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Linear Control system
Chapter # 8
Root Locus techniques

Hadil 10/3/1996

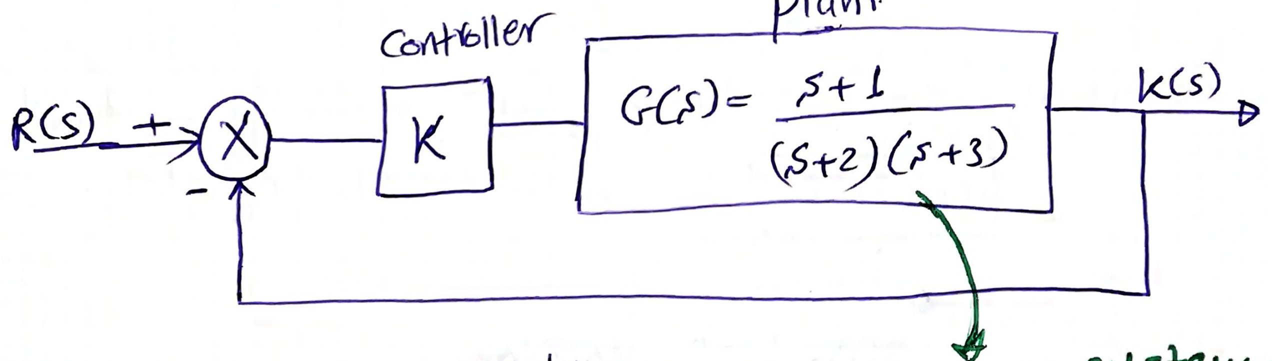
Root locus :-

Is agraphical presentation of the closed Loop poles as system parameter (k) is varied from 0 to infinity.

⇐ locus: المحل الهندسي الذي يحقق شرط معين.

⇐ من خلال تقنية الـ root locus بعمل دراسة لل closed loop system وبخلي أغير قيمة k من الصفر لحد الانفنتي (∞) لحتى أوصل لل specification إلي بدي إياها لل system.

* from these poles Location we can charactrize the feedback system if stable or not and the type of response in transient response



open loop system.

رمزها x

رمزها o

$KG(s) \Rightarrow$ open loop system.

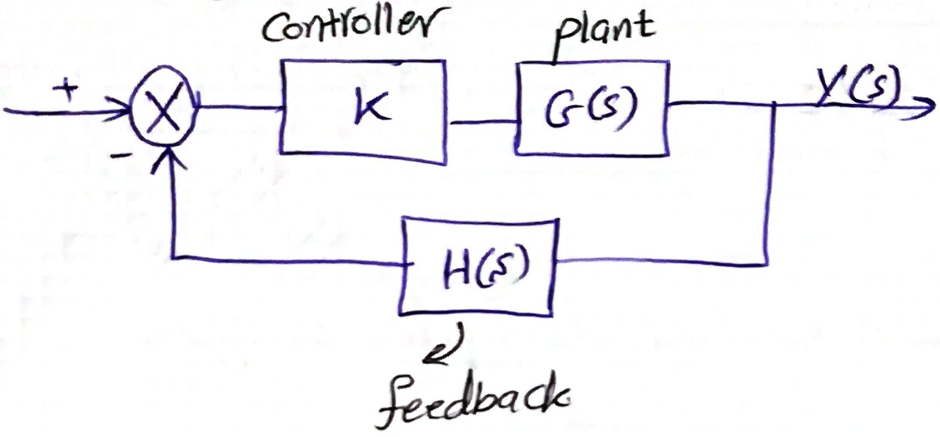
open Loops poles: $s = -2$, $s = -3$

open Loops Zeros: $s = -1$

$K \Rightarrow$ propotional Controller

"واجعل لي لسان صدق في الآخرين"

Sketching the Root locus :-



"negative feedback"

Steps :-

1. obtain the open Loop function

$$KG(s)\,H(s)$$

لو كانت H(s) تساوي 1 تكون ال open Loop هي عبارة عن KG(s)

to find the charactrstic equation , we find closed Loop system

$$\frac{C(s)}{R(s)} = \frac{K\,G(s)}{\underline{\underline{1+KG(s)H(s)}}}$$

characteristic equation

أو عكسها مباشرة دون حساب ال closed Loop system : نكتب المعادلة

$$1+\boxed{KG(s)H(s)} \Rightarrow$$ charactrstic equation.

open Loop function

2. From the open loop function Mark poles with (x) and Zero with (o)

3. Draw the Locus "Location" of the real axis to the Left of an odd number of real poles plus Zero

"لما يكون عدد ال Poles , Zeros فردي برسم ال Locus

$\boxed{4}$ The Root Locus orginates on the poles of $G(s)H(s)$ and terminate on the Zero of $G(s)H(s)$.

$\boxed{5}$ The Root Locus is symmetrical with respect to real axis.

$\boxed{6}$ Draw the Asymptotes. →

عبارة عن خطوط التقارب تكون
خارجة من poles ومتجهة نحو
الـ Zeros الموجود بـ $\underline{\alpha}$
وحيث الخطوط تؤول إلى "∞"

* $\alpha = n - m$

n → # of poles

m → # of Zeros

* $\delta_m = \dfrac{\sum P_i - \sum Z_i}{\alpha}$ intersection points with real axis

* $\theta_m = r\,\dfrac{180^\circ}{\alpha}$ is the Asymptotes angle

$r = \pm 1, \pm 3, \pm 5, \ldots$ تختار عدد $\underline{r}$ بناءً على عدد الـ $\underline{\alpha}$

Example #

$\alpha = 1 \quad , r = +1$

$\alpha = 2 \quad , r = \pm 1$

$\alpha = 3 \quad , r = \pm 1 , +3$

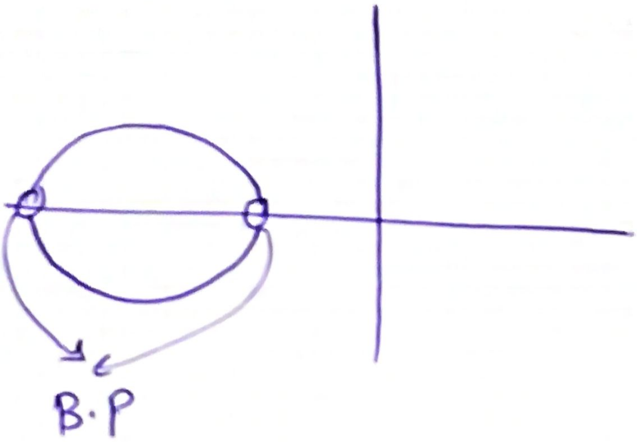
$\alpha = 4 \quad , r = \pm 1 , \pm 3$

$\vdots \qquad \vdots$

يُريدونَ أنْ يُطفئوا نورَ اللهِ بأفواهِهم ويأبى الله
إلا أنْ يُتمَّ نورَه ولو كرِهَ الكافرون
سورة التوبة
9/5/2020

$\boxed{7}$ The Break away points Will appear among the roots of Polynomial which obtain from:-

$$\frac{d}{ds}\left[G(s)H(s)\right] = 0 \quad \text{OR} \quad N(s)D'(s) - N'(s)D(s)$$



⇒ to find the K-Controller value at specific poles

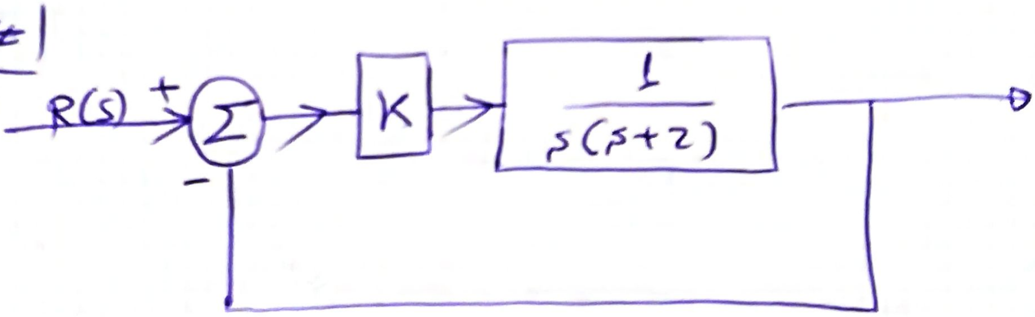
* from charactristic equation:

$$1 + KG(s)H(s) = 0$$

$$K = \frac{-1}{G(s)H(s)}\bigg|_{s=s_0}$$

بنجيب قيمة K بناءً على قيمة
ال poles المعطاة في
السؤال ، المطلوب حساب
قيمة K عندها لانو قيمة K تختلف عند كل pole

Example #1



Find RL ??

Sol :-

$\boxed{1}$ obtain open Loop function.

$$KG(s)H(s) = \frac{K}{s(s+2)}$$

poles : $s=0$, $s=-2$

Zeros : ϕ " لا يوجد Zeros "

$$\alpha = n-m \Rightarrow 2-0=2 \Rightarrow \boxed{\alpha=2}$$

$$\sigma_m = \frac{\sum P_i - \sum Z_i}{\alpha} = \frac{-2-0}{2} = \boxed{-1}$$

$$\theta_m = \frac{(\pm1)(180^\circ)}{\alpha} = \frac{(\pm1)(180)}{2} = \boxed{\pm 90^\circ}$$

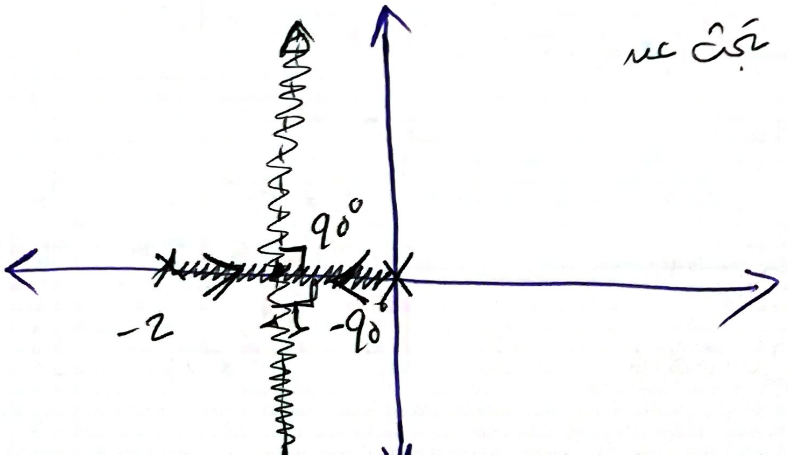
* The break away point :-

خطوط التقارب تحدد اتجاه الانطلاق
عند الزوايا التي تم حسابها
بينما B.P توضح نقطة الانطلاق

$$\frac{d}{ds}\left(G(s)H(s)\right) = 0$$

$$\frac{d}{ds}\left(\frac{1}{s(s+2)}\right) = \frac{-(2s+2)}{s^2(s+2)^2} = 0$$

$$-(2s+2)=0 \Rightarrow 2s=-2 \Rightarrow \boxed{s=-1}$$



كل poles تتحرك تجاه عند
ال Zero

* Sketch the Root Locus for the following transfer function:-

~ unity feedback with $G(s) = \dfrac{s+3}{s(s+1)(s+2)(s+4)}$

$\downarrow$ $H(s) = 1$

Sol:-

$\boxed{1}$ open loop system $KG(s)H(s) = \dfrac{K(s+3)}{s(s+1)(s+2)(s+4)}$

$P: 0, -1, -2, -4$

$Z: -3$

$\alpha = n - m = 4 - 1 = \boxed{3}$

$$\delta_m = \frac{\sum P_i - \sum Z_i}{\alpha} = \frac{(-4-2-1)-(-3)}{3} = \boxed{-\frac{4}{3}}$$

$\delta_m \Rightarrow$ لا تقع على ال R.L

$$\theta_m = (\pm 1, +3)\frac{(180)}{3} = \boxed{+60^\circ}, \boxed{-60^\circ}, \boxed{+180^\circ}$$

* The breakaway point:-

$$\frac{d}{ds}\big(G(s)H(s)\big) = \frac{d}{ds}\left(\frac{s+3}{s(s+1)(s+2)(s+4)}\right)$$

$$= \frac{-(3s^4 + 26s^3 + 77s^2 + 84s + 24)}{s^2(s+1)^2(s+2)^2(s+4)^2} = 0$$

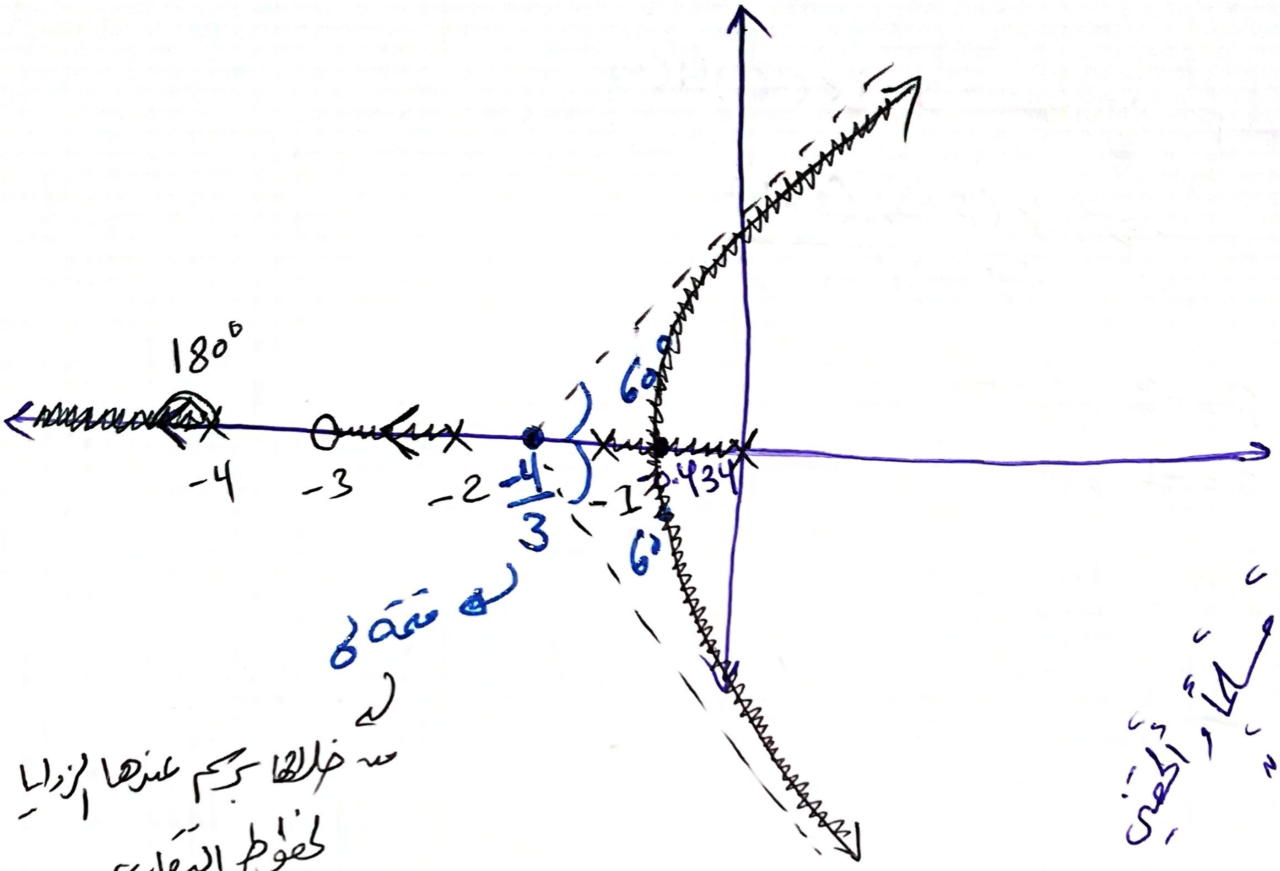
$s = -0.434$ ✓

$s = -1.609$ × مرفوضة

$s = -3.311 - J0.6812$ × مرفوضة

$s = -3.311 + J0.6812$ × مرفوضة

} لا تقع على R.L [illegible]



نقطة o

من خلالها نحكم عندها زوايا
لخطوط التقارب

"توضيح مسار المنحنى
بالتفصيل"
سوف نتعرف

B.P عامة عند -0.434 وهي توضح نقطة الانطلاق للابتعاد عن ال poles باتجاه ال ∞ وخطوط التقارب تحدد اتجاه الانطلاق بزوايا 60°, -60° و -180°

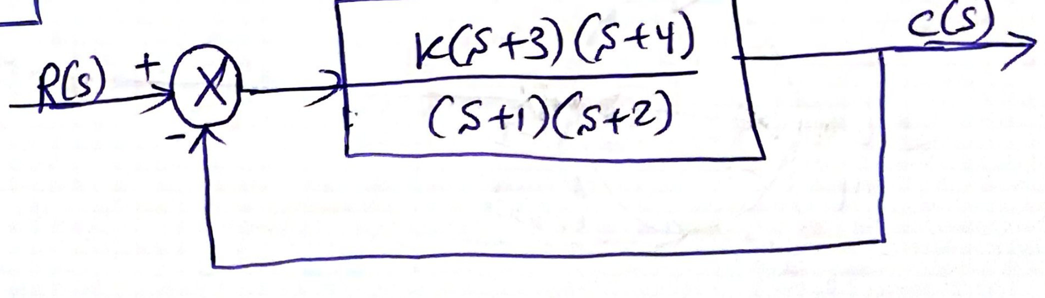
كل poles تتجه عند Zero

-2 تروح لعند $\underline{\underline{-3}}$ " zero"

-4 تروح لا Zero إلى ال $\underline{\underline{\infty}}$ بزاوية $\underline{\underline{180°}}$

-1 و 0 يروحوا لا Zero بال $\underline{\infty}$ بزوايا 60° و -60°

Example #1



Sol:-

$\boxed{1}$ open Loop system $\Rightarrow$ $KG(s)H(s)$

$$\Rightarrow \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

Poles: $s=-3$, $s=-4$

Zeros: $s=-1$, $s=-2$

$$\alpha = 2-2 = \boxed{0}$$

$$\sigma_m = \frac{\sum P_i - \sum Z_i}{\alpha} = \frac{(-3-4)-(-1-2)}{0} = \boxed{\infty}$$

$\Leftarrow$ ما حيكون في عندي زوايا " θ_m "

$\Rightarrow$ The break point :

