx_2 - kx_1 = D \dot{x}_1(t) + M_1 \ddot{x}_1(t)$$

$$f(t) = M_2 \frac{d^2 x_2(t)}{dt^2} + kx_2(t) - kx_1(t) \quad \dots (*)$$

$$0 = M_1 \frac{d^2 x_1(t)}{dt^2} + D \frac{dx_1(t)}{dt} + kx_1(t) - kx_2(t) \quad \dots (**)$$

$\ddot{x}_1(t), \ddot{x}_2(t)$ ← State variable ← متغيرات الحالة
 $\downarrow$ $\downarrow$
 2-state variable 2-state variable
 متغيران الحالة متغيران الحالة

عدد 4 State variable واحد

$$\dot{x}_1(t) = v_1(t) \quad \dots (1)$$

$$\dot{x}_2(t) = v_2(t) \quad \dots (2)$$

$$\ddot{x}_2(t) = \dot{v}_2(t)$$

$$\ddot{x}_1(t) = \dot{v}_1(t)$$

$$f(t) = M_2 \dot{v}_2(t) + kx_2(t) - kx_1(t)$$

$$M_2 \dot{v}_2(t) = f(t) - kx_2(t) + kx_1(t)$$

$$\dot{v}_2(t) = \frac{f(t)}{M_2} - \frac{kx_2(t)}{M_2} + \frac{kx_1(t)}{M_2} \quad \dots (3)$$

المجموع في معادلة (*) < (**) (*)

6

$$0 = M_1 \dot{v}_1(t) + D v_1(t) + k x_1(t) - k x_2(t)$$

$$M_1 \dot{v}_1(t) = -D v_1(t) - k x_1(t) + k x_2(t)$$

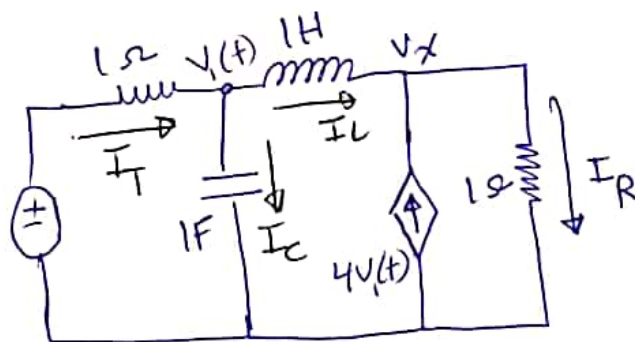
$$\dot{v}_1(t) = -\frac{D}{M_1} v_1(t) - \frac{k}{M_1} x_1(t) + \frac{k}{M_1} x_2(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/M_1 & -D/M_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ k/M_2 & 0 & -k/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ v_1(t) \\ x_2(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

* Represent the electrical network in state space where I_R is the output.

Sol:

عدد ال S.S. عدد ال
النظام الختمة للطاقة



* Current in inductor $I_L(t)$
* Voltage in capacitor $v_C(t)$

$$\frac{I}{C} = C \frac{dv_C}{dt}, \quad v_L = L \frac{di}{dt}$$

$$\frac{I}{T} = \frac{I}{C} + I_L, \quad \frac{I}{C} = \frac{I}{T} - I_L$$

$$\frac{I}{C} = C \frac{dv_C}{dt}$$

$$\frac{I}{T} - I_L = C \frac{dv_C}{dt} \Rightarrow \frac{dv_C}{dt} = \frac{I}{T} - \frac{I_L}{C}$$

$$\frac{I}{T} = \frac{v_i - v_1}{R} = \frac{v_i - v_1}{1}$$

فاخير على ما هو مطلوب
نفسه ال
وغيره
عنه
فانظر على ما هو مطلوب
نفسه ال
وغيره
عنه

$$\frac{dv_c}{dt} = \frac{v_i - v_1}{C} - \frac{I_L}{C}$$

$$\left[\frac{dv_c}{dt} = \frac{v_i}{C} - \frac{v_c}{C} - \frac{I_L}{C} \right] \quad \text{و. 1}$$

$$v_1 = v_c \quad \text{لأنه}$$

$$v_L = L \frac{di}{dt}$$

$$v_1 - \frac{I}{R} = L \frac{di}{dt}$$

$$v_1 - 4v_1 - \frac{I}{L} = L \frac{di}{dt}$$

$$-3v_1 - \frac{I}{L} = L \frac{di}{dt}$$

$$-\frac{3v_c}{L} - \frac{I_L}{L} = \frac{di_L}{dt}$$

$$v_1 = v_c$$

$$\Rightarrow \left[\frac{di_L}{dt} = -\frac{3v_c}{L} - \frac{I_L}{L} \right]$$

$$\begin{bmatrix} \dot{I}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{1}{L} & -\frac{3}{L} \\ -\frac{1}{C} & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} v_i(t)$$

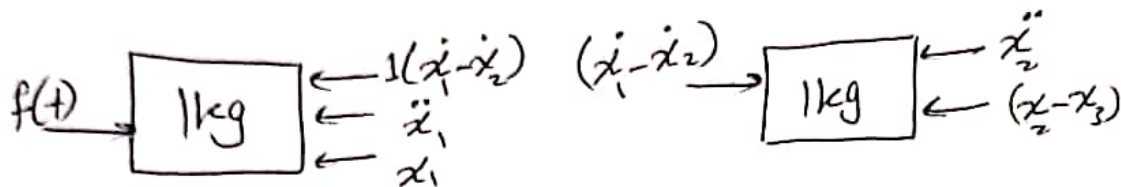
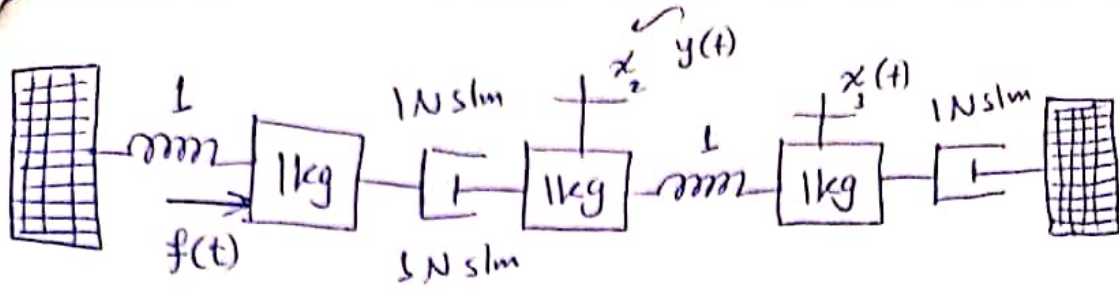
$$y(t) = \frac{I}{R} = 4v_1(t) + \frac{I}{L}(t)$$

$$y(t) = 4v_c + I_L$$

$$y = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix}$$

"ادخلوها بسلام ذلك يوم الخلود"

