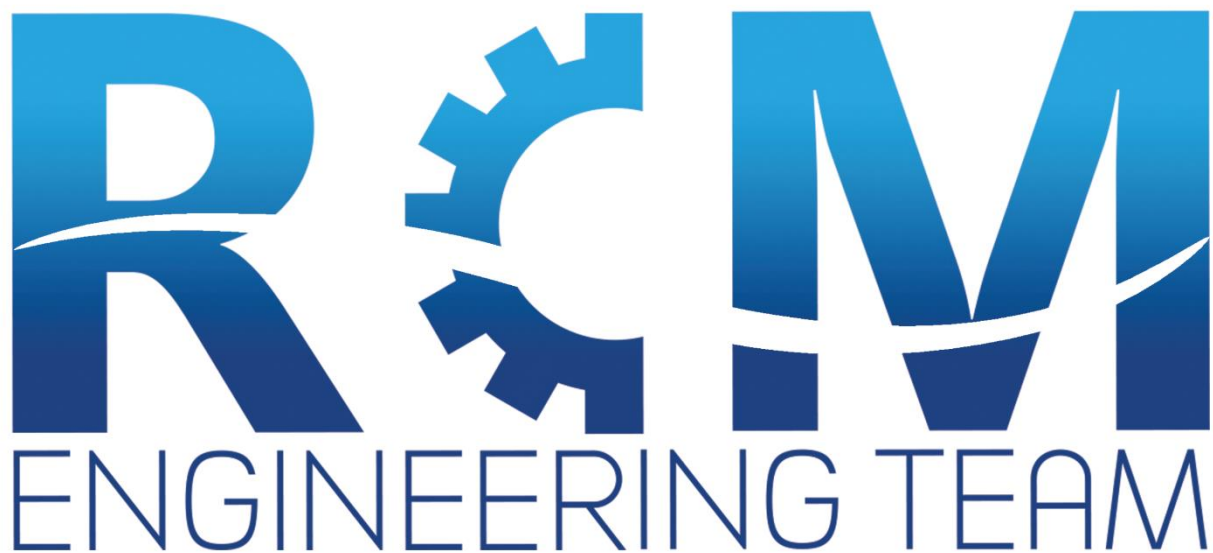




RCM Engineering Team “ Branch of the Islamic University ”

Chapter 5

for : Control Systems – EELE 3360



Preparation: Eng. Hadil Jamal Matter

Assistant teacher at the Islamic University of Gaza

The second semester of the academic year 2020 - 2021

* objectives :-

- Block diagram -

- 1] Reduce a block diagram of multiple subsystems to a single block representation the transfer function.
- 2] Convert the block diagram to signal flow graph.
- 3] Find the transfer function of multiple subsystems using Mason's rule.
- 4] Represent state equations as signal-flow graphs.
- 5] perform transformations between similar systems using transformation matrices, state space and diagonalize a system matrix

There are two methods to interconnections of many subsystems:

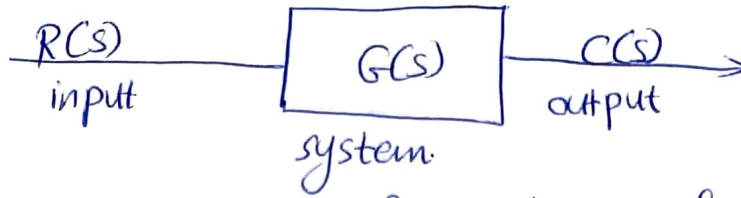
- 1] Block diagram
- 2] signal flow graphs (SFG)

The interconnection types :-

- 1] cascade form.
- 2] parallel form.
- 3] feedback form.

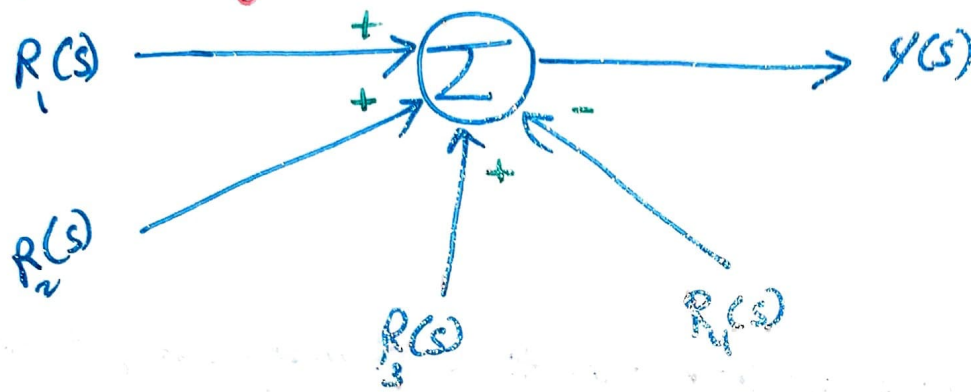
الترابط بين الأنظمة
المتعددة
الترابط التسلسلي
الترابط المتوازي
الترابط بالتغذية الراجعة
الترابط المتسلسل
الترابط المتوازي
الترابط بالتغذية الراجعة
الترابط المتسلسل
الترابط المتوازي
الترابط بالتغذية الراجعة

* Block diagram :-



The Block diagram represents of a single transfer function, so If you have multiple subsystem you can represent it in different methods and forms.

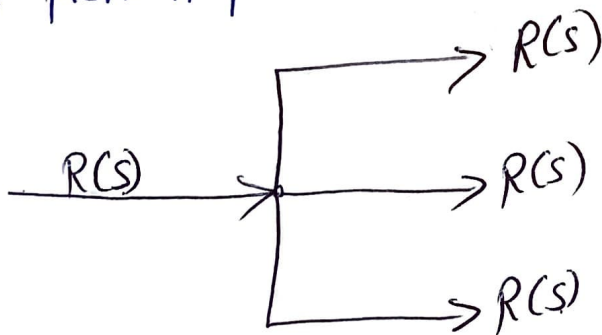
* Summing Junction :-



$$Y(s) = R_1(s) + R_2(s) + R_3(s) - R_4(s)$$

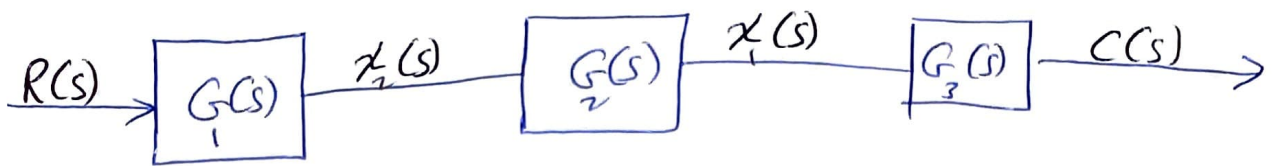
The output of summing Junction is equal of all input signals

* pick-off point :-



فاختلف الأثرين من بينهم
كفروا من مشركهم عظيم
وأنظروا يوم يلقون الله الظالمين
اليوم في جهنم فيه
سورة مريم

* Cascade form :-

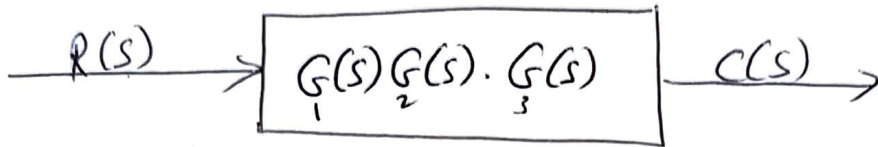


$$X(s) = R(s) \cdot G(s)$$

$$\begin{aligned} X_2(s) &= R(s) \cdot G_1(s) \\ X_1(s) &= X_2(s) \cdot G_2(s) \Rightarrow R(s) \cdot G_1(s) \cdot G_2(s) \\ X_1(s) &= X_2(s) \cdot G_2(s) \Rightarrow R(s) \cdot G_1(s) \cdot G_2(s) \cdot G_3(s) \end{aligned}$$

$$C(s) = X_1(s) \cdot G_3(s) \Rightarrow \underline{\underline{\text{input}}} R(s) G_1(s) G_2(s) G_3(s)$$

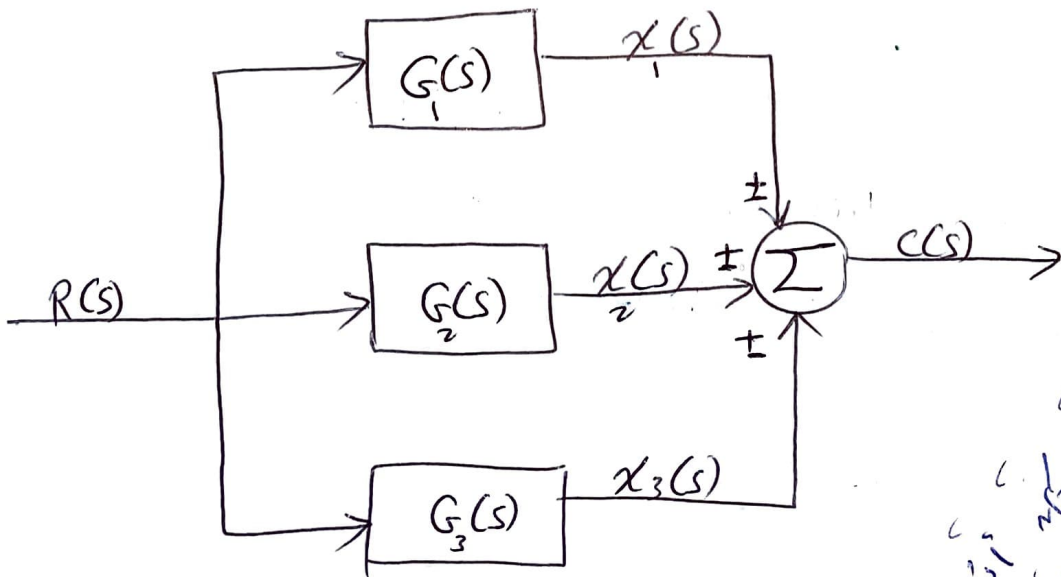
↓
output



$$\frac{C(s)}{R(s)} = G_1(s)G_2(s)G_3(s)$$

* parallel form :-

all subsystem have the same input signal, and the output equal the summation of all subsystem.



$$X(s) = R(s)G(s)$$

$$X_2(s) = R(s) \cdot G_2(s)$$

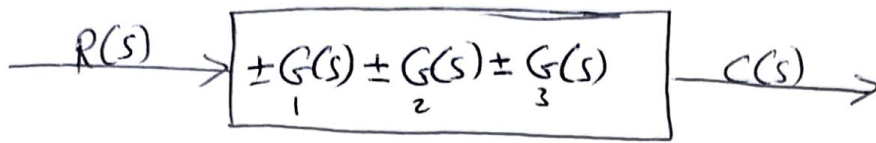
$$X_3(s) = R(s) \cdot G_3(s)$$



Handwritten notes in Arabic script, including the word "عز" (Ez) and "عز" (Ez), with a large arrow pointing right.

$$C(s) = [G_1(s) \pm G_2(s) \pm G_3(s)] \cdot R(s)$$

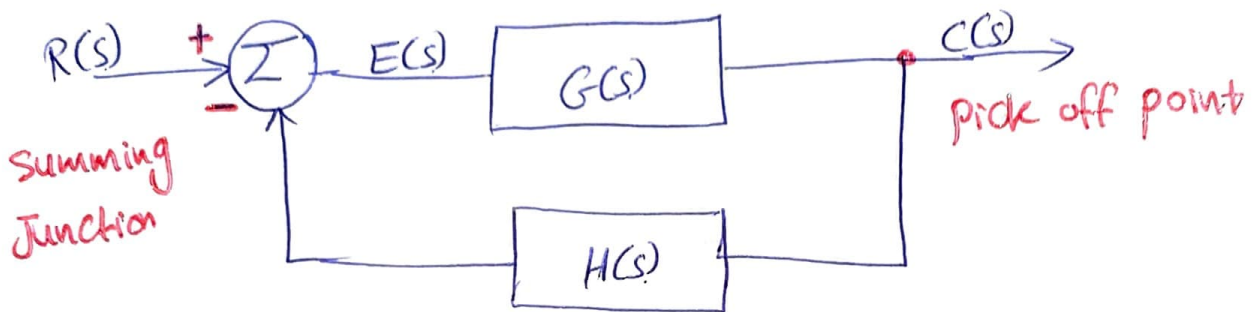
$$\frac{C(s)}{R(s)} = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$



#

* Feedback form :-

نظام مع تغذية راجعة feedback system



$E(s)$: error signal which is the output of summing junction
 $R(s)$: desired input for the closed loop system

$$C(s) = E(s) \cdot G(s) \quad \dots (1)$$

$$E(s) = R(s) - C(s) \cdot H(s) \quad \dots (2)$$

المعادلة (2) في (1)

$$C(s) = (R(s) - C(s) \cdot H(s)) \cdot G(s)$$

$$C(s) = R(s) \cdot G(s) - C(s) H(s) \cdot G(s)$$

$$C(s) + C(s) \cdot H(s) \cdot G(s) = R(s) \cdot G(s)$$

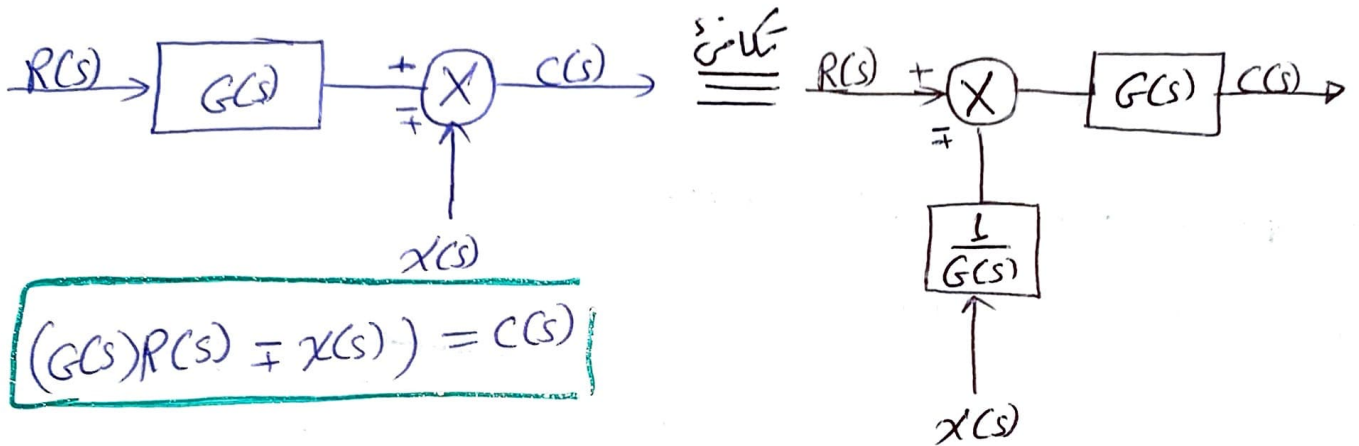
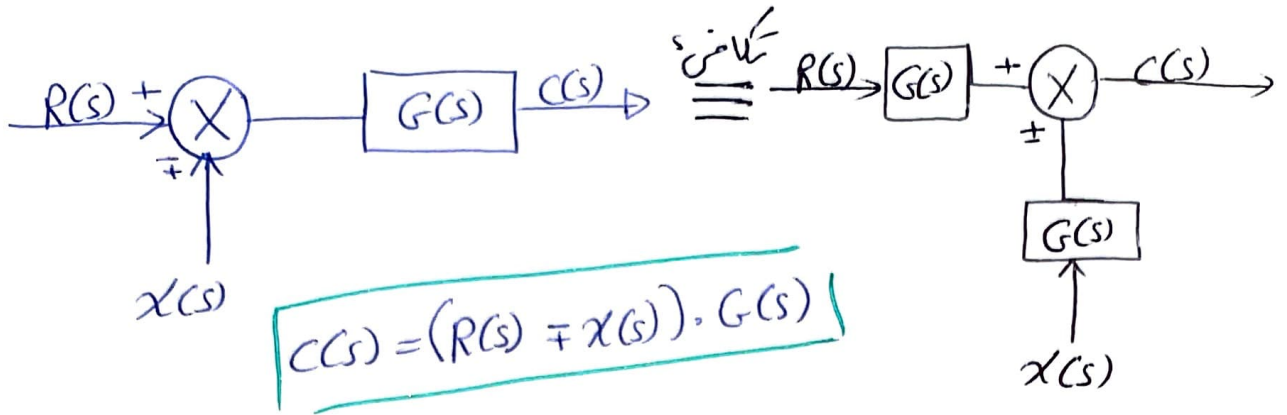
$$C(s) (1 + H(s) G(s)) = R(s) \cdot G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s) \cdot G(s)}}$$

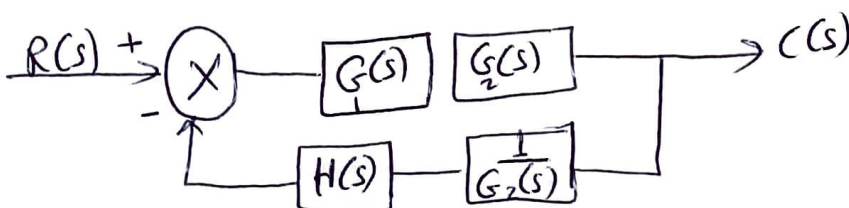
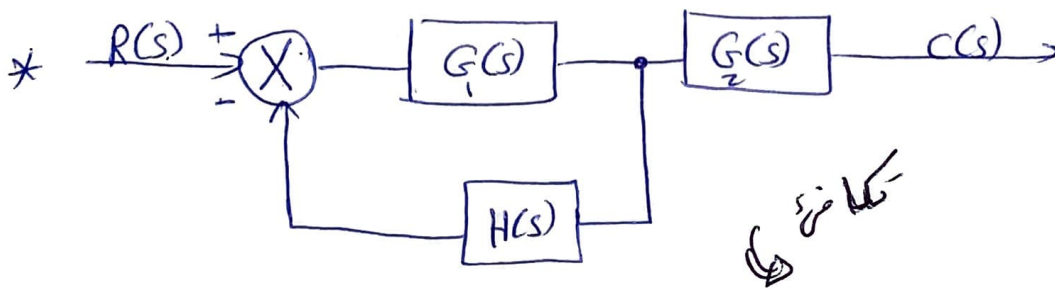
هذا هو TF لنظام مع تغذية راجعة
 closed loop system

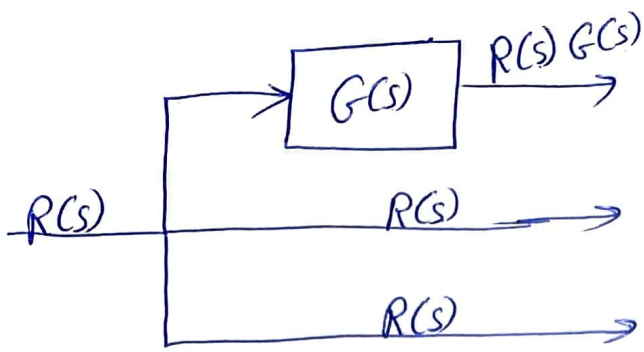
* Moving Blocks to Create Familiar Forms: -

⇒ Move blocks left and right past summing junctions and pick off points

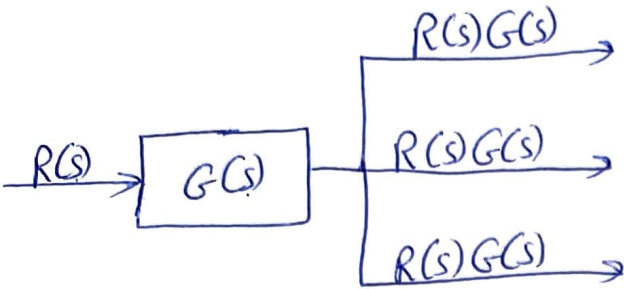
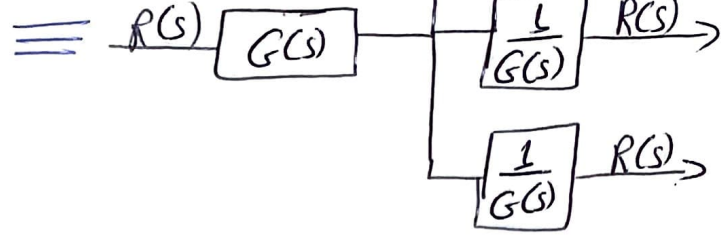


← خلاصه الكلام : لما انتقل ال Block قبل ال Summing junction
على حيز ، لما انتقل ال Block بعد ال Summing junction
على حيز مع معك ال Block

