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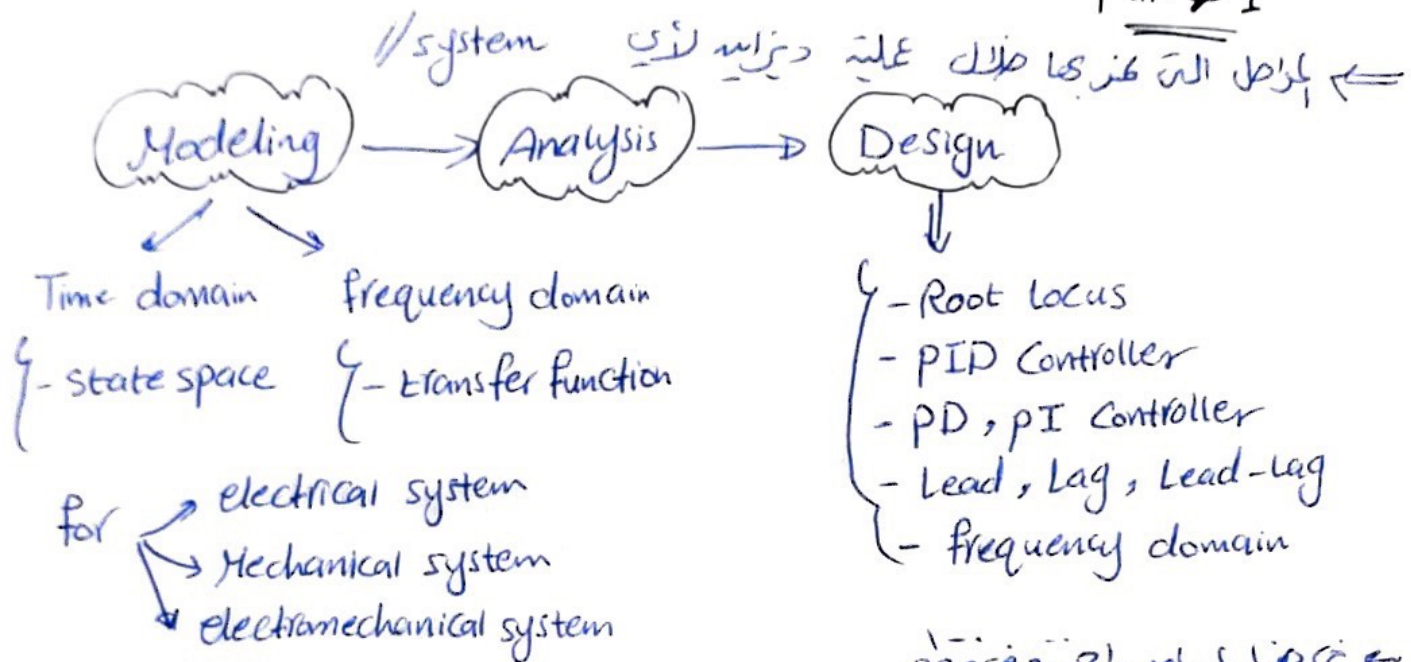
The Islamic University of Gaza (IUG)

Linear Control #

Chapter 4

Time response
(analysis and design)

Part #1



- Linearization
- Block diagram.

نظم التحكم

على الترتيب

Root Locus



⇒ Analysis

* time response

transient

steady state

* Stability

Routh - Hurwitz

Nyquist

CH #6



طرق محفص ال

stability

Digital Control

نظم التحكم

* Frequency response : Bode plot

Digital , PID

نظم التحكم

Control

"واهم شيء في التحكم هو الترتيب"

نظم التحكم

نظم التحكم

نظم التحكم

* Modeling: نموذج رياضي أو physical model من أجل النظام system.

* Analysis: We will be make analysis for this system to satisfy the main reason, which is make controllable system has specific point and satisfy our desired response.

- هياي لمرحلة بعمل قض لا system، ريثوت انا Stable اول
- ريثوت ال response تاع ال system

Time response $\Rightarrow$ equation that describes the response of the system and contain two type of Response.

Time response = transient response + steady state response

↓
Natural response

↓
Forced response

- هادا الجزء يتلاش بقدره " ريثوت لـ $t \rightarrow \infty$ "
 response بروج للعرض
 - هادا الجزء يعتمد على Initial condition
 - هادا الجزء يتلاش بقدره " ريثوت لـ $t \rightarrow \infty$ "
 input

ملحوظة // علنا احييت ال time response وقت ال system لازم يكون عني stable.

* لو مكان ال system ← stable هياي يكون عني

Final value
دعكون ال response تاع ال system راجع للانفني (∞)

والله اعلم ما ارسلتم من قوة ومن رسله
الكل يهتدون به عند الله، عندكم
الاتصال.

* poles and Zeros, system response:-

X ← Poles رمز ←

0 ← Zeros ←

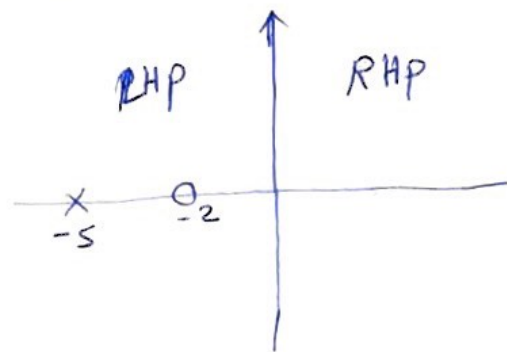
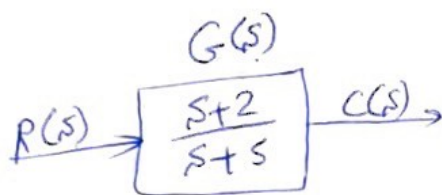
* If all poles lies on the LHP region $\Rightarrow$ system is stable



The output response will be contain two type of response, one of them goes to zero at $t \rightarrow \infty$

of them goes to zero at $t \rightarrow \infty$

Example #1 Given: $\frac{C(s)}{R(s)} = \frac{s+2}{s+5}$, $R(s) = \frac{1}{s}$ as shown below



$\Rightarrow$ system is stable

$$C(s) = R(s) \cdot \frac{(s+2)}{(s+5)} \Rightarrow C(s) = \frac{(s+2)}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$$

$$A = \frac{(s+2)s}{s(s+5)} \Big|_{s=0} = \boxed{\frac{2}{5}}$$

$$B = \frac{(s+2)(\cancel{s+s})}{s(\cancel{s+s})} \quad |_{s=-s} = \boxed{\frac{3}{s}}$$

$$C(s) = \frac{2/s}{s} + \frac{3/s}{s+5}$$

$$c(t) = \left(\frac{2}{s} + \frac{3}{s} e^{-st} \right) u(t)$$

Forced response
"input response"

Natural response
"transient response"

فَقُلْ إِنَّهُ لَا إِلَهَ إِلَّا اللَّهُ، اسْتَغْفِرُ
لِزَيْنٍ، لِلنَّفْسِ، لِقَوْلِكَ، اللَّهُ
يَوْمَ مَقْبَلِكُمْ وَسَوَاءٌ
أَصَوْتُ فِي