Example # | Find the state equation of the mechanical system



$$f(t) = \ddot{x}_1 - \dot{x}_2 + \ddot{x}_1 + x_1 \quad \dots (1)$$

$$\ddot{x}_1 - \dot{x}_2 = \ddot{x}_2 + x_2 - x_3 \quad \dots (2)$$

$$0 = -\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_2 + x_2 - x_3 \quad \dots (3)$$

* The highest order are $\ddot{x}_1, \ddot{x}_2, \ddot{x}_3$ 3 متغيرات
6-State variable $\ddot{x}_1, \ddot{x}_2, \ddot{x}_3$ كل متغير 2-state variable

$$y(t) = x_3(t) = v_5$$

اعتراف للناس حياهم ومعهم في
نظرة مدونة
سورة الانبياء

دالة الرمز v لـ
state variable

$$\begin{aligned} v_1 &= x_1, & v_2 &= \dot{x}_1 = v_1 \\ v_3 &= x_2, & v_4 &= \dot{x}_2 = v_3 \\ v_5 &= x_3, & v_6 &= \dot{x}_3 = v_5 \end{aligned}$$

$$\begin{aligned} f(t) &= v_2 - v_4 + v_2 + v_1 \\ \dot{v}_2 &= f(t) + v_4 - v_2 - v_1 \end{aligned}$$

بالجمع في المعادلات //

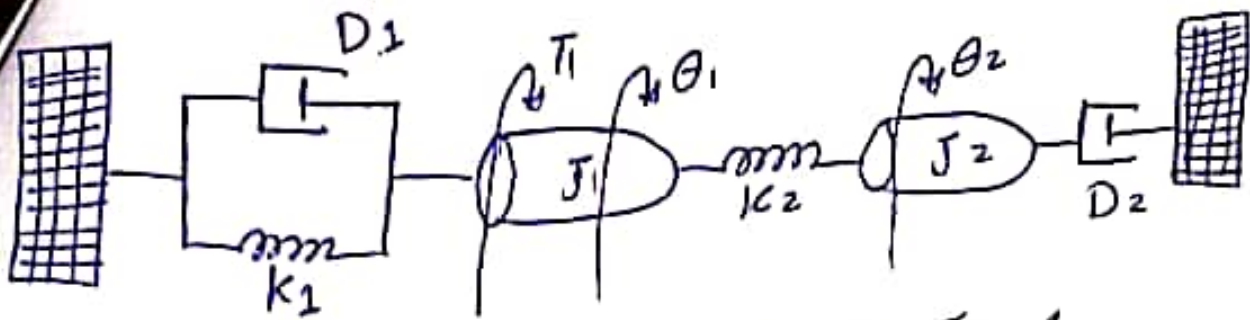
$$\begin{aligned} 0 &= -v_2 + v_4 + v_4 + v_3 - v_5 \\ \dot{v}_4 &= v_2 - v_3 - v_4 + v_5 \end{aligned}$$

$$\begin{aligned} 0 &= -v_3 + v_5 + v_6 + v_6 \\ \dot{v}_6 &= v_3 - v_5 - v_6 \end{aligned}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \\ \dot{v}_5 \\ \dot{v}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t)$$

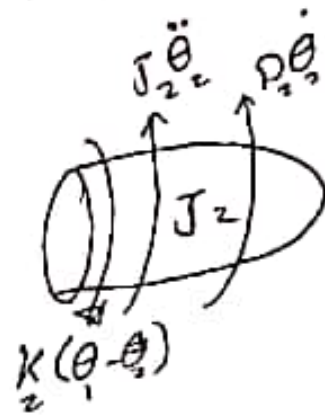
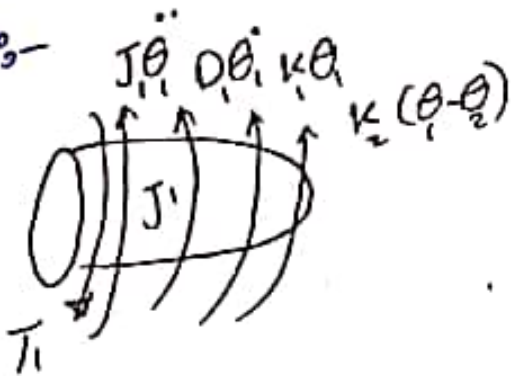
$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Find the state space equation for the mechanical Model:-



Given: $D_1 = D_2 = 1$, $k_1 = k_2 = 1$, $J_1 = J_2 = 1$

Sol:-



$$T_1(t) = J_1 \ddot{\theta}_1 + D_1 \dot{\theta}_1 + k_1 \theta_1 + k_2 \theta_1 - k_2 \theta_2$$

$$\boxed{T_1(t) = \ddot{\theta}_1 + \dot{\theta}_1 + \theta_1 + \theta_1 - \theta_2} \dots \textcircled{1}$$

$$k_2(\theta_1 - \theta_2) = J_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2$$

$$\boxed{0 = -k_2 \theta_1 + k_2 \theta_2 + J_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2} \Rightarrow \boxed{0 = -\theta_1 + \theta_2 + \ddot{\theta}_2 + \dot{\theta}_2} \dots \textcircled{2}$$

$\Rightarrow$ two variable $\ddot{\theta}_1, \ddot{\theta}_2 \Rightarrow$ four state space

$$v_1 = \theta_1, \quad v_1' = \dot{\theta}_1 = v_3$$

$$v_2 = \theta_2, \quad v_2' = \dot{\theta}_2 = v_4$$

$$v_3 = \dot{\theta}_1$$

$$v_4 = \dot{\theta}_2$$

بسم الله الرحمن الرحيم
الحمد لله الذي هدانا لهذا
الذي كنا لنهتدي لاه
للسقوى العظيم
سورة البقرة

$$\tau_1(t) = \ddot{\theta}_1 + \dot{\theta}_1 + 2\theta_1 + \theta_2 \quad \dots \textcircled{1}$$

$$0 = -\theta_1 + \theta_2 + \ddot{\theta}_2 + \dot{\theta}_2 \quad \dots \textcircled{2}$$

س.س مع ① من معادلة

$$\tau_1(t) = \dot{v}_3 + v_3 + 2v_1 - v_2$$

$$\Rightarrow \boxed{\dot{v}_3 = \tau(t) - v_3 - 2v_1 + v_2}$$

$$0 = -v_1 + v_2 + \dot{v}_4 + v_4$$

$$\Rightarrow \boxed{\dot{v}_4 = v_1 - v_2 - v_4}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tau(t)$$

#

converting the transfer function to state space:-

There are three ways:

- 1] Direct Decomposition.
- 2] Cascade Decomposition.
- 3] Parallel Decomposition.