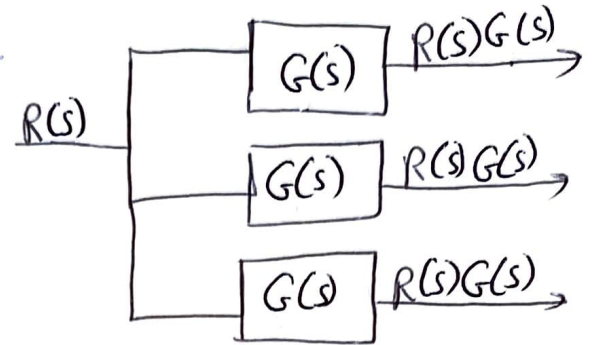




جانبی



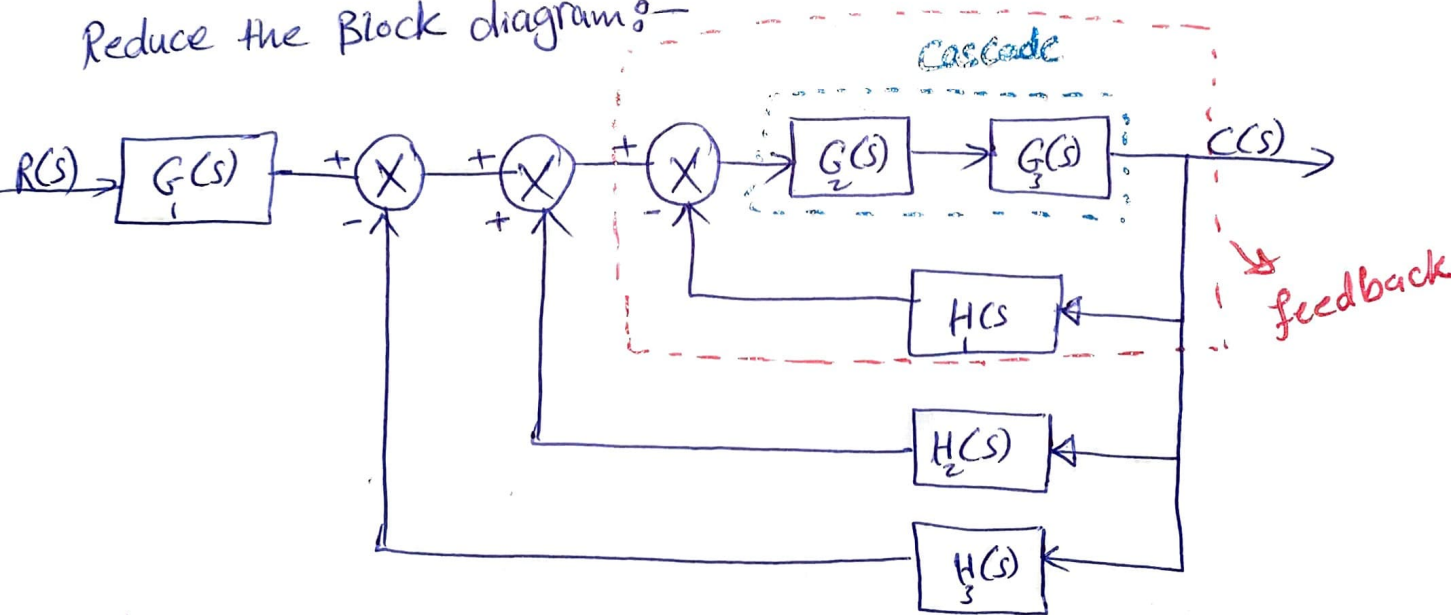
جانبی



Example # 5.1

مسئله الکترونیک

Reduce the block diagram:-



So:-

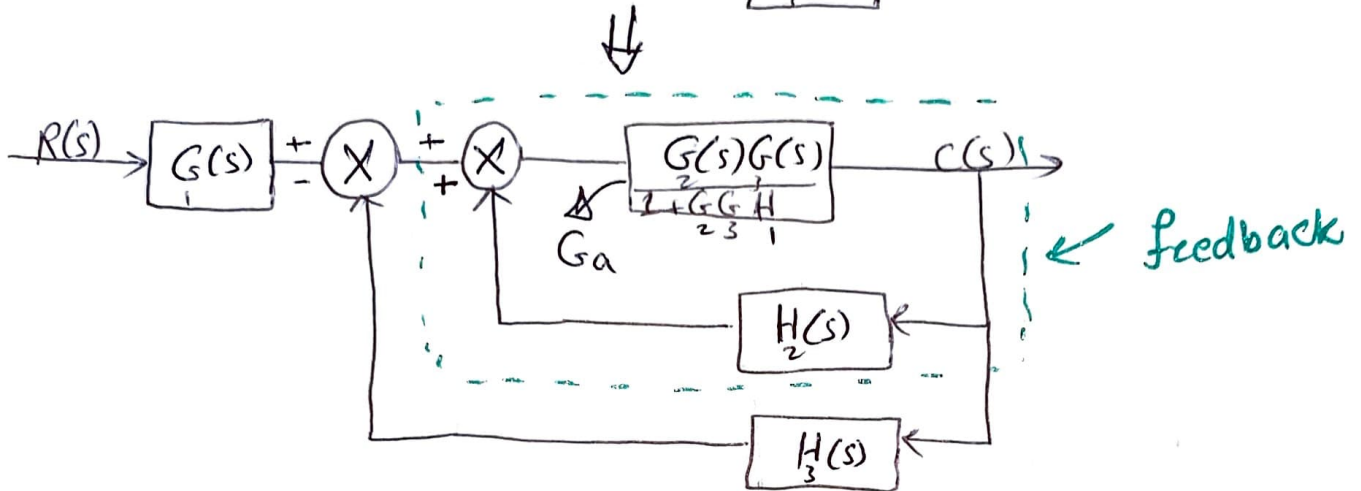
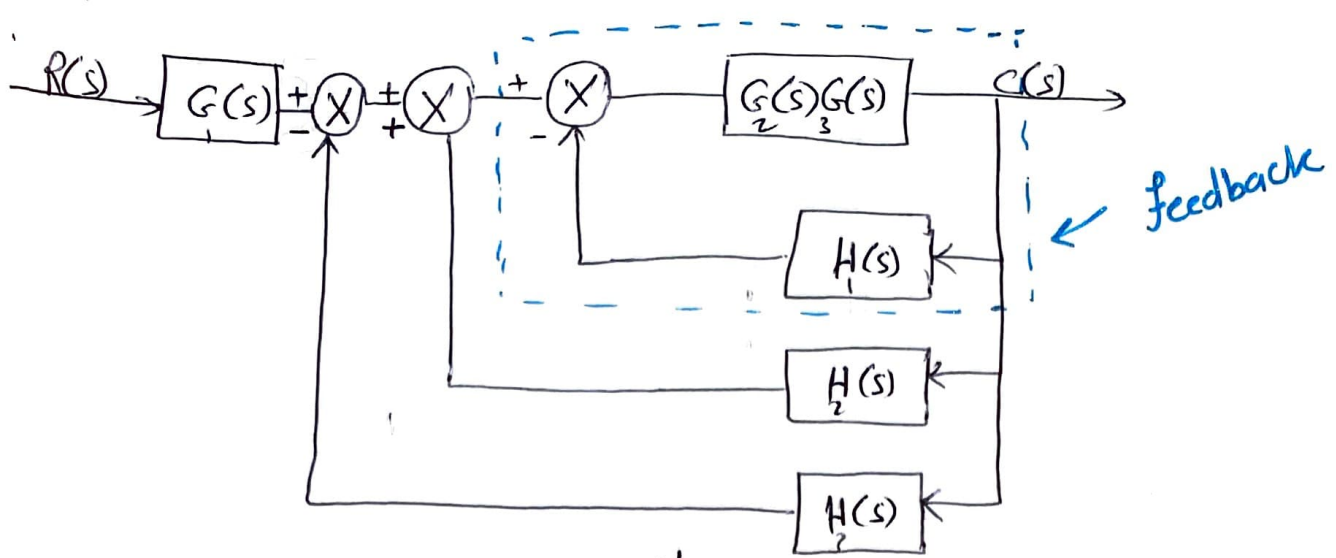
$G_2(s), G_3(s) \Rightarrow \text{Cascade}$

$$\boxed{G_2(s)G_3(s)}$$

از طرف دیگر به هم می‌زنند

$G_2(s)G_3(s), H_1(s) \Rightarrow \text{Feedback}$

$$\boxed{\frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_1(s)}}$$

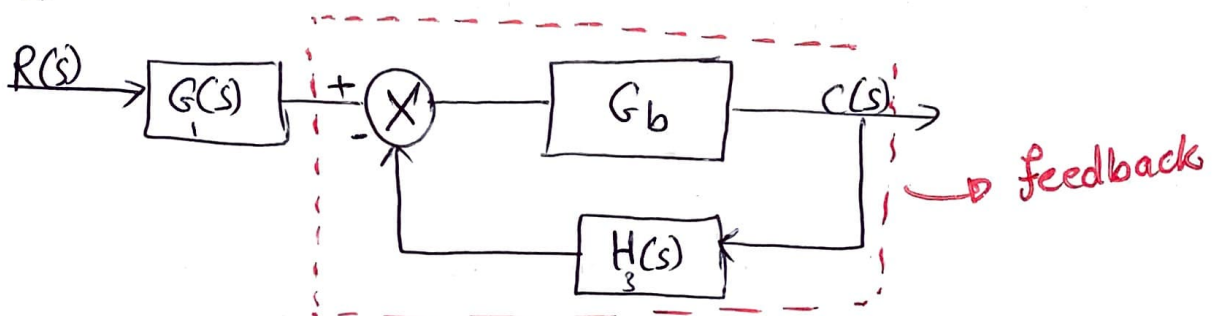


$G_a, H_2 \Rightarrow \text{feedback}$

$$G_a = \frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_1(s)}$$

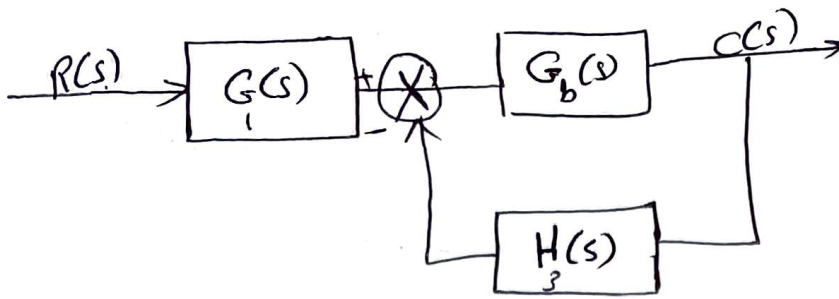
$$G_b = \frac{G_a}{1 + G_a H_2} \Rightarrow \frac{\frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_1(s)}}{1 + \frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_1(s)} H_2(s)} = \frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_1(s) - G_2(s)G_3(s)H_2(s)}$$

نقل الأجزاء إلى المقام
لأنه في المقام



$G_b, H_3 \Rightarrow \text{feedback}$

$$G_b = \frac{G_2 G_3}{1 + G_2 G_3 H_1 - G_2 G_3 H_2}$$



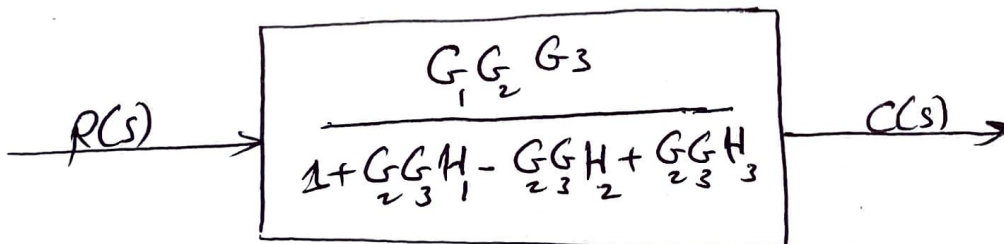
G_b, H_3

$$G_c = \frac{G_b}{1 + G_b H_3} \Rightarrow \frac{G_2 G_3}{1 + G_2 G_3 H_3} = \frac{G_2 G_3}{1 + G_2 G_3 H_3} \quad \#$$



$G_1(s), G_c(s) \Rightarrow \text{cascade.}$

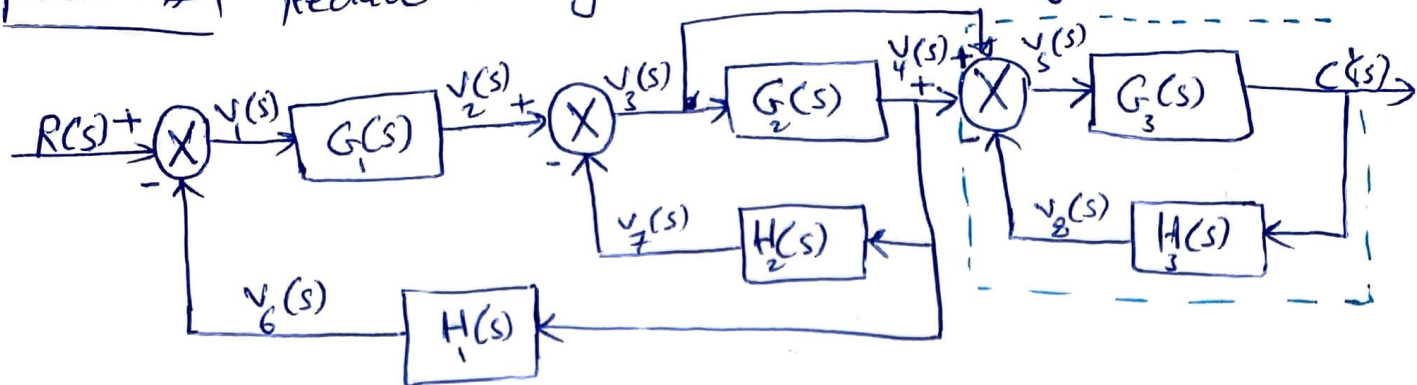
$$G_d = G_1(s) G_c(s) \Rightarrow \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_1 - G_2 G_3 H_2 + G_2 G_3 H_3}$$



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_1 - G_2 G_3 H_2 + G_2 G_3 H_3} \quad \#$$

هو المطلوب

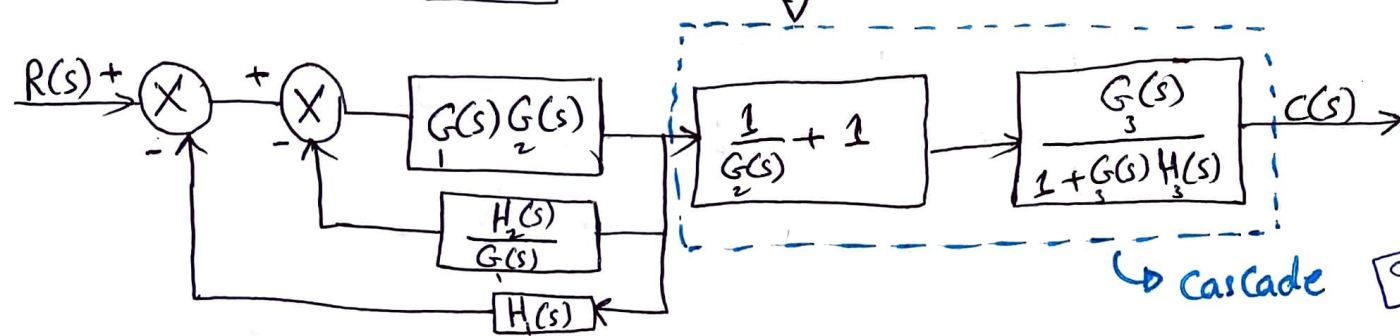
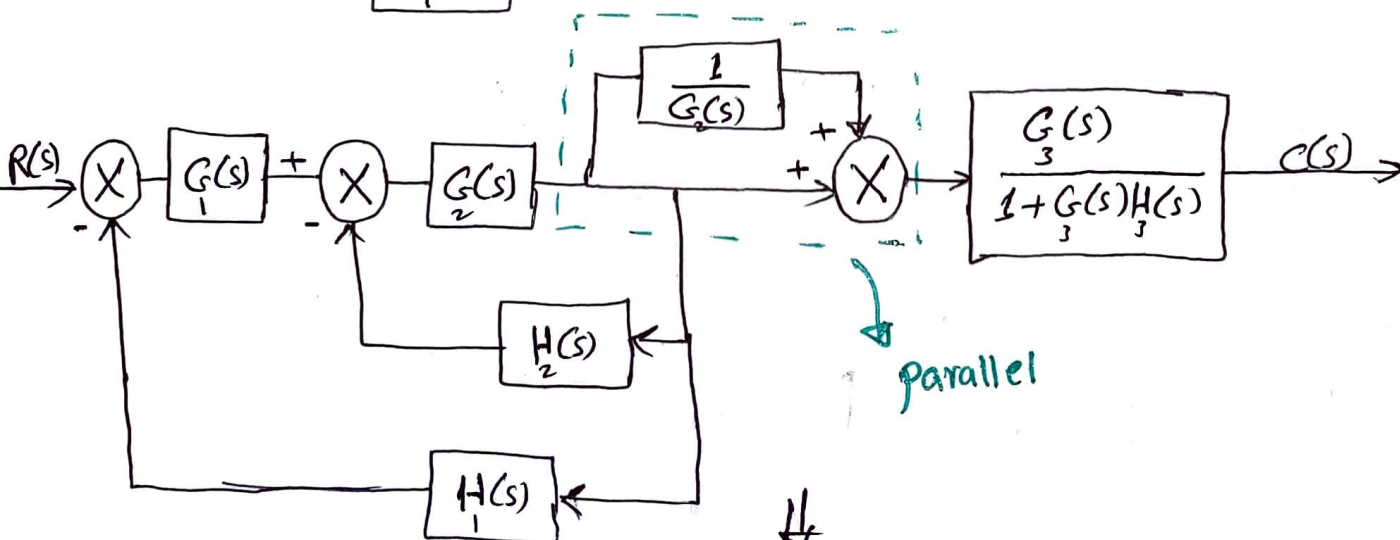
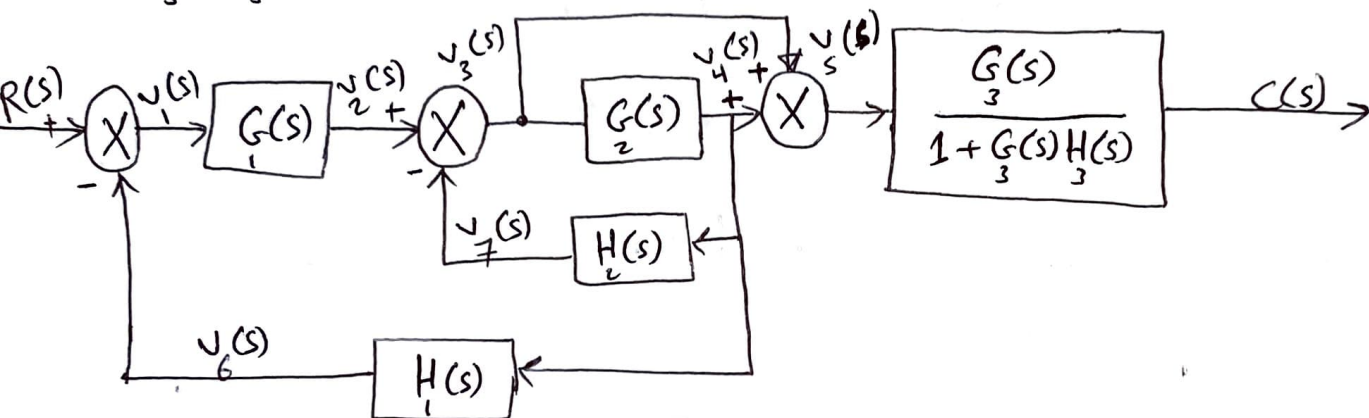
problem #1 Reduce the system shown to a single T.F

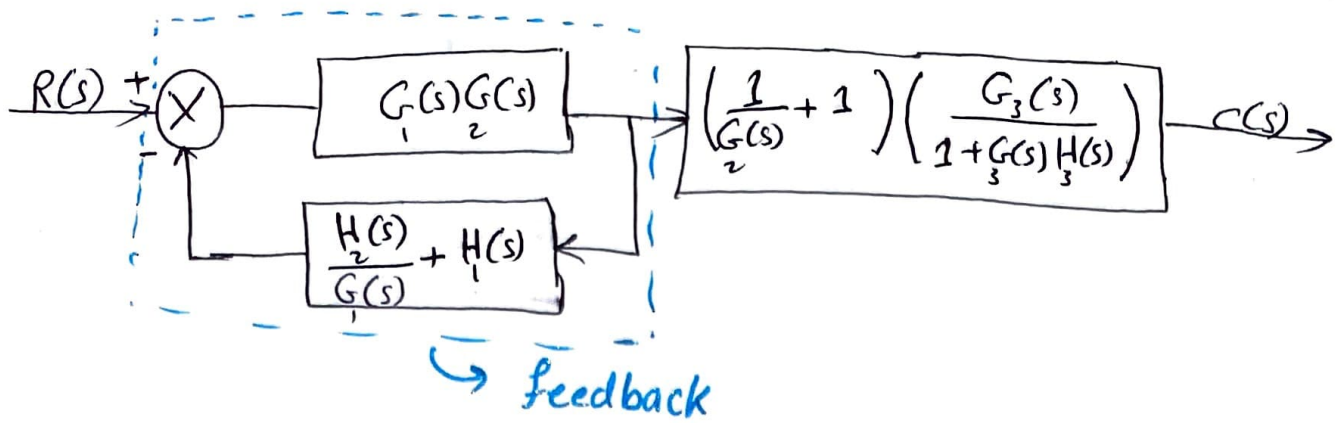


Sol:-

$G_3(s), H_3(s) \Rightarrow$ feedback

$$\frac{G_3(s)}{1 + G_3(s)H_3(s)} \quad \leftarrow \text{parallel}$$

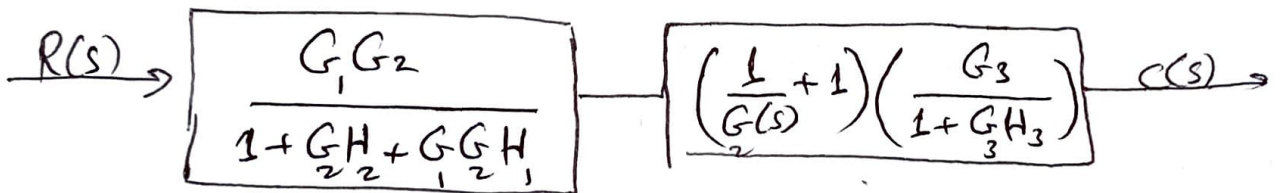




$G_1(s)G_2(s)$ feedback with $\frac{H_2(s)}{G_1(s)} + H_1(s)$

$$\frac{G_1 G_2}{1 + (G_1 G_2) \left(\frac{H_2}{G_1} + H_1 \right)} = \boxed{\frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_1}}$$