Example #1 المثال الأول

Given the system of figure below, write the output $C(t)$, in general terms, specify the forced and natural parts of the solution

Example 2) Given the system of figure below, write the output $C(s)$, in general terms, specify the forced and natural parts of the solution.

$$R(s) = \frac{1}{s} \rightarrow \left[\frac{(s+3)}{(s+2)(s+4)(s+5)} \right] \rightarrow C(s)$$

Sol: $\frac{C(s)}{R(s)} = \frac{(s+3)}{(s+2)(s+4)(s+5)}$

$$C(s) = \frac{(s+3)(1)}{s(s+2)(s+4)(s+5)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+4)} + \frac{D}{(s+5)}$$

$$A = \frac{s+3}{(s+2)(s+4)(s+5)} \Big|_{s=0} = \boxed{3/40}$$

$$B = \frac{(s+3)(s+2)}{s(s+2)(s+4)(s+5)} \Big|_{s=-2} = \boxed{-1/12}$$

$$= \frac{(s+3)(s+4)}{s(s+2)(s+4)(s+5)} \quad |_{s=-4} = \boxed{-1/8}$$

$$= \frac{(s+3)(s+5)}{s(s+2)(s+4)(s+5)} \quad |_{s=-5} = \boxed{2/15}$$

$$C(s) = \frac{3/40}{s} - \frac{1/12}{(s+2)} - \frac{1/8}{(s+4)} + \frac{2/15}{(s+5)}$$

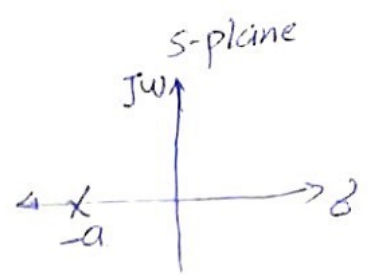
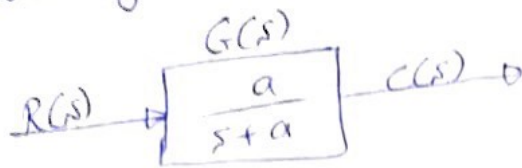
$$c(t) = \left(\frac{3}{40} - \frac{1}{12} e^{-2t} - \frac{1}{8} e^{-4t} + \frac{2}{15} e^{-5t} \right) u(t)$$

Forced Response

Natural response

"وَلَقَدْ خَلَقْنَاكَ مِنْ نَجَسٍ مُسْتَفِيزٍ" (فصل ١٥)
 "بِأَسْفُوفٍ" (فصل ١٥)
 "لَبَدٍ" (فصل ١٥)
 "وَلَقَدْ خَلَقْنَاكَ مِنْ نَجَسٍ مُسْتَفِيزٍ" (فصل ١٥)
 "بِأَسْفُوفٍ" (فصل ١٥)
 "لَبَدٍ" (فصل ١٥)

* First order system without zeros



If the input is unit step $\Rightarrow$

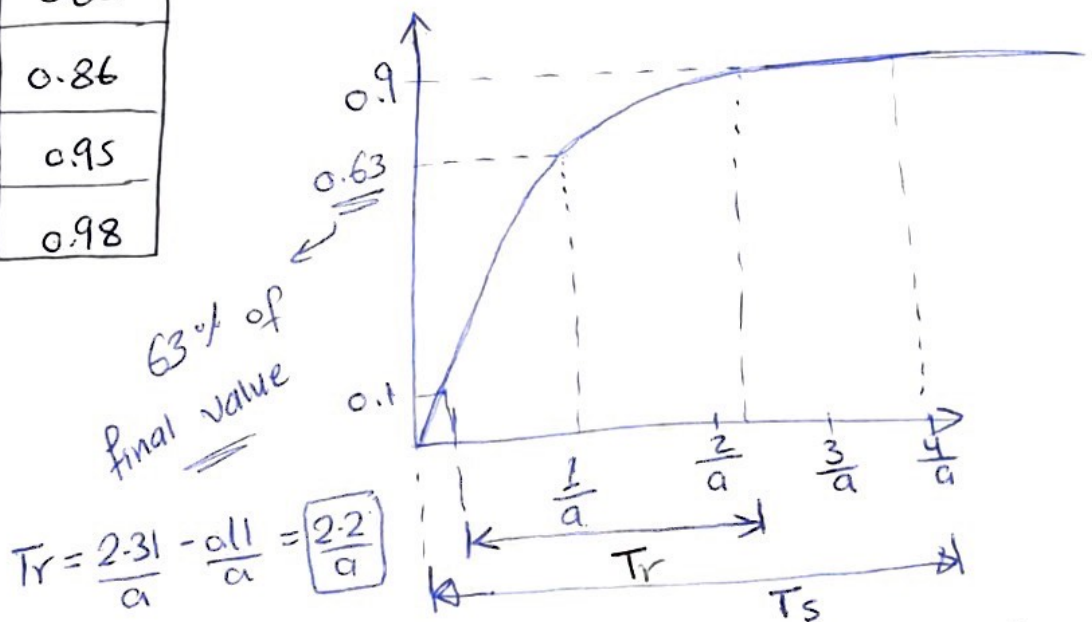
$$R(s) = \frac{1}{s}$$

$$C(s) = R(s) \cdot G(s) = \frac{a}{s(s+a)}$$

$$C(t) = C_f(t) + C_n(t) = \underline{\underline{1 - e^{-at}}}$$

t	C(t)
0	0
1/a	0.63
2/a	0.86
3/a	0.95
4/a	0.98

$\frac{1}{a} \Rightarrow$ time constant of the response



$$Tr = \frac{2.31}{a} - \frac{0.1}{a} = \boxed{\frac{2.2}{a}}$$

Tr:

Time rise τ_r is defined as the time for the waveform to go from 0.1 to 0.9 of its final value. $\boxed{Tr = \frac{2.2}{a}}$ قانون

Settling time $\underline{\underline{Ts}}$ is the time for the response to reach, and stay within 2% of its final value

$$\boxed{Ts = 4\tau = \frac{4}{a}}$$

قانون

Final value $\approx$ 2% response

* first order system :-

$$G(s) = \frac{a}{s+a}$$

الصيغة العامة لها

في كل ما يخص τ , T_s , dc gain

$$G(s) = \frac{1}{\frac{s}{a} + 1}$$

نفس

نفس الشيء $\frac{1}{a}$ هو $\frac{1}{a}$ في النهاية

$$\tau = \frac{1}{a}$$

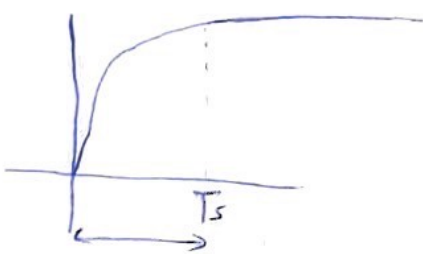
نفس الشيء τ هو $\frac{1}{a}$

$$T_s = \frac{4}{a}$$

$$K_{dc} = G(s) \Big|_{s=0} = \frac{\text{final value}}{\text{input value}}$$

$$K_{dc} = 1$$

steady state T_s هو الوقت الذي يستغرقه النظام للوصول إلى الحالة المستقرة



Example #1

$$G(s) = \frac{10}{s+5}, \text{ find Dc-gain, } \tau, T_s, T_r$$

$$\text{Sol} \Rightarrow G(s) = \frac{10/5}{\frac{s}{5} + 1}$$

$$\text{Dc-gain} \Rightarrow G(s) \Big|_{s=0} = \frac{10/5}{\frac{0}{5} + 1} = 2$$