* Direct Decomposition //

توضيح خطوات كل:

1) أول خطوة حدد درجة المقام لأننا نبدأ مع درجة المقام حدد عدد

2) انحلل المقام معادلة وحيدة غير مفككة ولو كانت مفككة للعوامل وفقاً
state variables

3) جعل المقادير أي s^{-n} من خلال سعة البسط، المقام على أكبر
أس في المقام.

$$\frac{x(t)}{x(t)}$$

4) نخرج T.F ب dummy variable

5) نرسم state diagram للمعادلة الأخيرة وذلك بعدة خطوات:

- نجد أن input, output ونحدد عدد ال integrator $(\frac{1}{s})$

وعددها يكون على عدد ال state variable

- نجد أن input, output لكل integrator

- نحدد تيم المقادير مع معادلة ال input

"راستقنوا بالصبر والصلابة، انما
كثرة الالام الحاسية"
- السيرة -

Example # | Convert T.F to S.S by using Direct

$$G(s) = \frac{5(s+1)}{s(s+2)(s+10)}$$

Sol: $G(s) = \frac{5(s+1)}{s^3 + 12s^2 + 20s}$

3 poles $\rightarrow$ State variable 3 عدد $\leftarrow$ 3 پoles

$$G(s) = \frac{5s + 5}{s^3 + 12s^2 + 20s}$$

قسمت على اولی مرتبه پoles

$$\frac{Y(s)}{R(s)} = G(s) = \frac{5s^{-2} + 5s^{-3}}{1 + 12s^{-1} + 20s^{-2}} \cdot \frac{X(s)}{X(s)}$$

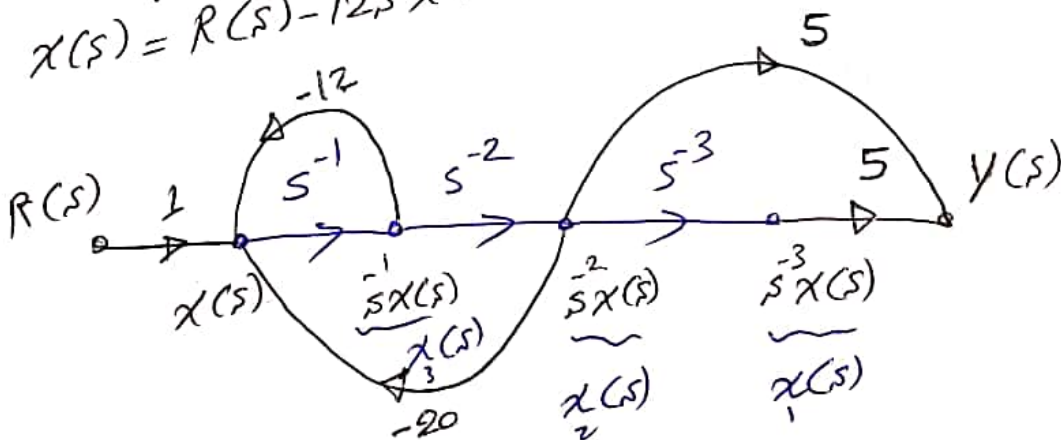
$$Y(s) = (5s^{-2} + 5s^{-3}) \cdot X(s)$$

$$R(s) = (1 + 12s^{-1} + 20s^{-2}) X(s)$$

$$\boxed{\begin{aligned} Y(s) &= 5s^{-2}X(s) + 5s^{-3}X(s) \\ R(s) &= X(s) + 12s^{-1}X(s) + 20s^{-2}X(s) \end{aligned}}$$

output 2 مرتبه
input 2 مرتبه

$$X(s) = R(s) - 12s^{-1}X(s) - 20s^{-2}X(s)$$



* آخر خطوة هي إيجاد متعة ال state variable من خلال لسميتك
 output لكل integrator

$$x_1(s) = s^{-3} x(s) \Rightarrow \dot{x}_1 = x_2$$

$$x_2(s) = s^{-2} x(s) \Rightarrow \dot{x}_2 = x_3$$

$$x_3(s) = s^{-1} x(s) \Rightarrow \dot{x}_3 = x(t) = R(t) - 12x_3(t) - 20x_2(t)$$

$$y(t) = 5x_2(t) + 5x_1(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -20 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Example #1 $\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$

T.F to SS
direct

Sol^o $\frac{24s^{-3}}{1 + 9s^{-1} + 26s^{-2} + 24s^{-3}} = \frac{C(s)}{R(s)}$

عدد ال state variable
 3 باري
 درجة المقام

$$\frac{C(s)}{R(s)} = \frac{24s^{-3}}{1 + 9s^{-1} + 26s^{-2} + 24s^{-3}} \cdot \frac{x(s)}{x(s)}$$

$$C(s) = 24s^{-3} x(s)$$

$$R(s) = (1 + 9s^{-1} + 26s^{-2} + 24s^{-3}) x(s)$$

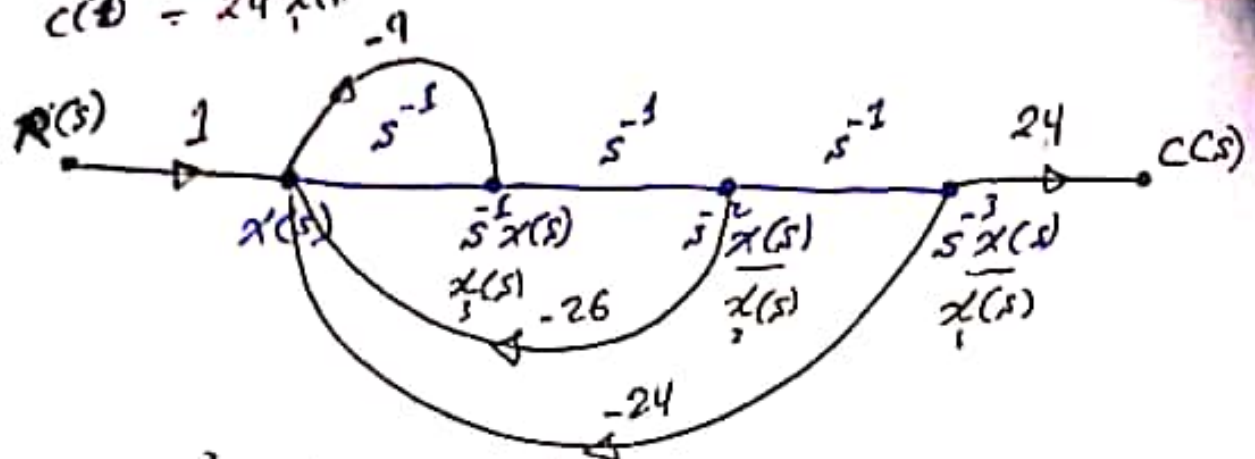
$$R(s) = x(s) + 9s^{-1} x(s) + 26s^{-2} x(s) + 24s^{-3} x(s)$$