المصطلح لهم ←

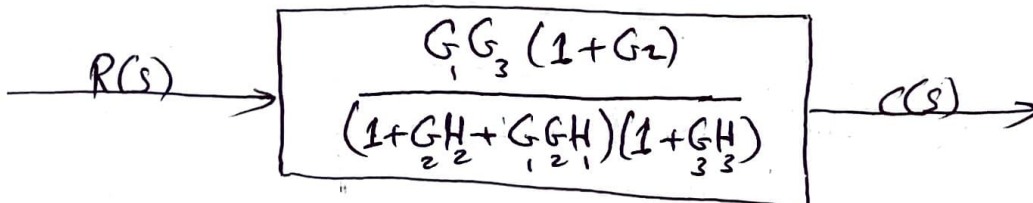


$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_2 + 1) G_3}{(1 + G_2 H_2 + G_1 G_2 H_1) G_2 (1 + G_3 H_3)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_3 (1 + G_2)}{(1 + G_2 H_2 + G_1 G_2 H_1) (1 + G_3 H_3)}}$$

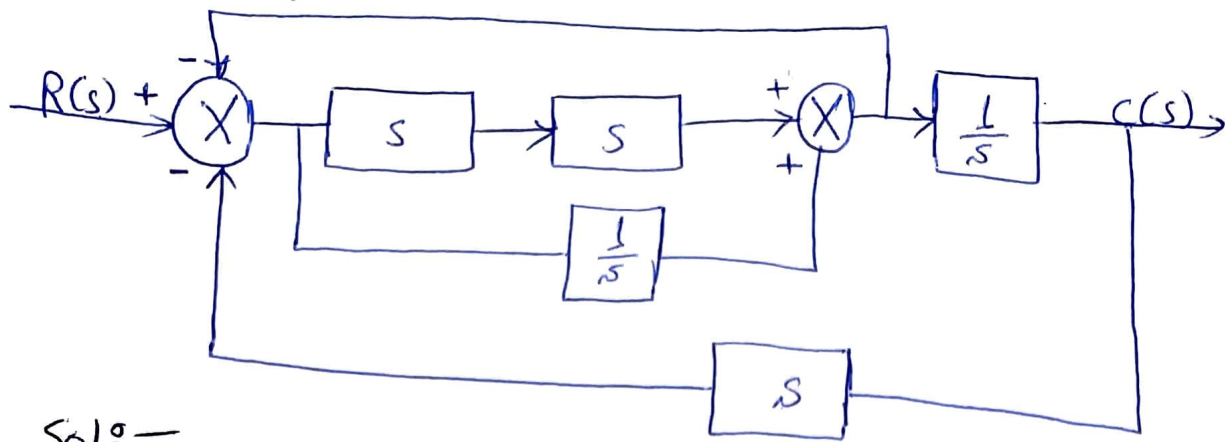
#

"بينا كلامنا يا اولادنا وبنينا"
 "الاسفل فالتبنا مع السهل"
 "م ال عول"



#

* Find the Equivalent transfer function:-

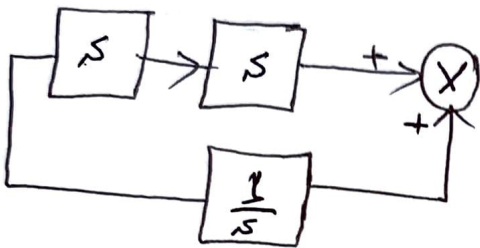


Sol:-



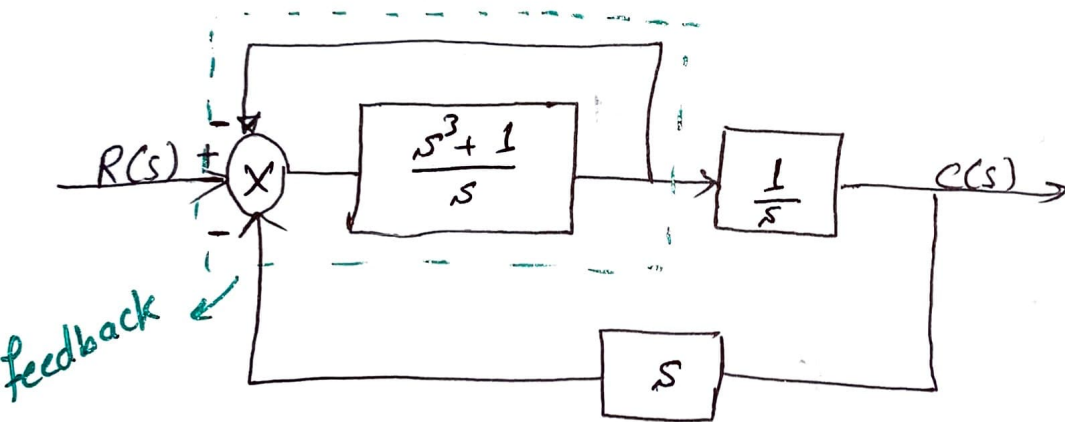
cascade

$$s \times s = s^2 \text{ i.e.,}$$



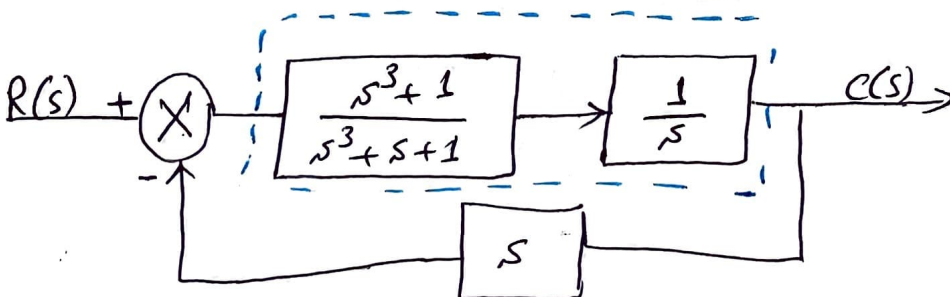
parallel

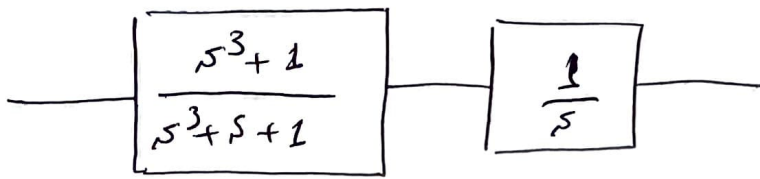
$$s^2 + \frac{1}{s} = \frac{s^3 + 1}{s}$$



$$\frac{\left(\frac{s^3 + 1}{s}\right)}{1 + \left(\frac{s^3 + 1}{s}\right)(1)} = \frac{s^3 + 1}{s + s^3 + 1}$$

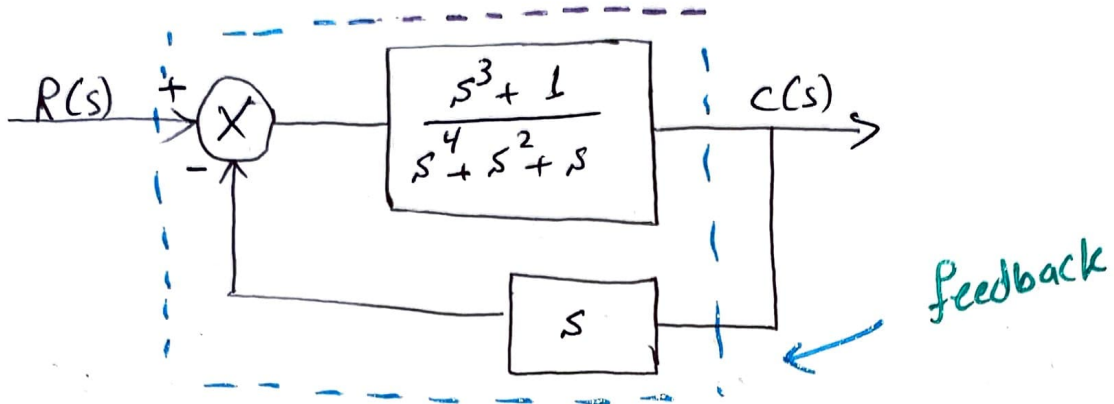
cascade





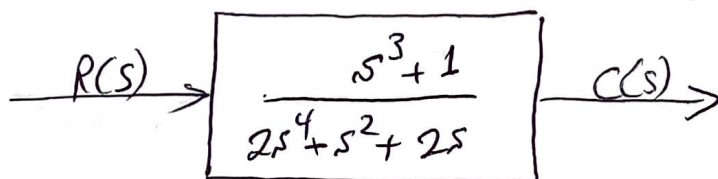
Cascade =

$$\left(\frac{s^3 + 1}{s^3 + s + 1} \right) \left(\frac{1}{s} \right) \Rightarrow \frac{s^3 + 1}{s(s^3 + s + 1)} = \boxed{\frac{s^3 + 1}{s^4 + s^2 + s}}$$



$$\frac{C(s)}{R(s)} = \frac{\left(\frac{s^3 + 1}{s^4 + s^2 + s} \right)}{1 + s \left(\frac{s^3 + 1}{s^4 + s^2 + s} \right)} \Rightarrow \frac{(s^3 + 1)}{s^4 + s^2 + s + s^4 + s}$$

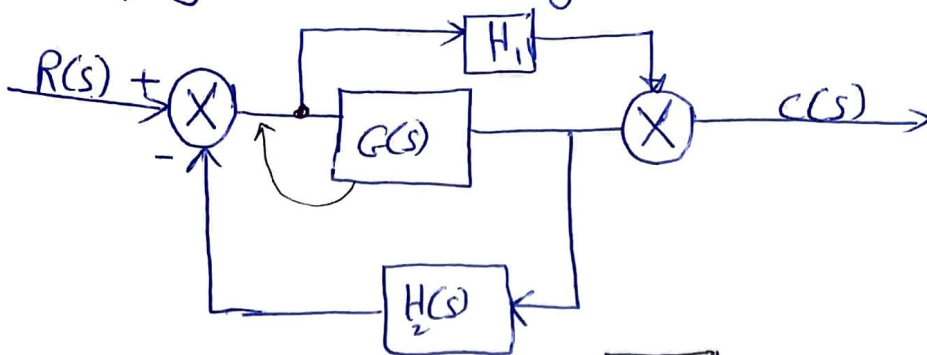
$$\boxed{\frac{C(s)}{R(s)} = \frac{s^3 + 1}{2s^4 + s^2 + 2s}} \quad \#$$



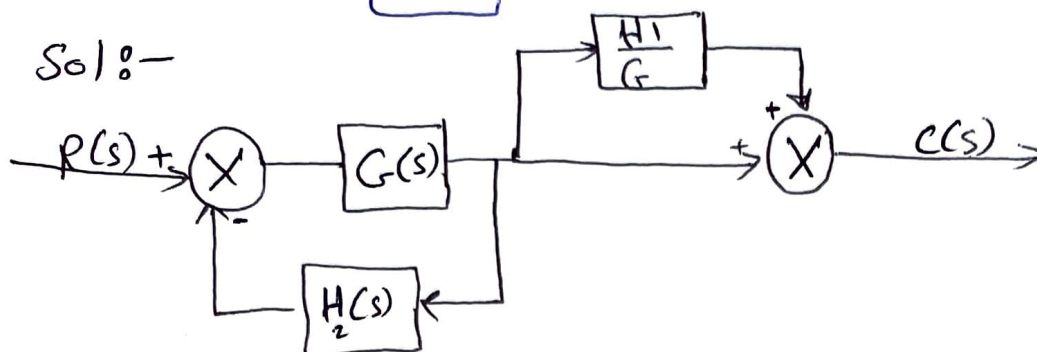
وهو المطلوب

أنا الذي دأبوا على الصلابة
التي دأبوا على الصلابة
التي دأبوا على الصلابة
التي دأبوا على الصلابة
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التي دأبوا على الصلابة

* Simplify the Block diagram:-



Sol:-



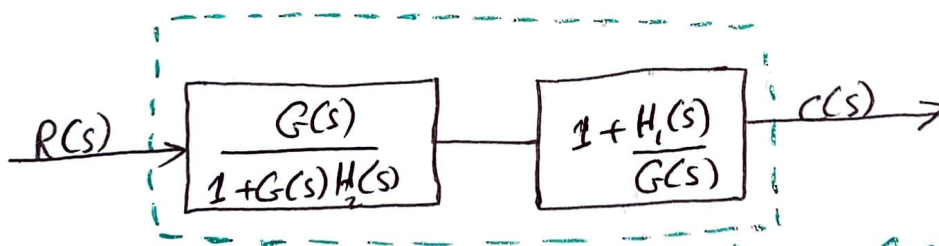
$G(s), H_2(s) \Rightarrow$ feedback

$$\frac{G(s)}{1 + G(s)H_2(s)}$$

parallel

$\frac{H_1(s)}{G(s)}, 1 \Rightarrow$ parallel

$\frac{H_1(s)}{G(s)} + 1 \Rightarrow$ series

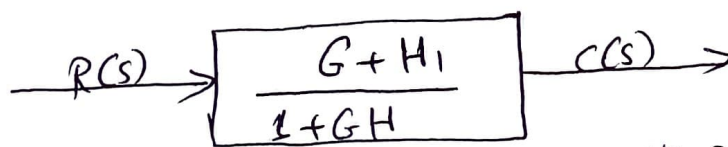


Cascade (series)

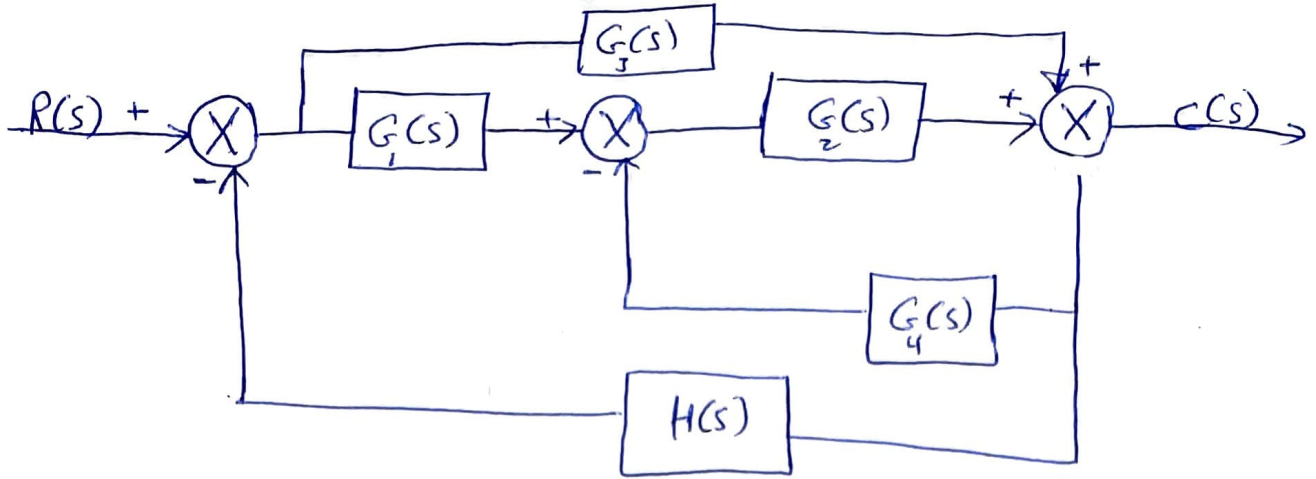
$$\frac{C(s)}{R(s)} = \left(\frac{G(s)}{1 + G(s)H_2(s)} \right) \cdot \left(1 + \frac{H_1(s)}{G(s)} \right)$$

$$\Rightarrow \left(\frac{G(s)}{1 + G(s)H_2(s)} \right) \left(\frac{G(s) + H_1(s)}{G(s)} \right)$$

$$\frac{C(s)}{R(s)} \Rightarrow \left[\frac{G(s) + H_1(s)}{1 + G(s)H_2(s)} \right]$$

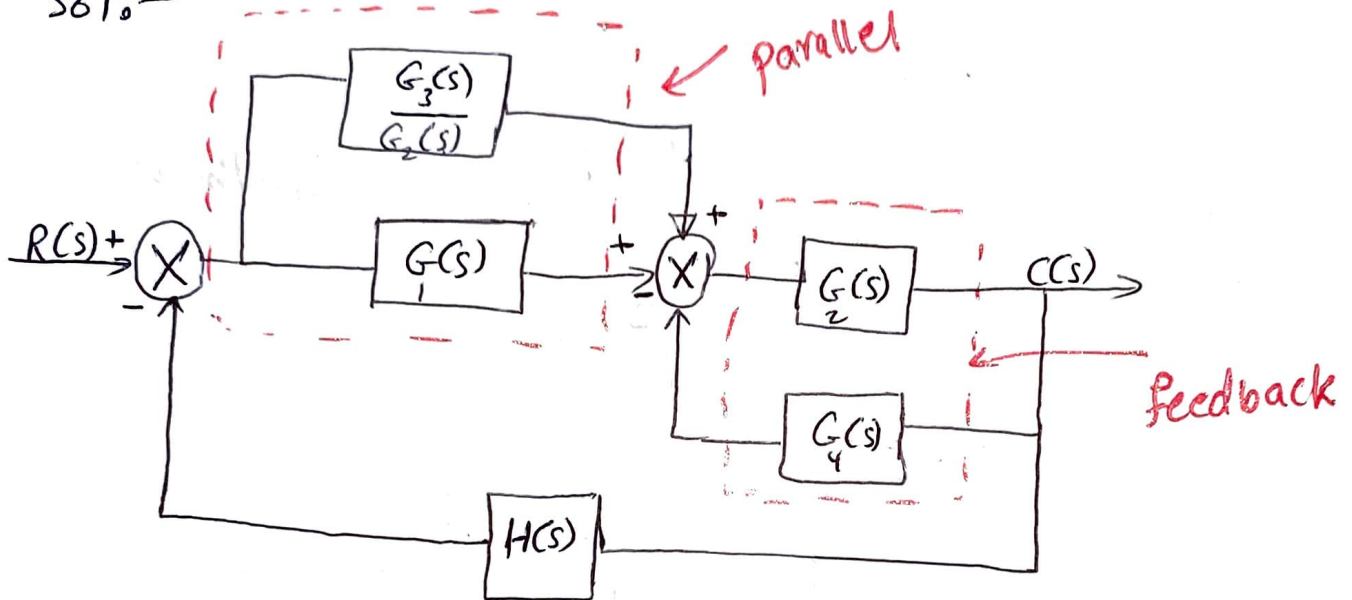


calc,



* Reduce the system to single transfer function:-

Sol:-

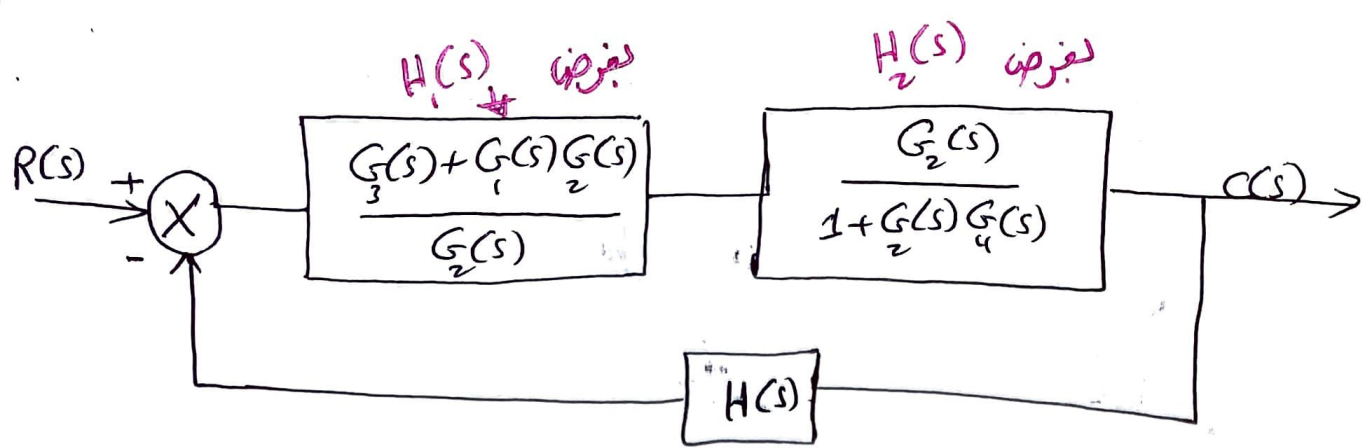


$G_2(s), G_4(s) \Rightarrow$ feedback

$$\boxed{\frac{G_2(s)}{1 + G_2(s)G_4(s)}} \leftarrow \text{feedback}$$

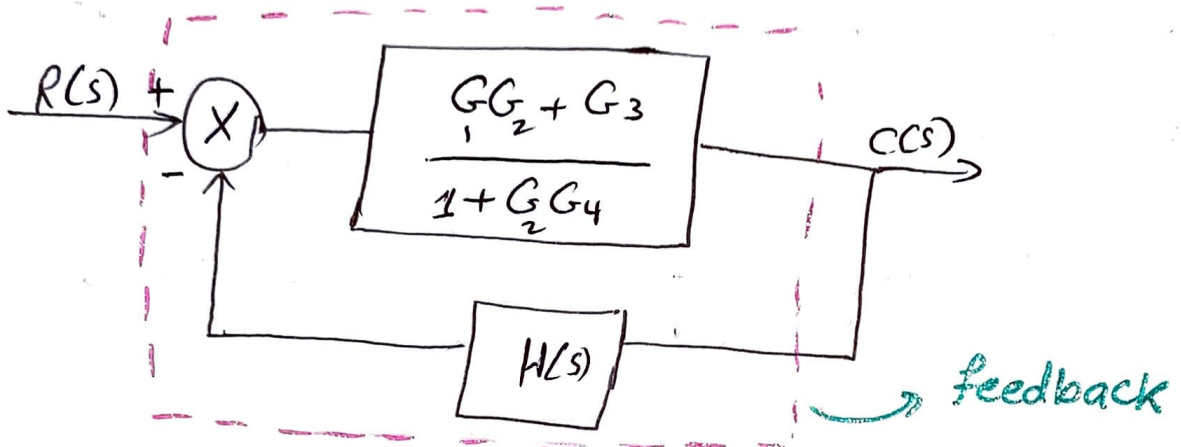
$\frac{G_3(s)}{G_2(s)}, G_1(s) \Rightarrow$ parallel

$$\frac{G_3(s)}{G_2(s)} + G_1(s) = \boxed{\frac{G_3 + G_1G_2}{G_2}} \leftarrow \text{parallel}$$