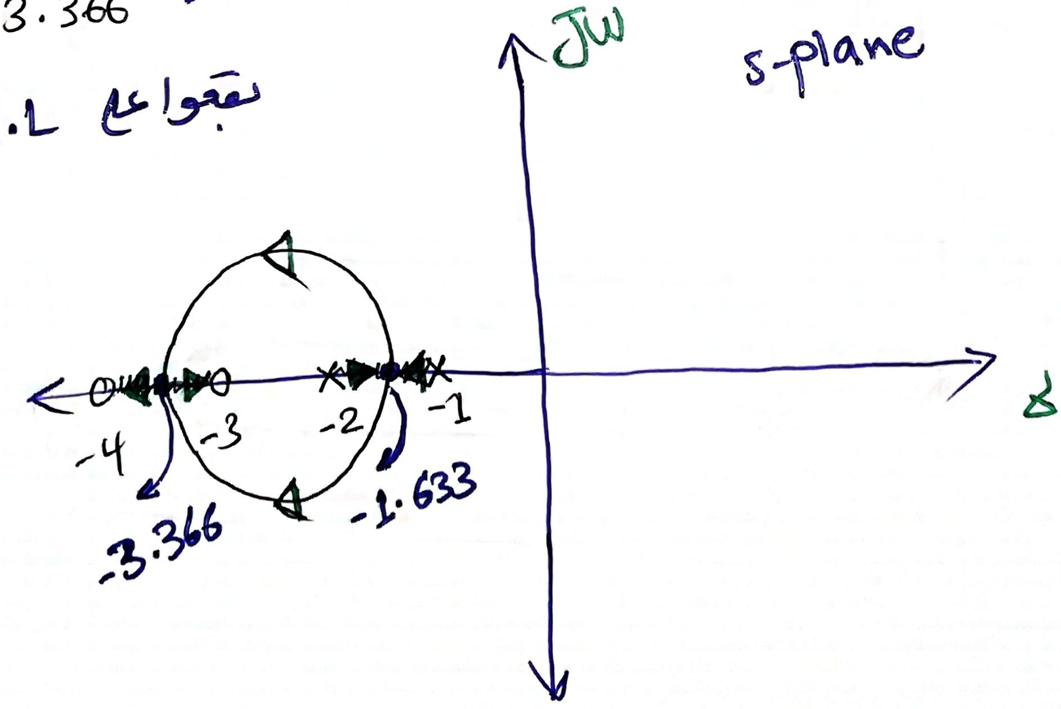
$$\frac{d}{ds}\left(G(s)H(s)\right) = \frac{d}{ds}\left(\frac{(s+3)(s+4)}{(s+1)(s+2)}\right)$$

$$= \frac{-(4s^2+20s+22)}{(s+1)^2(s+2)^2} = 0$$

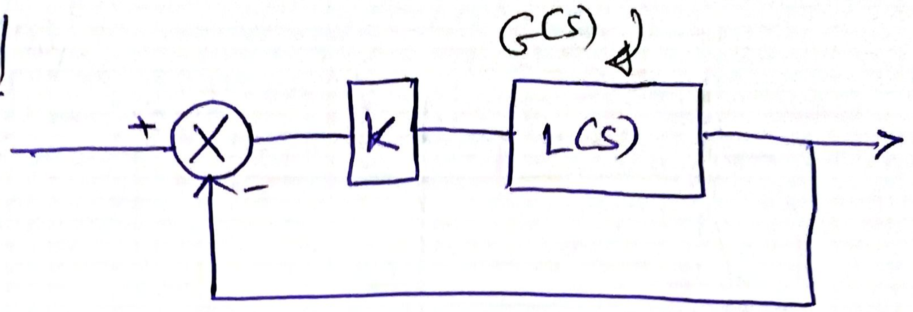
$s = -1.633$ ✓

$s = -3.366$ ✓

R.L يقعوا عل



Example # 1



$$L(s) = \frac{(s+1)}{s(s+2)(s+3)}$$, sketch R.L

Sol:-

open Loop system:- $$KG(s)H(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

poles: $s = 0, -2, -3$

Zeros: $s = -1$

$\alpha = 3-1 = \boxed{2}$

$\sigma_m = \frac{(-2-3)-(-1)}{2} = \boxed{-2}$

$\theta_m = \frac{(\pm 1)(180)}{2} = \boxed{\pm 90^\circ}$

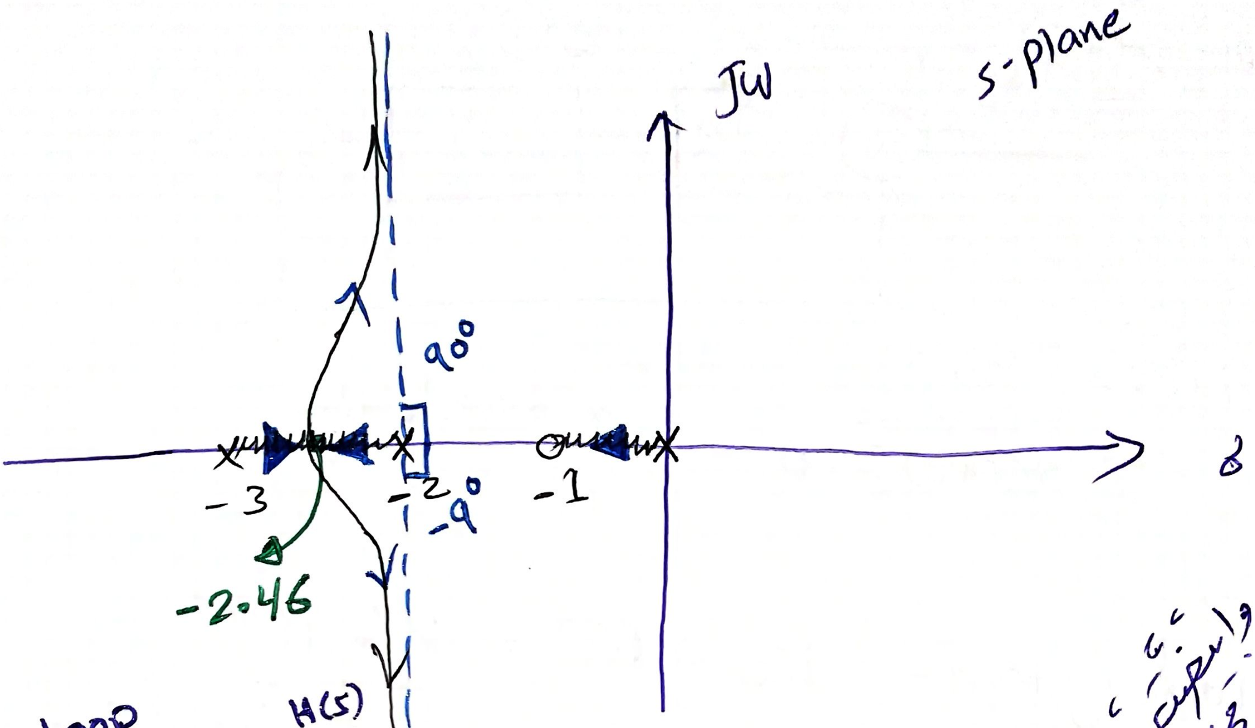
"وقضى ربك ألا تعبدوا إلا إياه
(٢٣) وبالوالدين إحسانا"
سورة الإسراء
10/5/2020

Break away points:-

$$\frac{d}{ds}\left(G(s)H(s)\right) = \frac{d}{ds}\left(\frac{s+1}{s(s+2)(s+3)}\right) \Rightarrow$$

$$\Rightarrow \frac{-(2s^3+8s^2+10s+6)}{s^2(s+2)^2(s+3)^2} = 0$$

$s = -0.767 + J0.7925$
$s = -0.767 - J0.7925$ } مرفوضة لأنها لها جزء complex فلا تقع ع R.L
$s = -2.465$ ← تقع على R.L



open Loop

$$L(s)=\frac{(s+1)}{s(s+2)(s+3)}\cdot\underset{H(s)}{(1)}$$

على الماتلاب الأمر سهل جداً

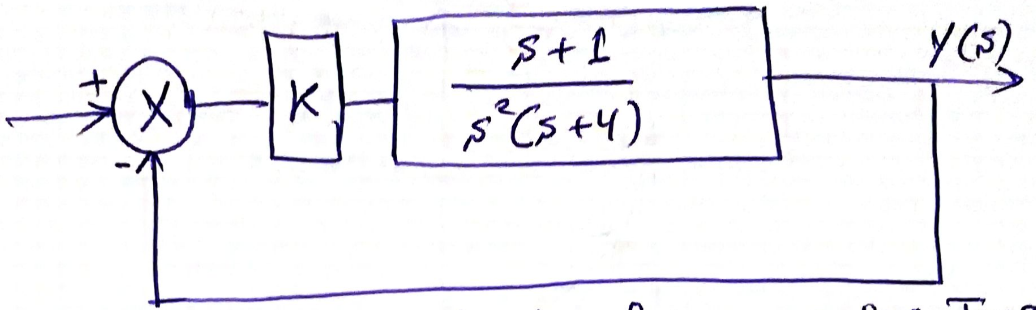
```
num= [1  1]
den= [1 5 6 0]
sys= tf (num, den)
rlocus (sys)
```

تقدر أمشي بالمؤشر وأعرف قيمة K إلي تخلي الـ system عالة عند stable وقيمة K عند Break away points

"وابيضت عيناه من الحزن فهو كظيم" سورة يوسف.

"إن الله وملائكته يصلون على النبي يا أيها الذين آمنوا صلوا عليه وسلموا تسليما" سورة الأحزاب

Example



sketch the Root Locus and check if it cross the Img-axis or not.

Sol:-

open Loop function: $KG(s)H(s) = \frac{K(s+1)}{s^2(s+4)}$

poles: $s=0,\ s=0,\ s=-4$

Zeros: $s=-1$

$\alpha = 3-1 = \boxed{2}$

$\delta_m = \frac{-4-(-1)}{2} = \boxed{-1.5}$

$\theta_m = \frac{(\pm 1)(180)}{2} = \boxed{\pm 90^\circ}$

"إن الله يأمر بالعدل والإحسان وإيتاء ذي القربى وينهى عن الفحشاء والمنكر والبغي" سورة النحل

Breakaway points: $\frac{d}{ds}\left(G(s) \cdot H(s)\right) = 0$

$$\Rightarrow \frac{d}{ds}\left(\frac{s+1}{s^2(s+4)}\right) = \frac{-(2s^2+7s+8)}{s^3(s+4)^2}$$

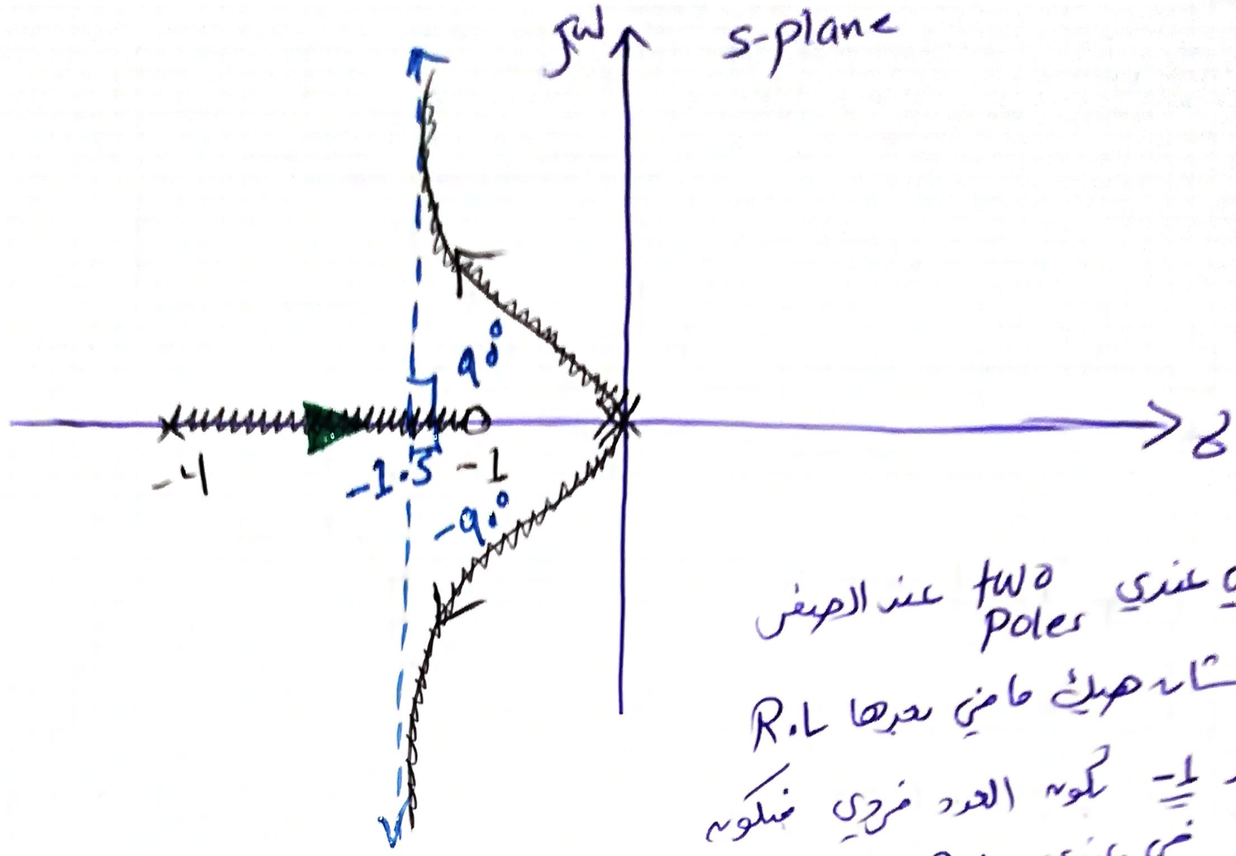
$-(2s^2+7s+8)=0$

$s = -1.75 + J0.9682$

$s = -1.75 - J0.9682$

there is no Break away point

" لأنهم كومبلكس poles "



في عندي two poles عند الصفر
عشان هيك ما في بعدها R.L
بعد 1- بكون العدد فردي فبكون
في عندي R.L

⇒ You can check if cross or not By using Routh Hurwitz

⇒ closed Loop ⇒ $\dfrac{KG(s)}{1+KG(s)H(s)}$

charactristic equation: $1+KG(s)H(s)$

$$1+\frac{K(s+1)}{s^2(s+4)} = s^3+4s^2+Ks+K=0$$

s^3	1	K
s^2	4	K
s^1	$\frac{3K}{4}$	
s^0	K	

بكون stable لو كان العمود الأول موجب

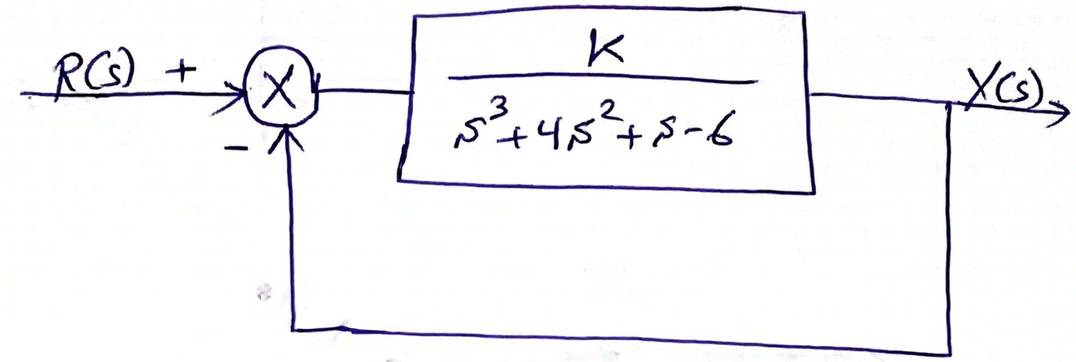
$\Rightarrow \frac{3K}{4} > 0 \Rightarrow K > 0$

$\Rightarrow K > 0$

the system stable for $K > 0$ ...

#

Example #1



Sketch the R.L and check if cross the σ-axis or not, if cross find the value of k at this point and poles at the same point.

sol:-

open Loop system: $KG(s)H(s) = \frac{K}{s^3+4s^2+s-6}$

$s^3+4s^2+s-6=0$

$\Rightarrow (s-1)(s+2)(s+3)=0$

poles : $s=1,\ s=-2,\ s=-3$

Zeros : ϕ " لا يوجد "

$\alpha = \boxed{3}$, $\sigma_m = \frac{-3-2+1}{3} = \boxed{\frac{-4}{3}} = -1.33$

$\theta_m = \frac{(\pm 1, +3)(180)}{3} = +60^\circ, -60^\circ, +180^\circ$

Break away point: $\frac{d}{ds}\left(\frac{1}{s^3+4s^2+s-6}\right) = 0$

$$\frac{-(3s^2+8s+1)}{(s^3+4s^2+s-6)^2} = 0$$

$s = -0.131$ ✓ تقع على R.L

$s = -2.535$ " لا تقع على R.L " مرفوضة

من خلال الرسم ت.ع ⟸

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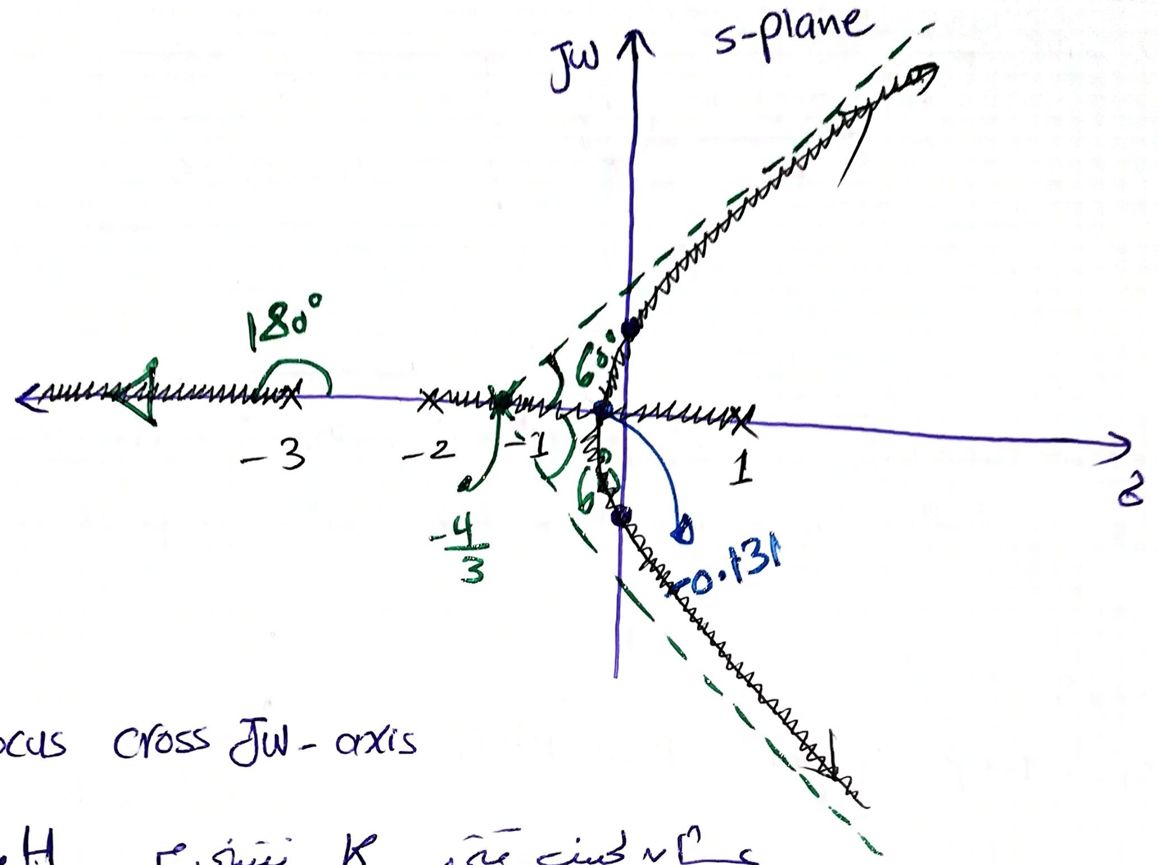