15

$$X(s) = R(s) - 9s^{-1}X(s) - 26s^{-2}X(s) - 24s^{-3}X(s)$$

$$C(s) = 24s^{-3}X(s)$$

$$C(t) = 24x_1(t)$$



$$X_1(s) = s^{-3}X(s)$$

$$\Rightarrow \dot{x}_1 = x_2$$

$$X_2(s) = s^{-2}X(s)$$

$$\Rightarrow \dot{x}_2 = x_3$$

$$X_3(s) = s^{-1}X(s)$$

$$\Rightarrow \dot{x}_3 = x(t) = r(t) - 24x_1 - 26x_2 - 9x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 24 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

"والله اعلم" ولله الحجة
والله اعلم بالله والآخرين
والله اعلم بالصواب
والله اعلم بالبين
والله اعلم بالحق

$$[3] \quad H(s) = \frac{7}{s^2 + 9s + 11}$$

$$\text{sol: } \frac{C(s)}{R(s)} = \frac{7s^{-2}}{1 + 9s^{-1} + 11s^{-2}} \cdot \frac{X(s)}{X(s)}$$

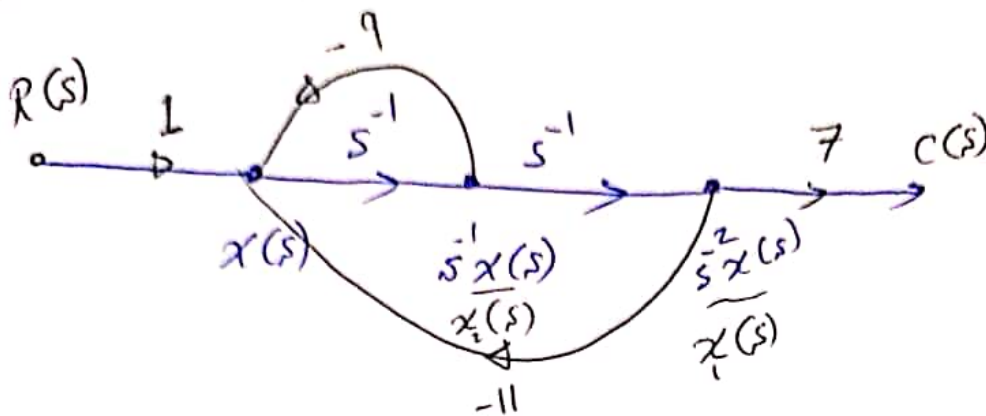
$$C(s) = 7s^{-2}X(s)$$

$$R(s) = X(s) + 9s^{-1}X(s) + 11s^{-2}X(s)$$

[16]

$$R(s) = R(s) - 9s^{-1}x(s) - 11s^{-2}x(s)$$

$$C(s) = 7s^{-2}x(s)$$



$$x_1(s) = s^{-2}x(s) \Rightarrow \dot{x}_1 = x_2$$

$$x_2(s) = s^{-1}x(s) \Rightarrow \dot{x}_2 = x(t) = r(t) - 9x_2(t) - 11x_1(t)$$

$$C(t) = 7x_1(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -11 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$C(t) = [7 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

* General form for direct //

$$\frac{C(s)}{R(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \dots + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$$

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_0 & -a_1 & \dots & -a_{n-1} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = [b_0 \quad b_1 \quad \dots \quad b_{n-1}] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + [0] r(t)$$

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
الحمد لله الذي هدانا لهذا
الذي كنا لنهتدي لولا أن هدانا الله

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
الحمد لله الذي هدانا لهذا
الذي كنا لنهتدي لولا أن هدانا الله

[7]

* By direct Method :-

$$H(s) = \frac{s+5}{s^2+9s+11}$$

Sol :-

$$H(s) = \frac{C(s)}{R(s)} = \frac{s^{-1} + 5s^{-2}}{1+9s^{-1}+11s^{-2}} = \frac{\chi(s)}{\chi(s)}$$

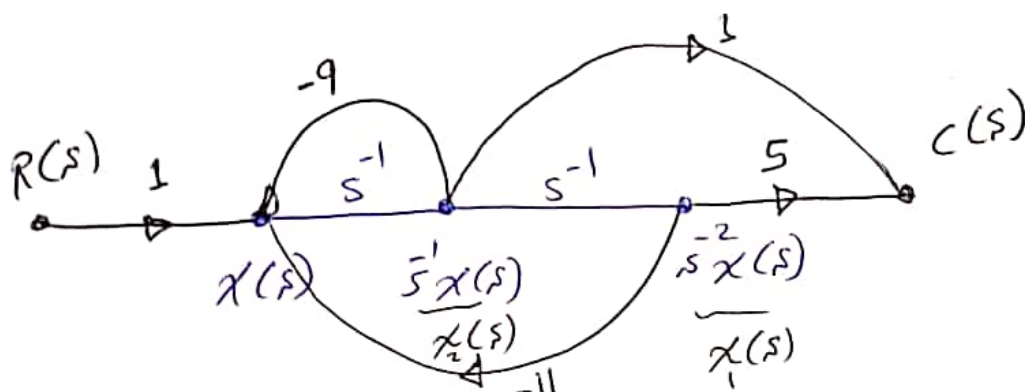
$$C(s) = (s^{-1} + 5s^{-2})\chi(s)$$

$$R(s) = (1+9s^{-1}+11s^{-2})\chi(s)$$

$$C(s) = s^{-1}\chi(s) + 5s^{-2}\chi(s)$$

$$R(s) = 1 - 9s^{-1}\chi(s) - 11s^{-2}\chi(s)$$

ماضوا بالله واسئله وانفقوا ما
حطلكم مستغلبه فيه نالينه
ماضوا منكم وانفقوا لهم
زجر الكبر
سوء الكبر



$$\chi_1(s) = s^{-2}\chi(s) \Rightarrow \dot{\chi}_1 = \chi_2$$

$$\chi_2(s) = s^{-1}\chi(s) \Rightarrow \dot{\chi}_2 = \chi(t) = r(t) - 9\chi_2 - 11\chi_1$$

$$C(t) = \chi_2(t) + 5\chi_1(t)$$

$$\begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -11 & -9 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}$$

