

* Chapter 2 $\Rightarrow$ Modeling in the Freq domain

* Introduction $\Rightarrow$

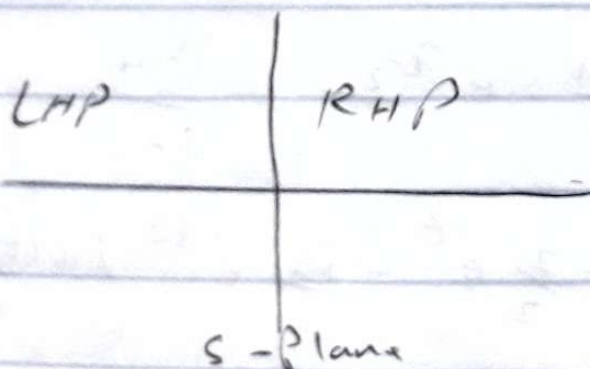
The first step in the design process is to transform the physical system into mathematical modeling to understand system, and design control systems to achieve a particular specification.

.. Modeling :- It's a translator that represent a physical device by its equivalent equation.

* Laplace transform $\Rightarrow$

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt \quad \text{unilateral}$$

$$s = \underbrace{\sigma}_{\text{real}} + j\underbrace{\omega}_{\text{imaginary}}$$



- Stable system when poles lies in LHP.

- To use Laplace must system $\boxed{\text{LTI}}$.

Ex: $x(t) = 3e^{-2t}u(t) + 4e^t u(-t)$

find $x(s)$??

$$X(s) = \int_0^{\infty} 3e^{-2t} \cdot e^{-st} dt + \int_{-\infty}^0 4e^t e^{-st} dt$$

$$= \frac{3}{s+2} - \frac{4}{s-1}$$

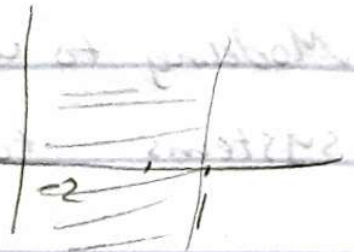
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$$\frac{3}{s+2}$$

$$-\frac{4}{s-1}$$

$$\operatorname{Re}\{s\} > -2$$

$$\operatorname{Re}\{s\} < 1$$



* Partial fraction $\Rightarrow$ $\frac{\text{num}}{\text{den}}$

order num $\leq$ order den $\Rightarrow$ proper

(i) Real distinct.

Ex: $x(s) = \frac{2s^3 + 2s - 10}{s^3 + 3s^2 - s - 3}$

by complex: $2 - \frac{3/4}{s-1} + \frac{7/2}{s+1} - \frac{3s/4}{s+3}$

$$\Rightarrow x(t) = 2\delta(t) \left(-\frac{3}{4}e^t + \frac{7}{2}e^{-t} - \frac{3s}{4}e^{-3t} \right) u(t)$$

Ex) Real & repeats,

$$X(s) = \frac{2s^2 + 3s - 1}{(s+1)(s+2)^3} \quad \text{find } x(t)??$$

$$X(s) = \frac{K_1}{s+1} + \frac{K_2}{(s+2)^3} + \frac{K_3}{(s+2)^2} + \frac{K_4}{s+2}$$

$$K_1 = \left. \frac{2s^2 + 3s - 1}{(s+2)^3} \right|_{s=-1} = [-2]$$

$$K_2 = \left. \frac{2s^2 + 3s - 1}{s+1} \right|_{s=-2} = [-1]$$

$$K_3 = \frac{1}{1!} \left. \frac{d}{ds} \left(\frac{2s^2 + 3s - 1}{s+1} \right) \right|_{s=-2} = [4]$$

$$K_4 = \frac{1}{2!} \left. \frac{d^2}{ds^2} \left(\frac{2s^2 + 3s - 1}{s+1} \right) \right|_{s=-2} = [2]$$

$$\Rightarrow x(t) = \left(-2e^{-t} - \frac{t^2}{2} e^{-2t} + 4te^{-2t} + 2e^{-2t} \right) u(t)$$

$$\frac{1}{s^n} \Leftrightarrow \frac{t^{n-1}}{(n-1)!} u(t)$$

$$t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}}$$

2.1

Ex: Complex & distinct

Ex: $X(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$ find $x(t)$?

$$X(s) = \frac{100(s+3)}{(s+6)(s+3-j4)(s+3+j4)}$$

$$= \frac{K_1}{s+6} + \frac{K_2}{s+3-j4} + \frac{K_2^*}{s+3+j4}$$

$$K_1 = \frac{100(s+3)}{s^2+6s+25} \Big|_{s=-6} = F(2)$$

$$K_2 = \frac{100(s+3)}{(s+6)(s+3+j4)} \Big|_{s=-3-j4} = 10 \angle -53.13^\circ$$

$$K_2^* = 10 \angle 53.13^\circ$$

$$\therefore x(t) = \left(-12e^{-6t} + 20e^{-3t} \cos(4t - 53.13^\circ) \right) u(t)$$

$$\mathcal{L}^{-1} \left\{ \frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta} \right\} \Leftrightarrow 2|K| e^{-\alpha t} \cos(\beta t + \theta)$$

(4) Complex & repeated.