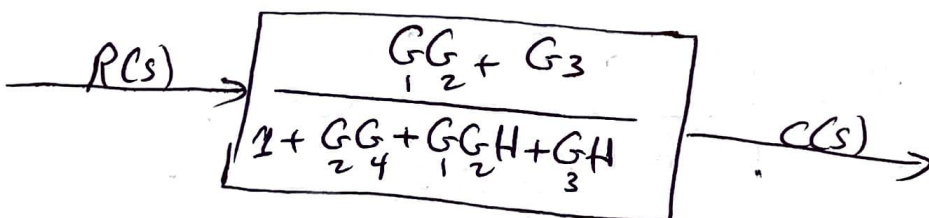
$H_1(s), H_2(s) \Rightarrow \text{Cascade}$

$$H_1 * H_2 = \left(\frac{G_3 + G_1 G_2}{G_2} \right) \left(\frac{G_2}{1 + G_2 G_4} \right) = \frac{G_1 G_2 + G_3}{1 + G_2 G_4}$$

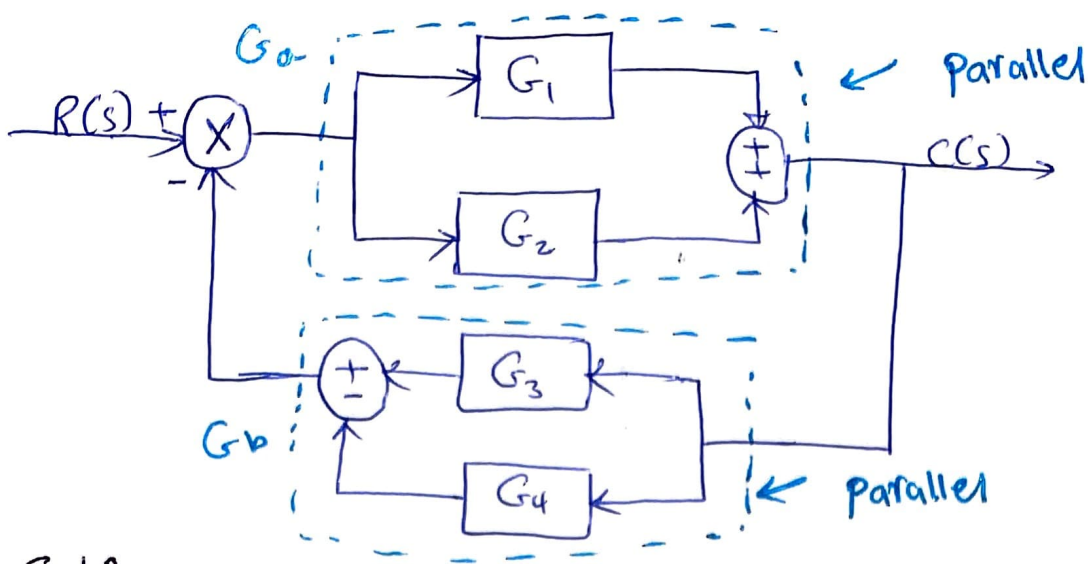


$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 + G_3}{1 + G_2 G_4}$$

$$\frac{1 + \left(\frac{G_1 G_2 + G_3}{1 + G_2 G_4} \right) (H(s))}{1 + G_2 G_4} \Rightarrow \frac{G_1 G_2 + G_3}{1 + G_2 G_4 + G_1 G_2 H + G_3 H} \quad \#$$



و در اینجا مطلوب



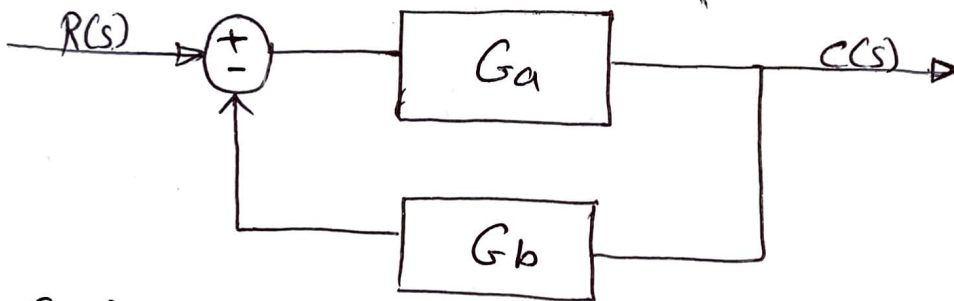
Sol:-

$G_1(s), G_2(s) \Rightarrow$ parallel

$$G_a = G_1 + G_2 \quad \leftarrow \text{المجموع}$$

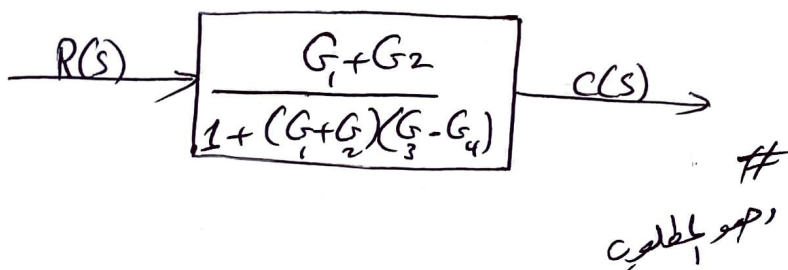
$G_3(s), G_4(s) \Rightarrow$ parallel

$$G_3(s) - G_4(s) \quad \leftarrow \text{الفرق}$$

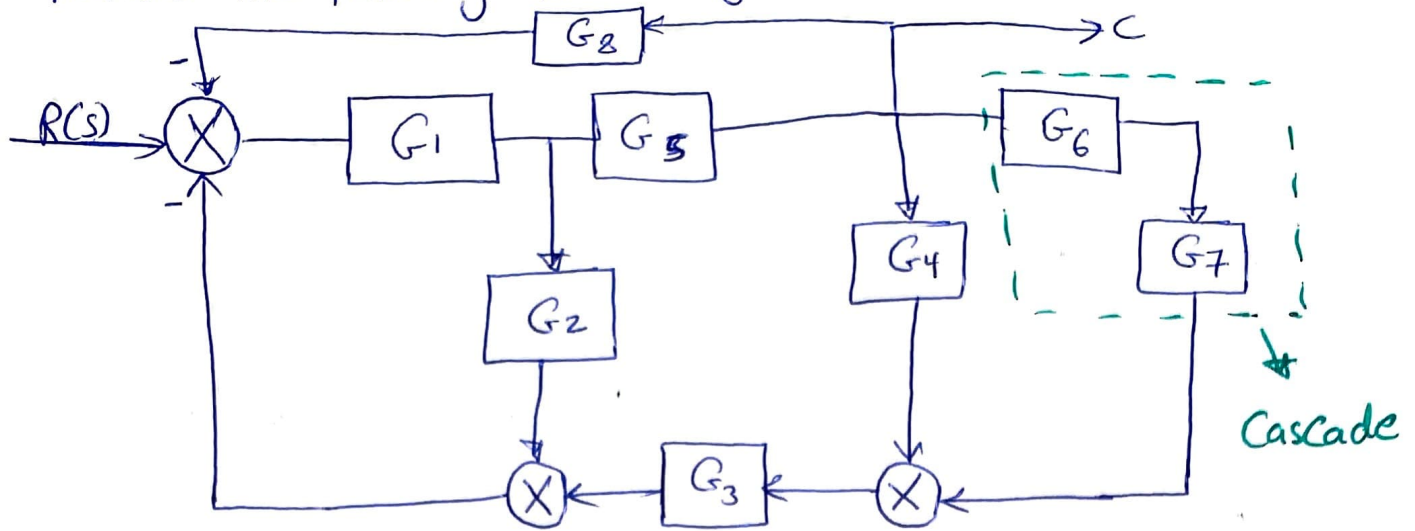


$G_a(s), G_b(s) \Rightarrow$ feedback

$$\frac{C(s)}{R(s)} = \frac{G_a}{1 + G_a G_b} = \left[\frac{G_1 + G_2}{1 + (G_1 + G_2)(G_3 - G_4)} \right] \quad \#$$



Reduce the following Block diagram :-

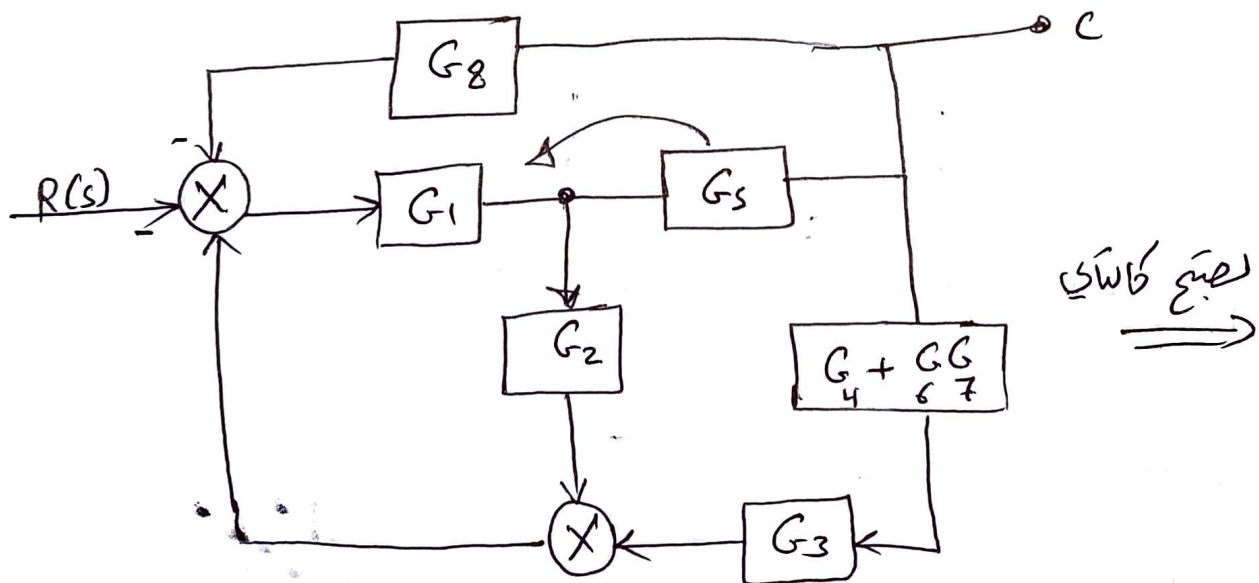


Sol :-

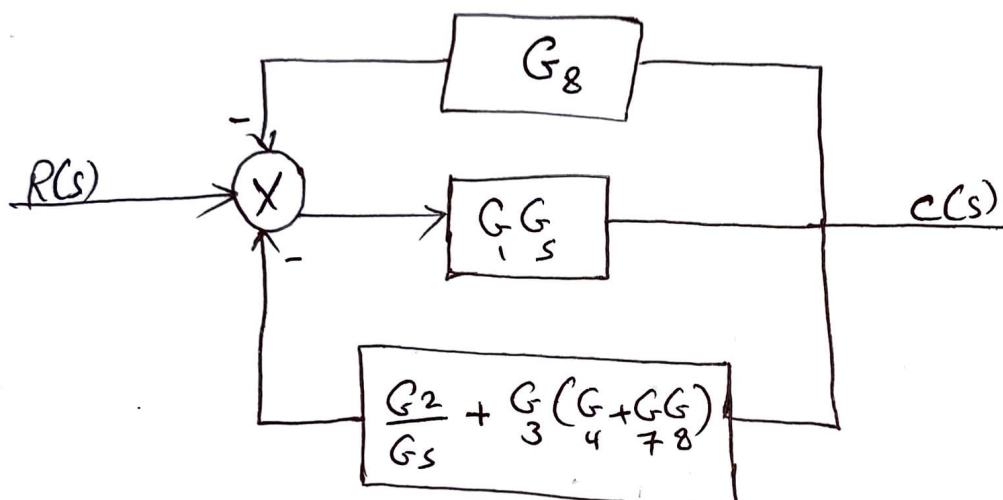
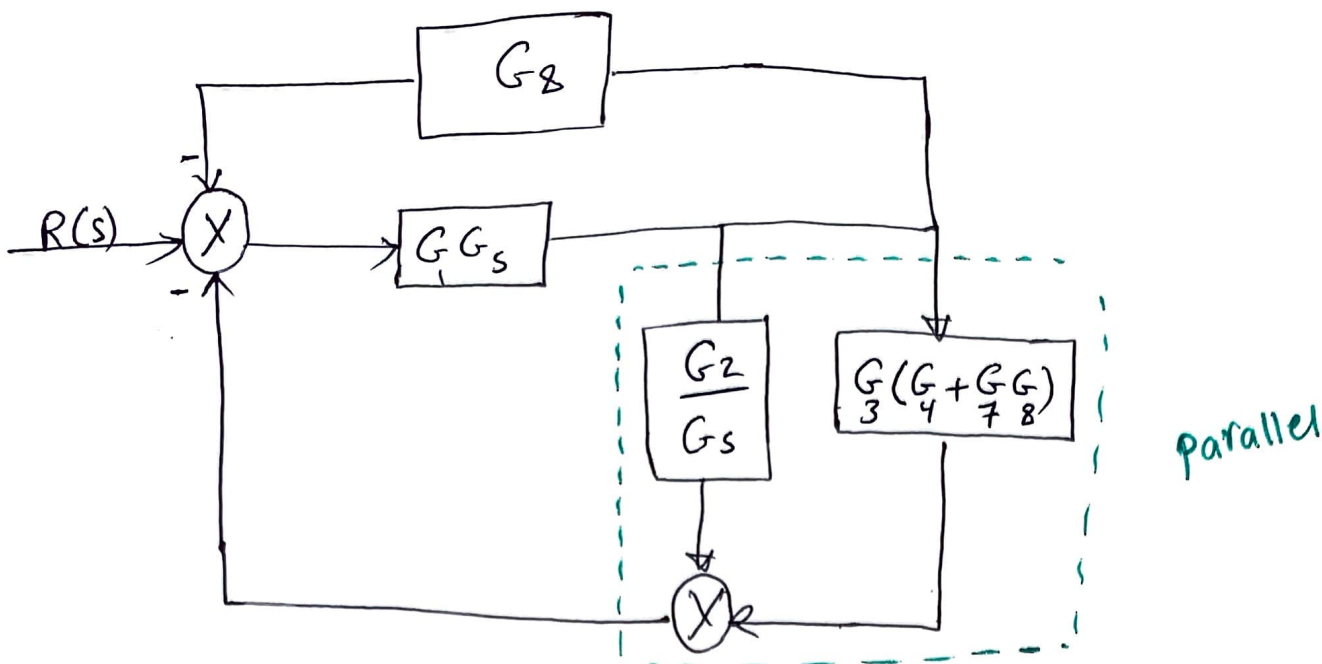
$G_6, G_7 \Rightarrow \text{cascade}$

(G_6, G_7) parallel with G_4

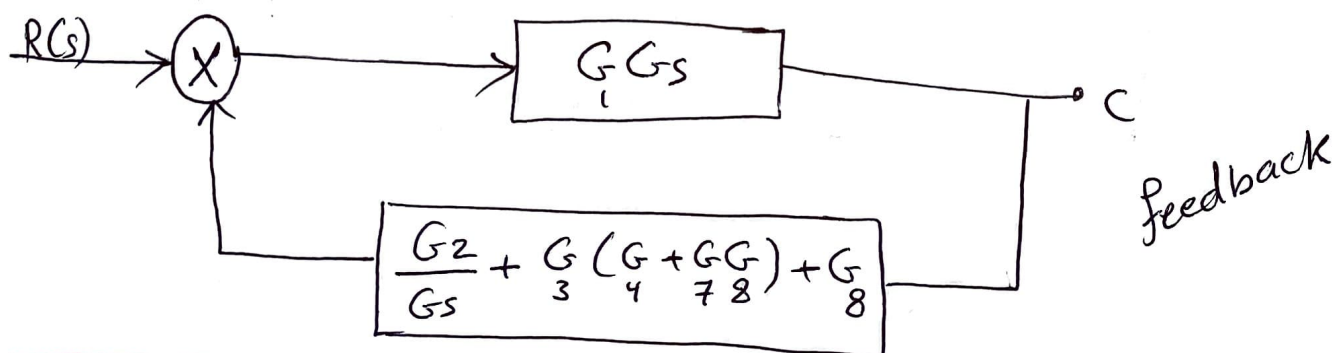
$$G_a = G_6 G_7 + G_4$$



نقل الكسور G_5 من ال node

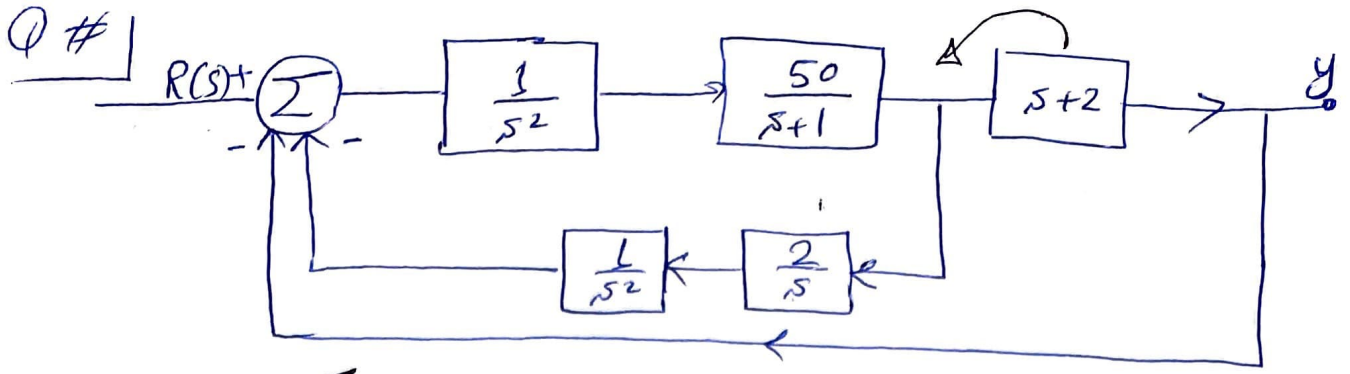


$$\left[\frac{G}{8} \right], \left[\frac{G_2}{G_s} + \frac{G}{3} \left(\frac{G}{4} + \frac{G G}{7 8} \right) \right] \Rightarrow \text{parallel}$$