$$T_c = \tau = \frac{1}{5} \text{ sec}, T_r = \frac{2.2}{5} = 0.44$$

$$T_s = \frac{4}{5} = 0.8 \text{ s}$$

الصيغة العامة ←

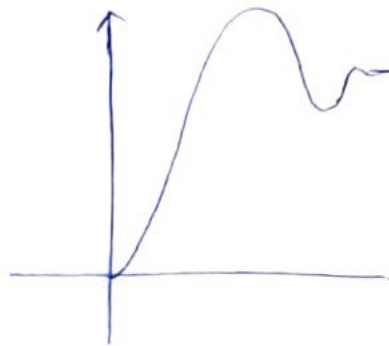
$$G(s) = \frac{s+C}{(s+\alpha+JB)(s+\alpha-JB)}$$

$$y(s) = \frac{k_1}{s} + \frac{k_2}{s+\alpha+JB} + \frac{k_2^*}{s+\alpha-JB}$$

Forced Response

Natural response

system response ←



الشكل العام للاستجابة

ملامحة للتوصيف //

Real part $\Rightarrow$ damping. يكون مسؤول عن

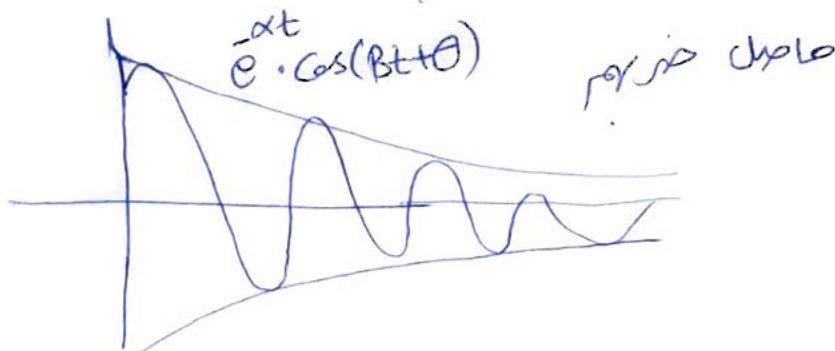
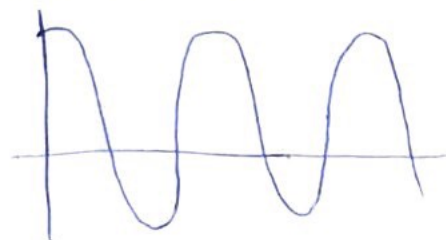
Imaginary part $\Rightarrow$ oscillation. يكون مسؤول عن

(الاختزازات) (الذبذبات)

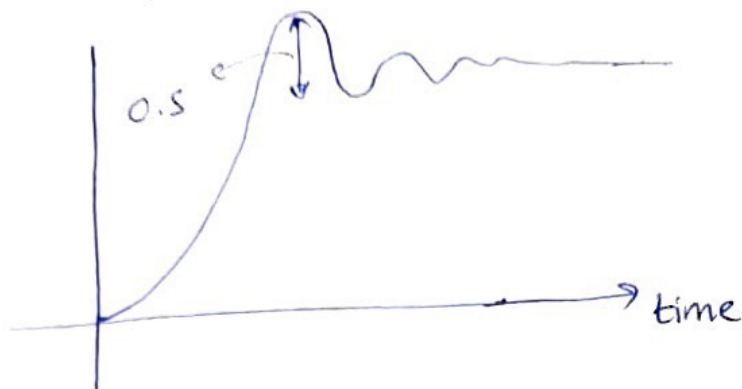
و للتوصيف أكثر $\Leftarrow$ كونه الكوبلاند في time domain

$$2|K|e^{-\alpha t} \cos(Bt+\theta)$$

damping ← $e^{-\alpha t}$ يكون مسؤول عن
oscillation ← $\cos(Bt+\theta)$ يكون مسؤول عن



The time response for this type :-



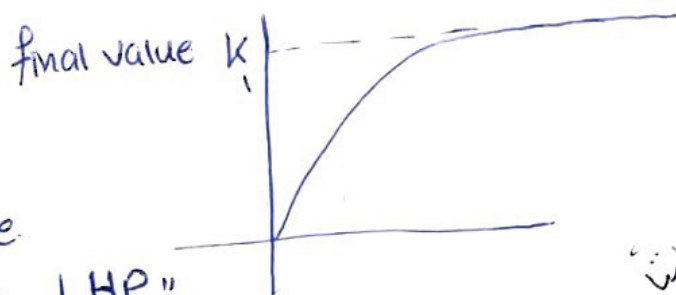
[2] overdamped :-

1. تكون سرعة أكبر من 1
 2. تكون ال poles ← real and distinct
 3. لا يوجد فيها overshoot
 4. الأقطاب معقدة
 5. الأقطاب حقيقية

$$G(s) = \frac{s+c}{(s+a)(s+b)}$$

$$y(s) = \underbrace{\frac{k_1}{s}}_{\text{forced response}} + \underbrace{\frac{k_2}{s+a} + \frac{k_3}{s+b}}_{\text{Natural response}}$$

الاستجابة النظام (response)



- No-overshoot.
- The system is stable.
- "because all poles in LHP"

$$y(t) = k_1 + \underbrace{k_2 e^{-at} + k_3 e^{-bt}}_{\text{transient response}}$$

final value = k

التي هي مقبولة
 زفوننا وضا عيدين النار
 ال عيدين

[3] Critically Damped :-

[1] تكون وحدة $\zeta = 1$

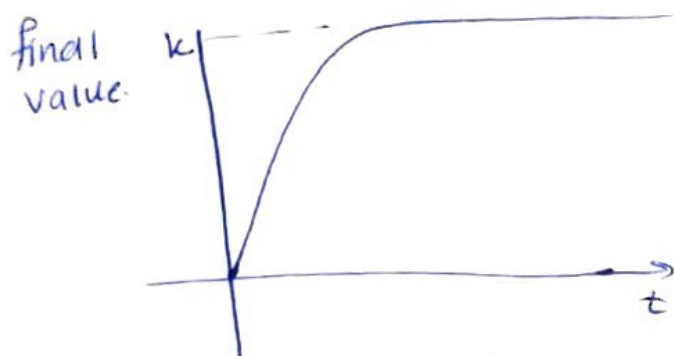
[2] تكون $\Leftarrow$ poles Real and Repeated