Ex: $X(s) = \frac{768}{(s^2 + 6s + 25)^2}$ find $x(t)$.

$$X(s) = \frac{K_1}{(s+3-j4)^2} + \frac{K_2}{s+3-j4} + \frac{K_3}{(s+3+j4)^2} + \frac{K_4}{s+3+j4}$$

$$K_1 = \frac{768}{(s+3+j4)^2} \Big|_{s=-3+j4} = -12$$

$$K_2 = \frac{1}{1!} \frac{d}{ds} \left(\frac{768}{(s+3+j4)^2} \right) \Big|_{s=-3+j4} = 3 \angle -90^\circ$$

$$\therefore x(t) = \left[-24(t) e^{-3t} \cos(4t) + 6 e^{-3t} \cos(4t - 90^\circ) \right] u(t)$$

النتيجة النهائية

* Transfer Function :-

relation between input and output of the system.

$$H(s) = \frac{Y(s)}{X(s)}$$

* All initial condition must be equal zero, if we give initial condition we will put it zero.

Ex. $2y''(t) + 3y'(t) + 5y(t) = x'(t) - 2x(t)$

Find T.F.

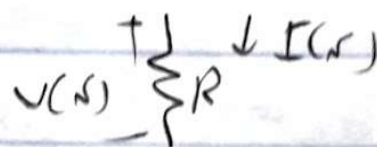
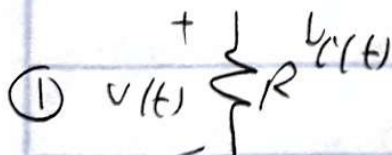
$$(2s^2 + 3s + 5) Y(s) = (s - 2) X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{s - 2}{2s^2 + 3s + 5}$$

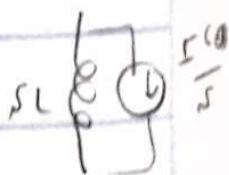
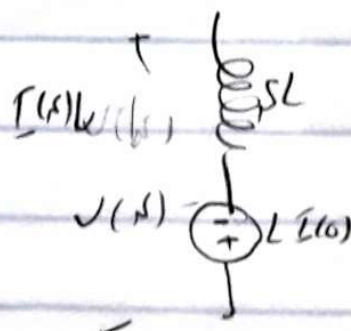
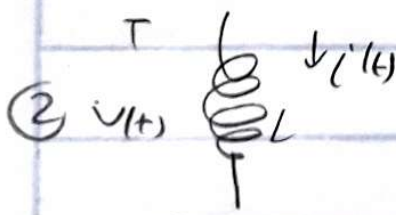
* Electrical network transfer function.

Time domain

Freq domain



$$V(s) = I(s) R$$



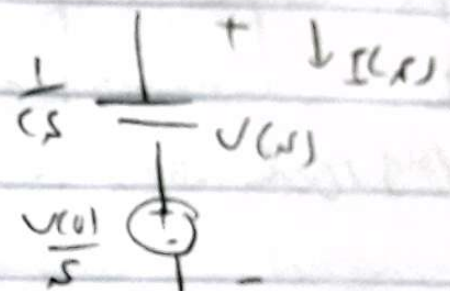
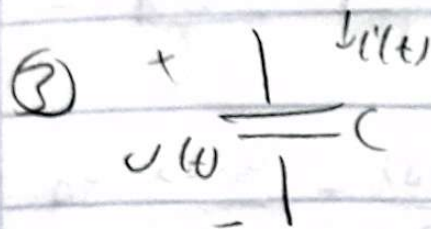
$$V(t) = L \frac{di}{dt}$$