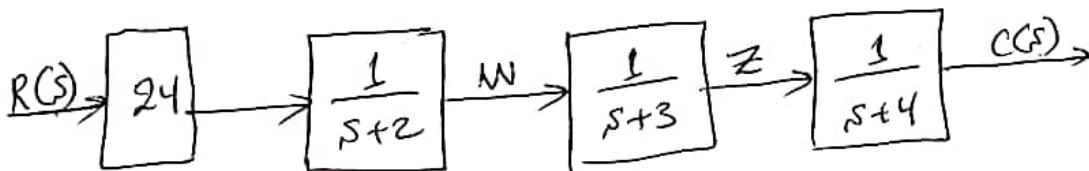
#

cascade methode :-

may applied to T.F that are written as product of simple first or 2nd order components (factored form)

Ex #1 $\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$

Sol: $\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24} = \frac{24}{(s+2)(s+3)(s+4)}$



$$\frac{C(s)}{Z(s)} = \frac{1}{s+4} = \frac{s^{-1}}{1+4s^{-1}} \cdot \frac{V_3}{V_3}$$

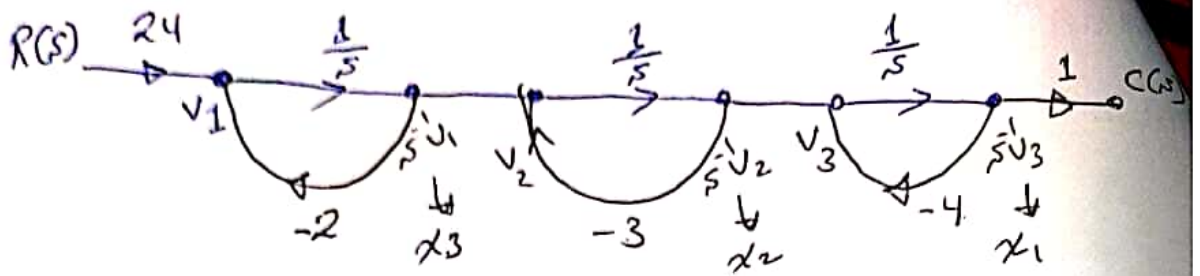
$$\boxed{C(s) = s^{-1} V_3} \quad Z(s) = V_3 + 4s^{-1} V_3 \Rightarrow \boxed{V_3 = Z(s) - 4s^{-1} V_3(s)}$$

$$\frac{Z}{W} = \frac{1}{s+3} = \frac{s^{-1}}{1+3s^{-1}} \cdot \frac{V_2}{V_2}$$

$$\boxed{Z(s) = s^{-1} V_2} \quad W(s) = V_2 + 3s^{-1} V_2 \Rightarrow \boxed{V_2 = W(s) - 3s^{-1} V_2(s)}$$

$$\frac{W}{24R} = \frac{1}{s+2} = \frac{s^{-1}}{1+2s^{-1}} \cdot \frac{V_1}{V_1}$$

$$\boxed{W = s^{-1} V_1} \quad 24R = V_1 + 2s^{-1} V_1 \Rightarrow \boxed{V_1 = 24R - 2s^{-1} V_1}$$



$$\dot{x}_1 = -4x_1 + x_2$$

$$\dot{x}_2 = -3x_2 + x_3$$

$$\dot{x}_3 = -2x_3 + 24r$$

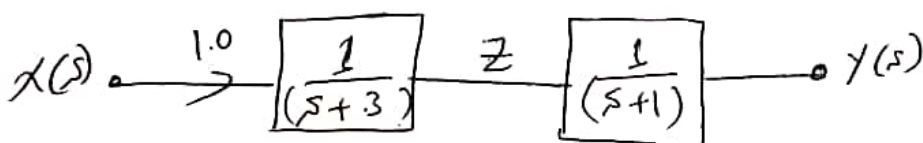
$$y = c(t) = x_1(t)$$

$$\dot{x} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r(t)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$H(s) = \frac{10}{s^2 + 4s + 3} = \frac{10}{(s+1)(s+3)}$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{10}{(s+3)} \cdot \frac{1}{(s+1)}$$



$$\frac{Z(s)}{X(s)} = \frac{10}{s+3} \cdot \frac{V_1}{V_2} = \frac{10s^{-1}}{1+3s^{-1}} \cdot \frac{V_1}{V_1}$$

$$U_1(s) = 10s^{-1}V_1$$

$$X(s) = (1+3s^{-1})V_1$$

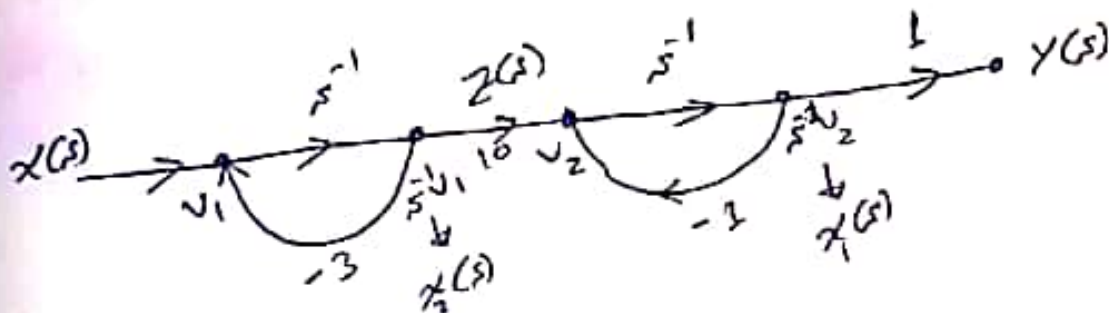
$$V_1 = X(s) - 3s^{-1}V_1$$

$$\frac{Y(s)}{Z(s)} = \frac{1}{(s+1)} = \frac{s^{-1}}{1+s^{-1}} \cdot \frac{V_2}{V_2}$$

$$V(s) = s^{-1}V_2(s)$$

$$Z(s) = (1+s^{-1})V_2(s)$$

$$V_2(s) = Z(s) - s^{-1}V(s)$$



$$\dot{x}_1 = 10x_2 - x_1$$

$$\dot{x}_2 = x_1 - 3x_2$$

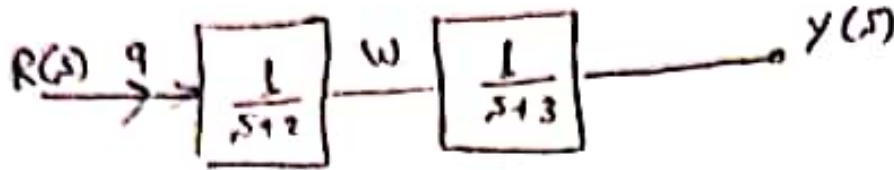
$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