[3] لا يكون فيها overshoot

$$G(s) = \frac{s+c}{(s+a)^2}$$

" الصيغة العامة "

$$y(s) = \underbrace{\frac{k_1}{s}}_{\text{Forced response}} + \underbrace{\frac{k_2}{(s+a)^2} + \frac{k_3}{(s+a)}}_{\text{Natural response}}$$



No overshoot

* ملاحظة // في حالة ال overdamped, critical damped تقريباً إهم نصف ال response به بفرق بسيطة وحدة ال critical أسرع من ال overdamped يعني " يصل ال final value " قبل ال overdamped

[4] undamped Response :-

[1] تكون وحدة $\zeta = 0$

[2] تكون $\Leftarrow$ poles Imaginary poles

$$G(s) = \frac{(s+c)}{(s-jB)(s+jB)}$$

" الصيغة العامة "

$$y(s) = \underbrace{\frac{k_1}{s}}_{\text{Forced response}} + \underbrace{\frac{k_2}{(s-jB)} + \frac{k_2^*}{(s+jB)}}_{\text{Natural response}}$$

Forced response

Natural response

$$y(t) = K_1 + 2|K| \cos(Bt + \theta)$$

oscillation → هذا الجزء مفقود

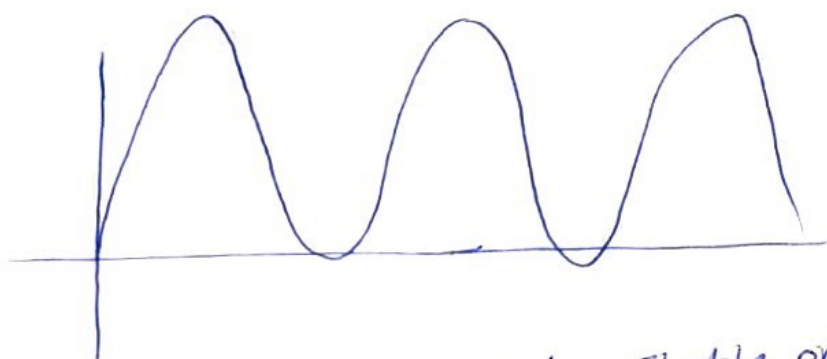
$e^{-x} \Rightarrow$ damping مؤلة

دھنا ماضي عني Real part دباي ماضي α ←

ساخنی عیسیٰ

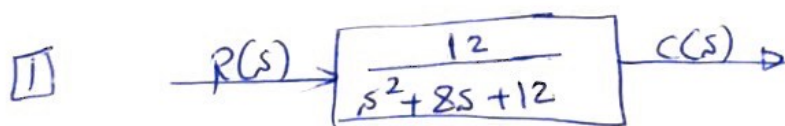
ملاؤں کا مارتا // فی حالت ال undamped لا گونہ فی عین

Final value



* This system called margin stable it may be stable or unstable.

Example #1 For each of the system shown below, find the value of ζ and report kind of response expected.



5010

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

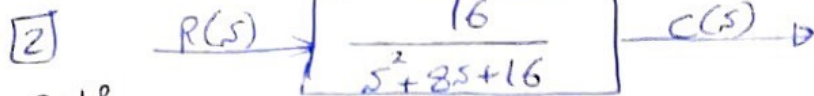
الصفحة العامة للموارد

$$\Rightarrow 2\zeta W_n = 8$$

$$\Rightarrow \omega_n^2 = 12 \Rightarrow \boxed{\omega_n = \sqrt{12}}$$

$$\epsilon = \frac{8}{2\omega_n} = \frac{8}{2\sqrt{12}} = 1.155$$

$$\epsilon > 1 \Rightarrow \text{overdamped}$$

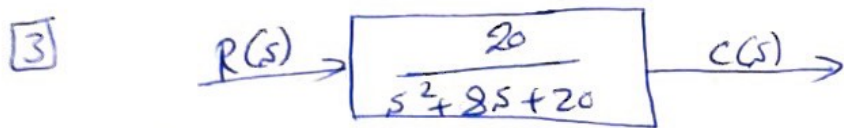



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + 16}$$

$$* \omega_n^2 = 16 \Rightarrow \boxed{\omega_n = 4}$$

$$* 2\zeta\omega_n = 8 \Rightarrow \zeta = \frac{8}{2\omega_n} = \frac{8}{(2)(4)} = \boxed{1}$$

$\boxed{\zeta = 1} \Rightarrow$ critically damped #



Sol: -

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$* \omega_n^2 = 20 \Rightarrow \boxed{\omega_n = \sqrt{20}}$$

$$* 2\zeta\omega_n = 8 \Rightarrow \zeta = \frac{8}{2\sqrt{20}} = \boxed{0.8944}$$

$\zeta < 1 \Rightarrow$ underdamped #

// also see response si di

[1] overdamped Response

poles: two real at $-\delta_1, -\delta_2$, $c(t) = k_1 e^{-\delta_1 t} + k_2 e^{-\delta_2 t}$

[2] underdamped Response

poles: two complex at $-\delta_d \pm j\omega_d$, $c(t) = A e^{-\delta_d t} \cos(\omega_d t - \phi)$

[3] undamped Response.

poles: two imaginary at $\pm j\omega_d$, $c(t) = A \cos(\omega_d t - \phi)$

[4] critically damped Response.

poles: two real at $-\delta_1$, $c(t) = k_1 e^{-\delta_1 t} + k_2 t e^{-\delta_1 t}$

Find the response for each system:-

$$\boxed{1} \quad C(s) = \frac{9}{s(s^2 + 9s + 9)}$$

Sol :-

$$C(s) = \frac{9}{s(s^2 + 9s + 9)} = \frac{A}{s} + \frac{B}{(s + 7.854)} + \frac{C}{(s + 1.146)}$$

$$C(s) = \frac{A}{s} + \frac{B}{(s + 7.854)} + \frac{C}{s + 1.146}$$

$$A = \frac{9s}{s(s^2 + 9s + 9)} \Big|_{s=0} = \boxed{1}$$

$$B = \frac{(9)(s + 7.854)}{s(s + 7.854)(s + 1.146)} \Big|_{s = -7.854} = \boxed{0.1708}$$

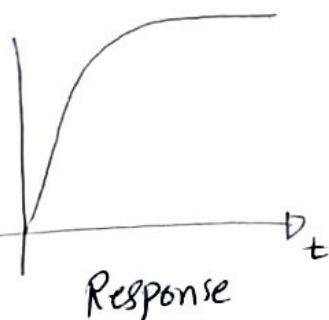
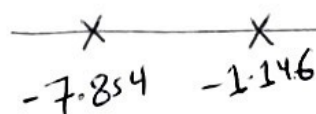
$$C = \frac{9(s + 1.146)}{s(s + 7.854)(s + 1.146)} \Big|_{s = -1.146} = \boxed{-1.1707}$$

$$C(s) = \frac{1}{s} + \frac{0.1708}{s + 7.854} - \frac{1.1707}{s + 1.146}$$

$$\boxed{c(t) = \left(1 + 0.1708e^{-7.854t} - 1.1707e^{-1.146t} \right) u(t)} \quad \#$$