$$V(s) = L(sI(s) - I(0))$$

$$V(s) = sL I(s) - I(0)L$$

OR Current source

$$I(s) = \frac{V(s)}{sL} + \frac{I(0)}{s}$$



$$i = C \frac{dV}{dt}$$

$$V(s) = \frac{I(s)}{Cs} + \frac{V(0)}{s}$$

$$V(t) = \frac{1}{C} \int i'(t) dt + V_0$$

* Ex 2

$$V(0) = 1V, \quad i'(0) = 2A$$

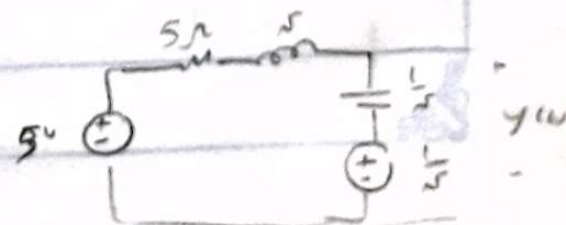
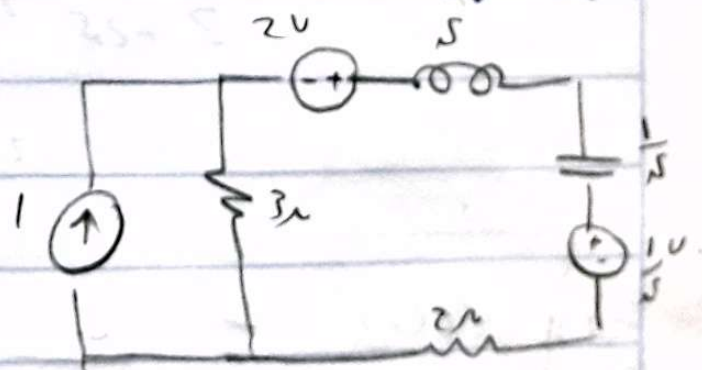
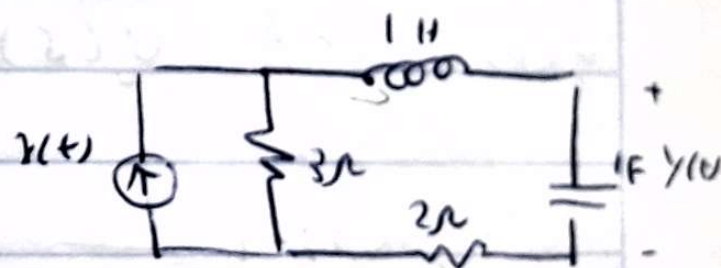
$$x(t) = \delta(t) \quad \text{find } y(t)$$

$$I(s) = \frac{5 - \frac{1}{s}}{s + s + \frac{1}{s}} =$$

$$\Rightarrow y(s) = \frac{1}{s} I(s) + \frac{1}{s}$$

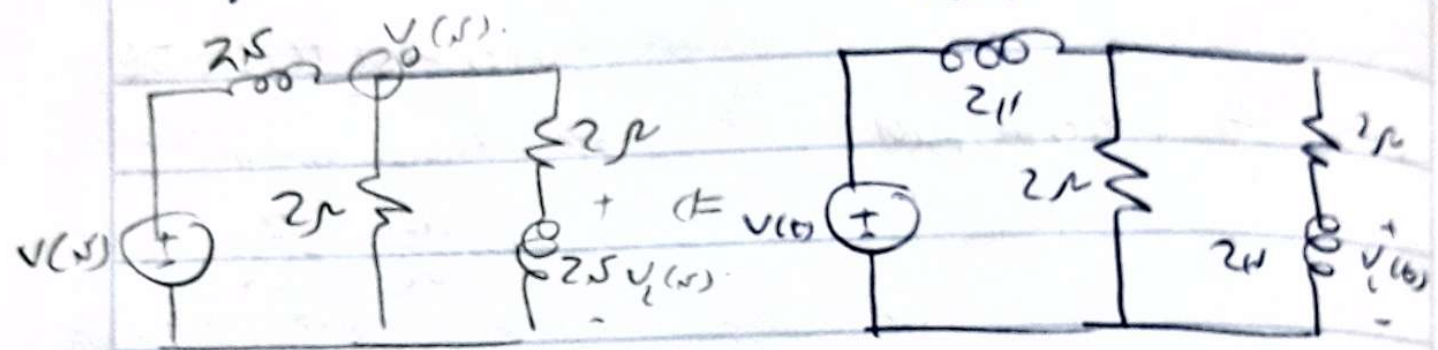
$$= \frac{s+10}{s^2 + s + 1}$$

$$\Rightarrow y(t)$$



Fr, P 2.17

Find T.F $G(s) = \frac{V_o(s)}{V(s)}$



By node voltage: $\frac{V_o(s) - V(s)}{2s} + \frac{V_o(s)}{2} + \frac{V_o(s)}{2+2s} = 0$

$$\Rightarrow V_o(s) \left(\frac{1}{2s} + \frac{1}{2} + \frac{1}{2+2s} \right) = \frac{V(s)}{2s}$$