The Root Locus cross Jw - axis

علشان نحسب متى K نستخدم R.H

"Routh Hurwitz"

$$1 + kG(s)H(s) = 0$$

$$1 + \frac{k}{(s-1)(s+2)(s+3)} = 0$$

$\boxed{s^3 + 4s^2 + s - 6 + k = 0}$ → charactristic equation

s^3	1	1
s^2	4	k-6
s^1	10-K	0
s^0	k-6	

Rang stability :-

$10 - K > 0 \Rightarrow 10 > K$

$K - 6 > 0 \Rightarrow K > 6$

$\boxed{6 < k < 10}$

← نعمل تعويض في ال Zeros نعوض منه $10 - K = 0$

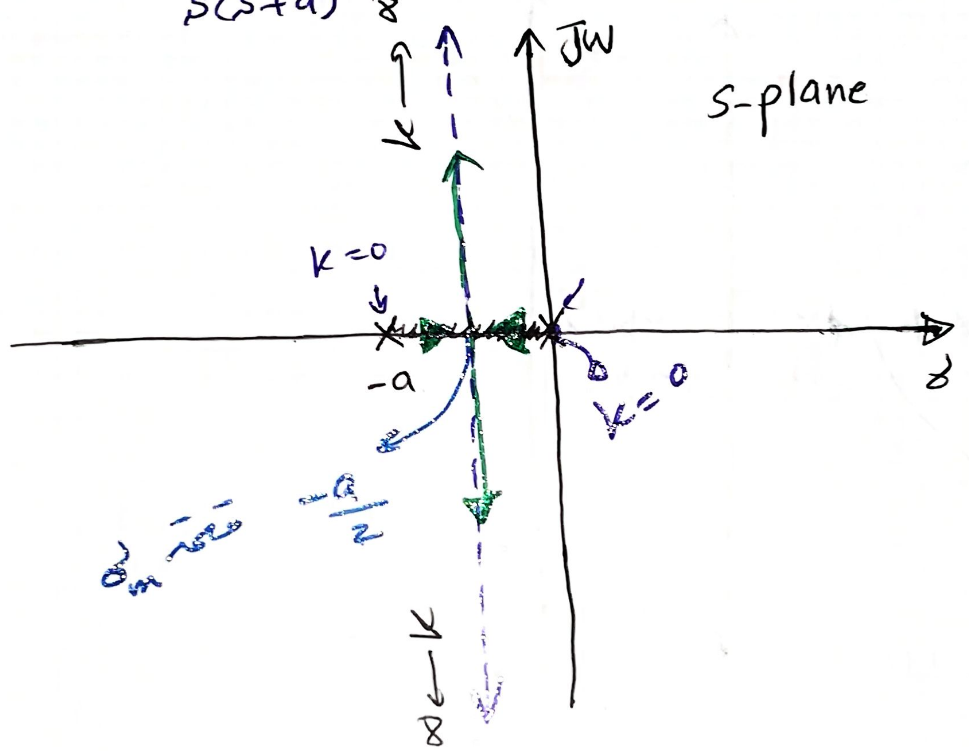
⇒ system is margin at $\boxed{k = 10}$

to find the poles take the row which is before row of Zeros:

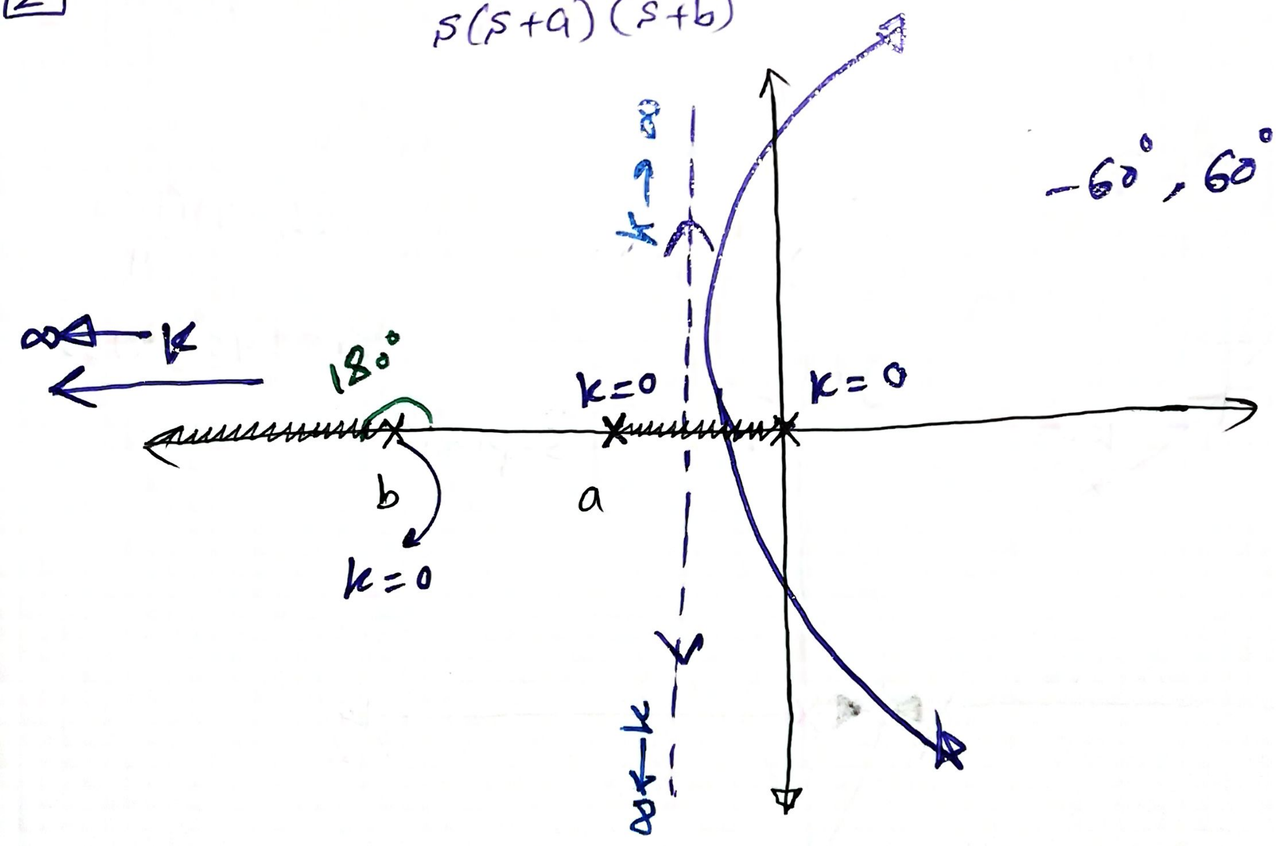
$4s^2 + k - 6 = 0 \Rightarrow 4s^2 + 4 = 0 \Rightarrow s^2 = -1$, $\boxed{s = \pm J}$ #

* Root Locus diagrams that show the effects of adding poles to G(s).H(s).

$\boxed{1}$ $G(s).H(s) = \dfrac{K}{s(s+a)}$, $a > 0$



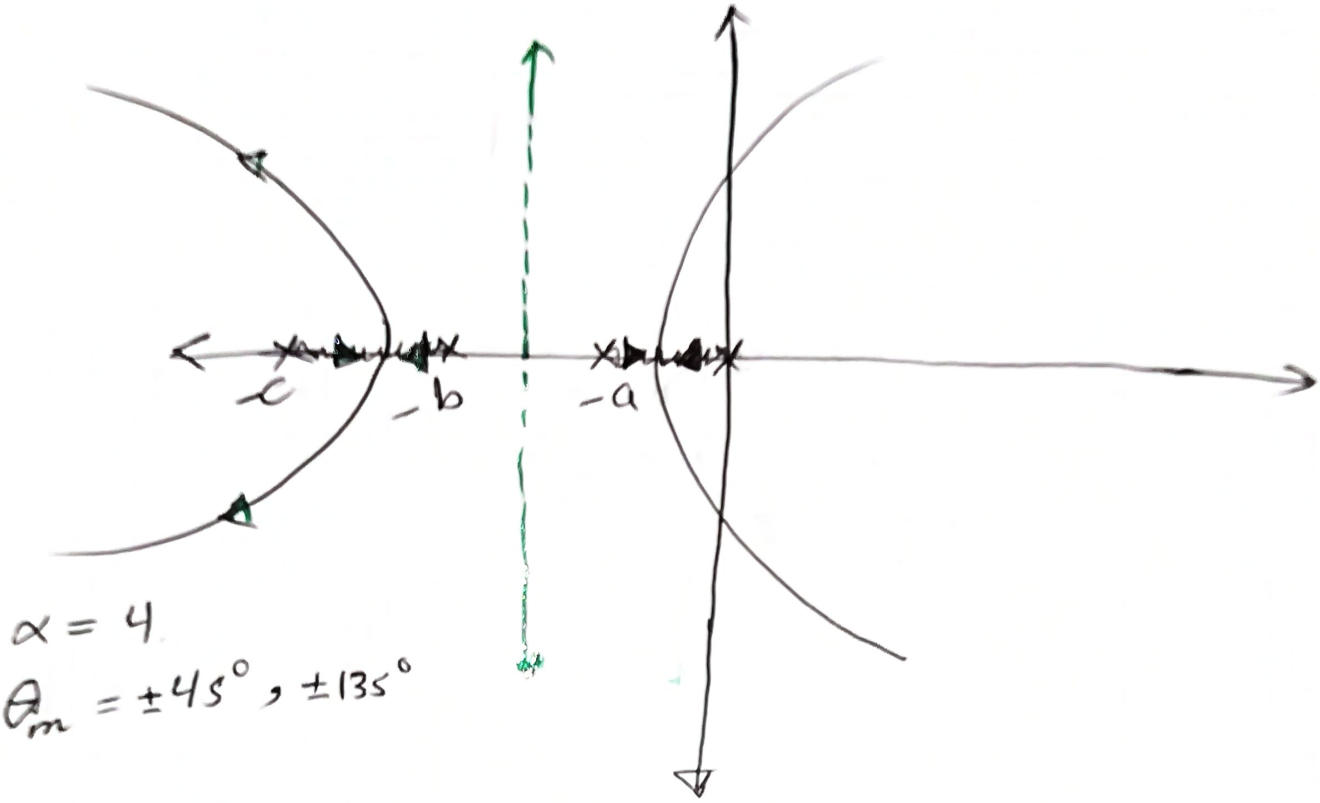
$\boxed{2}$ $G(s).H(s) = \dfrac{K}{s(s+a)(s+b)}$ $b > a > 0$



بزوايا $-60°, 60°$

$\boxed{3}$ another pole is added to G(s).H(s) at $s=-c$

$$G(s)\,H(s) = \frac{K}{s(s+a)(s+b)(s+c)}$$



$\alpha = 4$

$\theta_m = \pm 45^\circ, \pm 135^\circ$

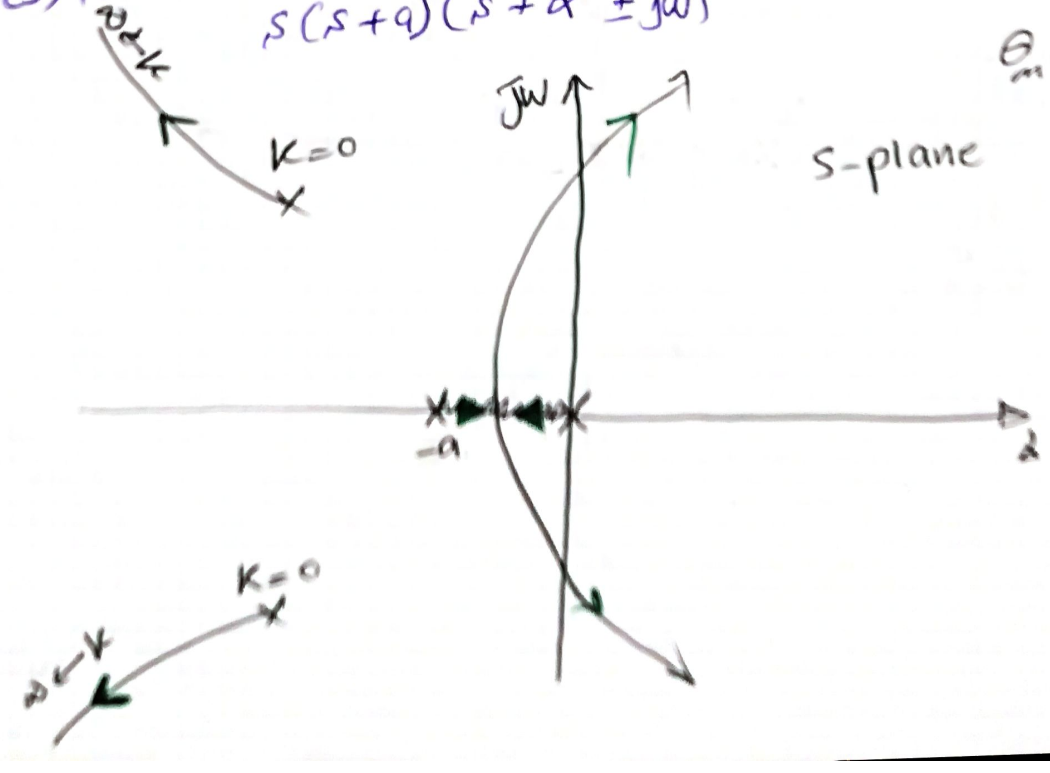
$\boxed{4}$ addition of pair of complex conjugate poles to the transfer function.

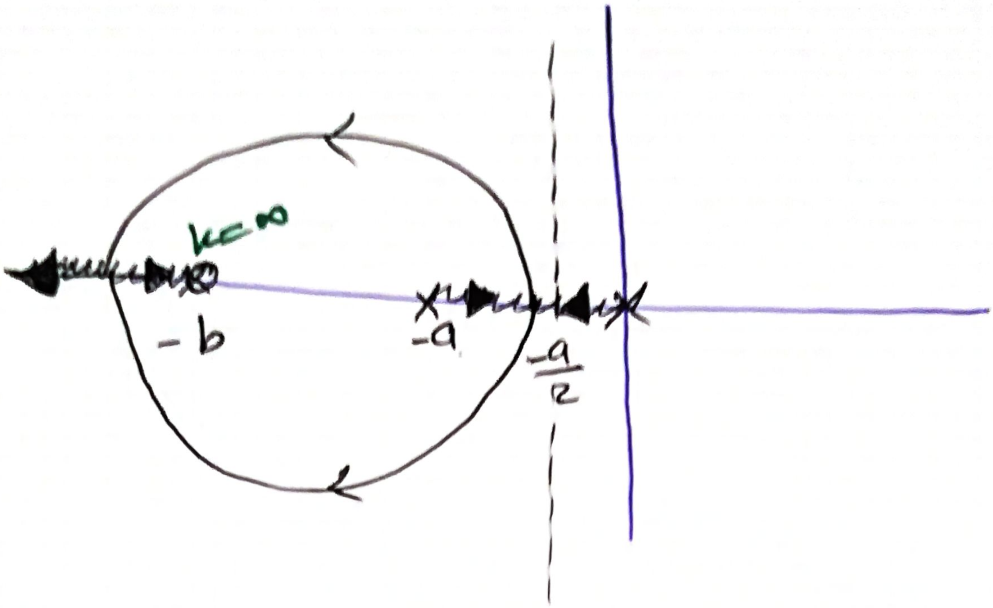
$$G(s)\,H(s) = \frac{K}{s(s+a)(s+\alpha \pm jw)}$$