$\Rightarrow$ Real poles and distinct $\Rightarrow$ overdamped Response

s-plane



مستقيم، سريع، غير متذبذب
- غير متذبذب

$$\boxed{2} \quad C(s) = \frac{9}{s(s^2 + 2s + 9)} \Rightarrow \frac{A}{s} + \frac{B}{s+1-j2.82} + \frac{B^*}{s+1+j2.82}$$

$$A = \frac{9s}{s(s^2 + 2s + 9)} \Big|_{s=0} = \boxed{1}$$

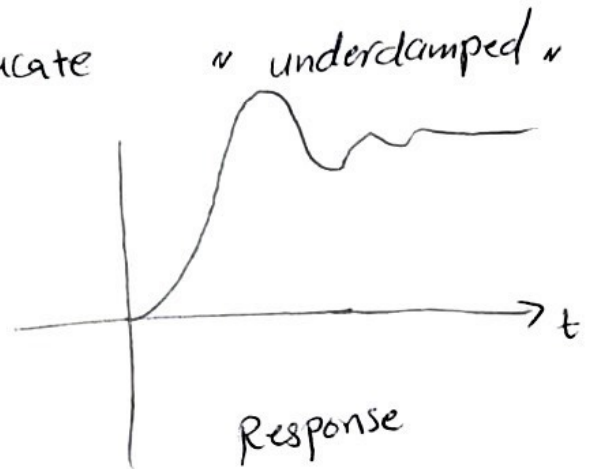
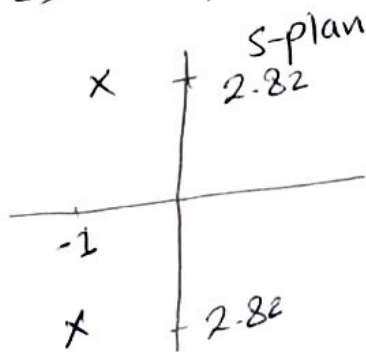
$$B = \frac{(9)(s+1-j2.82)}{s(s+1-j2.82)(s+1+j2.82)} \Big|_{s=-1+j2.82} = 0.533 \angle 160^\circ$$

$$B^* = 0.533 \angle -160^\circ$$

$$C(t) = \frac{1}{s} + \frac{0.533 \angle 160^\circ}{s+1-j2.82} + \frac{0.533 \angle -160^\circ}{s+1+j2.82}$$

$$c(t) = \left[1 + 1.066 e^{-t} \cos(2.82t + 160^\circ) \right] u(t) \quad \#$$

poles $\Rightarrow$ complex and conjugate "underdamped"



$$\boxed{3} \quad C(s) = \frac{9}{s(s^2 + 9)} = \frac{A}{s} + \frac{B}{s+j3} + \frac{B^*}{s-j3}$$

$$A = \frac{9s}{s(s^2 + 9)} \Big|_{s=0} = \boxed{1}$$

$$B = \frac{9(s-j3)}{s(s-j3)(s+j3)} = \boxed{-0.5} = 0.5 \angle 180^\circ$$

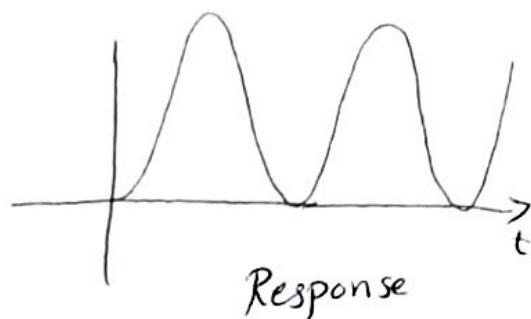
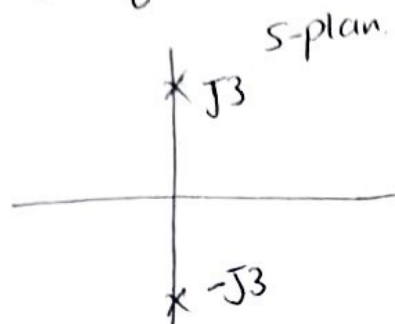
$$B^* = \boxed{-0.5}$$

$$C(s) = \frac{1}{s} + \frac{0.5 \angle 180^\circ}{s-j3} + \frac{0.5 \angle -180^\circ}{s+j3}$$

$\boxed{14}$

$$c(t) = (1 + \cos(3t + 180)) u(t)$$

poles $\Rightarrow$ imaginary ~ undamped ~



$$\boxed{4} \quad c(s) = \frac{9}{s(s^2 + 6s + 9)} = \frac{A}{s} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)}$$

$$A = \frac{9s}{s(s^2 + 6s + 9)} \Big|_{s=0} = \boxed{1}$$

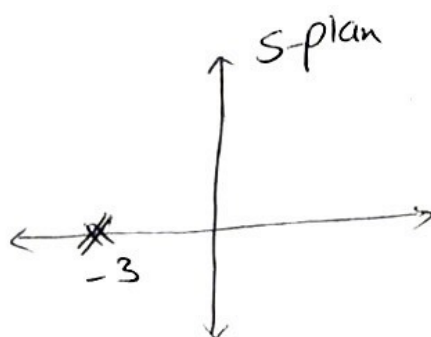
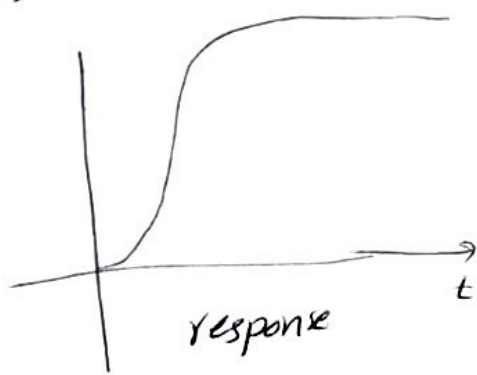
$$B = \frac{9(s+3)^2}{s(s+3)^2} \Big|_{s=-3} = \boxed{-3}$$

$$C = \frac{d}{ds} \left(\frac{9}{s} \right) \Big|_{s=-3} = \frac{(-9)}{s^2} \Big|_{s=-3} = \boxed{-1}$$