$$\Rightarrow V_o(s) = V(s) \left(\frac{1}{1+s+\frac{2s}{2+2s}} \right)$$

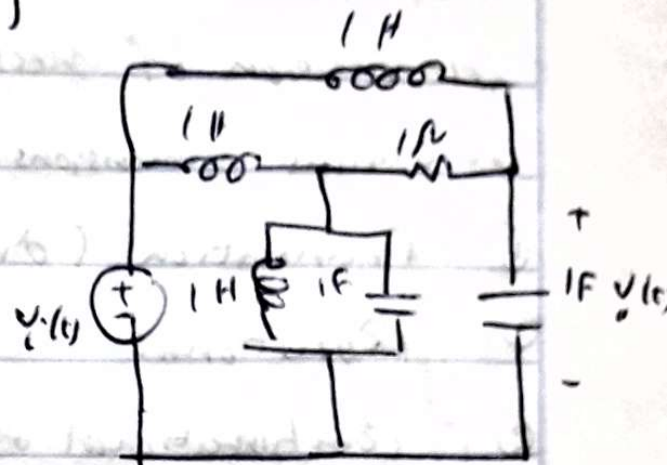
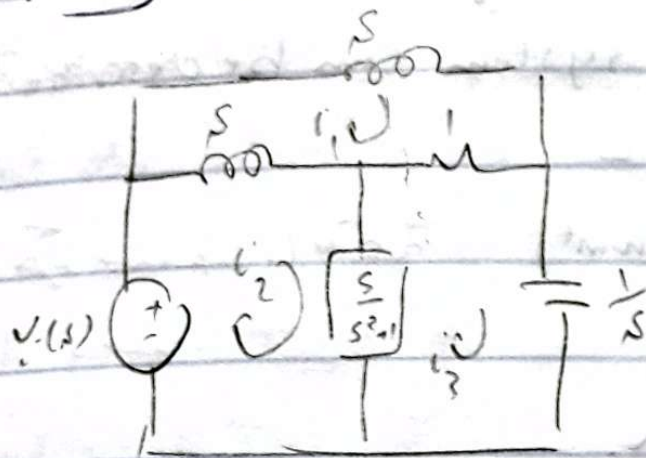
$$V(s) = \frac{2s}{2s+2} V_o(s)$$

$$= V(s) \left(\frac{2s}{2+2s} \right) \left(\frac{1}{1+s+\frac{2s}{2+2s}} \right)$$

$$= 1 \quad \quad \quad 2+2s+2s+2s^2+2s = 2s^2+6s+2$$

$$\Rightarrow \frac{V_o(s)}{V(s)} = \frac{s}{s^2+3s+1}$$

[P. 2.18] Find T.F $G(s) = \frac{V_o(s)}{V_i(s)}$.



$$(2s+1)I_1'(s) - sI_2'(s) - I_3'(s) = 0.$$

$$-sI_1'(s) + \left(s + \frac{s}{s^2+1}\right)I_2'(s) - \frac{s}{s^2+1}I_3'(s) = V_i(s).$$

$$-I_1'(s) - \frac{s}{s^2+1}I_2'(s) + \left(1 + \frac{1}{s} + \frac{s}{s^2+1}\right)I_3'(s) = 0.$$

* Translation Mechanical System, T.F. :-

The motion of Mechanical system can be describe in various dimensions as ; -

- ① Translation (displacement) انتقال (displacement)
- ② Rotational دوراني
- ③ Combination of both.

* Mechanical system have three Passive linear system.

- 1) Spring $\Rightarrow$ Energy stored elements.
- 2) Mass $\Rightarrow$ Energy stored elements.
- 3) Viscous damper $\Rightarrow$ dissipated energy.

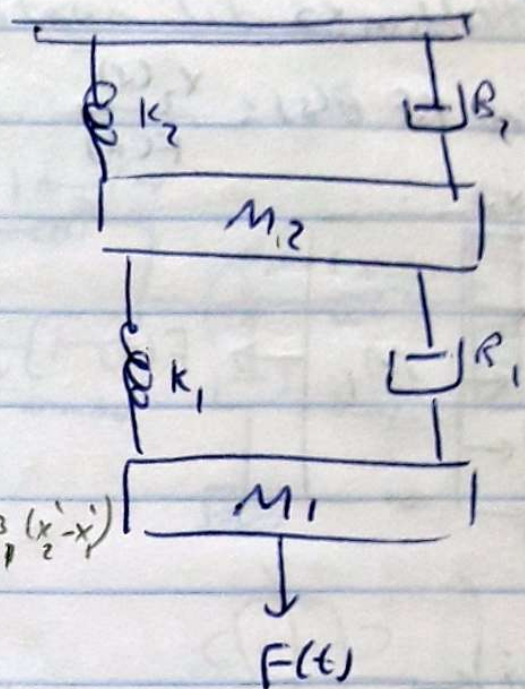
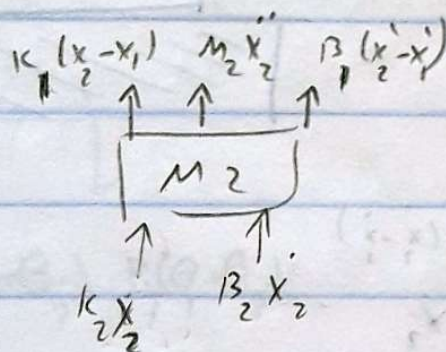
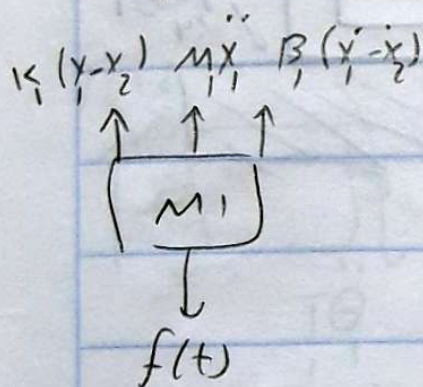
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Notes :- In translational Mechanical system the input of the system is a force $F(s)$ and the output is a displacement $X(s)$.