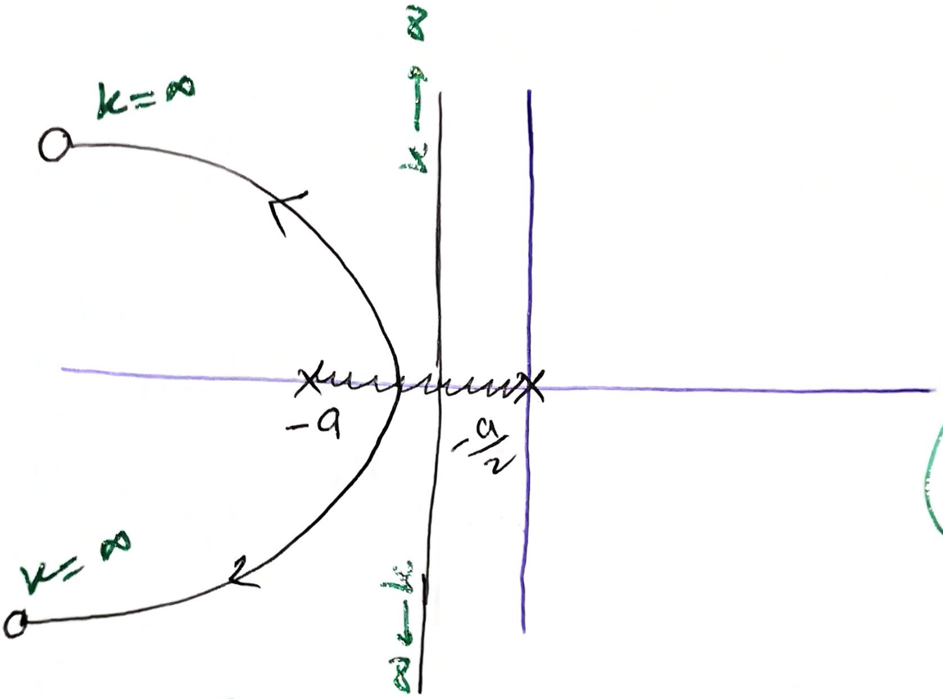
$\alpha = 4$

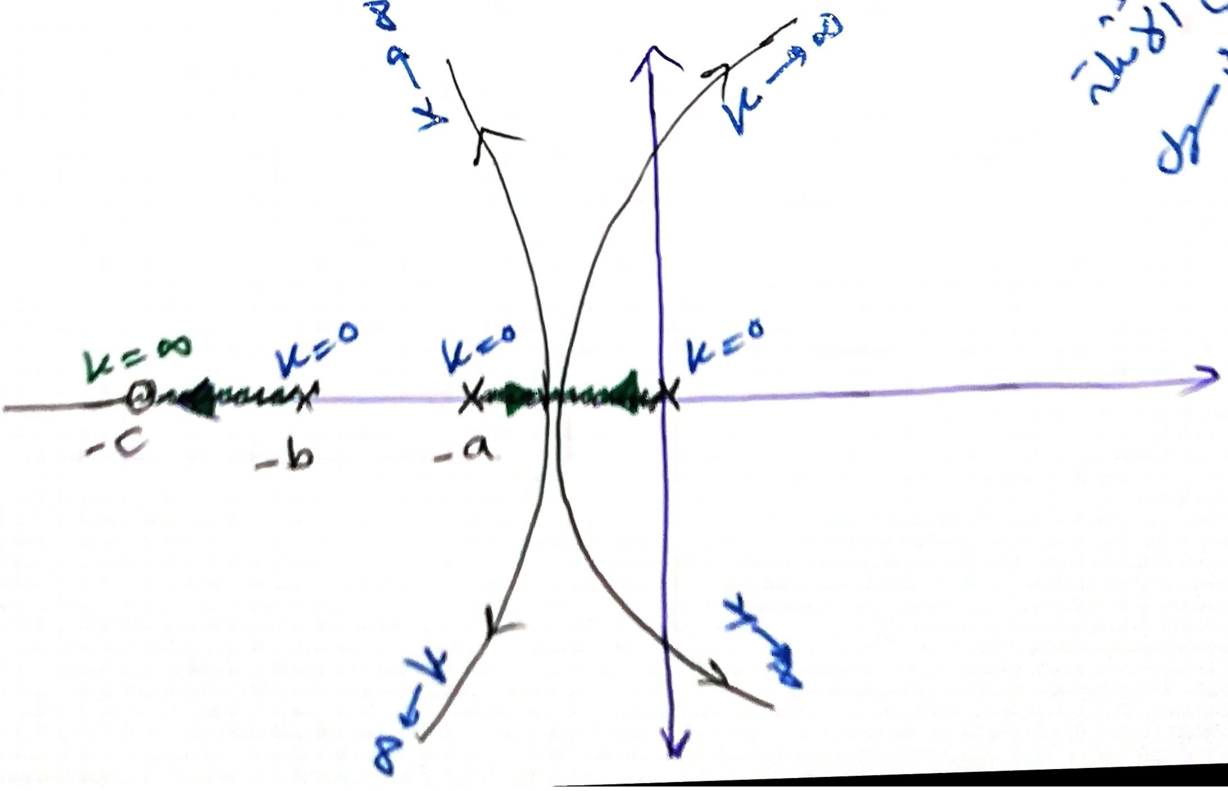
$\theta_m = (\pm 1, \pm 3)\frac{(180)}{4}$

$\theta_m = \pm 45^\circ, \pm 135^\circ$









حالات ال R.L

ع ا. تقاطعها مع الأسئلة
مكونة أ. سؤال

Example #1 Given $G(s)H(s) = \frac{k(s+b)}{s^2(s+a)}$, find R-L

when $b = 1$

i) $a = 10$, ii) $a = 9$, iii) $a = 8$, iv) $a = 3$, v) $a = 1$

Sol: at $b = 1$, $a = 10$

$$G(s)H(s) = \frac{k(s+1)}{s^2(s+10)}$$

poles: $s = 0$, $s = 0$, $s = -10$

Zeros: $s = -1$

$$\alpha = 3 - 1 = \boxed{2}$$

$$\delta m = \frac{(-10) - (-1)}{2} = \boxed{-4.5}$$

$$\theta m = \frac{(\pm 1)(180)}{2} = \pm 90°$$

Break away point: $\frac{d}{ds}\left(\frac{s+1}{s^2(s+10)}\right) = 0$

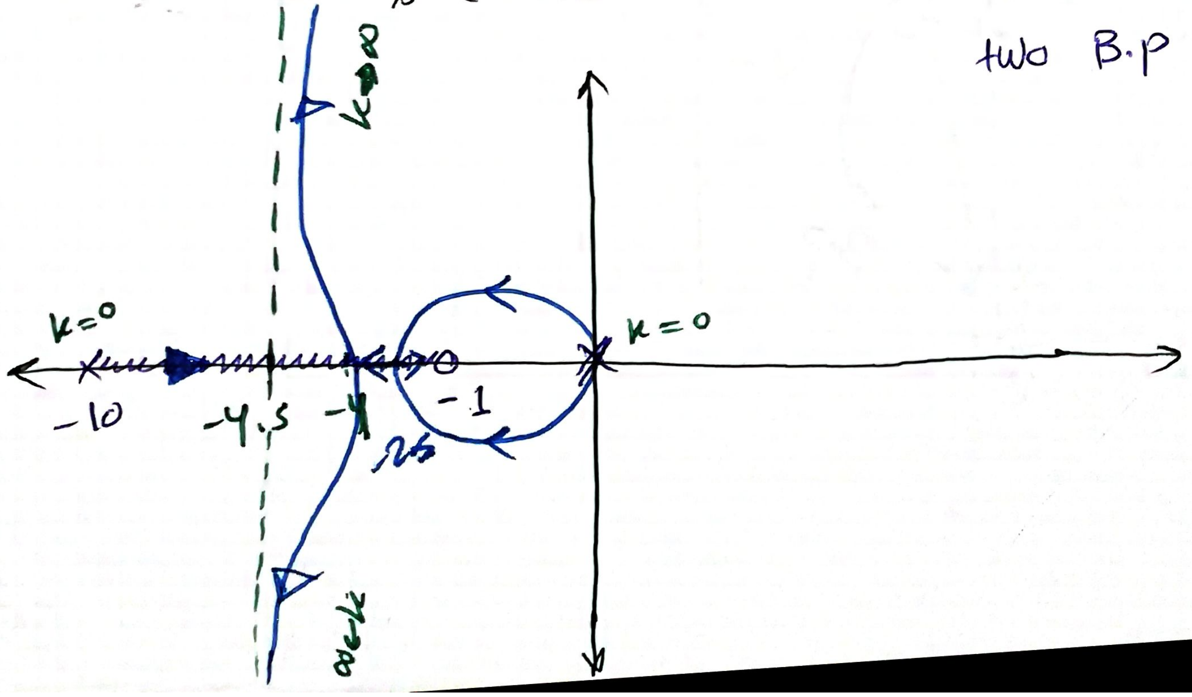
$$-\frac{(2s^2 + 13s + 20)}{s^3(s+10)^2} = 0$$

$$\left\{ \begin{array}{l} s = -2.5 \checkmark \\ s = -4 \checkmark \end{array} \right.$$

two B.p

"إنما الصدقات للفقراء والمساكين والعاملين
عليها والمؤلفة قلوبهم وفي الرقاب
والغارمين وفي سبيل الله وابن السبيل
فريضة من الله والله عليم حكيم"
سورة الأنفال

11/5/2020



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ii) at $b = 1$, $a = 9$

$$K\,G(s)\,H(s) = \frac{K(s+1)}{s^2(s+9)}$$

Poles: $s = -9$, $s = 0$, $s = 0$

Zeros: $s = -1$

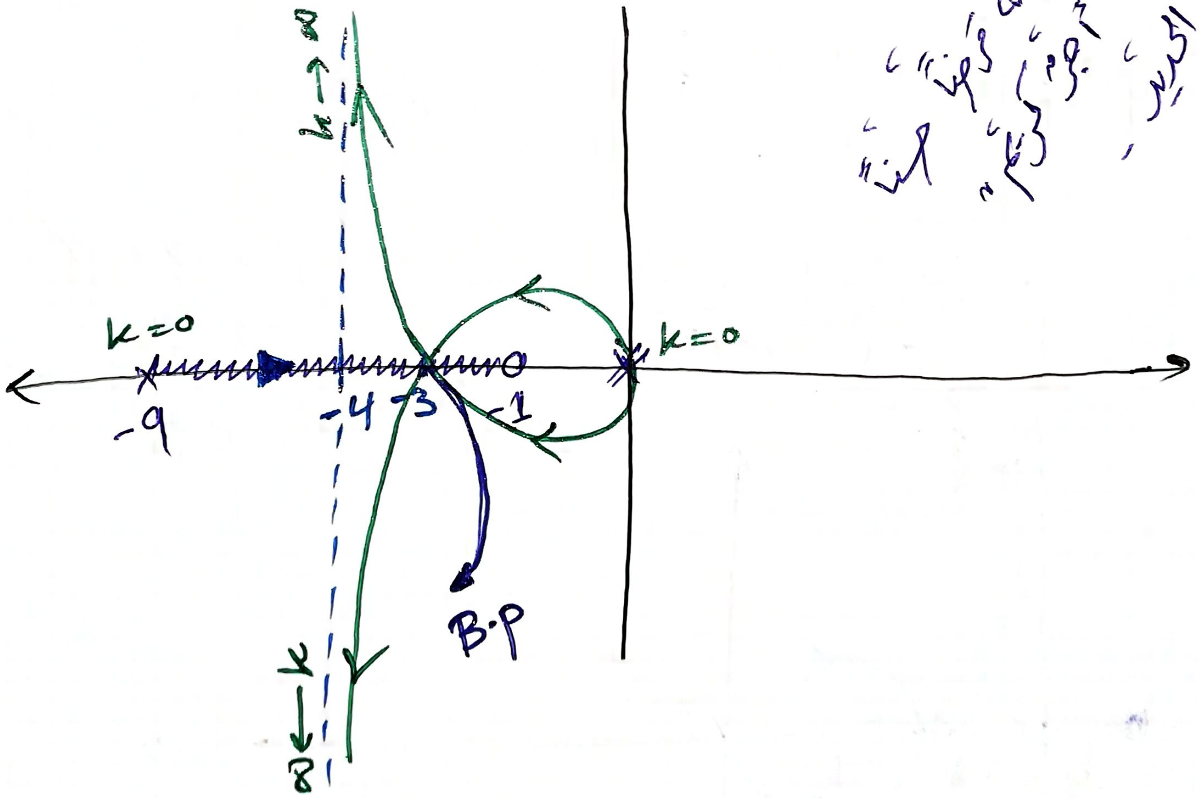
$\alpha = 3 - 1 = \boxed{2}$, $\delta_m = \frac{-9-(-1)}{2} = \boxed{-4}$

$$\theta_m = \frac{(\pm 1)(180)}{2} = \pm 90^\circ$$

Break away point: $\frac{d}{ds}\left(\frac{s+1}{s^2(s+9)}\right) = 0$

$$\frac{-(2s^2+12s+18)}{s^3(s+9)^2} = 0$$

$\boxed{s = -3}$, $\boxed{s = -3}$



"مَن ذَا الَّذِي يُقرِضُ اللَّهَ قَرضًا حَسَنًا
فَيُضَاعِفَهُ لَهُ وَلَهُ أَجرٌ كَرِيمٌ"
سورة الحديد

iii) at $b = 1$, $a = 8$

$$G(s)H(s) = \frac{k(s+1)}{s^2(s+8)}$$

Poles : $s = 0$, $s = 0$, $s = -8$

Zeros : $s = -1$

$\alpha = 3 - 1 = \boxed{2}$, $\delta_m = \frac{-8-(-1)}{2} = \boxed{-3.5}$

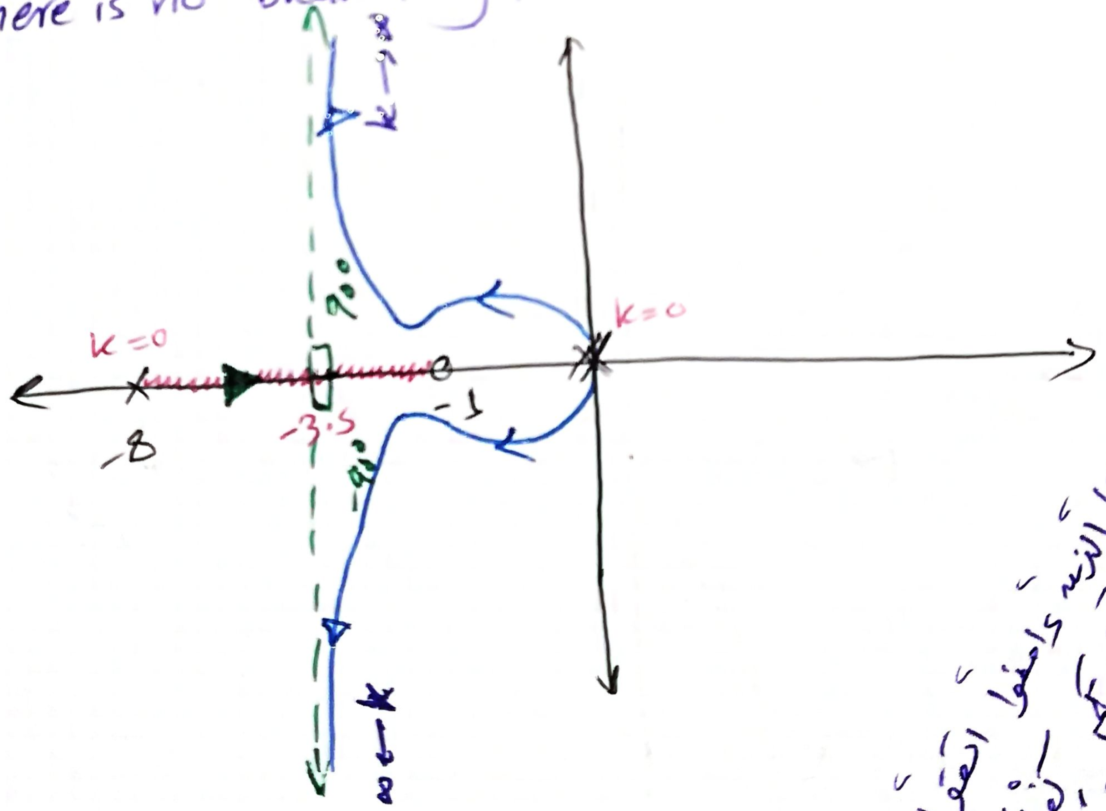
$$\theta_m = \frac{(\pm 1)(180)}{2} = \pm 90^\circ$$

Break away point :- $\frac{d}{ds}\left(\frac{s+1}{s^2(s+8)}\right) = 0$

$$\frac{-(2s^2 + 11s + 16)}{s^3(s+8)^2} = 0$$

$s = -2.75 + J0.6614$ ×
$s = -2.75 - J0.6614$ ×

there is no break away point "complex"



يَا أَيُّهَا الَّذِينَ آمَنُوا اتَّقُوا اللَّهَ وَآمِنُوا
بِرَسُولِهِ يُؤْتِكُمْ كِفْلَيْنِ مِن رَّحْمَتِهِ
وَيَجْعَل لَّكُمْ نُورًا تَمْشُونَ بِهِ
وَيَغْفِرْ لَكُمْ وَاللَّهُ غَفُورٌ
رَّحِيمٌ
سورة الحديد

(iv) $a = 3$, $b = 1$

$$G(s)H(s) = \frac{K(s+1)}{s^2(s+3)}$$

poles: $s = 0$, $s = 0$, $s = -3$

Zeros: $s = -1$

$\alpha = 3 - 1 = 2$, $\delta_m = \frac{-3-(-1)}{2} = \boxed{-1}$

$$\theta_m = \frac{(\pm 1)(180)}{2} = \pm 90^\circ$$

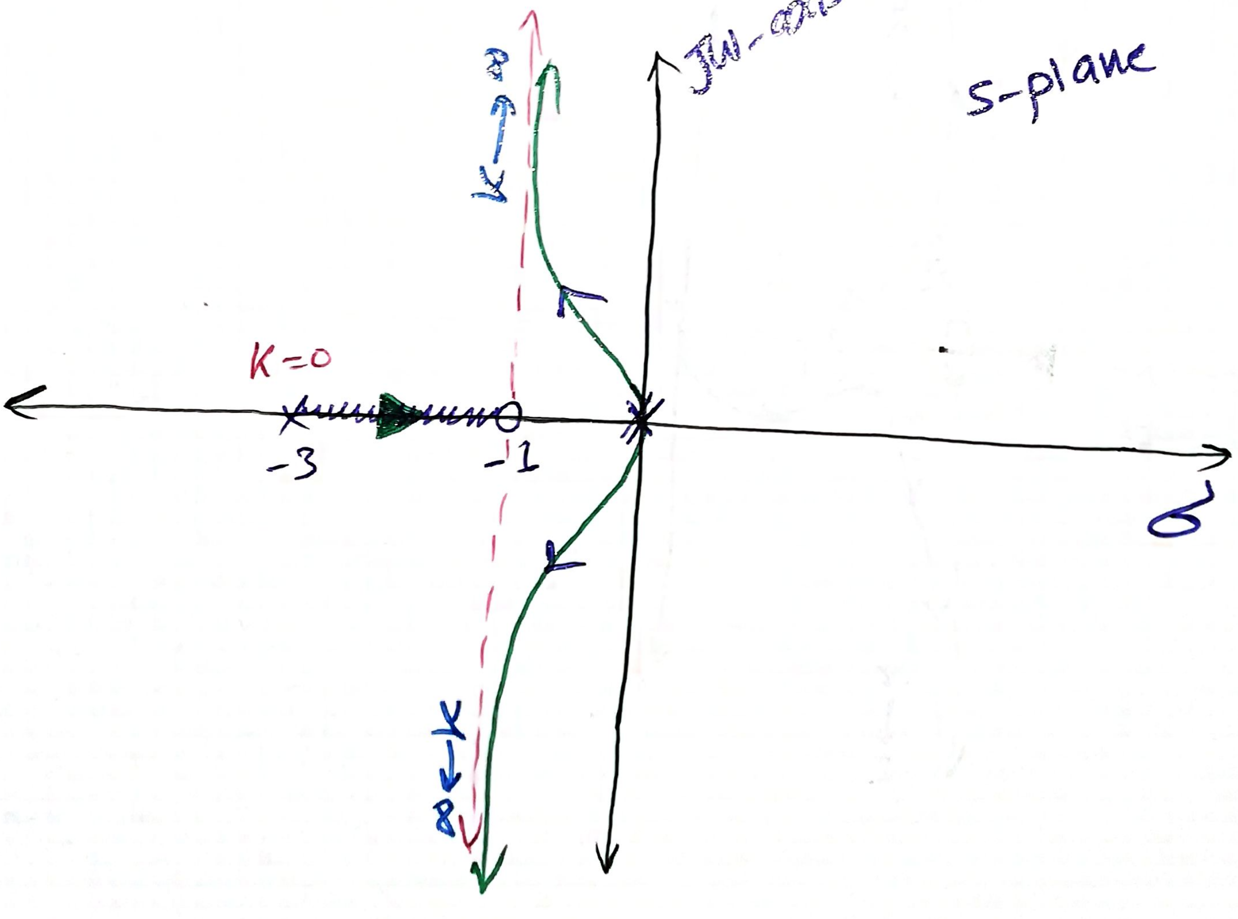
Break away point: $\frac{d}{ds}\left(\frac{s+1}{s^2(s+3)}\right) = 0$

$$-\frac{(2s^2+6s+6)}{s^3(s+3)^2} = 0$$

$s = -1.5 + j0.866$ ✗

$s = -1.5 - j0.866$ ✗

there is no break away point



v) $b = 1$, $a = 1$

$$K\,G(s)\,H(s) = \frac{K\,\cancel{(s+1)}}{s^2\,\cancel{(s+1)}}$$

The poles at $s = -1$ and Zeros at $s = -1$ Cancle each other out.