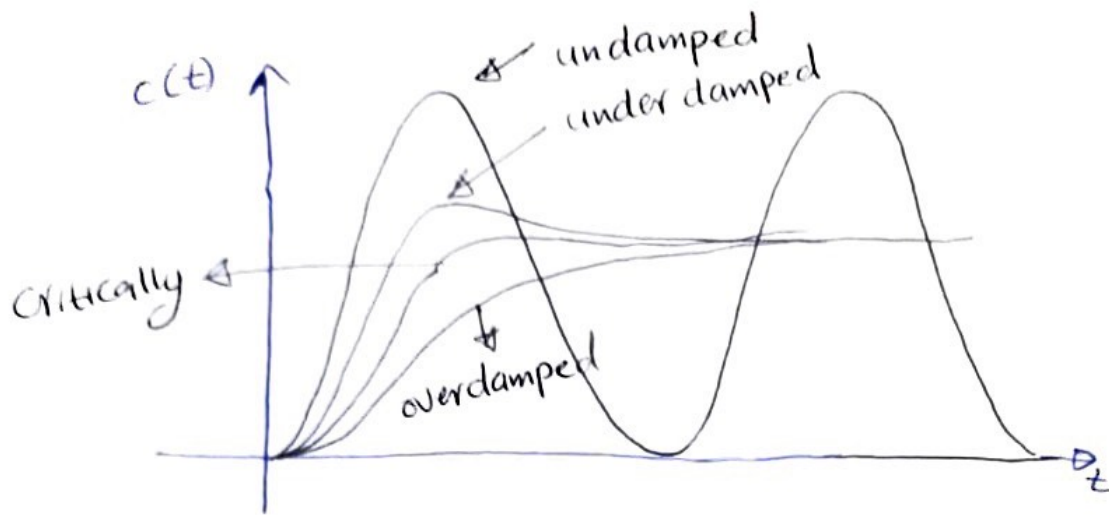
$$c(s) = \frac{1}{s} - \frac{3}{(s+3)^2} - \frac{1}{(s+3)}$$

$$\boxed{c(t) = (1 - 3t e^{-3t} - e^{-3t}) u(t)}$$

poles $\Rightarrow$ repeated ~ critical ~



قال الأستاذ الذي أدرك أن الذي هو
حزير، الصفيط، طر فانه طر
شالتم
القبرة



* تعریف بجعل $\zeta = 1$, التوافق

Natural frequency :-

frequency of oscillation of the system without damping.

damping ratio :-

is responsible for damping in oscillation

⇒ formula of characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

⇒ formula of response

$$CG(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

characteristic equation
(معادلة الجذور)

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2}$$

أكبر الجذور حتمًا ζ

If $\zeta = 0 \Rightarrow s_{1,2} = j\omega_n$

"un damped"

If $\zeta = 1 \Rightarrow s_{1,2} = -\zeta\omega_n$

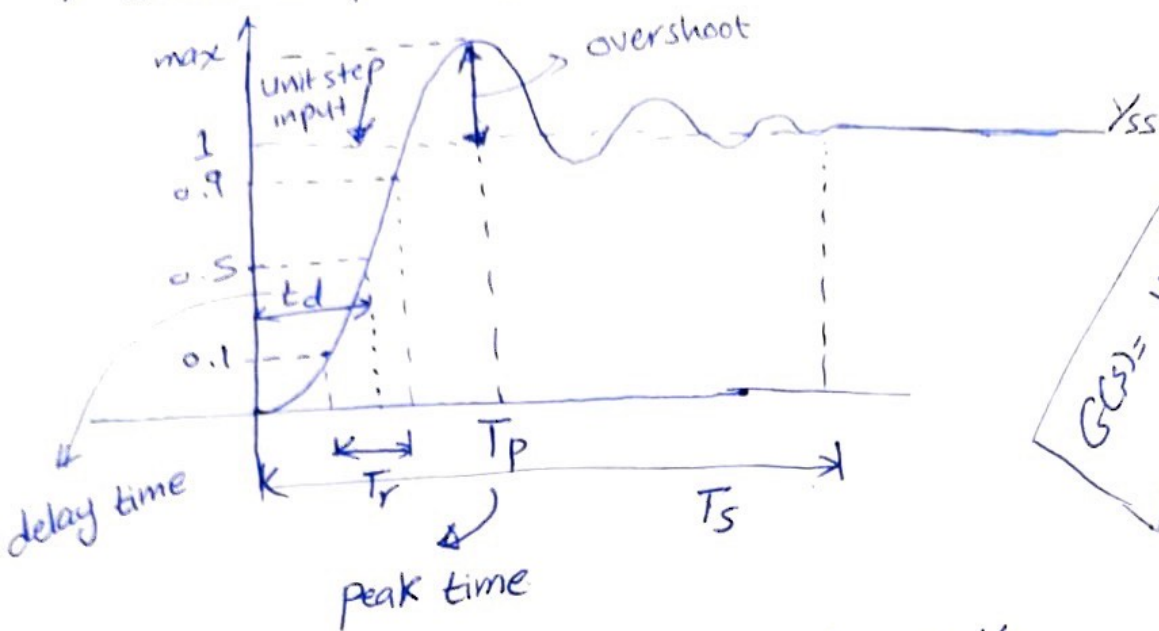
"critically"

If $\zeta > 1 \Rightarrow s_{1,2} = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}$

"overdamped"

If $0 < \zeta < 1 \Rightarrow s_{1,2} = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2}$

* under damped response :-



* Delay time t_d : time to reach 0.5 Y_{ss}

* Rise time t_r : time to rise from 0.1 Y_{ss} to 0.9 Y_{ss}

* Settling time t_s : time to settle within 5% of Y_{ss}

* Peak time $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$

* peak value = $1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$

* percent overshoot = $100 e^{-\xi\pi/\sqrt{1-\xi^2}}$

* settling time = $\frac{3}{\xi\omega_n}$ or $\frac{4}{\xi\omega_n}$

final value $\sim 2\%$ is $\sim 5\%$ is final value

* steady state error = input value - final value

* Time Constant = $\frac{1}{\xi\omega_n}$

for $\xi > 1$, we cannot define peak time, peak value, percent overshoot (no overshoot)

os% = $\frac{\text{maximum value} - \text{final value}}{\text{final value}} \times 100\%$

بَارَكَا اللَّهُ مَا أَتَوْا لَمْ يَنْفَعُوا لَنَا شَيْءٌ
سَقَطَ فِي يَدَيْهِمْ شَيْءٌ مِنْهُ
فَلَمْ يَكُنْ لَهُمْ نَصْرٌ مِنْ رَبِّهِمْ
وَلَمْ يَكُنْ لَهُمْ الْكَافِرِينَ

11/2/2019

Example # | Given the transfer function

$$\text{I)} \quad G(s) = \frac{100}{s^2 + 15s + 100}$$

Find T_p , %OS, T_s and T_r .

So:

$$\omega_n^2 = 100 \Rightarrow \boxed{\omega_n = 10}$$

$$2\zeta\omega_n = 15 \Rightarrow \zeta = \frac{(15)}{(2)(10)} = \boxed{0.75}$$

$$* T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{(10)\sqrt{1-(0.75)^2}} = \boxed{0.475}$$

$$* \%OS = 100 e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 100 e^{-\frac{(0.75)(\pi)}{\sqrt{1-(0.75)^2}}} = \boxed{2.838\%}$$

$$* T_s = \frac{4}{\zeta\omega_n} = \frac{4}{(0.75)(10)} = \boxed{0.533 \text{ s}} \#$$