#

$$H(s) = \frac{9}{s^2 + 5s + 6} = \frac{9}{(s+2)(s+3)}$$



$$\frac{W(s)}{R(s)} = \frac{9}{s+2} \cdot \frac{V_1}{V_2} = \frac{9s^{-1}}{1+2s^{-1}} \cdot \frac{V_1}{V_2}$$

$$\Rightarrow \boxed{W(s) = 9s^{-1}V_1(s)}$$

$$R(s) = V_2 + 2s^{-1}V_2$$

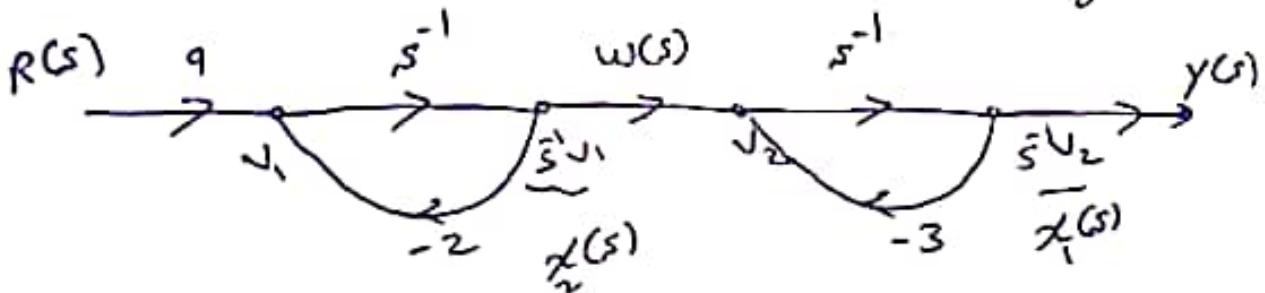
$$\boxed{V_2(s) = R(s) - 2s^{-1}V_2(s)}$$

$$\frac{Y(s)}{W(s)} = \frac{1}{s+3} \cdot \frac{V_2}{V_2} \Rightarrow \frac{Y(s)}{W(s)} = \frac{s^{-1}}{1+3s^{-1}} \cdot \frac{V_2}{V_2}$$

$$\Rightarrow Y(s) = s^{-1}V_2(s)$$

$$\Rightarrow W(s) = V_2(s) + 3s^{-1}V_2$$

$$V_2(s) = W - 3s^{-1}V_2$$



$$\dot{x}_1 = 9y(t) - 2x_1(t)$$

$$\dot{x}_2 = w(t) - 3x_2(t)$$

parallel decompositions :-

In this Method we need to use partial fraction to expand the T.F, put it in parallel form which have the same input and the output is equal the sum of all subsystems.

Parallel Method

Ex # | $H(s) = \frac{s+1}{s^3+9s^2+26s+24}$

Sol :

$$H(s) = \frac{s+1}{(s+2)(s+3)(s+4)}$$

By using partial fraction :

$$H(s) = \frac{k_1}{(s+2)} + \frac{k_2}{(s+3)} + \frac{k_3}{(s+4)}$$

$$k_1 = \frac{(s+1)}{(s+2)(s+3)(s+4)} \cdot (s+2) \Big|_{s=-2} = \boxed{-0.5}$$

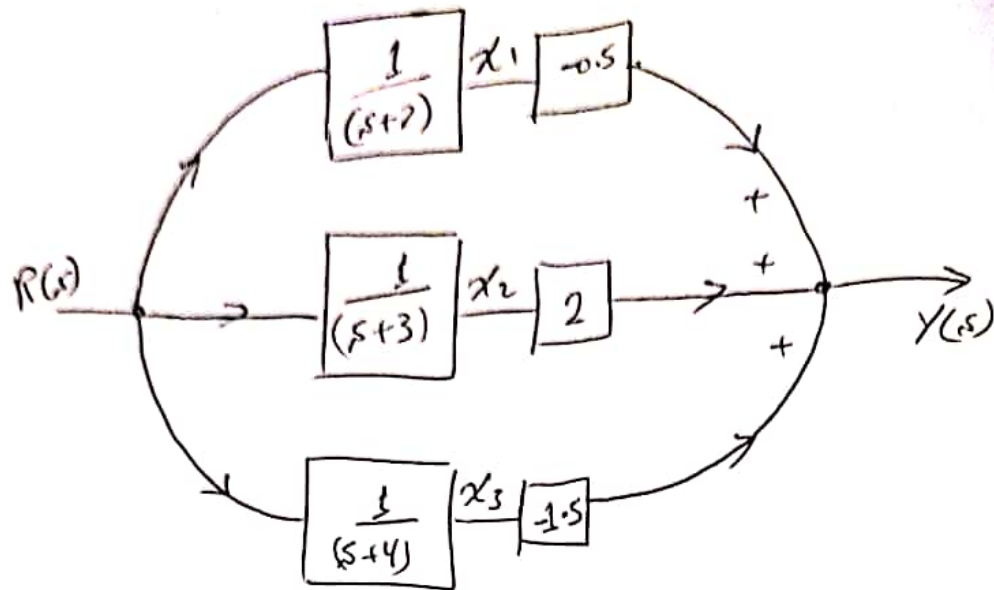
$$k_2 = \frac{(s+1)(s+4)}{(s+2)(s+3)(s+4)} \Big|_{s=-3} = \boxed{2}$$

$$k_3 = \frac{(s+1)(s+3)}{(s+2)(s+3)(s+4)} \Big|_{s=-4} = \boxed{-1.5}$$

$$H(s) = \frac{-0.5}{(s+2)} + \frac{2}{(s+3)} + \frac{-1.5}{(s+4)}$$

الاستجاب لازم تكون موجبة على السطح

أريد هذا التمرين
المفيدة التي تعلموها
أجبركم على ذلك
بالتفكير
أعتدنا لهم
أسئلة الاسرار
😊



$$\frac{x_1(s)}{R(s)} = \frac{1}{s+2} \Rightarrow s x_1(s) + 2 x_1(s) = R(s)$$

$$\boxed{\dot{x}_1 = -2x_1 + r(t)} \dots \textcircled{1}$$

$$\frac{x_2(s)}{R(s)} = \frac{1}{s+3} \Rightarrow s x_2(s) + 3 x_2(s) = R(s)$$

$$\boxed{\dot{x}_2 = -3x_2 + r(t)} \dots \textcircled{2}$$

$$\frac{x_3(s)}{R(s)} = \frac{1}{s+4} \Rightarrow s x_3(s) + 4 x_3(s) = R(s)$$

$$\boxed{\dot{x}_3 = -4x_3 + r(t)} \dots \textcircled{3}$$

$$\boxed{y = -0.5x_1 + 2x_2 - 1.5x_3} \quad \text{output equation}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r(t)$$

$$y = [-0.5 \quad 2 \quad -1.5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