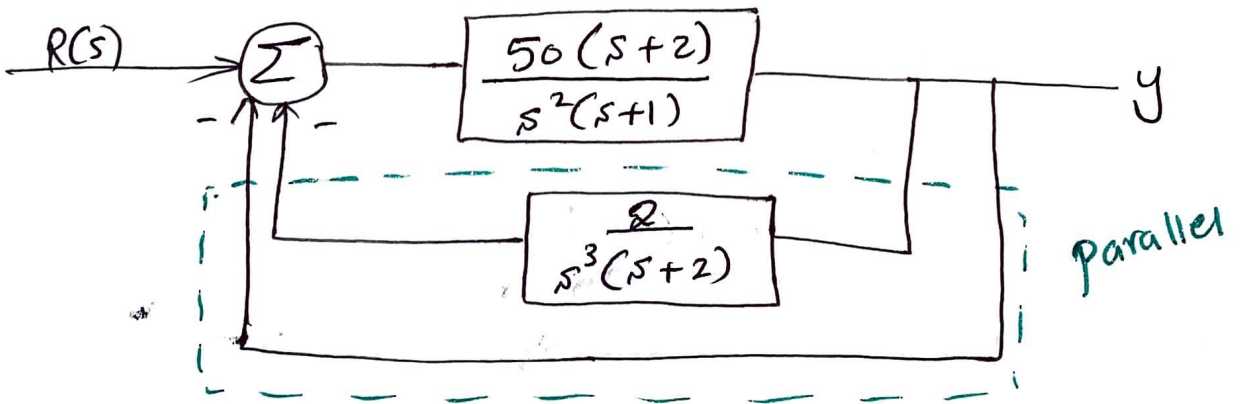
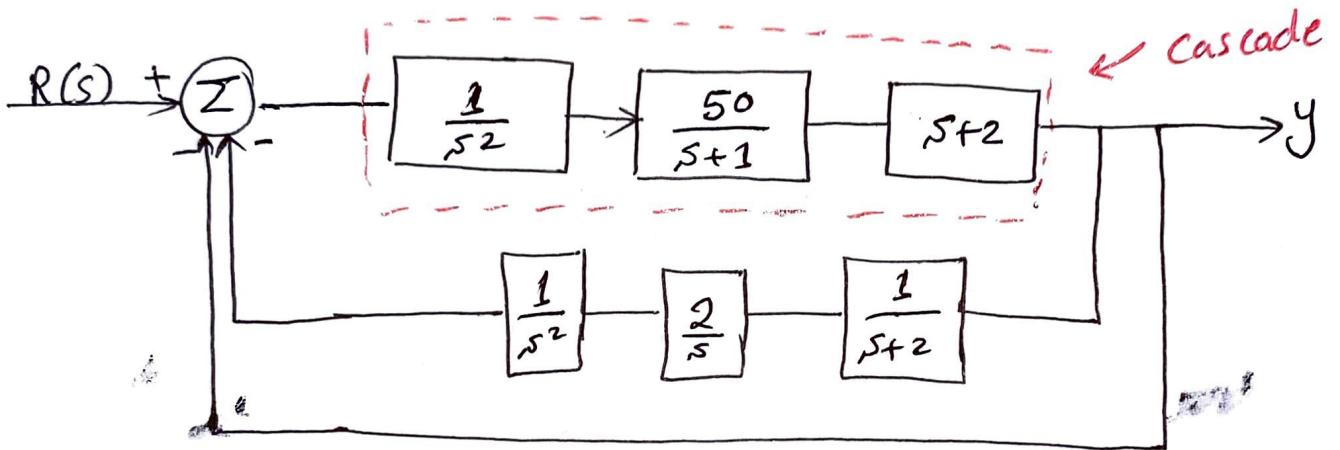
$$\frac{C(s)}{R(s)} = \frac{G_1 G_s}{1 + G_1 G_s \left(\frac{G_2}{G_s} + \frac{G}{3} \left(\frac{G}{4} + \frac{G G}{7 8} \right) + G_8 \right)}$$

المطلوب



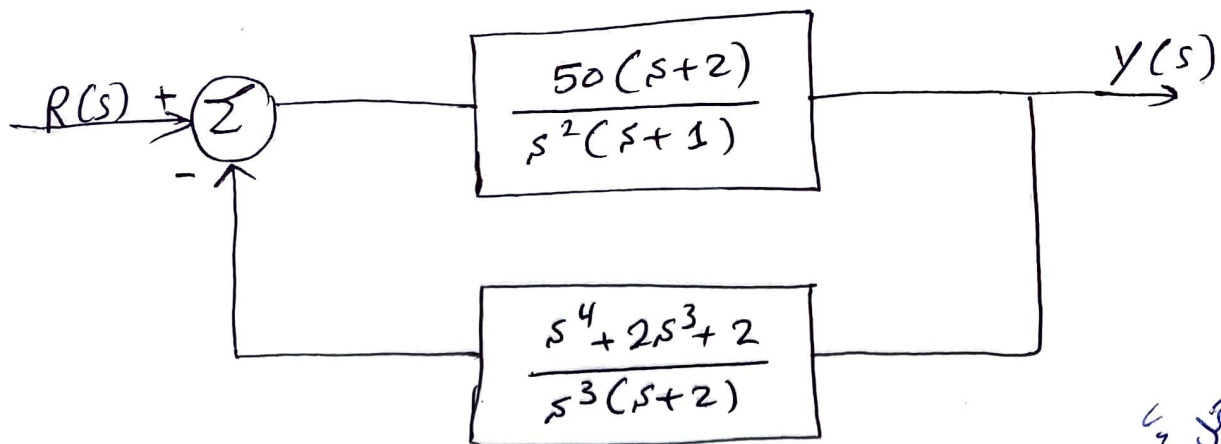
Sol: في هذا النوع من المسئلة (Block diagram) بشكل عام
 اكد بعينه حسب الشخص " عظم التفكير "

← هذا السؤال عليه حله بطريقتين ، لكن لا اختصار الخطوات عليه نقل
 السلسلة $[s+2]$ قبل ان node آ بعد ما نعمل feedback بعدها cascade



$\frac{2}{s^3(s+2)}$ parallel with 1

$$\frac{2}{s^3(s+2)} + 1 = \frac{s^4 + 2s^3 + 2}{s^4 + 2s^3}$$

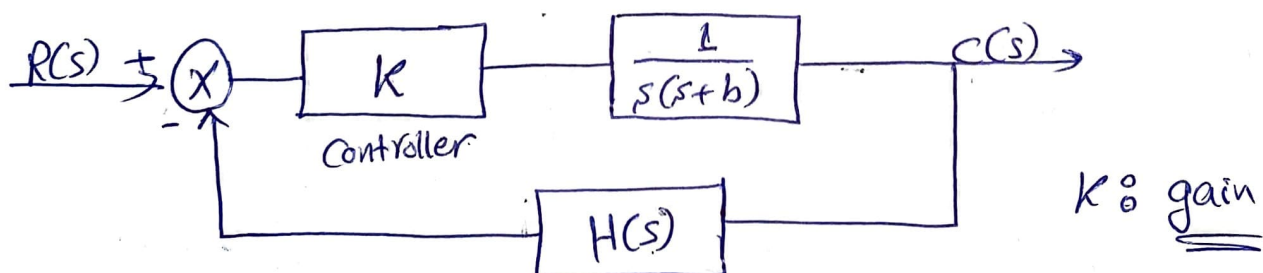


$$\frac{Y(s)}{R(s)} = \frac{50(s+2)}{s^3 + s^2} \cdot \frac{1}{1 + \left(\frac{50(s+2)}{s^3 + s^2} \right) \left(\frac{s^4 + 2s^3 + 2}{s^3(s+2)} \right)}$$

حل این معادله
برای پیدا کردن
فرکانس
سیستم

$$\boxed{\frac{Y(s)}{R(s)} = \frac{50s^3(s+2)}{s^6 + s^5 + 50s^4 + 100s^3 + 100}} \quad \#$$

* Analysis and design of feedback system:-



$$T(s) = \frac{C(s)}{R(s)} = \frac{K}{s(s+b)} \cdot \frac{1}{1 + \frac{K}{s(s+b)} \cdot H(s)} = \frac{K}{s(s+b) + K \cdot H}$$

$$T(s) = \frac{K}{s^2 + bs + K \cdot H}$$

If the feedback is unity ($H(s) = 1$)

$$\boxed{T(s) = \frac{K}{s^2 + bs + K}}$$

$$T(s) = \frac{K}{s^2 + bs + k} = \frac{k_{dc} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + bs + k = 0 \quad \text{"called characteristic equation"}$$

As K varies, the poles move through the three ranges of operation of a second-order system. :

- over damped
- critically damped, and
- underdamped.

$K \rightarrow$ dc-gain or proportional gain

from equation :

$$b = 2\zeta\omega_n$$

If $\underline{K}$ between $0 \rightarrow \frac{b^2}{4}$

$$0 < K < \frac{b^2}{4} \Rightarrow 0 < K < \frac{(2\zeta\omega_n)^2}{4}$$

$$0 < K < \zeta^2 \omega_n^2$$

$$, K = \underline{\omega_n^2}$$

$$0 < K < \zeta^2 \omega_n^2$$

$$\Rightarrow \zeta^2 > 1 \Rightarrow \boxed{\zeta > 1} \quad \text{overdamped}$$

poles are real and distinct //

$$s_{1,2} = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4K}}{2}$$

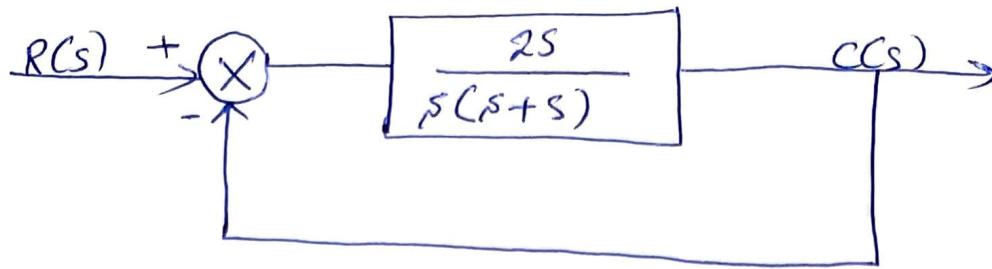
$$\text{If } K = \frac{b^2}{4} \Rightarrow \boxed{s_{1,2} = -\frac{b}{2}}$$

$\Rightarrow$ poles real and repeated $\Rightarrow$ critically damped ($\zeta = 1$)

* If $K > \frac{b^2}{4} \rightarrow$ the system is underdamped (poles complex and conjugate)

$$\zeta < 1, \quad s_{1,2} = -\frac{b}{2} \pm j \frac{\sqrt{4K - b^2}}{2}$$

Problem #1 for the system shown, find the peak time, percent overshoot, and settling time.



Soln:-

The closed loop transfer function:-

$$T(s) = \frac{\frac{2s}{s(s+5)}}{1 + \frac{2s}{s(s+5)}} \Rightarrow \frac{\frac{2s}{s^2+5s}}{\frac{s^2+5s+2s}{s(s+5)}}$$

$$T(s) = \frac{2s}{s^2+5s+2s} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n^2 = 2s \Rightarrow \boxed{\omega_n = 5}$$

$$2\zeta\omega_n = 5 \Rightarrow \zeta = \frac{5}{(2)(5)} = \boxed{0.5}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{5 \sqrt{1-(0.5)^2}} = \boxed{0.726}$$

$$\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$$

$$= e^{-\pi(0.5)/\sqrt{1-(0.5)^2}} \times 100\% = \boxed{16.303\%}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{(0.5)(5)} = \boxed{1.6 \text{ s}}$$

problem #1 | If you have transfer function of plant $G(s) = \frac{16}{s(s+a)}$

Asked to Design a closed loop system with $0.5\% = 5\%$

← أول خطوة (كاد صفة) closed loop transfer func.

$$T(s) = \frac{\frac{16}{s(s+a)}}{1 + \frac{16}{s(s+a)}} \Rightarrow \boxed{\frac{16}{s^2 + as + 16}}$$

$$W_n^2 = 16 \Rightarrow \boxed{W_n = 4}$$

$$2\varepsilon W_n = a \Rightarrow a \Rightarrow ??$$

$$0.5\% = 5\% = e^{-\pi \varepsilon / \sqrt{1-\varepsilon^2}}$$

$$\Rightarrow \zeta = \frac{-\ln\left(\frac{0.5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{0.5\%}{100}\right)}} = \frac{-\ln\left(\frac{5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{5\%}{100}\right)}} = \boxed{0.69}$$

$$a = 2\varepsilon W_n = (2)(0.69)(4) = \boxed{5.52}$$

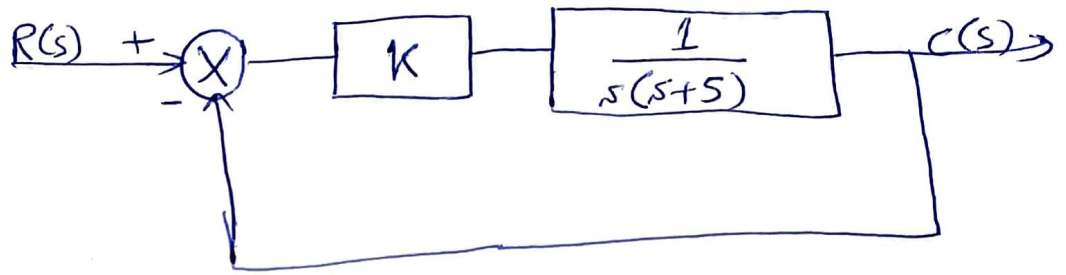
$$\boxed{\zeta = 0.69} < \underline{1} \Rightarrow \text{underdamped response.}$$

$$k > \frac{a^2}{4} \Rightarrow k > \frac{(5.52)^2}{4}$$

$$K > 7.6176$$

er dampen : 221

Example #



Design K to get overshoot 10%

Sol :

⇒ closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K \left(\frac{1}{s(s+5)} \right)}{1 + \frac{K}{s(s+5)}} \Rightarrow \frac{K}{s^2 + 5s + K}$$

$$\zeta = \frac{-\ln \left(\frac{0.5\%}{100} \right)}{\sqrt{\pi^2 + \ln^2 \left(\frac{0.5\%}{100} \right)}} = 0.591$$

$$2\zeta\omega_n = 5 \Rightarrow \omega_n = \frac{5}{(2)(0.591)} = 4.23$$

$\zeta = 0.591 < 1 \Rightarrow$ underdamped response.

$$K = \omega_n^2 = (4.23)^2 = 17.89 \neq$$

طريقة // AS أدت إلى overshoot
على شكل $(\zeta, \%)$

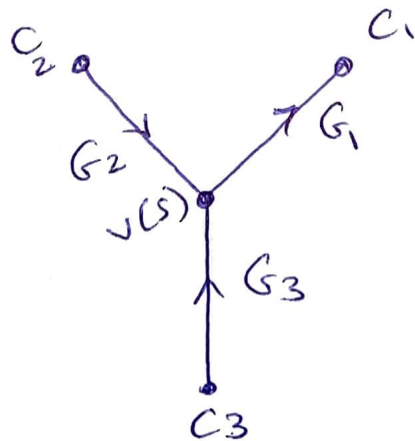
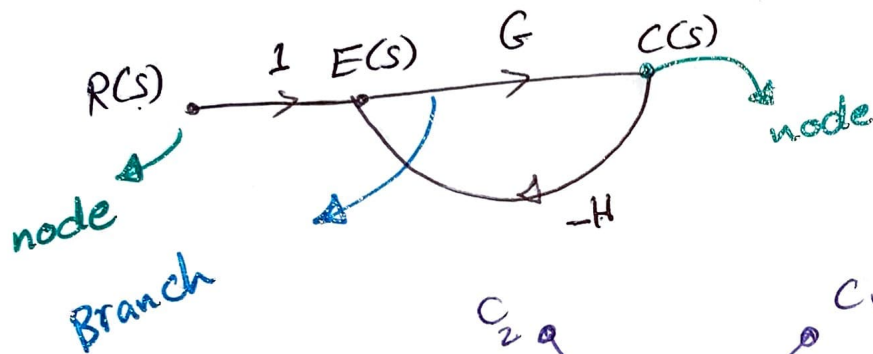
* Signal flow graphs (SFG):-

SFG are an alternative to block diagram representation.

* SFG Consist only of :-

* Branches : Which used to represent system.

* Nodes : which used to represent signal.



$$V(s) = G_2 C_2 + G_3 C_3$$

بناخذ فقط الأسماء المرفقة على

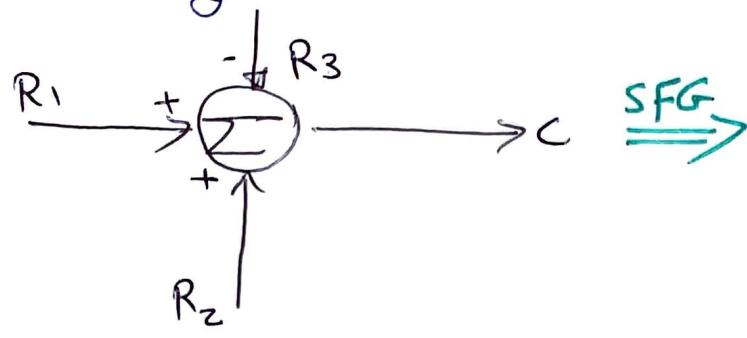
$$C_1 = V(s) G_1(s)$$

مضاجع

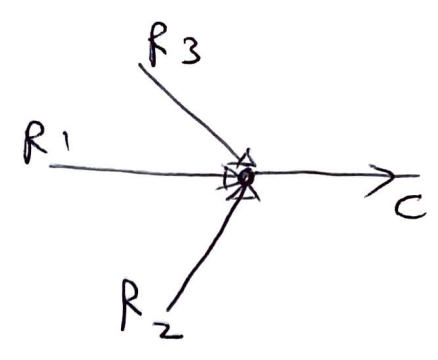
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* How we can convert the Block diagram to signal flow graph.