II Find the value of ζ , ω_n , T_s , T_r , T_p , %OS, k_{dc}

$$G(s) = \frac{361}{s^2 + 16s + 361}$$

$$* k_{dc} = G(s) \Big|_{s=0} = \frac{361}{361} = \boxed{1}$$

$$* \omega_n^2 = 361 \Rightarrow \omega_n = \sqrt{361} = \boxed{19}$$

$$* 2\zeta\omega_n = 16 \Rightarrow \zeta = \frac{16}{(2)(19)} = \boxed{0.421}$$

$$* T_s = \frac{4}{\zeta\omega_n} = \frac{4}{(0.421)(19)} = \boxed{0.5 \text{ s}}$$

$$* T_r = \frac{2.2}{\omega_n} = \frac{2.2}{19} = \boxed{0.115 \text{ s}}$$

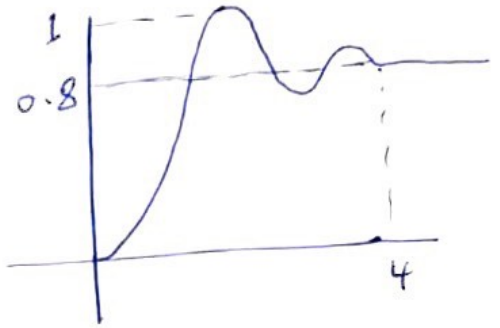
بسم الله الرحمن الرحيم
الحمد لله الذي هدانا لهذا
ما كنا لنهتدي لولا أن هدانا الله
والله ذو الفضل العظيم

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{19 \sqrt{1-(0.421)^2}} = \boxed{0.1822 \text{ s}}$$

$$\begin{aligned} 0.5\% &= 100 * e^{-\pi \zeta / \sqrt{1-\zeta^2}} \\ &= 100 * e^{-(\pi)(0.421) / \sqrt{1-(0.421)^2}} = \boxed{23.267\%} \end{aligned}$$

* Find the transfer function from this step time response

Sol:-



* Input value = 1

* final value = 0.8

$$\begin{aligned} * \text{S.S.E} &= \text{Input value} - \text{final value} \\ &= 1 - 0.8 = \boxed{0.2} \end{aligned}$$

$$* K_{dc} = \frac{\text{final value}}{\text{input value}} = \frac{0.8}{1} = \boxed{0.8}$$

$$* T_s = \boxed{4.5}$$

$$\begin{aligned} * \text{OS\%} &= \frac{\text{maximum value} - \text{final value}}{\text{final value}} * 100\% \\ &= \frac{1 - 0.8}{0.8} * 100\% = \boxed{\%25} \end{aligned}$$

$$\begin{aligned} * \zeta &= \frac{-\ln\left(\frac{\text{OS\%}}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\text{OS\%}}{100}\right)}} = \frac{-\ln\left(\frac{25\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{25\%}{100}\right)}} = \boxed{0.403} \end{aligned}$$

$$* T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{(\zeta)(T_s)} = \frac{4}{(0.403)(4)} = \boxed{2.481}$$

$$G(s) = \frac{K_{dc} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{(0.8)(2.481)^2}{s^2 + 2 * 0.403 * 2.481 s + (2.481)^2}$$

$$G(s) = \frac{4.92}{s^2 + 2s + 6.15}$$

المطلوب

* How to find ζ , ω_n from poles :-

Sol :-

$$\chi = \sqrt{(-\zeta\omega_n)^2 + (\omega_n\sqrt{1-\zeta^2})^2}$$

$$\chi^2 = \zeta^2\omega_n^2 + \omega_n^2(1-\zeta^2)$$

$$\chi^2 = \cancel{\zeta^2\omega_n^2} + \omega_n^2 - \cancel{\zeta^2\omega_n^2}$$

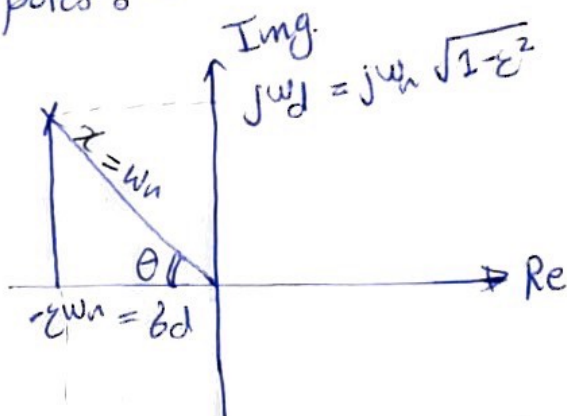
$$\chi^2 = \omega_n^2 \Rightarrow \chi = \omega_n$$

$$\theta = \tan^{-1} \left(\frac{\omega_n\sqrt{1-\zeta^2}}{-\zeta\omega_n} \right)$$

$$\cos\theta = \zeta$$

$$\omega_n = \sqrt{(-\mu\omega_n)^2 + (\omega_n\sqrt{1-\zeta^2})^2}$$

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



الزوايا بين المحاور
تؤخذ من
سوى الزوايا

Example :-

Find ζ , ω_n

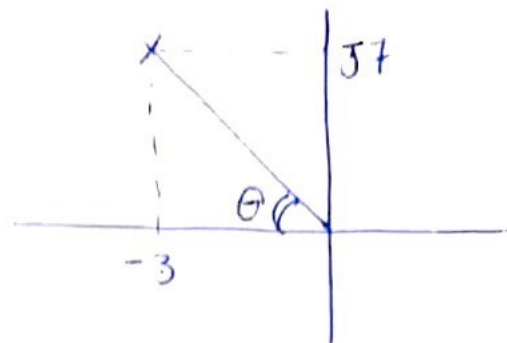
$$\omega_n = \sqrt{(7)^2 + (3)^2} = \boxed{7.615}$$

$$\zeta = \cos \theta$$

$$\theta = \tan^{-1}\left(\frac{7}{-3}\right) \Rightarrow \boxed{-66.80^\circ}$$

$$\zeta = \cos(-66.80) = \boxed{0.394}$$

$\mu < 1 \Rightarrow$ underdamped



نأخذ الزاوية التي يكوها المتجه مع المحور الحقيقي
نأخذ القيمة التي يكوها المتجه مع المحور الحقيقي
نأخذ القيمة التي يكوها المتجه مع المحور الحقيقي

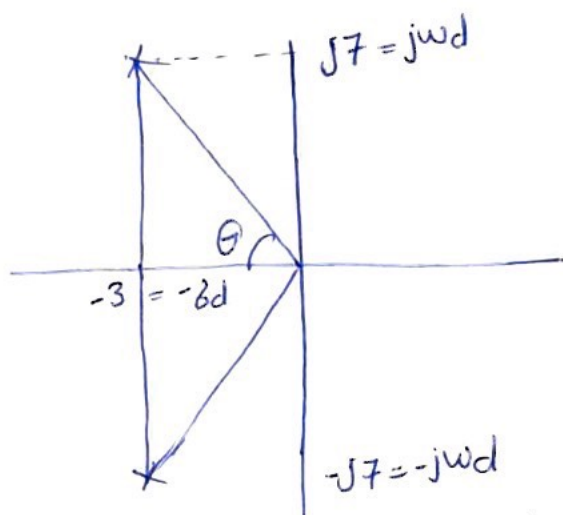
problem #

في التيارات الكثر

Given the pole plot shown in figure below, find ζ and ω_n , T_p , %OS and T_s

$$\begin{aligned} \zeta &= 0.394 \\ \omega_n &= 7.615 \end{aligned} \quad \left. \begin{array}{l} \text{من السؤال السابق} \\ \text{تم حسابهم} \end{array} \right\}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{7} = \boxed{0.4495}$$