Ex: >

Find the differential equations that describe the system.

1) free body diagram.



$$f(t) = K_1(x_1 - x_2) + B_1(\dot{x}_1 - \dot{x}_2) + M_1 \ddot{x}_1$$

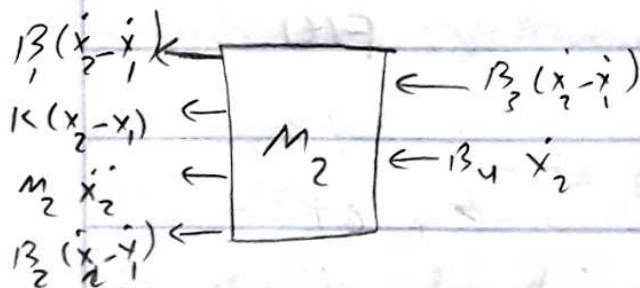
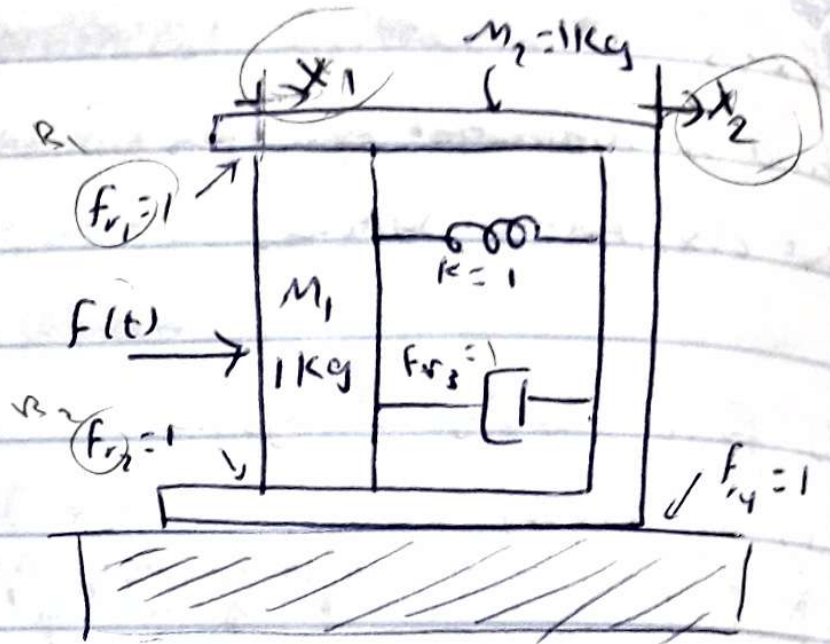
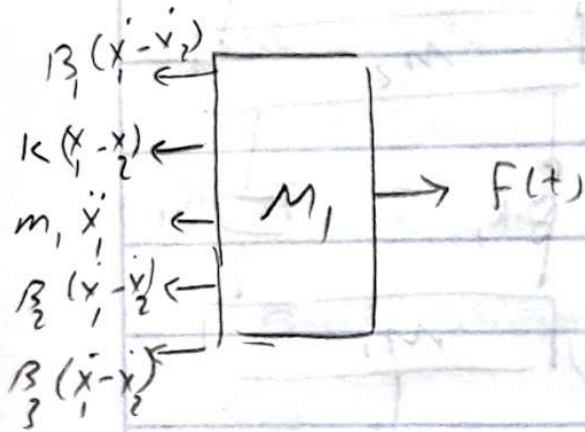
$$K_1(x_2 - x_1) + M_2 \ddot{x}_2 + B_1(\dot{x}_2 - \dot{x}_1) + K_2 x_2 + B_2 \dot{x}_2 = 0.$$