"أفهم الصورة للكون المصنوع من الخشب"
 "أفهم الصورة للكون المصنوع من الخشب"
 "أفهم الصورة للكون المصنوع من الخشب"
 "أفهم الصورة للكون المصنوع من الخشب"

$$\frac{x_1(s)}{R(s)} = \frac{1}{s+2} \Rightarrow s x_1(s) + 2 x_1(s) = R(s)$$

$$\dot{x}_1(t) + 2 x_1(t) = r(t)$$

$$\boxed{\dot{x}_1(t) = r(t) - 2 x_1(t)}$$

$$\frac{x_2(s)}{R(s)} = \frac{1}{s+3} \Rightarrow s x_2(s) + 3 x_2(s) = R(s)$$

$$\dot{x}_2(t) + 3 x_2(t) = r(t)$$

$$\boxed{\dot{x}_2(t) = r(t) - 3 x_2(t)}$$

$$\frac{x_3(s)}{R(s)} = \frac{1}{s+4} \Rightarrow s x_3(s) + 4 x_3(s) = R(s)$$

$$\dot{x}_3(t) + 4 x_3(t) = r(t)$$

$$\boxed{\dot{x}_3(t) = r(t) - 4 x_3(t)}$$

$$\boxed{y(t) = 12 x_1(t) - 24 x_2(t) + 12 x_3(t)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r(t)$$

$$C = [12 \quad -24 \quad 12] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

parallel method to find the state space Model for the system :-

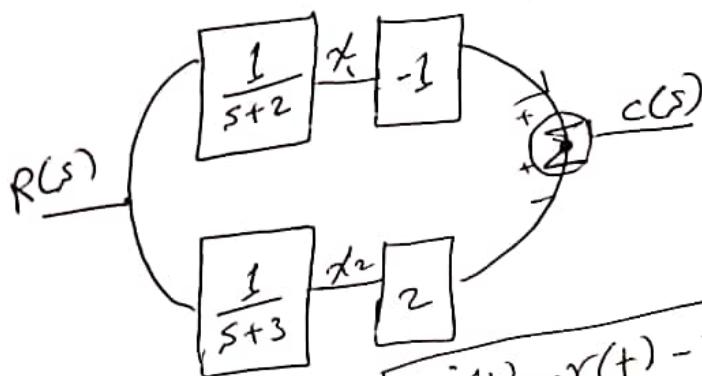
$$H(s) = \frac{s+1}{s^2+s+6}$$

$$H(s) = \frac{s+1}{s^2+s+6} \Rightarrow \frac{s+1}{(s+2)(s+3)} = \frac{k_1}{(s+2)} + \frac{k_2}{(s+3)}$$

$$k_1 = \frac{(s+1)(s+3)}{(s+2)(s+3)} \bigg|_{s=-2} = \boxed{-1}$$

$$k_2 = \frac{(s+1)(s+2)}{(s+2)(s+3)} \bigg|_{s=-3} = \boxed{2}$$

$$H(s) = \frac{-1}{(s+2)} + \frac{2}{(s+3)}$$



$$\frac{x_1(s)}{R(s)} = \frac{1}{s+2}$$

$$\frac{x_2(s)}{R(s)} = \frac{1}{s+3}$$

$$\Rightarrow \dot{x}_1(t) = r(t) - 2x_1(t) \quad \dots (1)$$

$$\Rightarrow \dot{x}_2(t) = r(t) - 3x_2(t) \quad \dots (2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(t)$$

$$y = x_1(t) + 2x_2(t)$$

$$y = \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \#$$

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صفة مهمة // بقدر أعرف إذا كان A matrix
 or stable or unstable
 في حالة التحويل من ال T.F إلى S.S بطريقة
 بالعادة بعد حل السؤال يقال إذا كان stable أو لا .
 الصيغة العامة للتحويل باستخدام parallel method

$$G(s) = \frac{k_1}{(s+a_0)} + \frac{k_2}{(s+a_1)} + \dots + \frac{k_n}{s+a_{n-1}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_0 & 0 & \dots & 0 \\ 0 & -a_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} r(t)$$

$$y(t) = [k_1 \ k_2 \ \dots \ k_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

* Converting from state-space to T.F :-

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

time domain

Convert to

$$\frac{y(s)}{u(s)}$$

s-domain

$$sX(s) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

$$sX(s) - AX(s) = BU(s)$$

$$X(s)(sI - A) = BU(s)$$

$$X(s) = (sI - A)^{-1} BU(s)$$

تحويل إلى
time domain

$$X(s) = (sI - A)^{-1} B u(s)$$

$$Y(s) = C X(s) + D u(s)$$

$$Y(s) = C (sI - A)^{-1} B u(s) + D u(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D}$$

Example #

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$C = [24 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + or(t)$$

Find T.F :

Sol :-

$$\frac{Y(s)}{U(s)} = C (sI - A)^{-1} B + D$$

$$(sI - A) \Rightarrow \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 24 & 26 & s+9 \end{bmatrix}$$

$$s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ we take}$$

$$(sI - A)^{-1} \text{ هاتين بديا كسبت}$$

$$(sI - A)^{-1} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 24 & 26 & s+9 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{s^2 + 9s + 26}{s^3 + 9s^2 + 26s + 24} & \frac{s+9}{s^3 + 9s^2 + 26s + 24} & \frac{1}{s^3 + 9s^2 + 26s + 24} \\ \frac{-24}{s^3 + 9s^2 + 26s + 24} & \frac{s^2 + 9}{s^3 + 9s^2 + 26s + 24} & \frac{s}{s^3 + 9s^2 + 26s + 24} \\ \frac{-24s}{s^3 + 9s^2 + 26s + 24} & \frac{-26s - 24}{s^3 + 9s^2 + 26s + 24} & \frac{s^2}{s^3 + 9s^2 + 26s + 24} \end{bmatrix}$$

$$(sI - A)^{-1} B = \begin{bmatrix} \frac{1}{s^3 + 9s^2 + 26s + 24} \\ \frac{s}{s^3 + 9s^2 + 26s + 24} \\ \frac{s^2}{s^3 + 9s^2 + 26s + 24} \end{bmatrix}$$

$$C(sI - A)^{-1} B = \begin{bmatrix} 24 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s^3 + 9s^2 + 26s + 24} \\ \frac{s}{s^3 + 9s^2 + 26s + 24} \\ \frac{s^2}{s^3 + 9s^2 + 26s + 24} \end{bmatrix}$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24} \quad \#$$

Example # | Convert the state and output equation to transfer function.