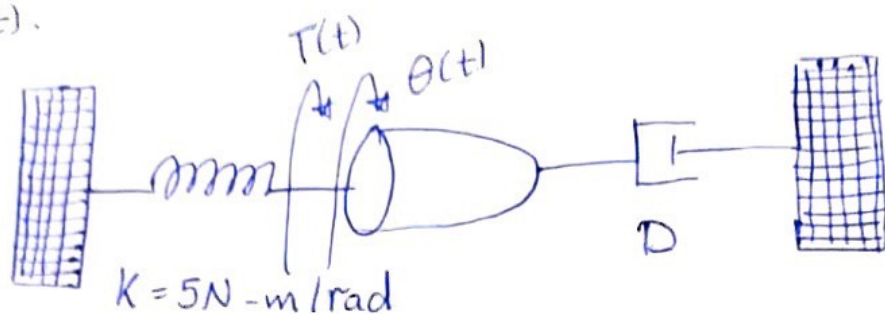
$$\begin{aligned} \%OS &= e^{-\pi \zeta / \sqrt{1-\zeta^2}} \times 100\% \\ &= e^{-\pi \times 0.394 / \sqrt{1-(0.394)^2}} \times 100\% = \boxed{26\%} \end{aligned}$$

$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{\sigma_d} = \frac{4}{3} = \boxed{1.333 \text{ s}}$$

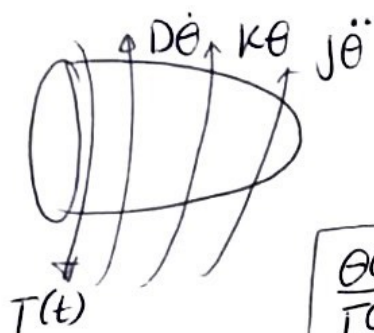
الله نور السموات والأرض قبل نوره
كليلة فيها حبس، حبس، حبس
نصحية في النور

Problem # المشكلة رقم

Given the system shown below, find J and D to yield 20% overshoot and a settling time of 2 seconds for a step input of torque $T(t)$.



Sol:



$$T(t) = J\ddot{\theta} + D\dot{\theta} + K\theta$$

$$T(s) = (Js^2 + Ds + K)\theta(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + K}}$$

نظام ثنائي القطب مع 1 صفر، حيث نريد أن يكون

$$\frac{\theta(s)}{T(s)} = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

$$\boxed{\omega_n = \sqrt{\frac{K}{J}}}, \quad \boxed{2\zeta\omega_n = \frac{D}{J}}$$

$$\zeta = \frac{-\ln\left(\frac{0.5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{0.5\%}{100}\right)^2}} = \frac{-\ln\left(\frac{20}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{20}{100}\right)}} = \boxed{0.456}$$

$$2\zeta\omega_n = \frac{D}{J}, \quad T_s = \frac{4}{\zeta\omega_n} = 2 \Rightarrow \omega_n = \frac{4}{(2)(0.456)} = \boxed{4.385}$$

$$\omega_n^2 = \frac{K}{J} \Rightarrow J = \frac{5}{(4.385)^2} = \boxed{0.26 \text{ kg.m}^2}$$

$$D = 2\zeta\omega_n J \Rightarrow (2)(2)(0.26) = \boxed{1.04 \text{ N.m}}$$

#

Example # Given $G(s) = \frac{25k}{s^2 + (5 + 5000k_1)s + 25k}$

0.5% = 10%, $T_s = 0.05s$, for unit step input

Find the value of k, k_1, ω_n, ζ , Type of response

Sol:-

$$G(s) = \frac{25k}{s^2 + 5s + 5000k_1 + 25k}$$

$$\zeta = \frac{-\ln\left(\frac{0.5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{0.5\%}{100}\right)^2}} = \frac{-\ln\left(\frac{10}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{10}{100}\right)^2}} = \boxed{0.5911}$$

$$\zeta < 1 \Rightarrow \boxed{\text{underdamped}}$$

$$T_s = 0.05 = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{(0.05)(0.5911)} = \boxed{135.32}$$

$$25k = \omega_n^2 \Rightarrow \omega_n = \sqrt{25k} \Rightarrow k = \frac{\omega_n^2}{25} = \frac{(135.32)^2}{25} = \boxed{732.54}$$

$$5 + 5000k_1 = 2\zeta\omega_n$$

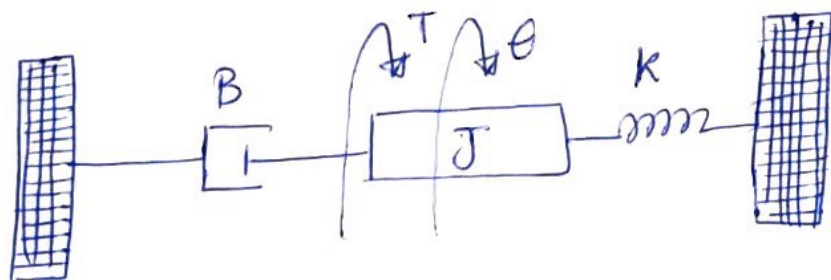
$$5 + 5000k_1 = (2)(0.5911)(135.32)$$

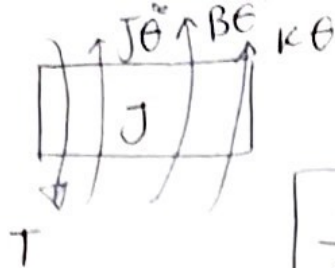
$$5 + 5000k_1 = 159.975$$

$$5000k_1 = 154.975$$

$$\Rightarrow \boxed{k_1 = 0.0311} \quad \#$$

Example # Design for $\zeta = 0.5$, $T_s = 8\text{sec}$, $k_{dc} = 1$





$$T(t) = J\ddot{\theta} + B\dot{\theta} + K\theta$$

$$T(s) = (Js^2 + Bs + K)\theta(s)$$

$$\left[\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K} \right]$$

$$G(s) = \frac{1/J}{s^2 + \frac{B}{J}s + \frac{K}{J}} = \frac{k_d \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\bar{T}_s = \frac{4}{\xi\omega_n} = 8 \Rightarrow \xi\omega_n = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$(0.5)\omega_n = \frac{1}{2} \Rightarrow \boxed{\omega_n = 1}$$

$$\frac{1}{J} = k_d \omega_n^2 \Rightarrow J = \frac{1}{k_d \omega_n^2} = \frac{1}{(1)(1)} \Rightarrow \boxed{J=1}$$

$$\frac{B}{J} = 2\xi\omega_n \Rightarrow B = J \cdot 2\xi\omega_n = (1)(0.5)(1) = 1$$

$$\boxed{B=1}$$

$$\frac{K}{J} = \omega_n^2 \Rightarrow K = (1)(1) = \boxed{1} \neq$$

$$\boxed{J=B=K=1}$$