Poles : $s = 0$, $s = 0$

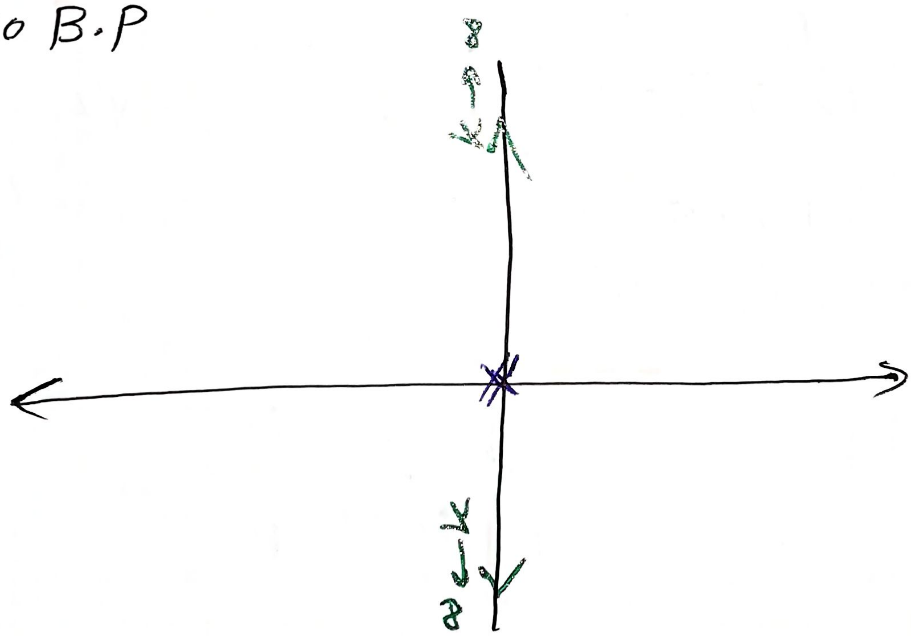
Zeros : ϕ

$\alpha = 2$, $\delta_m = 0$

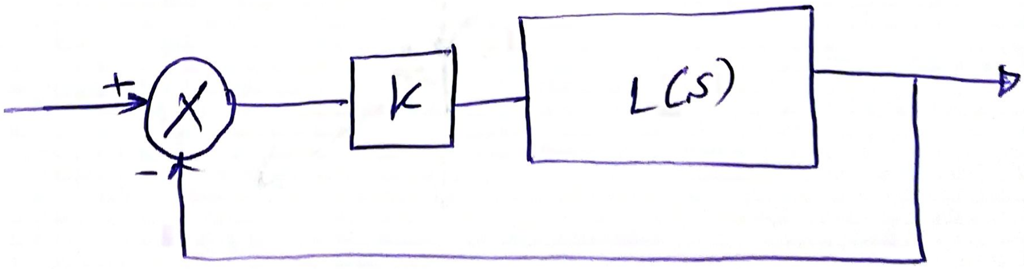
$$\theta_m = \frac{(\pm 1)(180^\circ)}{2} = \pm 90^\circ$$

$$B.P \Rightarrow \frac{d}{ds}\left(\frac{1}{s^2}\right) = -\frac{3}{s^4} \neq 0$$

No B.P



Example # 1 Find R-L and find K for critical stability.



$$L(s) = \frac{1}{s(s+1)(s+5)}$$

$$KG(s)H(s) = \frac{K}{s(s+1)(s+5)}$$

Poles : $s = 0$, $s = -1$, $s = -5$

Zeros : ϕ " لا يوجد "

$\alpha = 3 - 0 = \boxed{3}$, $\delta_m = \frac{(-5-1-0)-0}{3} = \boxed{-2}$

$$\theta_m = (\pm 1, +3)\frac{(180)}{3} = \pm 60°, 180°$$

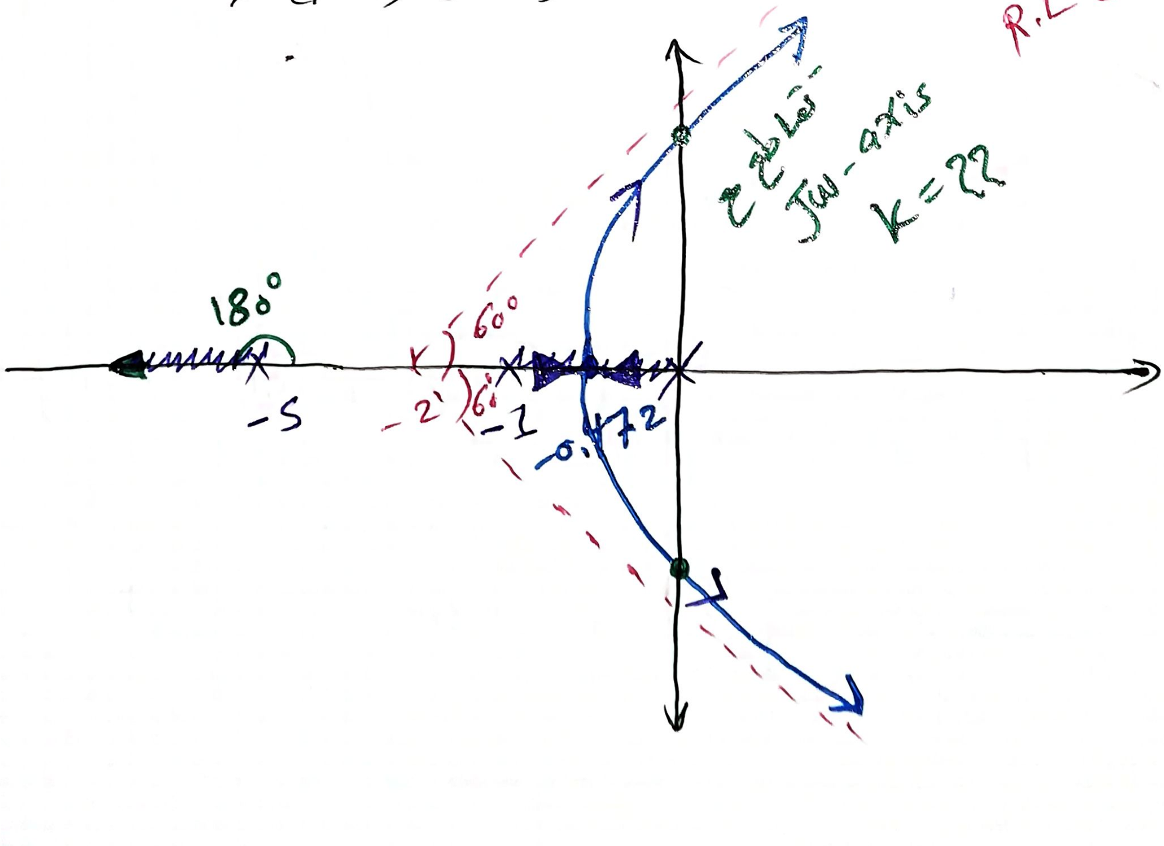
Break away point : $\frac{d}{ds}\left(G(s).H(s)\right) = 0$

$$\frac{d}{ds}\left(\frac{1}{s(s+1)(s+5)}\right) = 0$$

$$\frac{-(3s^2+12s+5)}{s^2(s+1)^2(s+5)^2} = 0$$

$s = -0.472$ ✓

$s = -3.527$ ✗ مرفوضة لا تقع على R.L



characteristic equation #

$$1 + kG(s)H(s) = 0$$

$$1 + \frac{k}{s(s+1)(s+5)} = 0 \Rightarrow s^3 + 6s^2 + 5s + k = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 6 & k \\ s^1 & \frac{30-k}{6} & 0 \\ s^0 & k & 0 \end{array}$$

system stable for:

$\rightarrow \frac{30-k}{6} > 0$

$\rightarrow k > 0$

$$\boxed{0 < k < 30}$$ stability condition #

⇐ المطلوب حساب قيمة $\underline{\underline{K}}$ لما ال R.L بتقطع JW-axis

تكون margin يعني poles ع JW-axis لما تكون من عندي صف كامل أصفار

$$\frac{30-k}{6} = 0 \Rightarrow \boxed{k = 30}$$

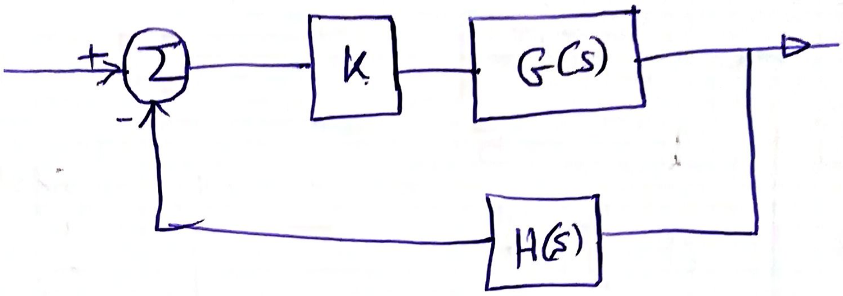
$$6s^2 + k = 0 \Rightarrow 6s^2 + 30 = 0$$

$$\boxed{s = \pm J2.236} \quad \#$$

$$\underline{\underline{s = \pm J\sqrt{5}}}$$

Example # Find R-L, check if the R-L Cross the Img. axes.

$G(s) = \frac{s+1}{s^2(s+4)}$, $H(s) = 1$



Sol :-

$KG(s)H(s) \Rightarrow \frac{K(s+1)}{s^2(s+4)}$

Poles : $s=0, s=0, s=-4$

Zeros : $s=-1$

$\alpha = 3-1 = \boxed{2}$, $\sigma_m = \frac{-4-1}{2} = \boxed{-2.5}$

$\theta_m = \frac{(\pm1)(180)}{2} = \pm 90°$

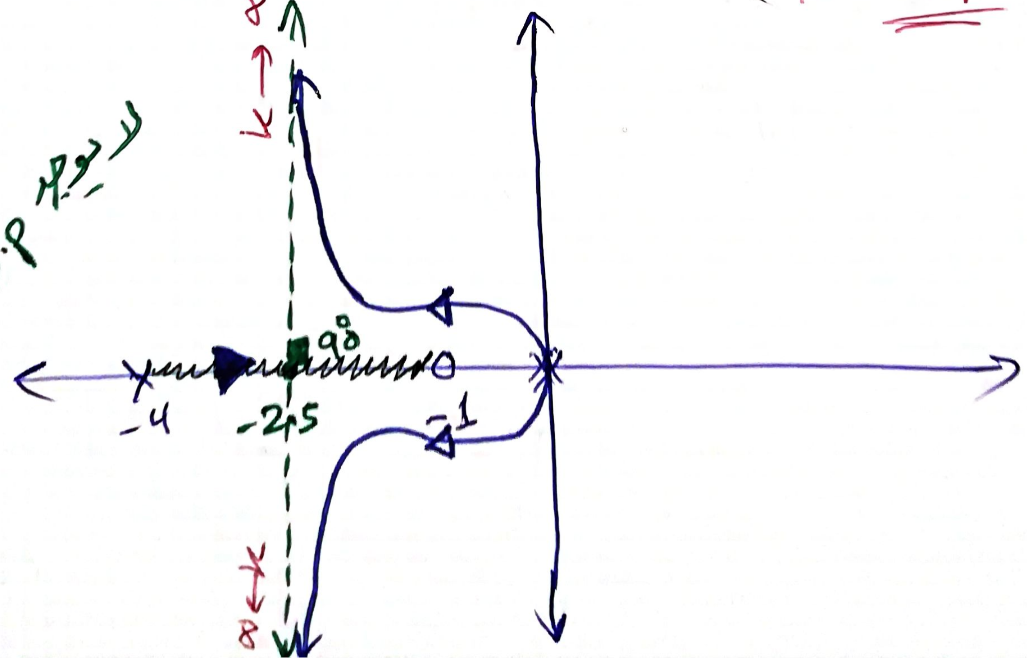
$B.P \Rightarrow \frac{d}{ds}\left(\frac{(s+1)}{s^2(s+4)}\right) \Rightarrow \frac{-(2s^2+7s+8)}{s^3(s+4)^2} = 0$

$-(2s^2+7s+8) = 0$

$s = -1.75 + J0.9682$ ✗

$s = -1.75 - J0.9682$ ✗

No B.P مرفوض



B.P لا يوجد

Example # 1 sketch the R.L with respect to k for the following system and given the imaginary axis crossing if any

ⓐ $kG(s)=\frac{k(s^2+2s+12)}{s(s^2+2s+10)}=\frac{k(s+1+J3.316)(s+1-J3.316)}{s(s+1+J3)(s+1-J3)}$

poles : $s=-1-J3$, $s=-1+J3$, $s=0$

Zeros : $s=-1-J3.316$, $s=-1+J3.316$

$\alpha = 3-2=\boxed{1}$

$\delta_m=\frac{(-1\cancel{-J3}\;-1\cancel{+J3})-(-1\cancel{-J3.31}\;-1\cancel{+J3.31})}{1}=\boxed{0}$

$\theta_m=\frac{(+1)(180)}{1}=\boxed{180^\circ}$

$B.P=\frac{d}{ds}\left(\frac{s^2+2s+12}{s(s^2+2s+10)}\right)=\frac{-(s^4+4s^3+30s^2+48s+120)}{s^2(s^2+2s+10)^2(s^2+2s+10)^2}=0$

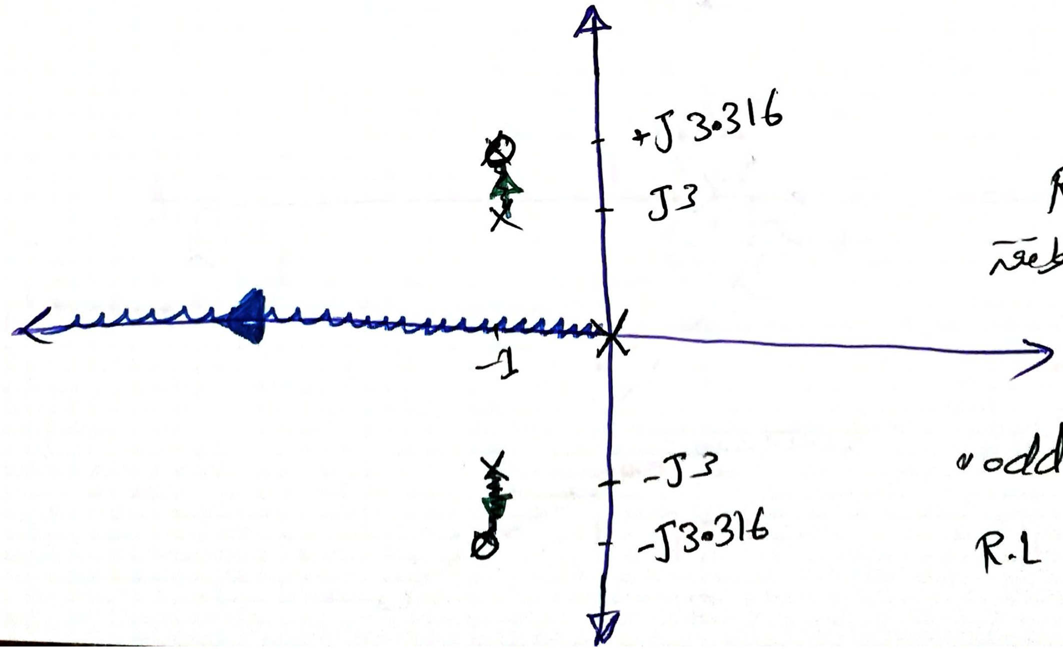
$-(s^4+4s^3+30s^2+48s+120)=0$

$s=-0.852-J2.2882$ × مرفوضة

$s=-0.852+J2.2882$ × مرفوضة

$s=-1.147+J4.3367$ × مرفوضة

$s=-1.147-J4.3367$ × مرفوضة



عشان نحدد ال R.L

بنعد ال poles الطعقة

ع Real axis

وعددها $\underline{\underline{1}}$ يعني odd

وبعد odd في R.L

b) $G(s) = \dfrac{1}{s(s+1\pm J3)}$

$$KG(s)H(s) = \frac{K}{s(s+1-J3)(s+1+J3)}$$

poles : $s=0$, $s=-1+J3$, $s=-1-J3$

Zeros : ϕ

$\alpha = 3-0 = \boxed{3}$, $\sigma_m = \dfrac{(-1+\cancel{J3}-1-\cancel{J3})-0}{3} = \boxed{\dfrac{-2}{3}}$

$$\theta_m = (\pm 1, +3)\frac{(180)}{3} = \pm 60^\circ, 180^\circ$$

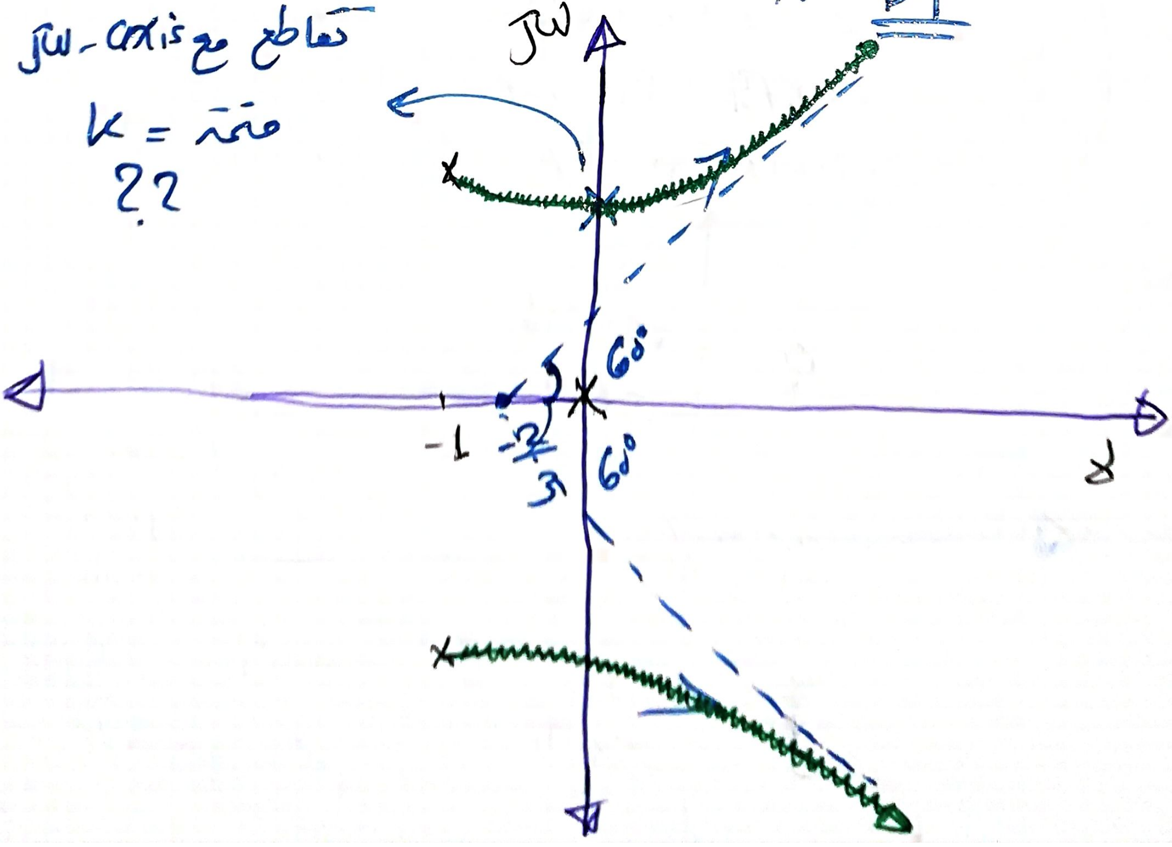
Breakaway point : $\dfrac{d}{ds}\left(\dfrac{1}{s(s\pm 1\pm J3)}\right)$

$$\frac{d}{ds}\left(\frac{1}{s(s^2+2s+10)}\right) = \frac{-(3s^2+4s+10)}{s^2(s^2+2s+10)^2} = 0$$

$$-(3s^2+4s+10)=0 \quad \begin{cases} s = -0.666 - J1.6996 \\ s = -0.666 + J1.6996 \end{cases}$$