$$(M_1 s^2 + B_1 s + K_1) X_1(s) - (B_1 s + K_1) X_2(s) = F(s).$$

$$-(B_1 s + K_1) X_1(s) + (M_2 s^2 + (B_1 + B_2) s + (K_1 + K_2)) X_2(s) = 0.$$

Ex 80 problem 2

Find F.F $G(s) = \frac{Y_2(s)}{F(s)}$



$$\therefore f(t) = B_1(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2) + M_1 \ddot{x}_1 + B_2(\dot{x}_1 - \dot{x}_2) + B_3(x_1 - x_2)$$

$$M_2 \ddot{x}_2 + B_4 \dot{x}_2 = B_1(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2) + B_2(\dot{x}_1 - \dot{x}_2) + B_3(x_1 - x_2)$$

$$\begin{aligned} \textcircled{1} \quad F(s) &= 3(\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) + \ddot{x}_1 \\ &= \ddot{x}_1 + 3\dot{x}_1 + x_1 - 3\dot{x}_2 - x_2 \end{aligned}$$

$$F(s) = (s^2 + 3s + 1) X_1(s) - (3s + 1) X_2(s) \quad \text{--- (1)}$$

$$\textcircled{2} \quad \ddot{x}_2 + \dot{x}_2 = 3(\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) \Rightarrow \ddot{x}_2 + \dot{x}_2 + 3\dot{x}_2 + x_2 = 3\dot{x}_1 + x_1$$

$$\Rightarrow (s^2 + 4s + 1) X_2(s) = (3s + 1) X_1(s)$$

$$\Rightarrow X_1(s) = \frac{s^2 + 4s + 1}{3s + 1} X_2(s)$$

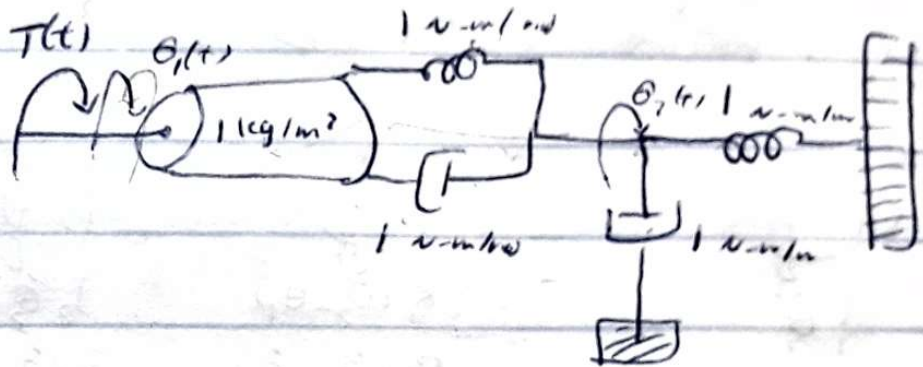
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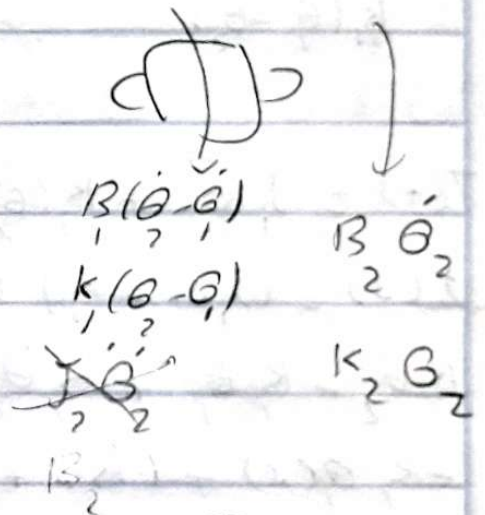
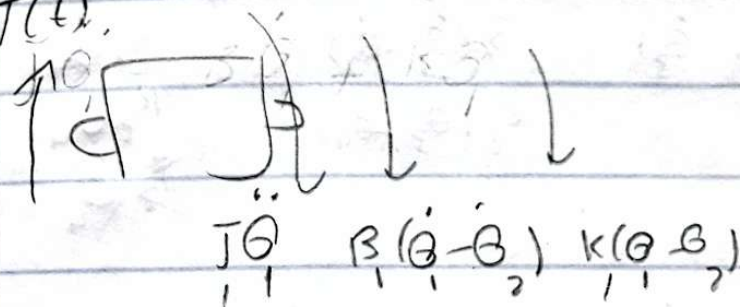
* Rotational Mechanical System T.F is