⇒ Summing Junctions:-

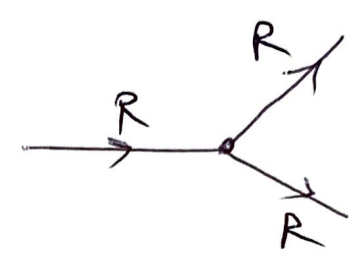
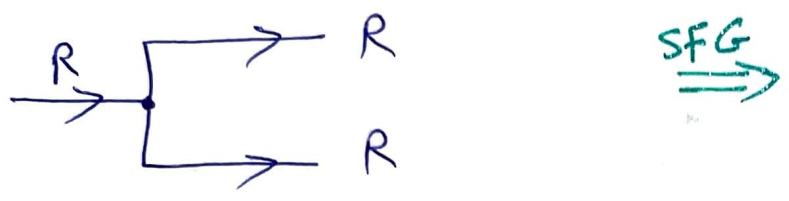


$$C = R_1 + R_2 - R_3$$

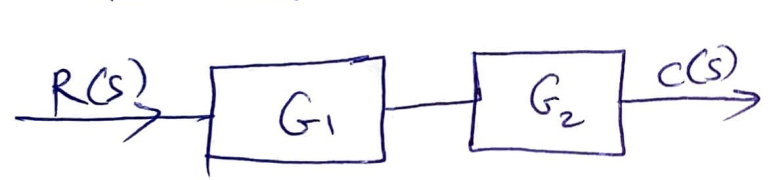


$$C = R_1 + R_2 - R_3$$

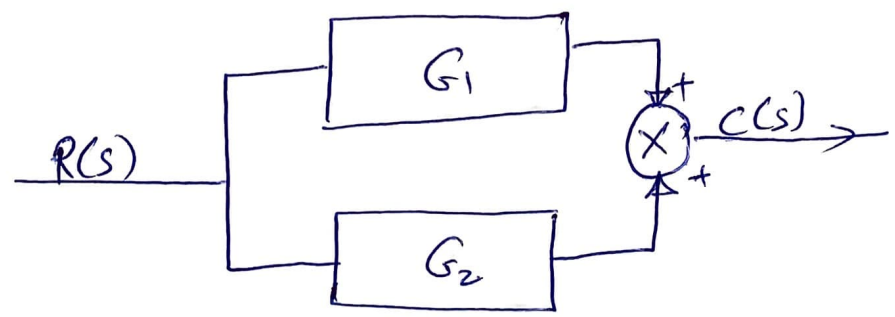
⇒ Pick-off point :-



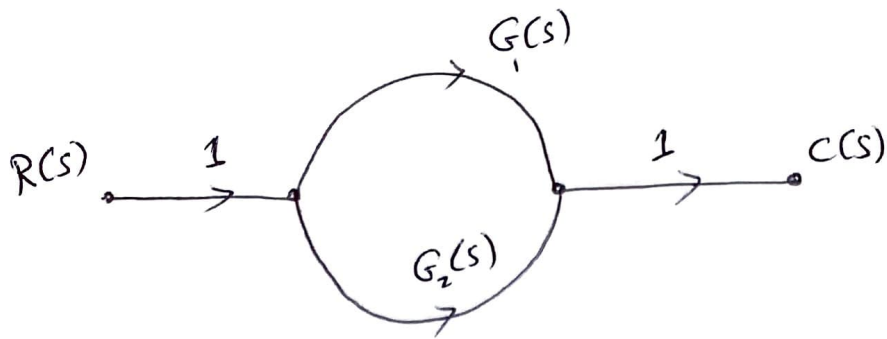
⇒ Cascade :-



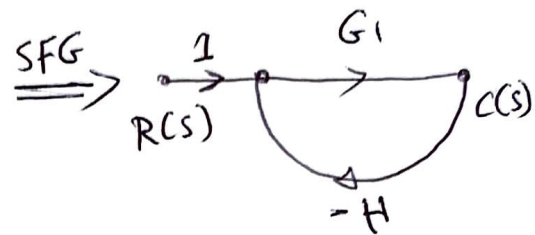
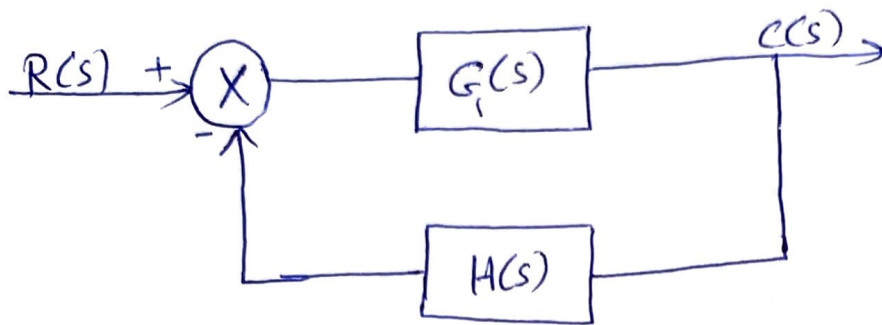
⇒ parallel :-



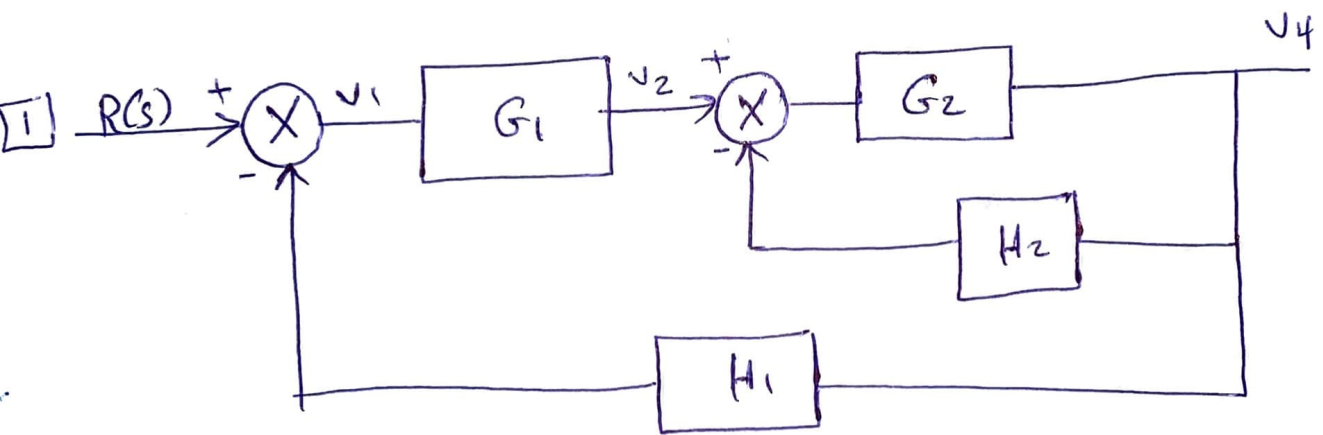
الضغطة الآتية
⇒



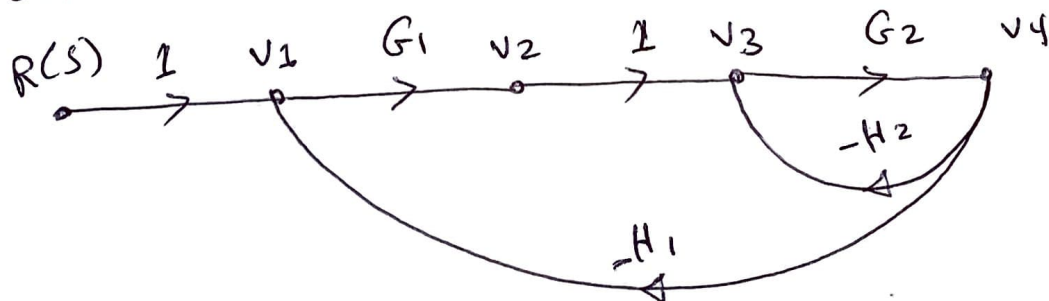
[3] Feedback #:

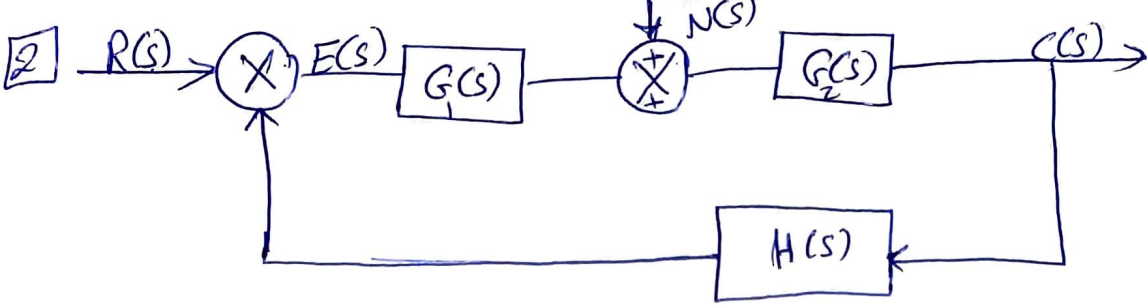


Example #1 Convert the Block diagram to SFG:-

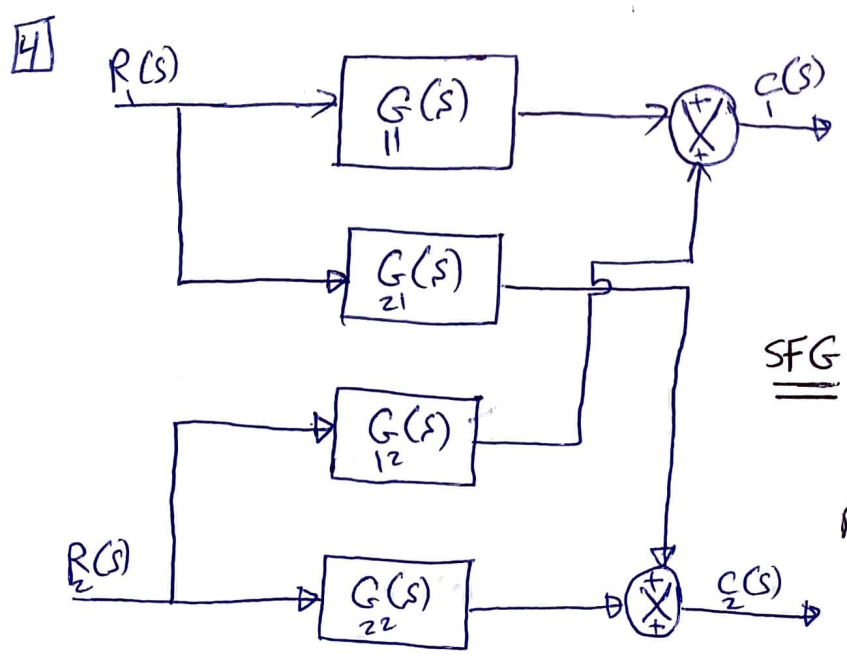
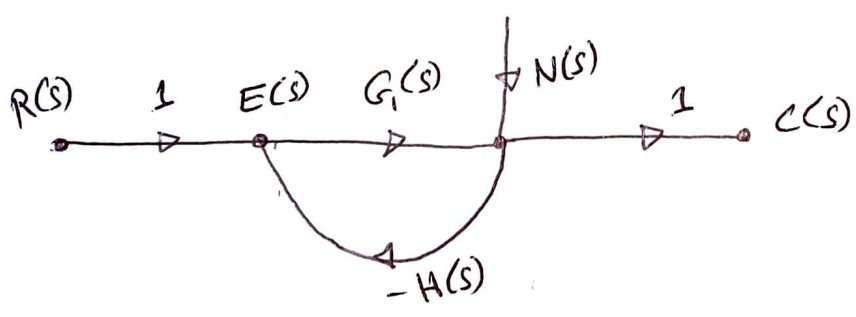
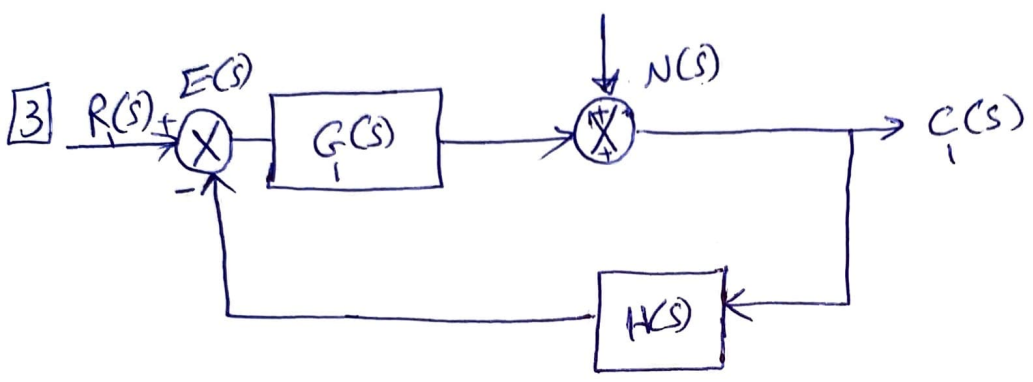
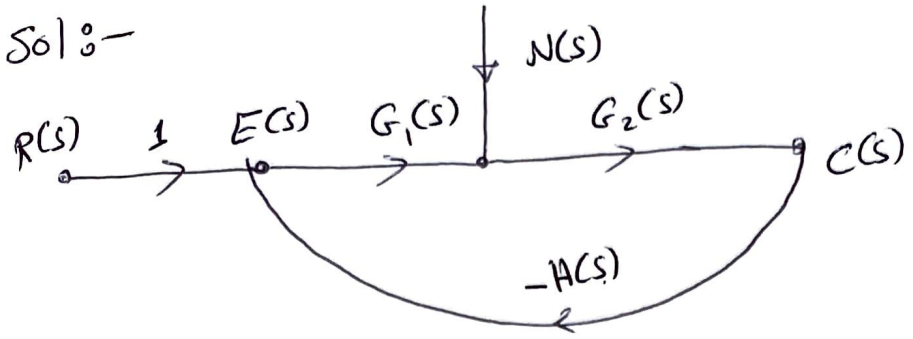


Sol:-

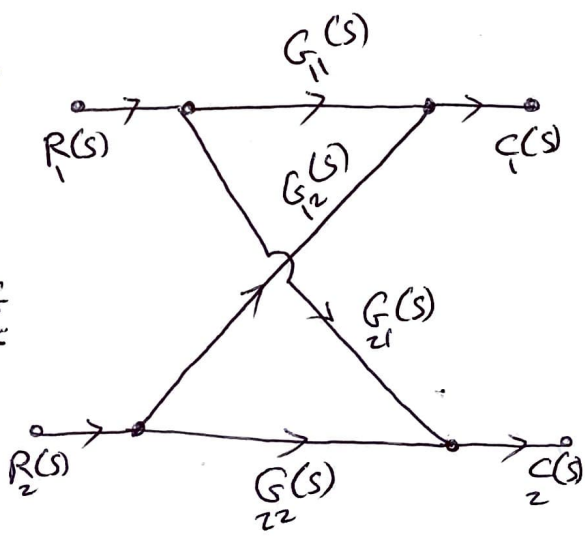




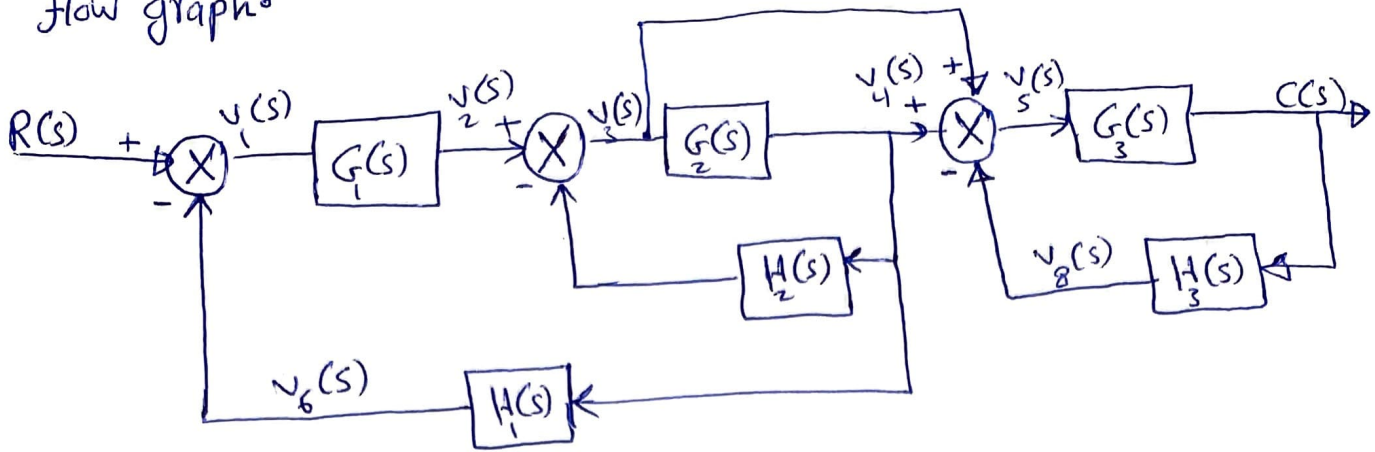
Sol:-



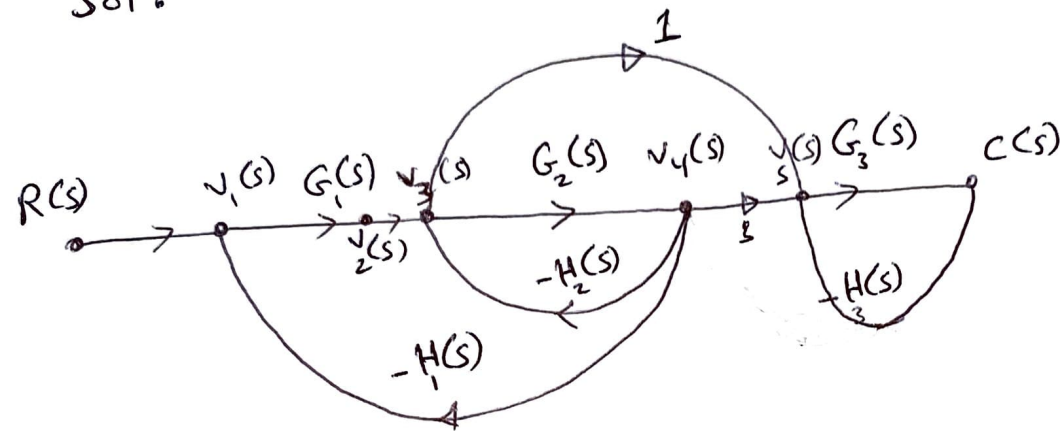
SFG



problem #1 Convert the Block diagram shown to a signal flow graph.



Sol:—



* Mason's formula: used for reduce SFG to single T.F.

Definitions:-

⇒ Input node (source):
is a node that has only outgoing branches.
كل عقدة التي لها فروع خارجة فقط هي عقدة مصدر
source

⇒ output node (sink):
is a node that has only incoming branches
كل عقدة التي لها فروع داخلية فقط هي عقدة مستهلكة
sink

$\Rightarrow$ path :

⇒ path :
Continuous connection of branches from one node to another
with arrowhead in the same direction

⇒ forward path :

is a path connects a source node to a sink node

"input, output" source, sink

⇒ Loop :

Loop: is closed path (originate and terminates on the same node)

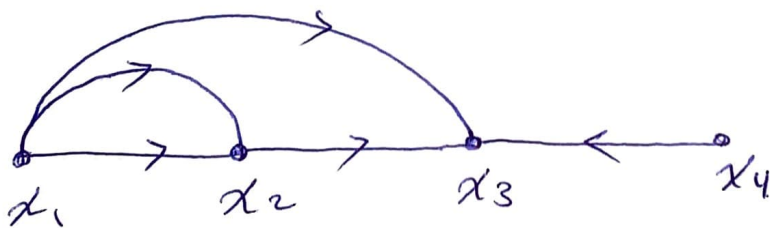


$\Rightarrow$ path gain :

→ path gain: is the product of T.F of all branches that form path

$\Rightarrow$ Loop gain :

→ Loop gain is the product of T.F of all branches that form Loop.


$$x_1 \rightarrow x_2$$

- 2-paths -

$$x_1 \rightarrow x_3$$

- 3-paths -

$$x_1 \rightarrow x_4$$

- no paths -

* No forward path

* No Loops

x_4

أخيراً
أنا خضعت
لرؤوف
إله الأوصياء
فقال له ملك
البرية
والتواضع

Mason's Rule: —

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum_{k=1}^P P_k \Delta_k$$

P : numbers of forward paths

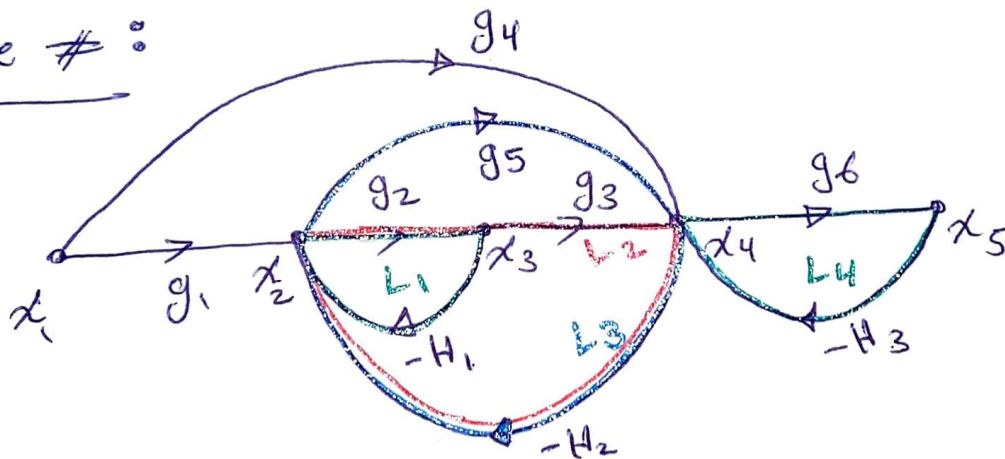
P_k : path gain or transmittance of k^{th} forward path

Δ : $1 - \sum \text{single Loop gains} + \sum \text{non-touching Loop gain taken 2 at time}$

$- \sum \text{non-touching Loop gain taken three at time} + \sum \dots$

Δ_k : $\Delta - \sum \text{Loop gain terms in } \Delta \text{ that touch the } k^{\text{th}} \text{ forward path.}$

Example #:



Find the transfer functions—

Sol: —

of forward path: 3-forward path

$$P_1 = g_1 g_2 g_3 g_6$$

$$P_2 = g_1 g_5 g_6$$

$$P_3 = g_4 g_6$$

لحل المسألة
نستخدم قاعدة
ماسون
لأنه لا يمكن
حلها بالطرق
التي تعلمها
الآن

of loops :-

$$\boxed{L_1 = -g_2 H_2} \quad , \quad \boxed{L_2 = -g_2 g_3 H_2} \quad , \quad \boxed{L_3 = -g_5 H_2} \quad , \quad \boxed{L_4 = -g_6 H_3}$$

$$\Delta = 1 - \sum \text{single loop gains} + \sum \text{non-touching loop gain taken 2 at time}$$

- $\Sigma_{\text{non-touching}}$ Loop gain taken 3 at time.

$$\Delta = 1 - \binom{L_1 + L_2 + L_3 + L_4}{1} + \binom{L_1 L_4}{1}$$

$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - 0 = \boxed{1}$$

$$A_3 = 1 - \frac{2}{1}$$

$$G(s) = \frac{L}{\Delta} \sum_{k=1}^p p_k \Delta_k$$

$$G(s) = \frac{p_1 \Delta_1 + p_2 \Delta_2 + p_3 \Delta_3}{\Delta}$$

$$G(s) = \frac{g_{12}g_{36} + g_{15}g_6 + g_{46}(1 + g_{21})}{1 - (-g_{21} - g_{23}g_{32} - g_{52}) + (-g_{21} \cdot -g_{63})}$$

$$G(s) = \frac{g_{123}g_6 + g_{15}g_6 + g_{46}(1 + g_2H_1)}{1 + g_2H_1 + g_2g_3H_2 + g_5H_2 + g_2H_1g_6H_3}$$

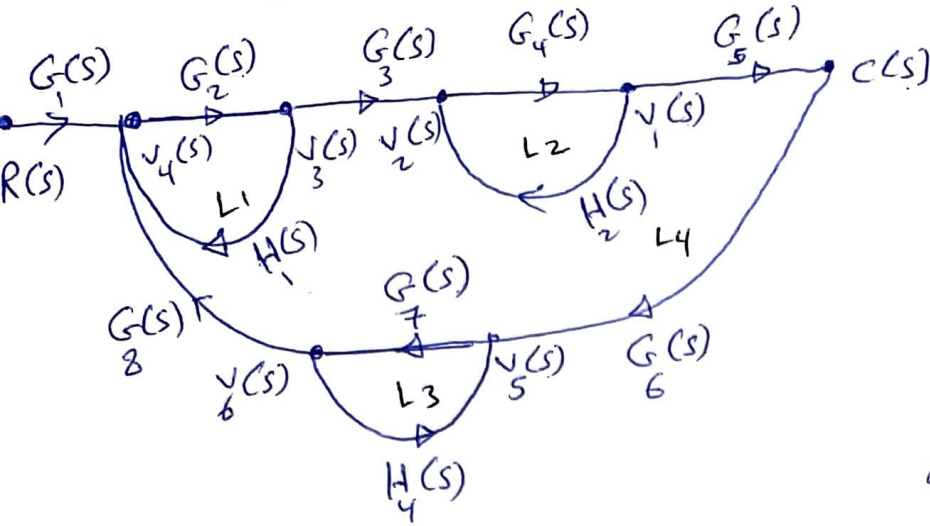
فانفس
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سوف
سوف
سوف

#

١٠ أمهٕ جعل الأذن عذرإ
١١ عذرإ، جعل لها
١٢ الله يل بحرسه صامرإ
١٣ صوت الفل، أنشزم لا سلووهٕ
١٤ جعل خلها
١٥ جعل
١٦ جعل
١٧ جعل
١٨ جعل
١٩ جعل
٢٠ جعل

Example # 5.7 | Find the transfer function $C(s)/R(s)$

for the signal flow graph:-



السلسلة الأولى، الثانية، الثالثة، الرابعة.
التي هي مسارات التغذية الراجعة.