× مرفوضة
× مرفوضة

No B.P



تقاطع مع jw-axis
K = قيمة
??

characteristic equation:-

$s^3 + 2s^2 + 10s + k = 0$

s^3	1	10
s^2	2	k
s^1	$\frac{20-k}{2}$	0
s^0	k	

system stable for:-

→ $\frac{20-k}{2} > 0 \Rightarrow \boxed{20 > k}$

→ $k > 0$

Range of stability

$\boxed{0 < k < 20}$

system margin and cross JW-axis when

« صف كامل أصفار »

$\frac{20-k}{2} = 0 \Rightarrow \boxed{k = 20}$

، لحساب قيمة s اللي عندها بصير تقاطع مع JW-axis

بناخذ الصف اللي قبل

$2s^2 + k = 0$

$2s^2 + 20 = 0 \Rightarrow s = \pm J\,3.1622$

or $\underline{\underline{s = \pm\sqrt{10}}}$

The value of k will be:-

$1 + kG(s)H(s) = 0$

$k = \frac{-1}{G(s)(1)} \Rightarrow k = \frac{-1}{\frac{1}{s(s^2+2s+10)}} \Big|_{s=\pm J\sqrt{10}}$

$\boxed{k = 20}$ #

c) $KG(s) = \dfrac{K}{s(s^2+4s+5)}$

$$KG(s)H(s) = \frac{K}{s(s^2+4s+5)} = \frac{K}{(s+2+J)(s+2-J)}$$

poles : $s=0$, $s=-2-J$, $s=-2+J$

Zeros : ϕ

$\alpha = 3-0 = \boxed{3}$

$$\sigma_m = \frac{(-2-\cancel{J}-2+\cancel{J})-0}{3} = \boxed{\frac{-4}{3}}$$

$$\theta_m = \frac{(\pm 1, 3)(180)}{3} = \pm 60^\circ, 180^\circ$$

$$B.P \Rightarrow \frac{d}{ds}\left(\frac{1}{s(s^2+4s+5)}\right) = 0$$

$$\frac{-(3s^2+8s+5)}{s^2(s^2+4s+5)^2} = 0 \qquad \begin{cases} s = -1 \\ s = -1.666 \end{cases}$$

$$1 + KG(s)H(s) = 0$$

$$1 + \frac{K}{s(s^2+4s+5)} = 0 \quad \Rightarrow \quad s^3 + 4s^2 + 5s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 4 & K \\ s^1 & \frac{20-K}{4} & 0 \\ s^0 & K & \end{array}$$

system stable for

$$\begin{cases} \frac{20-K}{4} > 0 \Rightarrow K < 20 \\ K > 0 \end{cases}$$

$$0 < K < 20$$

$K = 20$ make rows of Zero so, system is marginal

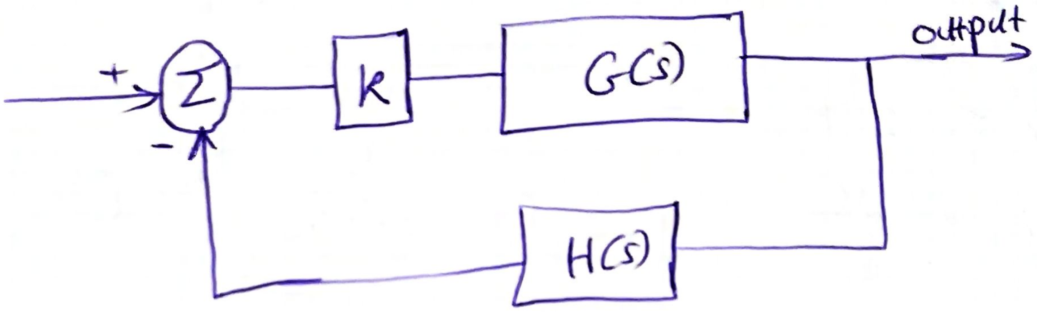
$4s^2 + 20 = 0$

$s_{1,2} \Rightarrow ??$ $\qquad s = \pm J\sqrt{5}$ #

Example #1 $\quad G(s)=\dfrac{10}{s^2+3s+7}$, $H(s)=1$

Find R-L and the value of k that satisfy the design

$O.S\% < 20\%$, $t_s \leq 2.7$ sec



Sol:-

$$KG(s)H(s)=\frac{10k}{s^2+3s+7}$$

Poles: $s=-1.5+J2.1794$, $s=-1.5-J2.1794$

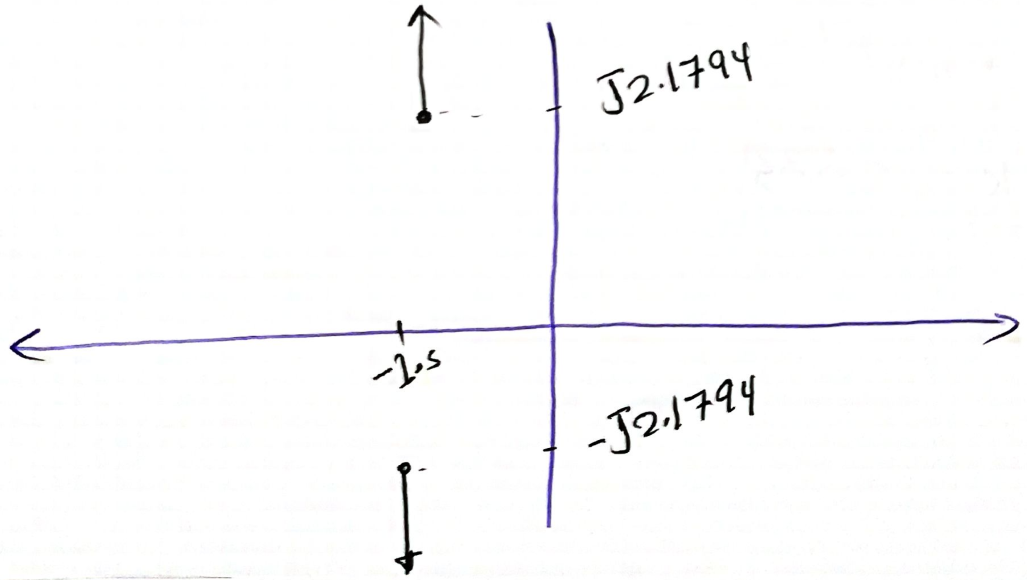
Zeros: ϕ

$\alpha = 2-0 = \boxed{2}$, $\sigma_m = \dfrac{-1.5+J2.1794-1.5-J2.1794-0}{2}$

$\boxed{\sigma_m = -1.5}$, $\theta_m = \dfrac{(\pm 1)(180)}{2} = \boxed{\pm 90^\circ}$

$$B.P = \frac{d}{ds}\left(\frac{10}{s^2+3s+7}\right) = 0 \Rightarrow \frac{-10(2s+3)}{(s^2+3s+7)^2} = 0$$

$$-10(2s+3)=0 \Rightarrow \boxed{s=\frac{-3}{2}}$$



$O.S = 20\%$

$$\xi = \frac{-\ln\left(\frac{0.5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{0.5\%}{100}\right)^2}} = \frac{-\ln\left(\frac{20\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{20\%}{100}\right)^2}} = \boxed{0.45}$$

$t_s \leq 2.7$

$$t_s = \frac{4}{\xi \omega_n} \Rightarrow 2.7 = \frac{4}{(0.45)(\omega_n)} \Rightarrow \boxed{\omega_n = 3.29}$$

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$= (-0.456)(3.29) \pm 3.29\sqrt{(0.45)^2 - 1}$$

$$s_{1,2} = -1.5 \pm J2.93$$

The value of k will be:

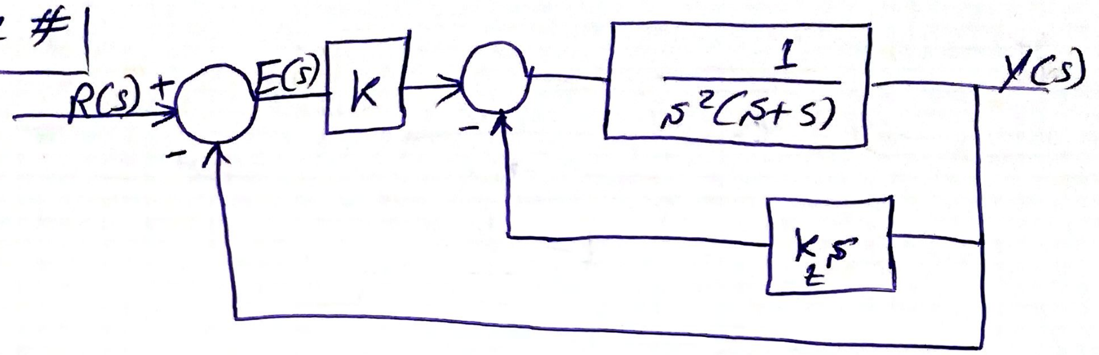
$$1 + kG(s)H(s) = 0$$

$$k = -\frac{1}{G(s).1}\Bigg|_{s=s_0}$$

$$k = \frac{-1}{\left(\frac{10}{s^2+3s+7}\right)}\Bigg|_{s=-1.5\pm J2.93}$$

$$\boxed{k = 0.383}$$ #

Example #1



a) set $K_t = 0$, Draw R.L for $K > 0$

b) set ~~K_t~~ $= 10$, Draw R.L for $K_t > 0$

c) set $K = 5$, Draw R.L for $K_t > 0$

sol :

at $K_t = 0$

open Loop $\Rightarrow$ $KG(s)H(s) = \frac{K}{s^2(s+5)}$

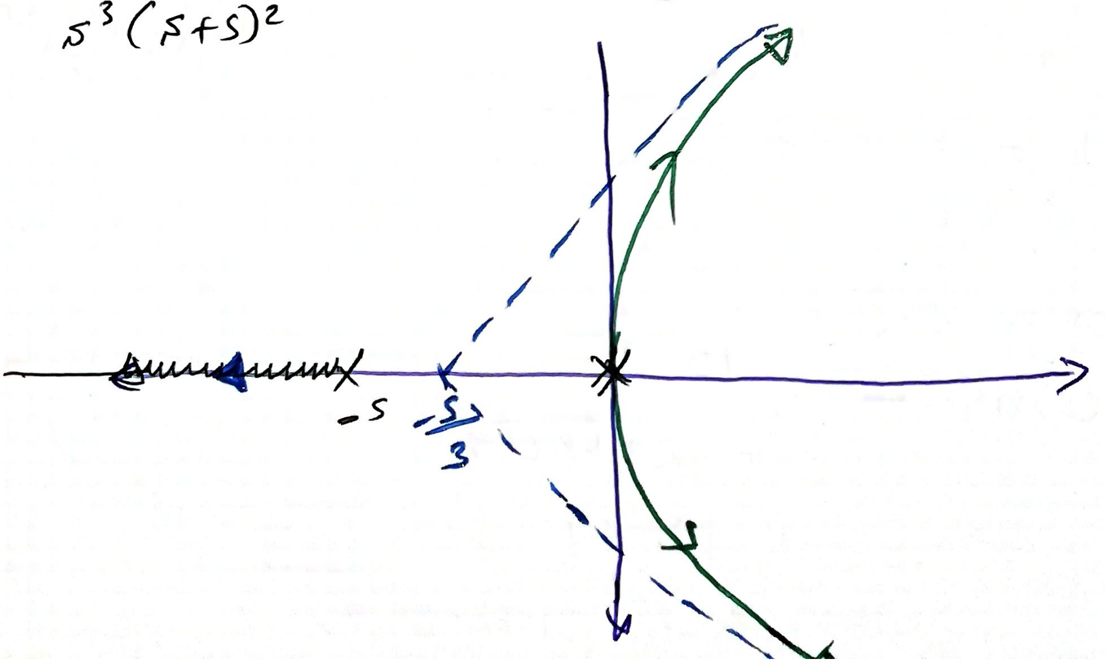
poles : $s = 0, s = 0, s = -5$

Zeros : ϕ

$\alpha = 3$, $\delta_m = \frac{-5}{3}$, $\theta_m = \frac{(\pm 1, 3)(180)}{3} = \pm 60°, 180°$

B.p $\Rightarrow$ $\frac{d}{ds}\left(\frac{1}{s^2(s+5)}\right) = 0$

$\frac{-(3s+10)}{s^3(s+5)^2} = 0$ $\Rightarrow$ $\boxed{s = \frac{-10}{3}}$ مرفوضة لاتقع R.L



b) at $K = 10$

$$KG(s)H(s) \Rightarrow K\cdot\left(\frac{\frac{1}{s^2(s+5)}}{1+\frac{K_t s}{s^2(s+5)}}\right) \Rightarrow \frac{K}{s^2(s+5)+K_t s}$$

open loop $\Rightarrow KG(s)H(s) = \dfrac{10}{s^2(s+5)+K_t s}$

charactristic equation // $\quad 1+KG(s)H(s) = 0$

$$\Rightarrow 1+\frac{10}{s^2(s+5)+K_t s} = 0$$

$$s^3+5s^2+K_t s+10 = 0 \Longrightarrow$$

$$\frac{1}{\cancel{s^3+5s^2+10}} \cdot \frac{\cancel{s^3+5s^2+10}}{\cancel{s^3+5s^2+10}} + \frac{K_t s}{s^3+5s^2+10} = 0$$

$$\boxed{1+K_t\frac{s}{s^3+5s^2+10} = 0}$$

By using Routh Hurwitz

s^3	1	K_t
s^2	5	10
s^1	K_t-2	
s^0	10	

system stable for

$K_t - 2 > 0$

$\underline{\underline{K_t > 2}}$

open loop $K_t\dfrac{s}{s^3+5s^2+10}$

when $K_t = 2 \Rightarrow$ system margin.

$5s^2+10 = 0 \Rightarrow \boxed{s = \pm j\sqrt{2}}$

$\Rightarrow$ open loop :

$$KG(s)H(s) = \frac{s}{s^3+5s^2+10}$$

قيمة $\underline{\underline{K_t>2}}$ راح نشوف ال R.L حسب قيمة K_t

و ال stability تبع ال system

open loop :- $KG(s)H(s) = \frac{s}{s^3+5s^2+10}$

Poles : $s = 0.174 + J1.356$, $s = 0.174 - J1.356$, $s = -5.349$

Zeros : $s = 0$

$\alpha = 3-1 = \boxed{2}$, $\delta_m = \frac{0.174 + \cancel{J1.356} + 0.174 - \cancel{J1.356} - 5.349}{2}$

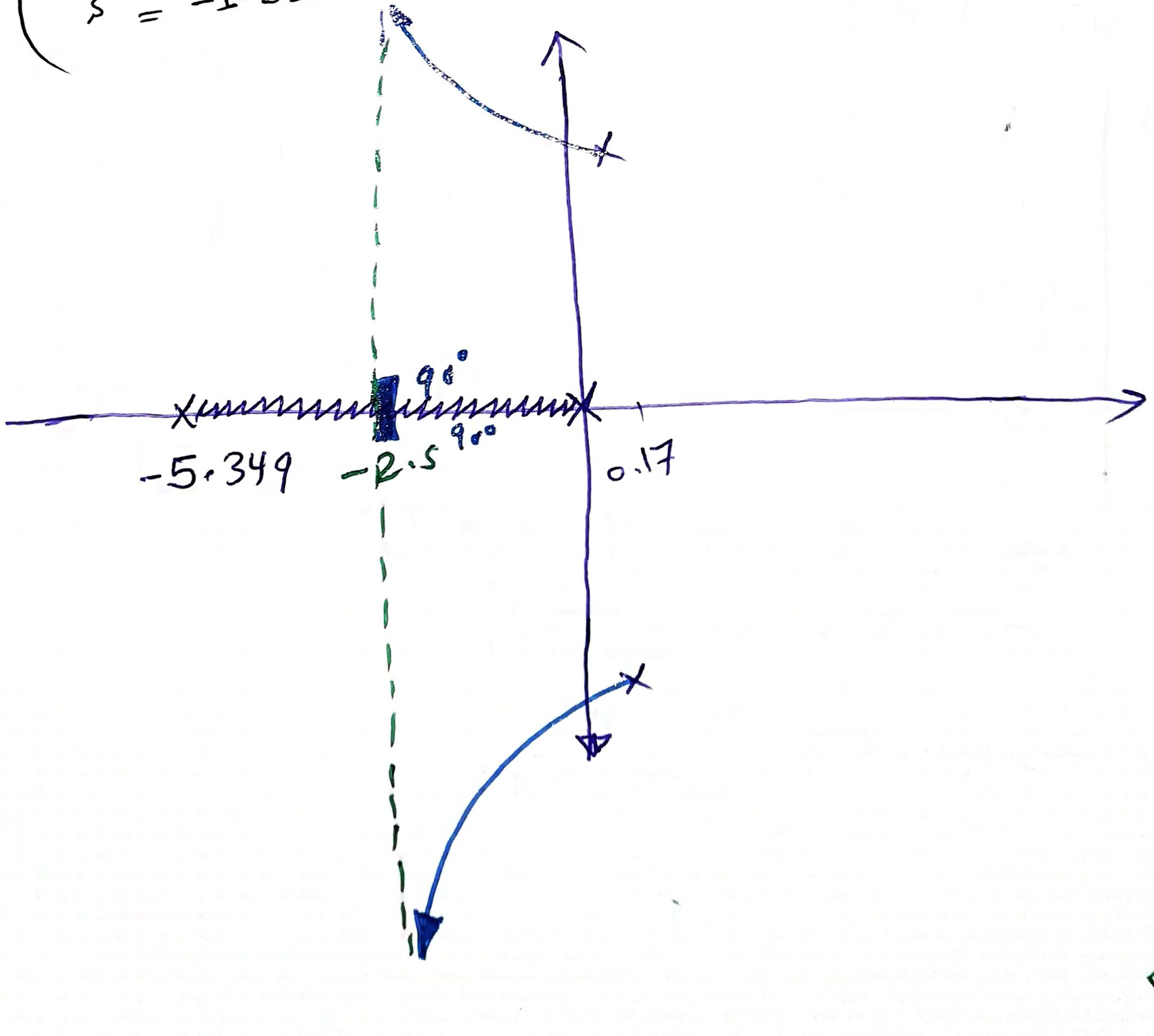
$\boxed{\delta_m = -2.5}$

$\theta_m = \frac{(\pm 1)(180)}{2} = \boxed{\pm 90^\circ}$

$B.P = \frac{d}{ds}\left(\frac{s}{s^3+5s^2+10}\right) \Rightarrow \frac{-(2s^3+5s^2-10)}{(s^3+5s^2+10)^2} = 0$