Exercise 2.9 :-

Find
 $\frac{\theta_2(s)}{T(s)}$



$T(t)$



$$T(t) = J_1 \ddot{\theta}_1 + B_1(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

$$T(t) = (\ddot{\theta}_1 + \dot{\theta}_1 + \theta_1) - (\dot{\theta}_2 + \theta_2)$$

$$T(s) = (s^2 + s + 1) \theta_1(s) - (s + 1) \theta_2(s) \quad \text{--- (1)}$$

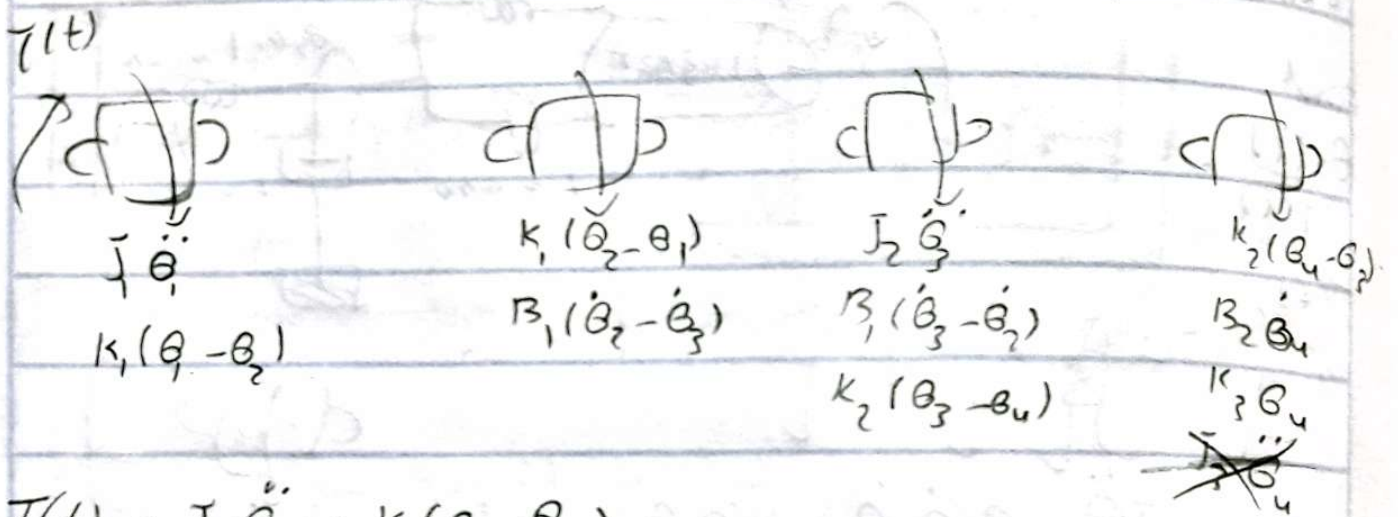
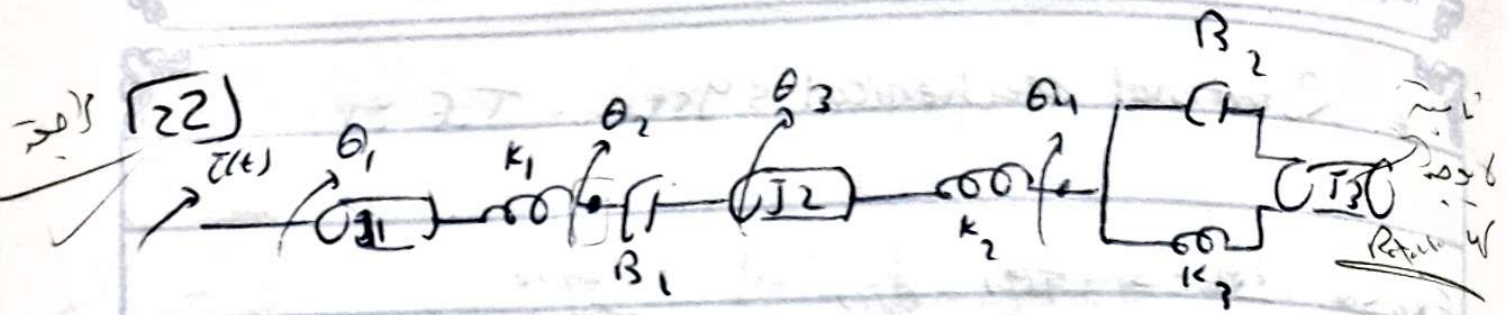
$$B_2 \dot{\theta}_2 + K_2 \theta_2 = B_1(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

$$\dot{\theta}_2 + \theta_2 + \dot{\theta}_1 + \theta_1 = \dot{\theta}_1 + \theta_1$$

$$(2s + 2) \theta_2(s) = (s + 1) \theta_1(s)$$

$$\Rightarrow \theta_1(s) = \frac{2(s+1)}{s+1} \theta_2(s) \Rightarrow \theta_1(s) = 2 \theta_2(s)$$