Sol:-

of forward paths:- $\underline{\underline{1}}$

$$P_1 = G_1(s) G_2(s) G_3(s) G_4(s) G_5(s)$$

of Loops:-

$$L_1 = G_2(s) H_1(s) \quad , \quad L_2 = G_4(s) H_2(s) \quad , \quad L_3 = \frac{G_7(s) H_4(s)}{7}$$

$$L_4 = G_2(s) \cdot G_3(s) \cdot G_4(s) G_5(s) \cdot G_6(s) G_7(s) \cdot G_8(s)$$

* non-touching Loops taken two at time.

$$L_1 \text{ and } L_2 = G_2 H_1 G_4 H_2$$

$$L_1 \text{ and } L_3 = G_2 H_1 G_7 H_4$$

$$L_2 \text{ and } L_3 = G_4 H_2 G_7 H_4$$

* non-touching Loops taken three at time.

$$L_1, L_2 \text{ and } L_3 = G_2 H_1 G_4 H_2 G_7 H_4$$

لا يوجد مسارات غير متلامسة في الرسم قد يتبين ذلك من الرسم.
من الرسم.

$$\Delta = 1 - [G_2H_1 + G_4H_2 + G_7H_4 + G_2G_3G_4G_5G_6G_7G_8] + [G_2H_1G_4H_2 + G_2H_1G_7H_4 + G_4H_2G_7H_4] - [G_2H_1G_4H_2G_7H_4]$$

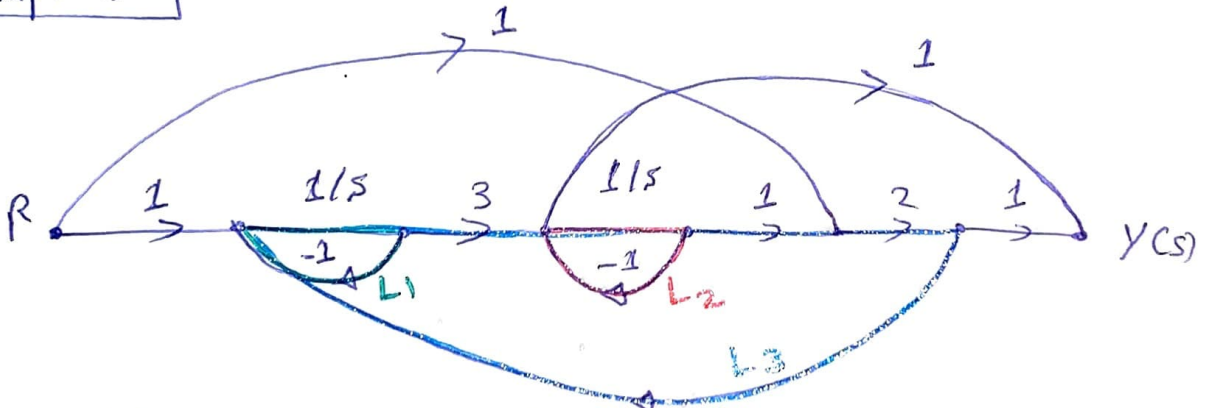
$$\Delta_1 = 1 - G_7H_4$$

$$G(s) = \frac{P_1 \Delta_1}{\Delta}$$

$$G(s) = \frac{(G_1G_2G_3G_4G_5) - G_7H_4}{\Delta}$$

#

Example #1



Find $\frac{Y(s)}{R(s)}$ By using Mason's Rule.

Sol:

of paths: 4

$$P_1 = 1 \cdot \frac{1}{s} \cdot 3 \cdot \frac{1}{s} \cdot 1 \cdot 2 \cdot 1 = \boxed{\frac{6}{s^2}}$$

$$P_2 = 1 \cdot 2 \cdot 1 = \boxed{2}$$

$$P_3 = 1 \cdot \frac{1}{s} \cdot 3 \cdot 1 = \boxed{\frac{3}{s}}$$

$$P_4 = 1 \cdot 2 \cdot -1 \cdot \frac{1}{s} \cdot 3 \cdot 1 = \boxed{-\frac{6}{s}}$$

نوہم لائن سے شری طوا کیسے لفظ
 سے پہلے لے کر علی ریاضی میں
 آؤں گے ہم عزت میں
 سوچ لیا۔

of Loops :-

$$\frac{L}{1} = \frac{1}{s} \cdot -1 = \boxed{-\frac{1}{s}}$$

$$L_2 = \frac{L}{s} \cdot -1 = \boxed{-\frac{L}{s}}$$

$$\frac{1}{3} = \frac{1}{5} \cdot 3 \cdot \frac{1}{5} \cdot 1 \cdot 2 \cdot -1 = \boxed{\frac{-6}{5^2}}$$

$$\Delta = 1 - \sum (L_1 + L_2 + L_3) + \sum (L_1 L_2)$$

$$= 1 - (L_1 + L_2 + L_3) + \frac{L_1 L_2}{2}$$

$$\Delta = 1 - \left(-\frac{2}{s} - \frac{6}{s^2} \right) + \frac{1}{s^2} = \boxed{\frac{s^2 + 2s + 7}{s^2}}$$

$$\Delta_1 = 1$$

$$\Delta_1 = 1$$
$$\Delta_2 = 1 - (L_1 + \frac{L_2}{2}) + \frac{L_1 L_2}{2} = \frac{S^2 + 2S + 1}{S^2}$$

$$\Delta_3 = 1$$

$$\Delta_4 = 1$$

$$\frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4}{\Delta}$$

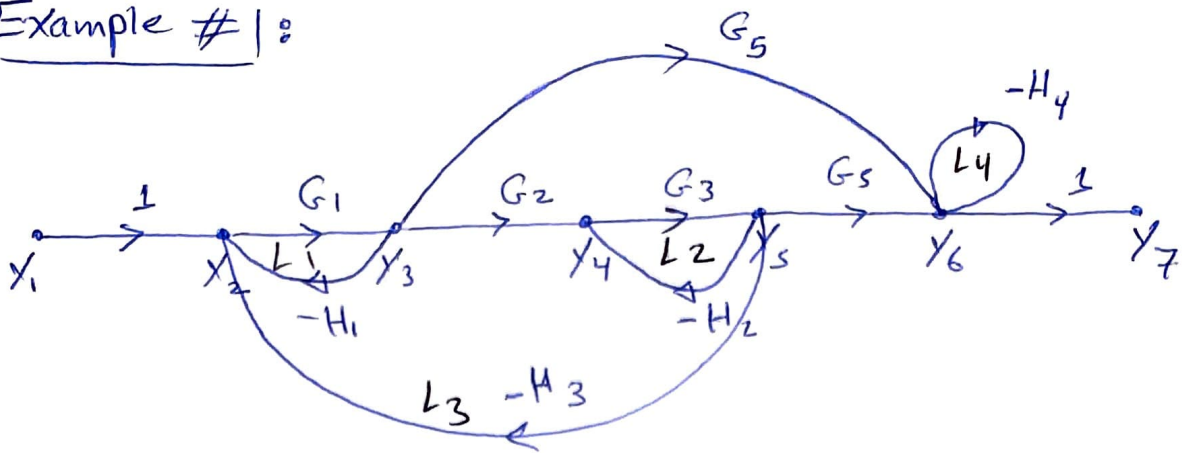
$$\frac{Y(s)}{R(s)} = \frac{\frac{6}{s^2} + 2\left(\frac{s^2 + 2s + 1}{s^2}\right) + \left(\frac{3}{s}\right)(1) + \left(\frac{-6}{s}\right)(1)}{\left(\frac{s^2 + 2s + 7}{s^2}\right)}$$

$$\frac{Y(s)}{R(s)} = \frac{2s^2 + s + 8}{s^2 + 2s + 7} \quad \#$$

١٠٠ لله عتو علم
 ورحم ما في الارض
 ما ذا تكتب عزاء وما تدرى نفسي
 في ترحم عتو
 صوت لعل

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Example #1:



Find $\frac{Y_2}{Y_1}$, $\frac{Y_4}{Y_1}$, $\frac{Y_6}{Y_1}$

Sol:—

$\frac{Y_2}{Y_1}$??

$P_1 = 1$

Loops :

$L_1 = -G_1 H_1$

$L_2 = -G_3 H_2$

$L_3 = -H_3 G_1 G_2 G_3$

$L_4 = -H_4$

non touching Loops (two loops):

$L_1 L_2$, $L_1 L_4$, $L_3 L_4$, $L_2 L_4$

non touching Loops (three loops):

$L_1 L_2 L_4$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2 + L_1 L_4 + L_3 L_4 + L_2 L_4) - (L_1 L_2 L_4)$

$\Delta_1 = 1 - (L_2 + L_4) + (L_2 L_4)$

$\frac{Y_2}{Y_1} = \frac{P_1 \Delta_1}{\Delta} = \frac{(1) [1 - (L_2 + L_4) + (L_2 L_4)]}{1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2 + L_1 L_4 + L_3 L_4 + L_2 L_4) - (L_1 L_2 L_4)}$

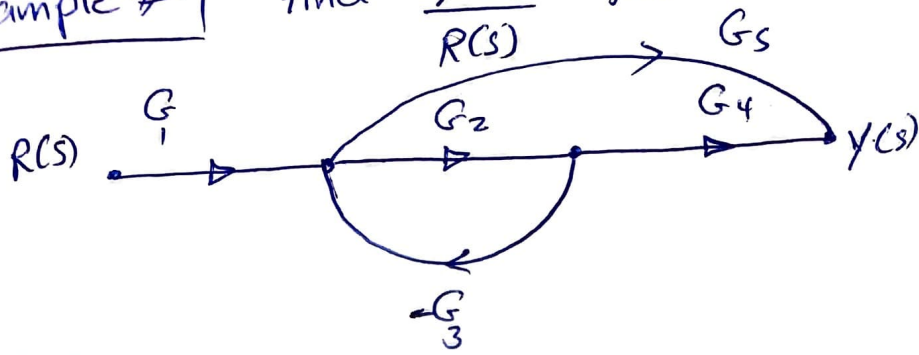
$\Rightarrow$

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Example # | Find $\frac{Y(s)}{R(s)}$ for.



$\in \text{Laplace}$

$$G_1 = \frac{1}{s+2}$$

$$G_2 = \frac{s}{s+3}$$

$$G_3 = +3s$$

$$G_4 = 1$$

$$G_5 = 1/s$$

Sol:-

of forward paths: 2

$$P_1 = G_1 G_2 G_4 = \left(\frac{1}{(s+2)} \right) \left(\frac{s}{s+3} \right) (1) = \boxed{\frac{s}{(s+2)(s+3)}}$$

$$P_2 = G_1 G_5 = \left(\frac{1}{s+2} \right) \left(\frac{1}{s} \right) \Rightarrow \boxed{\frac{1}{s(s+2)}}$$

of loops: 1

$$L_1 = G_2 G_3 = \left(\frac{-s}{(s+3)} \right) (3s) = \boxed{\frac{-3s^2}{s+3}}$$

$$\Delta = 1 - (L_1)$$

$$\Delta = 1 + \frac{3s^2}{(s+3)} = \boxed{\frac{3s^2 + s + 3}{s+3}}$$

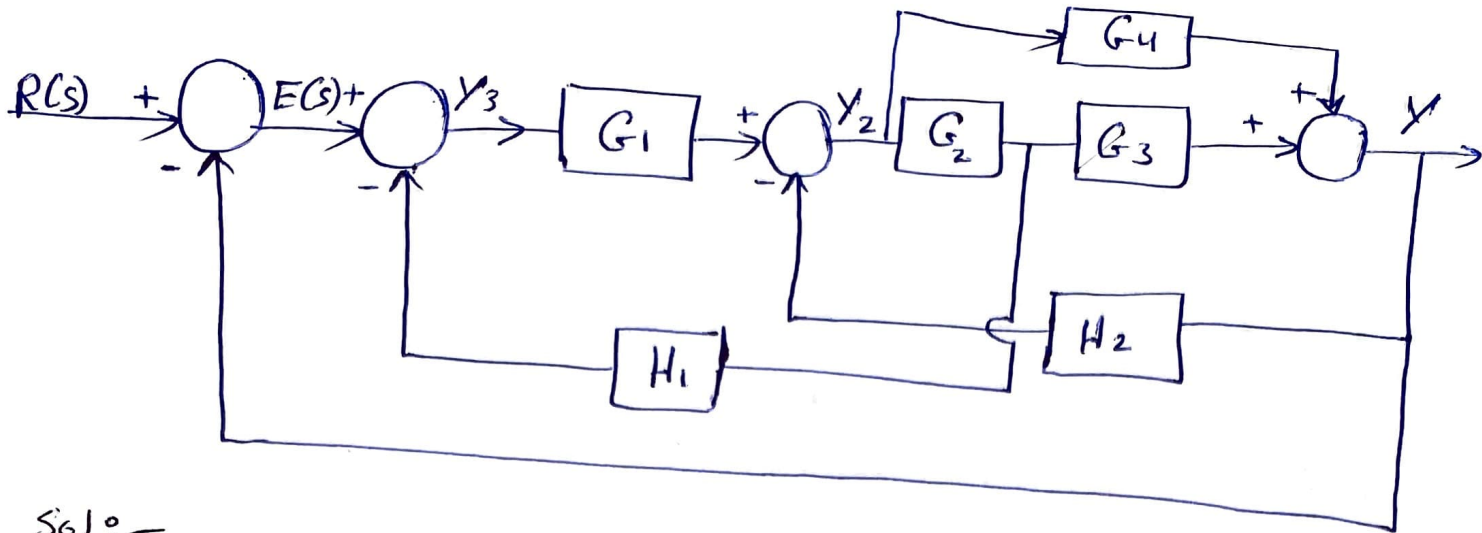
$$\Delta_1 = 1, \Delta_2 = 1$$

$$\frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{\left(\frac{s}{(s+2)(s+3)} \right) + \left(\frac{1}{s(s+2)} \right)}{\left(\frac{3s^2 + s + 3}{s+3} \right)}$$

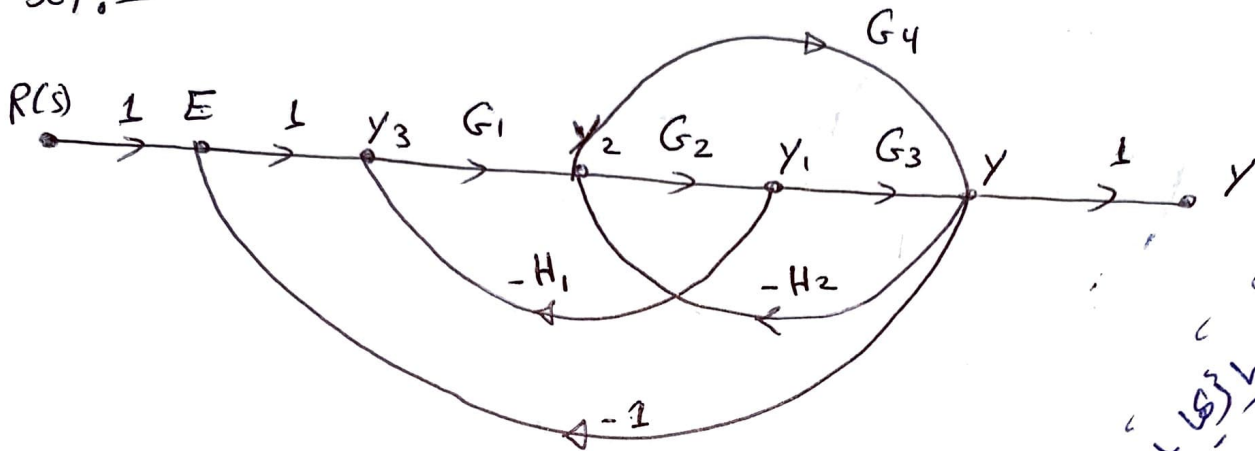
$$\frac{Y(s)}{R(s)} = \frac{\left(\frac{s^2 + s + 3}{s^3 + 5s^2 + 6s} \right)}{\left(\frac{3s^2 + s + 3}{s+3} \right)} = \frac{s^2 + s + 3}{(3s^2 + s + 3)(s^2 + 2s)}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{s^2 + s + 3}{3s^4 + 7s^3 + 5s^2 + 6s}} \quad \#$$

Example #1



Sol:-



of forward path:-

$$P_1 = G_1 G_2 G_3, \quad P_2 = G_1 G_4$$

of Loops:-

$$L_1 = -G_1 G_2 H_1, \quad L_2 = -G_2 G_3 H_2, \quad L_3 = -G_4 H_2$$

$$L_4 = -G_1 G_2 G_3, \quad L_5 = -G_1 G_4$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$$

$$\Delta = 1 - (-G_1 G_2 H_1 - G_2 G_3 H_2 - G_4 H_2 - G_1 G_2 G_3 - G_1 G_4)$$

$$\Delta = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2 + G_1 G_2 G_3 + G_1 G_4$$

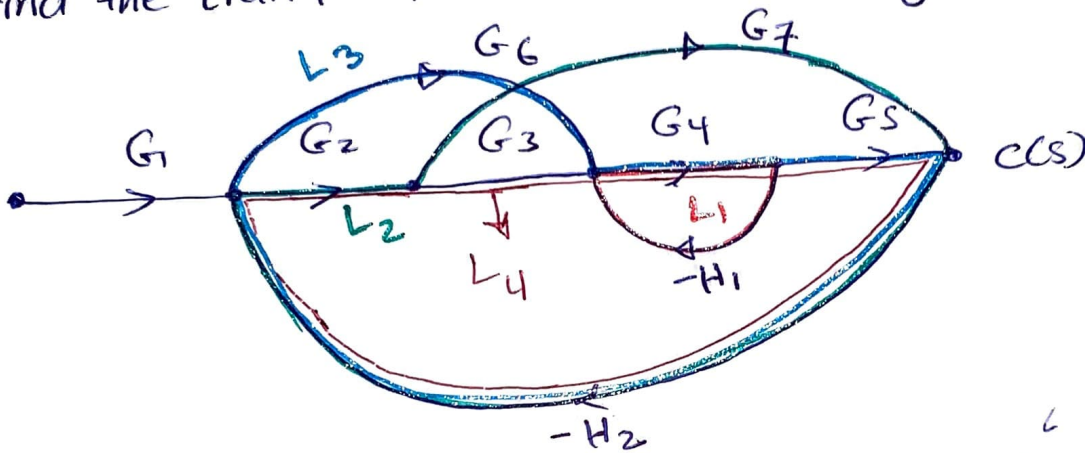
قال لي اني انا
كتاب كبري
بسم الله الرحمن الرحيم
سورة الفاتحة

$$\boxed{\Delta_1 = 1} \quad , \quad \boxed{\Delta_2 = 1}$$

$$\frac{Y(s)}{R(s)} = \frac{1}{\Delta} \sum_{k=1}^P P_k \Delta_k$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_1 G_4}} \quad \#$$

Find the transfer function $C(s)/R(s)$ by using Mason's formula.