$s = 1.1675$ ×
$s = -1.833 + J0.9588$ ×
$s = -1.833 - J0.9588$ ×

مرفوضين
No B.P



at $K = 5$

$$KG(s)H(s) = K\cdot\left(\frac{\frac{1}{s^2(s+5)}}{1+\frac{K_t s}{s^2(s+5)}}\right) \Rightarrow \frac{K}{s^2(s+5)+K_t s}$$

open Loop $\Rightarrow KG(s)H(s) = \dfrac{5}{s^2(s+5)+K_t s}$

charactrstic equation : $1+KG(s)H(s)=0$

$$1+\frac{5}{s^2(s+5)+K_t s}=0$$

$$s^3+5s^2+K_t s+5=0 \Rightarrow \frac{s^3+5s^2+5}{s^3+5s^2+5}+\frac{K_t s}{s^3+5s^2+5}=0$$

$$\Rightarrow 1+K_t\underbrace{G(s)H(s)}_{\text{open Loop}}=0$$

By using R.H //

s^3	1	K_t
s^2	5	5
s^1	K_t-1	
s^0	5	

system stable for:

$K_t - 1 > 0$

$K_t > 1$

When $K_t = 1 \Rightarrow$ system margin.

$5s^2+5=0 \Rightarrow \boxed{s=\pm J}$

$\Rightarrow$ open Loop: $\dfrac{s}{s^3+5s^2+5}$

poles: $s = 0.0929 - J0.9775$, $s = 0.0929 + J0.9775$, $s = -5.18$

Zeros: $s = 0$

$\underline{\underline{\alpha = 2}}$

$$\alpha_m = \frac{0.0929 - J0.9775 + 0.0929 + J0.9775 - 5.18}{2} = \boxed{-2.5}$$

$$\theta_m = \frac{(\pm 1)(180)}{2} = \pm 90^\circ$$

$$B.P \Rightarrow \frac{d}{ds}\left(\frac{s}{s^3 + 5s^2 + 5}\right) = 0$$

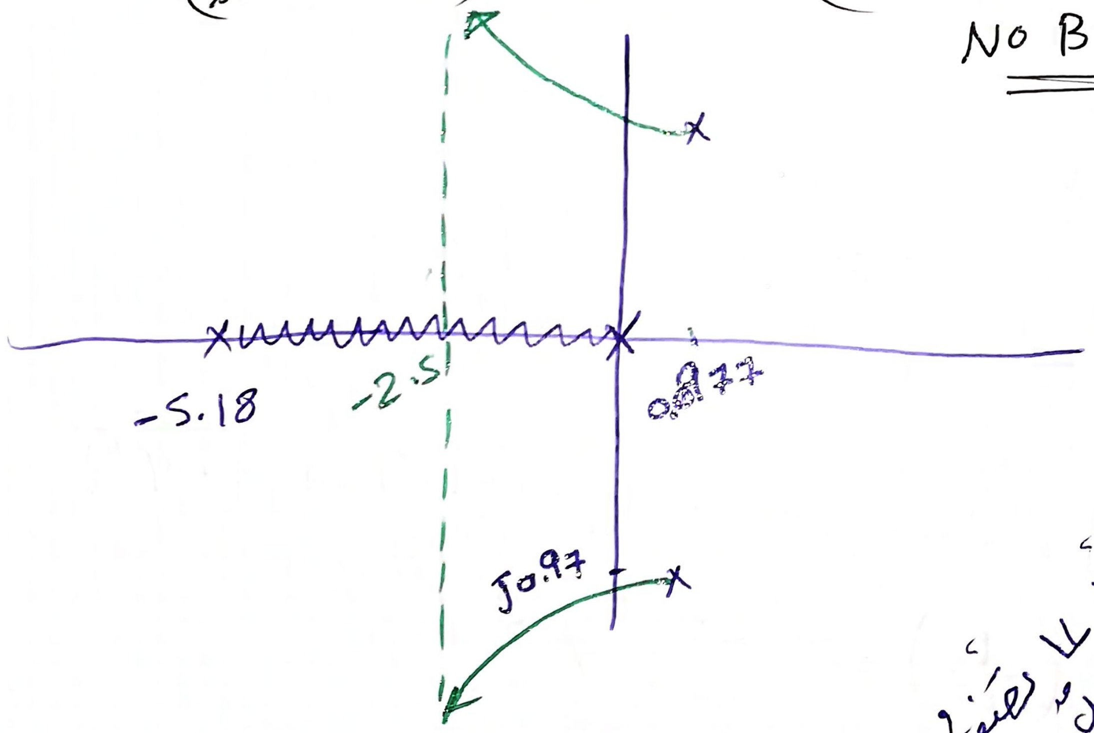
$$-\frac{(2s^3 + 5s^2 - s)}{(s^3 + 5s^2 + 5)^2} = 0$$

$s = 0.8622$ ×

$s = -1.681 - J0.2702$ ×

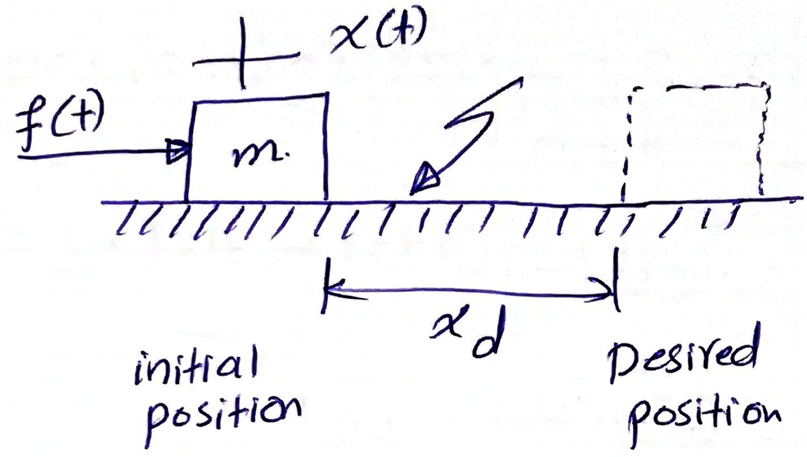
$s = -1.681 + J0.2702$ ×

No B.P

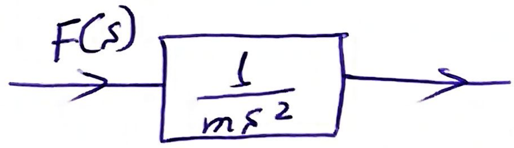


ولا تقولوا لما تصفُ ألسنتكم الكذبَ
هذا حلالٌ وهذا حرامٌ لتفتروا
ع الله الكذبَ "
سورة النحل -

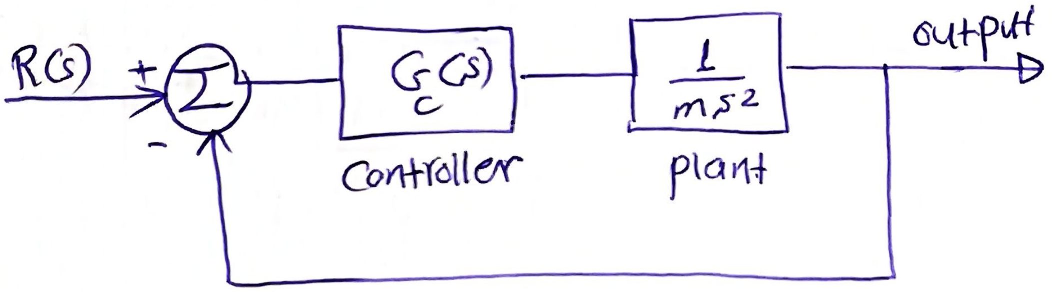
Example # 1



The plant



Feedback control system:-



$G_c(s) = K$, $m = 1$

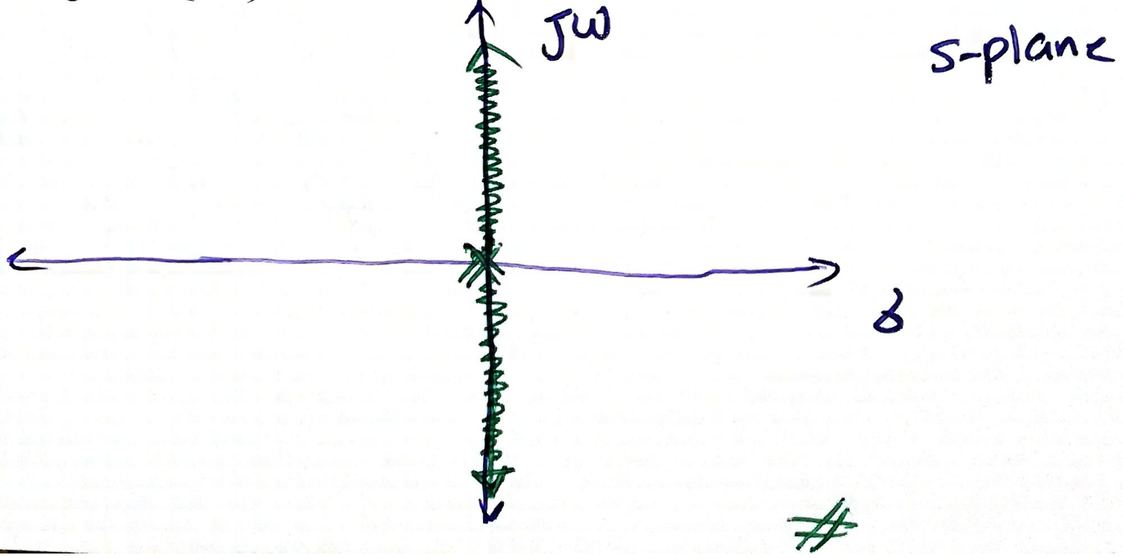
open Loop ⇒ $G_c(s) \cdot \frac{1}{ms^2}$ ⇒ $\boxed{\frac{K}{s^2}}$

poles: $s = 0$, $s = 0$

Zeros: ϕ

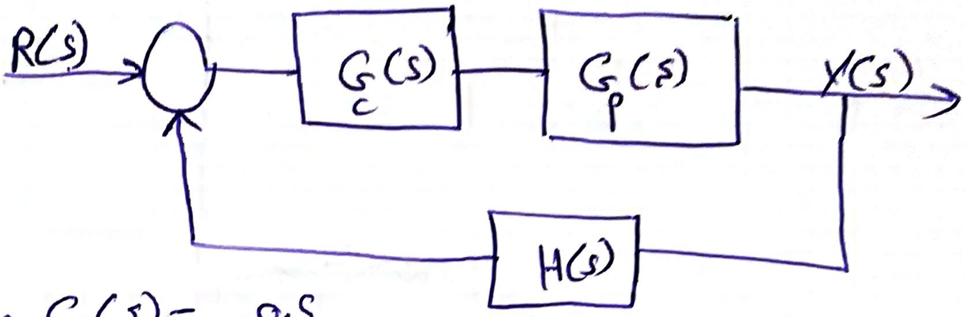
$\boxed{\alpha = 2}$, $\boxed{\sigma_m = 0}$, $\theta_m = \frac{(\pm 1)(180)}{2} = \boxed{\pm 90^\circ}$

B.P ⇒ $\frac{d}{ds}\left(\frac{1}{s^2}\right) = \frac{-2}{s^3} \neq 0$



#

Example #1



$G_c(s) = k(s+3)$, $G_p(s) = \frac{0.5}{s}$, $H(s) = \frac{1}{(s-1)}$

a) sketch the RootLocus With respect to k.

b) find the value of k with Locations of poles With R-L crosses the imaginary axis, if any?

c) Find the range of k, k>0 for which the root of the system characteristic equation are all real.

d) show that if the Controller can satisfy the following design specification, O.S% ≤ 4.32%, $T_s \leq 2.3$ sec

sol:

open Loop function: $kG_c(s)\,G_p(s)\,H(s) = k(s+3)\,\frac{(0.5)}{s} \cdot \frac{1}{(s-1)}$

poles: $s=0$, $s=1$

Zeros: $s=-3$

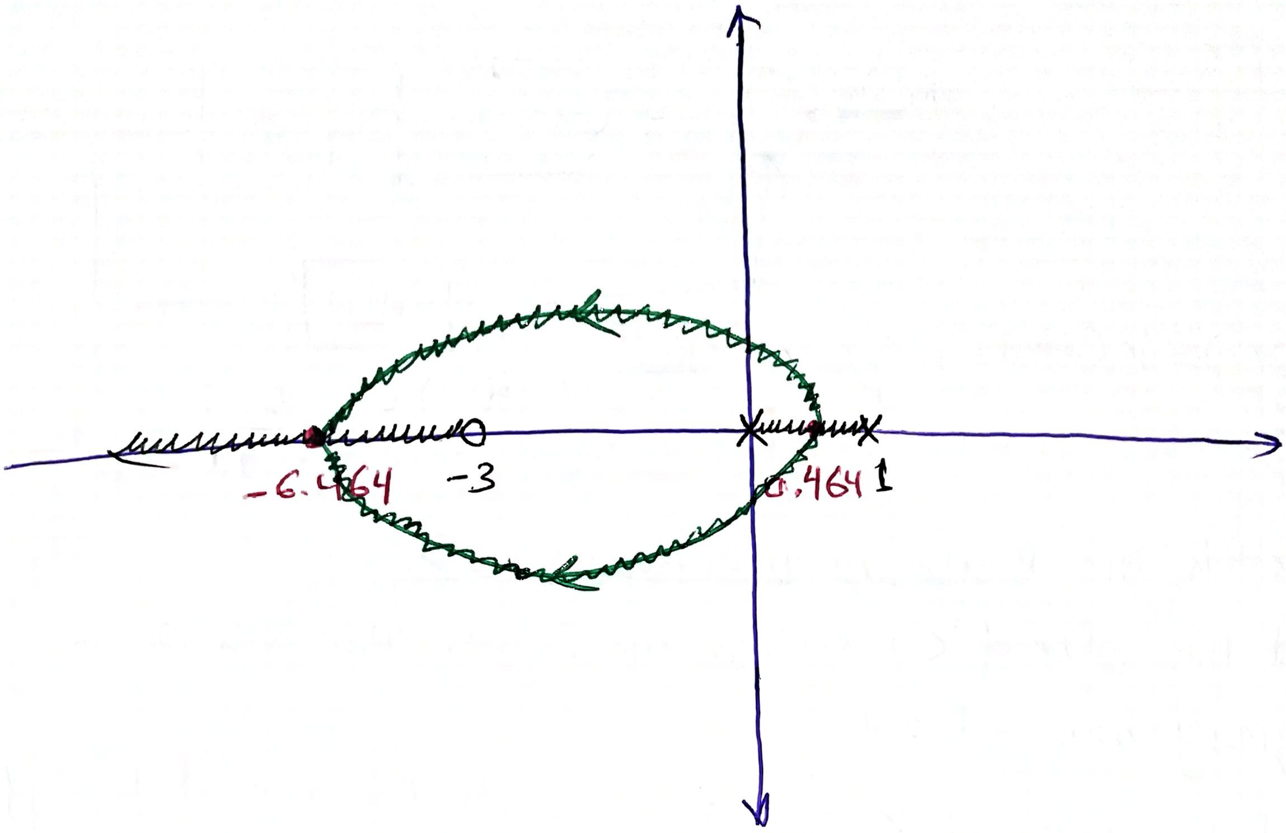
$\alpha = 2-1 = \boxed{1}$, $\delta m = \frac{1-(-3)}{1} = \boxed{4}$

$\theta_m = \frac{(+1)(180)}{1} = \boxed{+180^\circ}$

$B.p = \frac{d}{ds}\left(\frac{s+3}{2s(s-1)}\right) \Rightarrow \frac{-(s^2+6s-3)}{2s^2(s-1)^2} = 0$

$\Rightarrow \begin{cases} s = 0.4641 \\ s = -6.464 \end{cases}$ ✓ تقع ع R.L

✓ تقع على R.L



b) $1+KG(s)H(s)=0$

$$1+\frac{K(s+3)}{2s(s-1)}=0 \Rightarrow$$

$$2s^2-2s+ks+3k=0 \Rightarrow s^2-s+0.5ks+1.5k=0$$

$$\begin{array}{c|cc} s^2 & 1 & sk \\ s^1 & -1+\frac{1}{2}k & 0 \\ s^0 & 1.5k & \end{array}$$

system stable for:

$-1+\frac{1}{2}k>0 \Rightarrow k>2$

$1.5k>0 \Rightarrow k>0$

$\boxed{\text{at } k=2}$ ⇒ system is marginal

$s^2+1.5k=0 \Rightarrow s^2=-3$

$\boxed{s_{1,2}=\pm J\sqrt{3}}$ # at $\underline{k=2}$

$\underline{\underline{\text{or}}}$

$$K=\frac{-1}{G(s)H(s)}\Bigg|_{s=\pm J\sqrt{3}}$$

~k at Jω-axis crossing~

$k=\underline{\underline{2}}$

c) Range of k for the roots of thy system reals

نرجع لل Real poles إلي قاطعة فيه مع $\underline{\underline{B.P}}$ تقع ع R.L

$$s = -6.464$$

$$k = \frac{-1}{G(s)H(s)} \Bigg|_{s=-6.464}$$

$$k = -\frac{1}{\frac{(s+3)}{2s(s-1)}} \Bigg|_{s=-6.464} = \boxed{27.856} \ \#$$

so, at $k \geqslant 27.856$ roots are real #

d) O.S % < 4.32 , $T_s \leqslant 2.3$ sec

$$\varepsilon = \frac{-\ln\left(\frac{O.S\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{O.S\%}{100}\right)^2}} = \frac{-\ln\left(\frac{4.32\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{4.32\%}{100}\right)^2}} = 0.707$$

$$T_s = \frac{4}{\varepsilon \omega_n} \Rightarrow \omega_n = \frac{4}{(2.3)(0.707)} = \boxed{2.459 \text{ rad/sec}}$$

$$s_{1,2} = +\varepsilon\omega_n \pm J\omega_n\sqrt{1-\varepsilon^2}$$

$$s_{1,2} = -1.73 \pm J\,1.739$$

back to the value of marginal poles $s_{1,2} = \pm J\sqrt{3}$

$s_{1,2} = \pm J\,1.73$ at this point real value equal zero

so, there is no K controller can be satisfy these specification.