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y = [1.5 \quad 0.625] x$$

Sol:

$$\frac{Y(s)}{C(s)} = C(sI - A)^{-1} B + D$$

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, C = [1.5 \quad 0.625]$$

Compute $(sI - A)$

$$(sI - A) = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} \Rightarrow \text{قابض}$$

$$\begin{aligned} |sI - A| &= (s+4) \times s - (1.5 \times -4) \\ &= s^2 + 4s + 6 \end{aligned}$$

$$\text{adj}(sI - A)^{-1} = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$(sI - A)^{-1} \times B = \begin{bmatrix} \frac{2s}{s^2 + 4s + 6} \\ \frac{8}{s^2 + 4s + 6} \end{bmatrix}$$

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$$C(sI-A)^{-1} \times B = \left[\frac{5}{s^2+4s+6} + \frac{3s}{s^2+4s+6} \right]$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+4s+6}} \quad \#$$

* قبل ما نكتبنا شئ في system اللى عليه كقولنا الـ system اللى فيه
 ما بنفع الا للـ system اللى Linear, time invariant

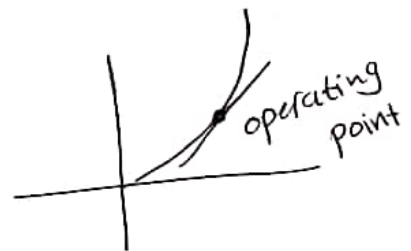
* اللى بنوينا عليه الـ system اللى non-linear ما بنفع الـ system اللى فيه
 اللى بشرط واحد وهو كقولنا الـ system اللى non-linear اللى فيه
 وهماي العملية بسببها "linearization"

* على خلاف state space بنحل الـ system اللى فيه
 Linearization اللى فيه

* Linearization Steps :-

- 1] get a non linear dynamic model of the system.
- 2] establish steady-state equilibrium (operating) cond.
- 3] Represent the non linear system as

$$\frac{dx(t)}{dt} = f[x(t), r(t)]$$



Example of non linear :-

$$\frac{dx_1(t)}{dt} = x_1(t) + x_2^2(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) + r(t)$$

$$\Delta \dot{x} = A^* \Delta x + B^* \Delta r$$

$$A^* = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \dots \\ \vdots & \vdots & \ddots \\ \frac{df_n}{dx_1} & \dots & \dots \end{bmatrix}_{x_0, r_0}$$

must be square matrix

$$B^* = \begin{bmatrix} \frac{df_1}{dx_1} & \dots & \frac{df_1}{dr_p} \\ \vdots & & \vdots \\ \frac{df_m}{dx_1} & \dots & \frac{df_m}{dr_p} \end{bmatrix}_{x_0, y_0}$$

بعض المصفوفات اما بتكون من اعداد و
او اكثر من اعداد، الـ input
العدد او input

Example #1

$$\begin{aligned} \dot{x}_1 &= x_1(t) + x_2^2(t) \\ \dot{x}_2 &= x_1(t) + r(t) \\ \dot{x}_3 &= \cos(x_2 x_1) \end{aligned}$$

"non-linear system"
لوصف النظام

Example #1 Given the non linear system below, find the linearized model:-

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\sin x_1 - 0.1x_2 + u(t)$$

$$y = \frac{1}{\sqrt{2}}, \quad x_{01} = \frac{\pi}{4}, \quad x_{02} = 0$$

Sol:-

$$\dot{x}_1 = x_2 = f_1$$

$$\dot{x}_2 = -\sin x_1 - 0.1x_2 + u(t) = f_2$$

$$A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_0, u_0} = \begin{bmatrix} 0 & 1 \\ -\cos x_1 & -0.1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.707 & -0.1 \end{bmatrix}$$

$-\cos \frac{\pi}{4}$

$$B^* = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{cases} \Delta \dot{x}_1 = x_2 \\ \Delta \dot{x}_2 = -0.707x_1 - 0.1x_2 + u \end{cases} \neq$$

"ولقد خلقنا الانسان وقلنا ما قولوا
به نفسه، فانه انزل اليه من اجل
الوحي" سورة الواقعة

Example #1 For the nonlinear system, find the linearized model:-

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 x_2 - x_2 + u(t), \quad x_0 = [x_{01} \ x_{02}] = [1 \ 0], \quad u_0 = 0$$

Sol:-

$$\dot{x}_1 = x_2 = f_1$$

$$\dot{x}_2 = -x_1 x_2 - x_2 + u(t) = f_2$$

$$A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_0} = \begin{bmatrix} 0 & 1 \\ -x_2 & -x_1 - 1 \end{bmatrix}_{x_0} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

$$B^* = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}_{x_0} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \Delta x_1^* \\ \Delta x_2^* \end{bmatrix} = \begin{bmatrix} x_2 \\ -2x_2 + u(t) \end{bmatrix} \quad \#$$

نلاحظ ان المصفوفة A* هي مصفوفة ثابتة، وهذا يعني ان النظام خطي.

$$y''(t) + 0.1y'(t) + \sin(y(t)) = u(t)$$

2-state space.

المعادلة هي الدرجة الثانية $\leftarrow$

$y_0 = \frac{1}{\sqrt{2}}, y'_0 = \frac{\pi}{4}, y''_0 = 0$

$$x_1 = y$$

$$\dot{x}_1 = \dot{y} = x_2$$

$$\Rightarrow \dot{x}_1 = x_2 = f_1$$

$$\Rightarrow \dot{x}_2 = \ddot{y} = -0.1\dot{y} - \sin(y) + u(t) = f_2$$

BS

$$A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.3x_1 & -0.1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.707 & -0.1 \end{bmatrix}$$

$$B^* = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left. \begin{aligned} \Delta x_1^* &= x_2 \\ \Delta x_2^* &= -0.707x_1 - 0.1x_2 + u(t) \end{aligned} \right\} \#$$

فريق RCM الهندسي

﴿ رَبَّنَا تَقَبَّلْ مِنَّا إِنَّكَ أَنْتَ السَّمِيعُ الْعَلِيمُ ﴾



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