Sol:-

of paths:-

$$P_1 = G_1 G_2 G_3 G_4 G_5$$

$$P_2 = G_1 G_2 G_4 G_5$$

$$P_3 = G_1 G_2 G_7$$

of loops:-

$$\boxed{L_1 = -G_4 H_1} \quad , \quad \boxed{L_2 = -G_2 G_7 H_2} \quad , \quad \boxed{L_3 = -G_1 G_2 G_4 H_2}$$

$$\boxed{L_4 = -G_2 G_3 G_4 H_2}$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2)$$

$$\Delta_1 = 1 - 0 = \boxed{1}$$

"سبارك الذي تزلج الفقيه عليه"
لكنه للفقيه تزلجاً
سورة الفرقان

"وعباد الرحمن الذين هم على صراط مستقيم"
صوت الفقيه
16

$$\Delta_2 = 1 - 0 = 1$$

$$\Delta_3 = 1 - L_1$$

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum_{k=1}^P P_k \Delta_k$$

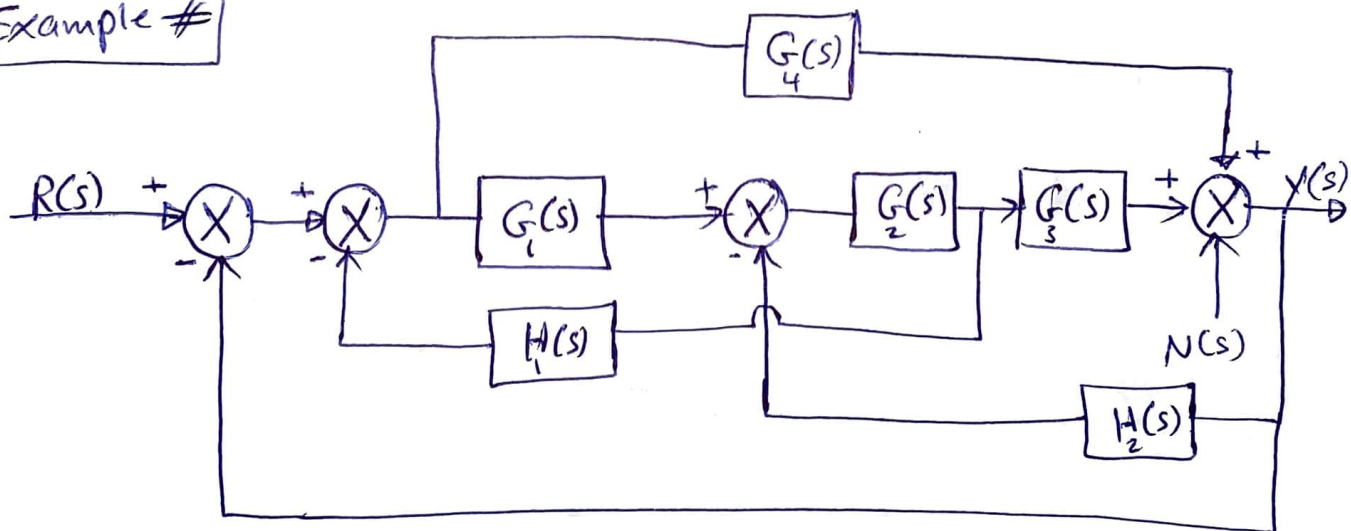
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_4 G_5 + G_1 G_2 G_7 (1 - L_1)}{1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_4 G_5 + G_1 G_2 G_4 G_5 + G_1 G_2 G_7 (1 + G_4 H_1)}{1 - (-G_1 H_1 - G_2 G_7 H_2 - G_2 G_4 G_5 H_2 - G_2 G_3 G_4 G_5 H_2) + G_4 H_1 G_2 G_7 H_2}$$

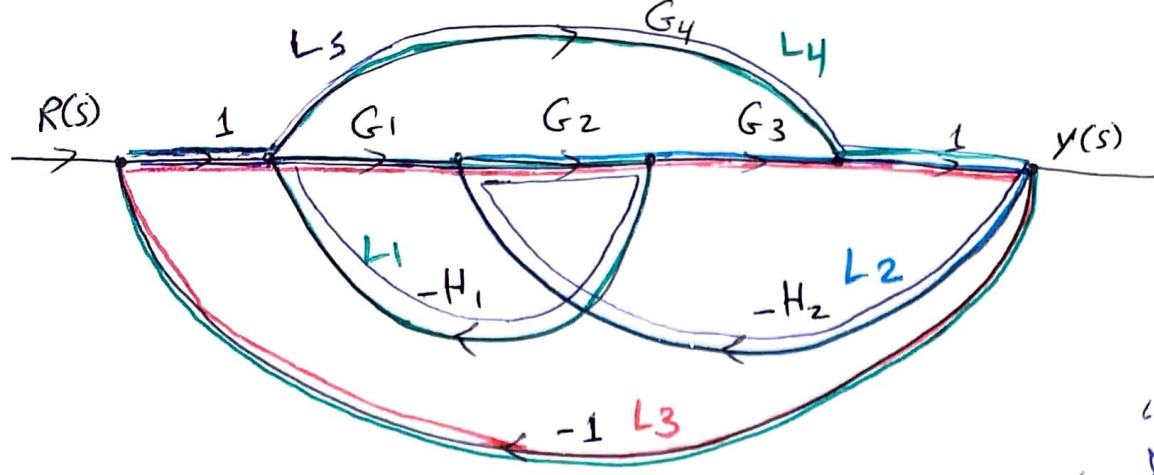
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_4 G_5 + G_1 G_2 G_4 G_5 + G_1 G_2 G_7 + G_1 G_2 G_4 G_7 H_1}{1 + G_1 H_1 + G_2 G_7 H_2 + G_2 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_4 H_1 G_2 G_7 H_2}$$

#

Example #



Find $\frac{Y(s)}{R(s)}$



of forward paths: —

2

$$P_1 = G_1 G_2 G_3$$

$$P_2 = (1)(G_4)(1) = G_4$$

of Loops:

$$L_1 = -G_1 G_2 H_1, L_2 = -G_2 G_3 H_2, L_3 = -G_1 G_2 G_3, L_4 = -G_4$$

$$L_5 = +G_4 H_2 G_2 H_1$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) = 0$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 + G_4 H_2 G_2 H_1}$$

"إنه يوم (أو من) يقول
ويعلم ما يكونه"

"ولقد آتينا داود
أقاليم الحمد لله الذي فضلنا
عليه من عباده المؤمنين
صلى الله عليه وسلم"

* Alternative Representation in state-space :-

As we see in chapter 3, we can represent the state space in different type like :-

- 1] direct form.
- 2] cascade form.
- 3] parallel form.

← نری با حلینا فی البایه انو کل system الی فقه transfer function
واحد فقه (unique) لینا ال state space بکونه ستر نه واپ

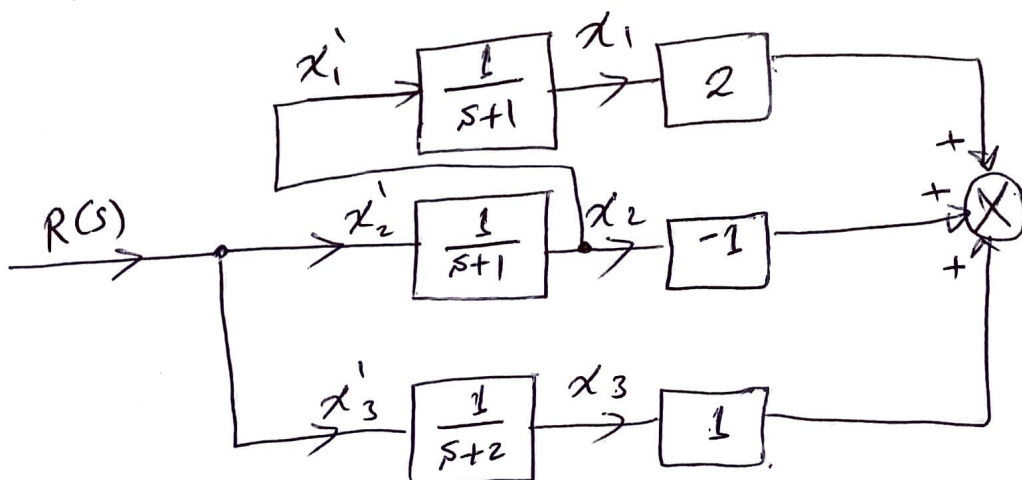
Example #1 $\frac{C(s)}{R(s)} = \frac{s+3}{(s+1)^2(s+2)}$

$$\frac{C(s)}{R(s)} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$$

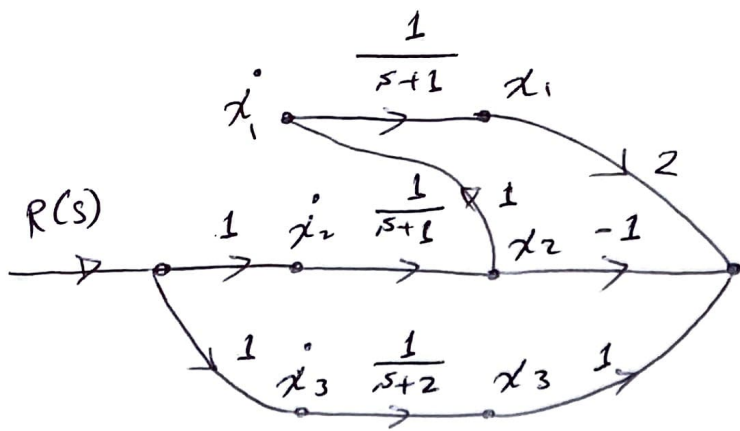
$$A = \frac{(s+3)(s+1)^2}{(s+1)^2(s+2)} \Big|_{s=-1} = \boxed{2}$$

$$B = \frac{d}{ds} \left(\frac{(s+3)(s+1)^2}{(s+1)^2(s+2)} \right) = \frac{-1}{(s+2)^2} \Big|_{s=-1} = \boxed{-1}$$

$$C = \frac{(s+3)(s+2)}{(s+1)^2(s+2)} \Big|_{s=-2} = \boxed{1}$$



Block diagram



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

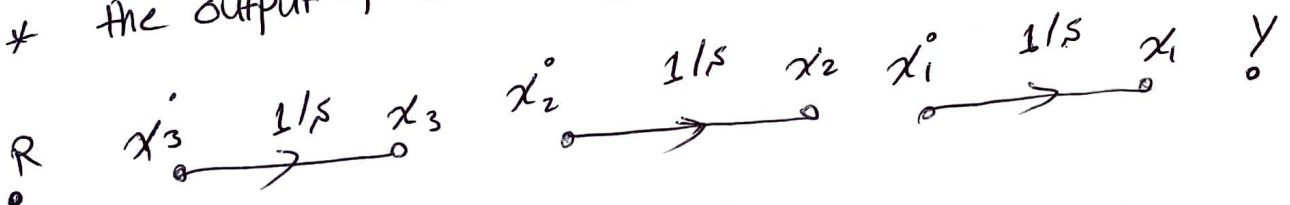
* Converting from state space model to signal flow graph.

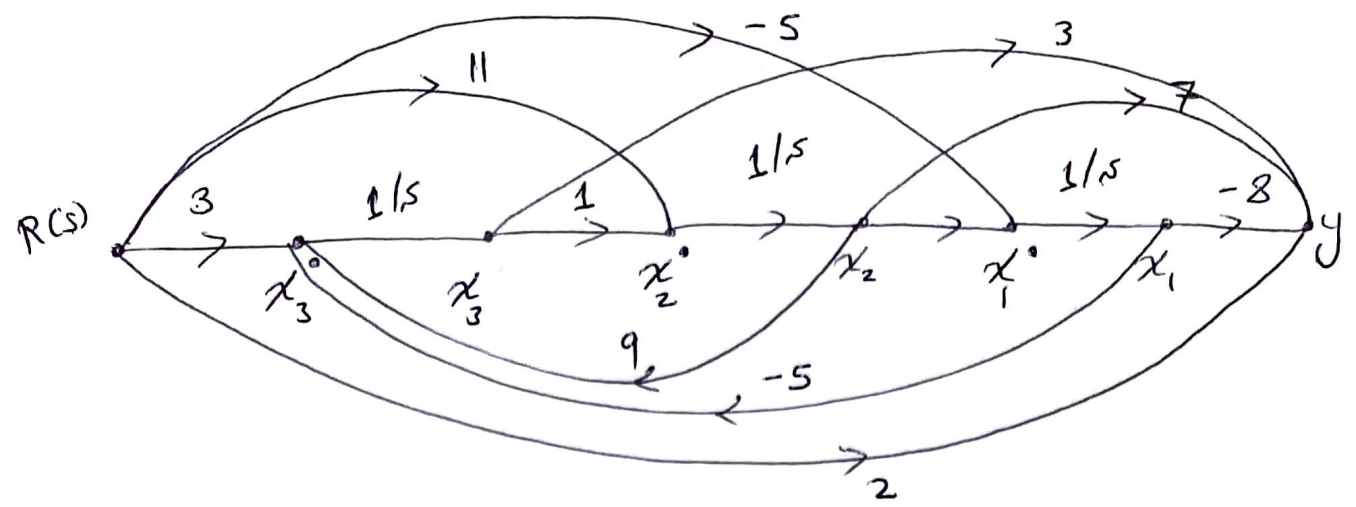
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & 9 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -5 \\ 11 \\ 3 \end{bmatrix} R$$

$$y = \begin{bmatrix} -8 & 7 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \end{bmatrix} R$$

Sol:-

- * Three state-space variables $\Rightarrow$ 3-integrators
- * the output of each integrator is a state variable.





* Similarity transformation:-

- We saw that system can be represented with different state variable, even though they have the same T.F
- These systems are called similar systems

↙ ↘
 diff. T.F, state space representation of the same system

- Transformation between similar system from one set of state equations to another without using the transfer function and signal flow graphs

- A system represented in state space as:-

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

"given system"

- Can be transformed to a similar system.

$$\begin{aligned} \dot{Z} &= P^{-1}APZ + P^{-1}Bu \\ y &= CPZ + Du \end{aligned}$$

$$\left| \begin{array}{l} x = PZ \\ Z = P^{-1}x \end{array} \right.$$

$$P = \begin{bmatrix} u & u \\ z_1 & z_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$x = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad \text{and} \quad z = p^{-1}x$$

Example #1 | Given the system represented in state space

by eqs.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0 \ 0]x$$

Transform the system to new set of state variable z

$$z_1 = 2x_1$$

$$z_2 = 3x_1 + 2x_2$$

$$z_3 = x_1 + 4x_2 + 5x_3$$

Sol:-

$$z = p^{-1}x$$

$$p^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 0.5 & 0 & 0 \\ -0.75 & 0.5 & 0 \\ 0.5 & -0.4 & 0.2 \end{bmatrix}$$

← إيجاد المصفوفة العكسية

$$z' = p^{-1}Apz + p^{-1}Bu$$

$$z' = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -7 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ -0.75 & 0.5 & 0 \\ 0.5 & -0.4 & 0.2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u$$

$$Z' = \begin{bmatrix} -1.5 & 1 & 0 \\ -1.25 & 0.7 & 0.4 \\ -2.5 & 0.4 & -6.2 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad u$$

$P^{-1}B$



we take

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$y = CPZ + Du$$

$$CP = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ -0.75 & 0.5 & 0 \\ 0.5 & -0.4 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} + 0$$

* Diagonalizing a system matrix:-

→ A diagonal system matrix has the advantages that each state equation is a function of only one state variable.

→ That mean Convert the system into parallel form as we see the general form of parallel decomposition in Chapter #3

⇒ the diagonal matrix equal $P^{-1}AP$

P is eigen vector

* How we can find the p-matrix (eigenvector)

① we need to find the eigen value of the system

$$\det(\lambda_i I - A) = 0 \quad , \quad \lambda_i : \text{eigen value "constant"}$$

② Now substitute in this equation :

$$(\lambda_i - A) p_i = 0$$

After that we will have the p-matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & \dots \\ p_{21} & p_{22} & \dots \\ \vdots & & \end{bmatrix}$$

Example # Find the diagonal system that is similar?

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = [2 \ 3] x$$

Sol :

① Find the eigen value.

$$|(\lambda_i I - A)| = 0 \quad \Rightarrow \quad \det(\lambda_i I - A) = 0$$

$$\det \begin{bmatrix} \lambda+3 & -1 \\ -1 & \lambda+3 \end{bmatrix} = (\lambda+3)(\lambda+3) - (1) = 0$$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$(\lambda+4)(\lambda+2) = 0$$

$$\Rightarrow \boxed{\lambda_1 = -4}, \boxed{\lambda_2 = -2} \Rightarrow \text{eigen values.}$$

② Find the eigenvector:

at $\lambda_1 = -4$

$$\begin{bmatrix} \lambda+3 & -1 \\ -1 & \lambda+3 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(\lambda_i - A) p_i = 0$ ← *المصفوفة*

$\lambda_1 = -4$ ← *القيمة المعطاة*

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-p_{11} - p_{21} = 0 \Rightarrow \boxed{p_{11} = -p_{21}}$$

$$-p_{11} - p_{21} = 0$$

assume that $\boxed{p_{21} = 1} \Rightarrow \boxed{p_{11} = -1}$

at $\lambda_2 = -2$

$$\begin{bmatrix} \lambda+3 & -1 \\ -1 & \lambda+3 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$p_{12} - p_{22} = 0 \Rightarrow \boxed{p_{12} = p_{22}}$$

$$-p_{12} + p_{22} = 0$$

assume $\boxed{p_{22} = 1} \Rightarrow \boxed{p_{12} = 1}$

$$P = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \text{eigen vector as } \vec{e}_1, \vec{e}_2$$

$$\text{Diagonal matrix} \Rightarrow P^{-1}AP = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix}$$

If we find the diagonal system:-

$$\dot{Z}' = P^{-1}APZ + P^{-1}Bu$$

$$\dot{Z}' = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$\dot{Z}' = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} Z + \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} u$$

$$y = CPZ$$

$$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} Z$$

$$y = \begin{bmatrix} 1 & 5 \end{bmatrix} Z$$

#

Example #1

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

Find the diagonal matrix?

Sol:-

① Find the eigen values

$$\det(\lambda I - A) = 0$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix}$$

8, 25, 4, 1, 3, 6

$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix}$

Signs: $(-)$, $(+)$, $(-)$, $(+)$, $(-)$, $(+)$

$$(\lambda^2(\lambda+6)+6) - (-11\lambda) = 0$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\boxed{\lambda = -1}, \boxed{\lambda = -2}, \boxed{\lambda = -3}$$

$$P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Example

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

Find the diagonal matrix:-

$$\det(\lambda I - A) = 0$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix} = \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\boxed{\lambda = -1}, \boxed{\lambda = -2}, \boxed{\lambda = -3}$$

⇒ Find the eigenvector:-

at $\lambda = -1$

$$\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

بالقوة من $\lambda = -1$

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 6 & 11 & 5 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-p_{11} - p_{21} = 0$$

$$-p_{21} - p_{31} = 0$$

$$6p_{11} + 11p_{21} + 5p_{31} = 0$$

$$\Rightarrow \boxed{p_{11} = -p_{21}}$$

$$\boxed{p_{21} = -p_{31}}$$

$$p_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

assume $\boxed{p_{11} = 1}$, then

$$\boxed{p_{21} = -1}$$

$$\boxed{p_{31} = 1}$$

at $\lambda = -2$

$$\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix} \begin{bmatrix} p_{21} \\ p_{22} \\ p_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

at $\lambda = -2$

$$\begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 6 & 11 & 4 \end{bmatrix} \begin{bmatrix} p_{21} \\ p_{22} \\ p_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2p_{21} - p_{22} = 0 \Rightarrow \boxed{p_{22} = -2p_{21}}$$

$$-2p_{22} - p_{23} = 0 \Rightarrow \boxed{p_{23} = -2p_{22}}$$

$$6p_{21} + 11p_{22} + 4p_{23} = 0$$

assume $\boxed{p_{21} = 1}$, then $\Rightarrow p_{22} = -2$

$$p_{23} = (-2)(-2) = \boxed{+4}$$

$$p_2 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

at $\lambda = -3$

$$\begin{bmatrix} -3 & -1 & 0 \\ 0 & -3 & -1 \\ 6 & 11 & 3 \end{bmatrix} \begin{bmatrix} p_{31} \\ p_{32} \\ p_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3p_{31} - p_{32} = 0 \Rightarrow \boxed{p_{32} = -3p_{31}}$$

$$-3p_{32} - p_{33} = 0 \Rightarrow \boxed{p_{33} = -3p_{32}}$$

Let $\underline{p_{31} = 1}$ then $\therefore \boxed{p_{32} = -3}$

$$p_{33} = (-3)(-3) = \boxed{9}$$

$\Rightarrow$ 29

$$P_3 = \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

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فريق RCM الهندسي

﴿ رَبَّنَا تَقَبَّلْ مِنَّا إِنَّكَ أَنْتَ السَّمِيعُ الْعَلِيمُ ﴾



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