Notes #1

To test if a point in the s-plane is on the root Locus (given the poles & zeros of open Loop func. KGH)

The angle of Zeros − angles of poles add up to an odd multiple of 180° ($(2n+1)\,180°$)

Example #1 $G(s) = \dfrac{K(s+2)}{(s^2+4s+13)}$ Is the point $(-3+J0)$ on the R.L?

Sol:-

$Z = -2$

$p = s = -2+J3\ ,\ -2-J3$

$\theta_{zeros} = 180°$ “لأنها بالسالب Real”

$\theta_{poles_1} = \tan^{-1}\left(\frac{3}{-2}\right) = \boxed{-56.309°}$

$\theta_{poles_2} = \tan^{-1}\left(\frac{-3}{-2}\right) = \boxed{56.309°}$

$\sum\theta_{zeros} - \sum\theta_{poles} \Rightarrow 180° - (\cancel{-56.309° + 56.309})$

$= 180°$ #

So its on R.L

* for the following closed loop transfer function.

- sketch the Root Locus for the system
- Check if the Root Locus cross the J-axis or not

$$T(s) = \frac{10}{s^2+3s+7+10K}$$

Sol:—

Charactrstic equation $\Rightarrow s^2+3s+7+10K=0$

$$1+KG(s)H(s)=0$$

$$\underset{1}{\frac{\cancel{s^2+3s+7}}{\cancel{s^2+3s+7}}} + \frac{10K}{s^2+3s+7} = \frac{0}{s^2+3s+7}$$

$$1+K\underbrace{\left(\frac{10}{s^2+3s+7}\right)}_{G(s)H(s)}=0$$

open loop : $KG(s)H(s) = \dfrac{K(10)}{s^2+3s+7}$

poles : $s=-1.5+J2.1$, $s=-1.5-J2.1$

Zeros : $\emptyset$

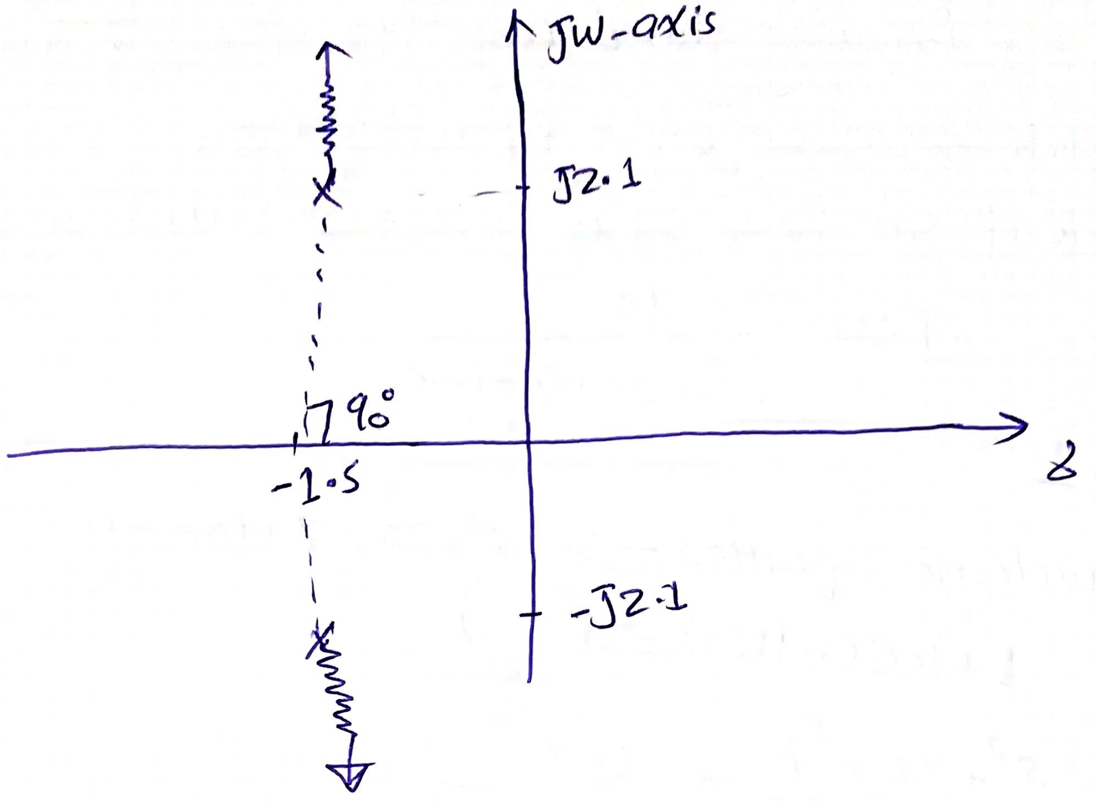
$\boxed{\alpha = 2}$, $\sigma_m = \dfrac{-1.5+\cancel{J2.1}-1.5-\cancel{J2.1}}{2} = \boxed{-1.5}$

$$\theta_m = \frac{(\pm 1)(180^\circ)}{2} = \boxed{\pm 90^\circ}$$

$$B.p \Rightarrow \frac{d}{ds}\left(\frac{10}{s^2+3s+7}\right) = \frac{-10(2s+3)}{(s^2+3s+7)^2} = 0$$

$\boxed{s = -\frac{3}{2}}$ #

" there is no B.p becouse there is no value on Real axis "



$1+kG(s)H(s)=0$

$s^2+3s+7+10k=0$

s^2	1	$7+10k$
s^1	3	0
s^0	$7+10k$	

R.L doesn't cross the jw-axis

system stable for:

$7+10k>0$

$10k>-7$

$k>-\frac{7}{10}$

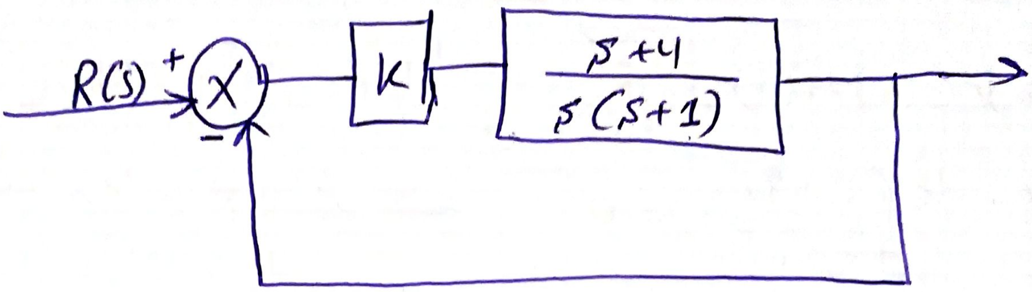
$k>-0.7$

k must be positive

so, system stable for $\boxed{k>0}$

"قل لن يصيبنا إلا ما كتب الله لنا
هو مولانا وعلى الله يتوكل
المؤمنون"
سورة التوبة

Example #1



Find the R.L and the range of K controller.

Sol:-

$$KG(s)H(s) = \frac{K(s+4)}{s(s+1)}$$

Poles: $s=0,\ s=-1$

Zeros: $s=-4$

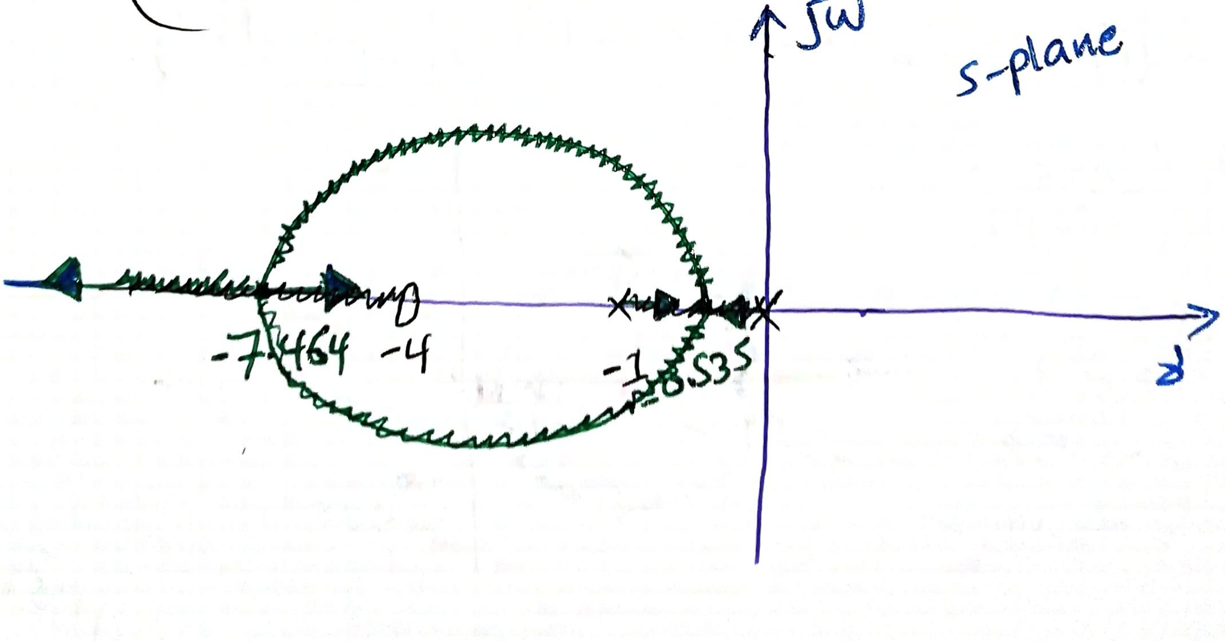
$\alpha = 2-1 = \boxed{1}$, $\delta_m = \frac{-1+4}{1} = \boxed{3}$

$$\theta_m = \frac{(\pm 1)(180)}{1} = 180^\circ$$

$$B.P = \frac{d}{ds}\left(\frac{s+4}{s(s+1)}\right) \Rightarrow \frac{-(s^2+8s+4)}{s^2(s+1)^2} = 0$$

$s = -0.535$ ✓

$s = -7.464$ ✓



to test the Stability :-

$$1 + kG(s)H(s) = 0$$

$$1 + \frac{k(s+4)}{s(s+1)} = 0 \Rightarrow s^2 + s + ks + 4k = 0$$

s^2	1	$4k$
s^1	$1+k$	0
s^0	$4k$	

system stable for

$1 + k > 0$

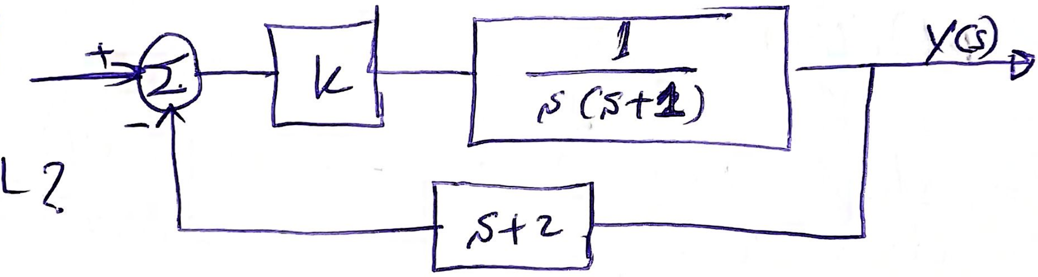
$\Rightarrow k > -1$

for $4k > 0$

$k > 0$

Range of stability $\boxed{k > 0}$

Example #1



sketch R.L?

Sol: $kG(s)H(s) = \frac{k}{s(s+1)}(s+2)$

Zeros: $s = -2$

poles: $s = 0,\ s = -1$

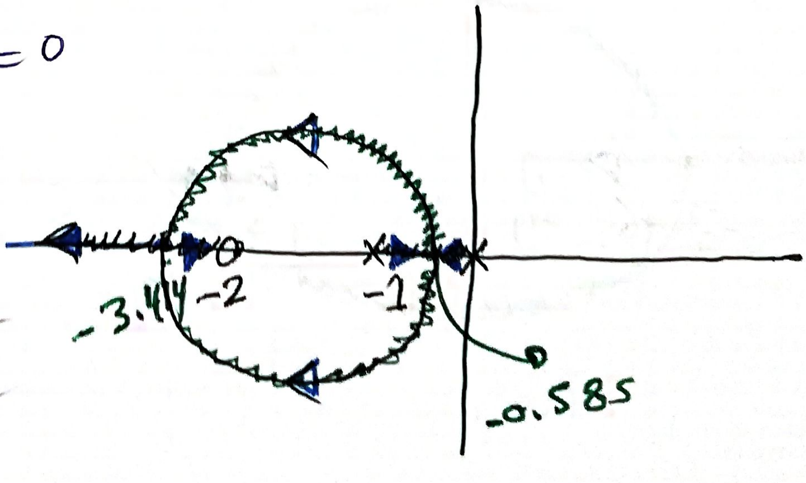
$\alpha = 2-1 = \boxed{1}$, $\delta_m = \frac{-1+2}{1} = \boxed{1}$, $\boxed{\theta_m = 180^\circ}$

$$B.P \Rightarrow \frac{d}{ds}\left(\frac{s+2}{s(s+1)}\right) = 0$$

$$-\frac{(s^2+4s+2)}{s^2(s+1)^2} = 0$$

$s = -0.585$ ✓

$s = -3.414$ ✓



Example # 1 for the following transfer function of plant $G(s)$, do the following :

$$G(s) = \frac{1}{s(s+1)}$$

for the following closed Loop poles which are Located at :

a) $s_o = -1 \pm J\sqrt{3}$

b) $s_o = -\frac{1}{2} \pm J\sqrt{3}$

show that if the specification can be acheived by the controller k or not

$$kG(s)H(s) = \frac{k}{s(s+1)}$$

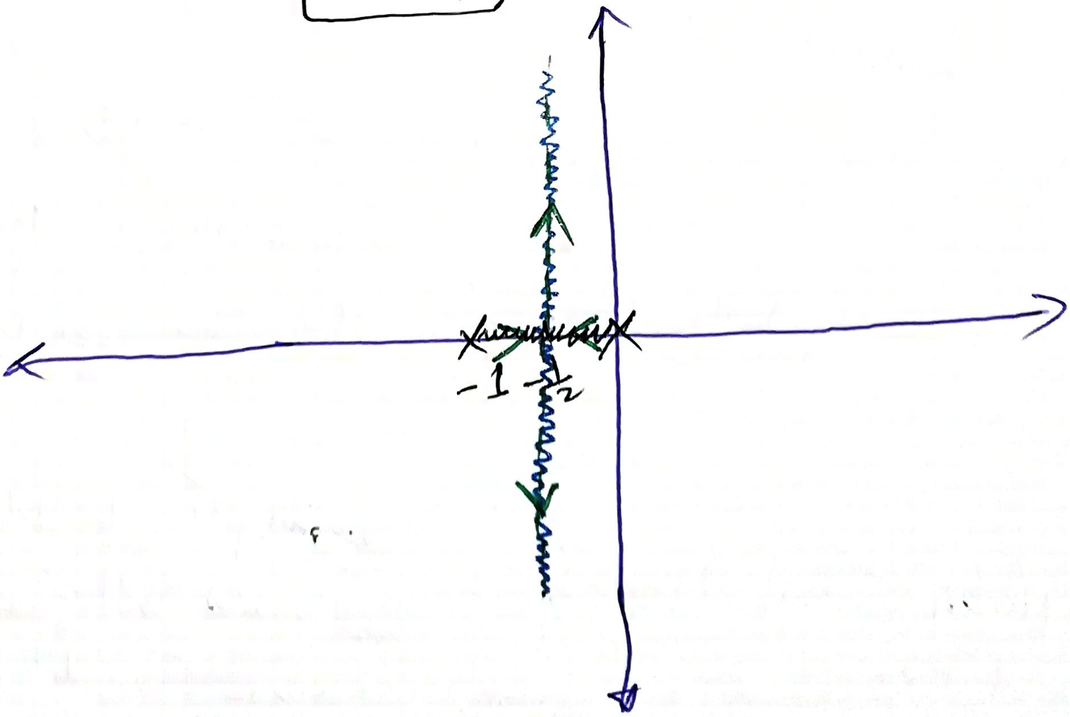
poles : $s = 0$, $s = -1$

Zeros : ϕ

$\boxed{\alpha = 2}$, $\boxed{\delta m = -\frac{1}{2}}$, $\theta m = \frac{(\pm 1)(180)}{2} = \pm 90°$

$$B.P \Rightarrow \frac{d}{ds}\left(\frac{1}{s(s+1)}\right) \Rightarrow \frac{-(2s+1)}{s^2(s+1)^2} = 0$$

$$\Rightarrow \boxed{s = -\frac{1}{2}}$$



for the closed Loop poles, first you need to check if it Lies on RL or not.

a) $S_o = -1 \pm j\sqrt{3}$

this poles doesn't Lie on the R.L, so that the k-controller Can't control the system to achieved this specification

b) $S_o = -\frac{1}{2} \pm j\sqrt{3}$

this poles Lie on the R.L, so you can find the value of k to achieve this specification

$$k = \frac{-1}{G(s)\,H(s)}\Bigg|_{s=s_o} \Rightarrow \frac{-1}{1/\left[\left(-\frac{1}{2}+j\sqrt{3}\right)\left(-\frac{1}{2}+j\sqrt{3}+1\right)\right]}$$

$\boxed{k = 3.25}$ # this is the value of controller

تم بحمد الله

نهاية: CH#8

Linear control system

" دعواهم فيها سبحانك اللهم وتحيتهم فيها سلام وآخر دعواهم أن الحمد لله رب العالمين "

" ربنا تقبل منا إنك أنت السميع العليم "

خالص الأمنيات لكم بالتوفيق
والنجاح في الحياة العملية والعلمية
ووفقنا الله وإياكم لما فيه الخير والصلاح
دمتم بود

Eng. Hadil Jamal Matter

13/5/2020