① ببقوة



$$T(t) = J_1 \ddot{\theta}_1 + k_1(\theta_1 - \theta_2)$$

$$T(s) = (J_1 s^2 + k_1) \theta_1(s) - k_1 \theta_2(s) \quad \text{--- (1)}$$

$$k_1 \theta_2 - k_1 \theta_1 + B_1 \dot{\theta}_2 - B_1 \dot{\theta}_1 = 0$$

$$-k_1 \theta_1(s) + (B_1 s + k_1) \theta_2(s) - B_1 s \theta_1(s) = 0 \quad \text{--- (2)}$$

$$J_2 \ddot{\theta}_3 + B_1 \dot{\theta}_3 - B_1 \dot{\theta}_2 + k_2 \theta_3 - k_2 \theta_4 = 0$$

$$(J_2 s^2 + B_1 s + k_2) \theta_3(s) - B_1 s \theta_2(s) - k_2 \theta_4(s) = 0 \quad \text{--- (3)}$$

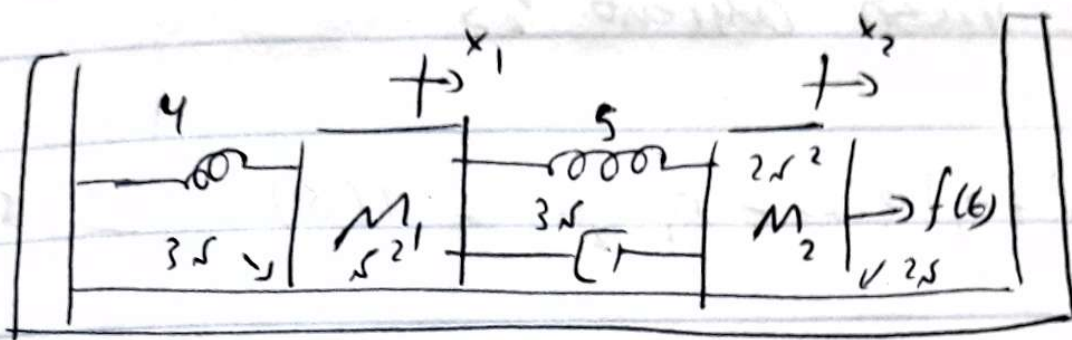
$$J_3 \ddot{\theta}_4 + B_2 \dot{\theta}_4 + k_3 \theta_4 + k_2 \theta_4 - k_2 \theta_3 = 0$$

$$(J_3 s^2 + (B_2 s + (k_3 + k_4)) \theta_4(s) - k_2 \theta_3(s) = 0 \quad \text{--- (4)}$$

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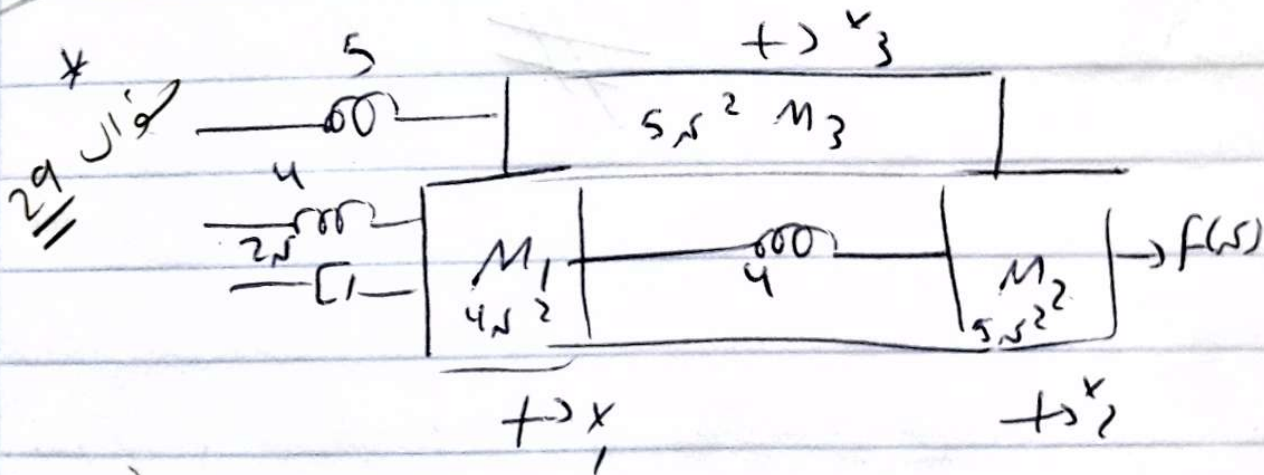
سوال 29



$$(s^2 + 6s + 9) X_1(s) - (3s + 5) X_2(s) = 0$$

$$-(5 + 3s) X_1(s) + (2s^2 + 5s + 5) X_2(s) = f(s)$$

سوال 29



$$(4s^2 + 4s + 4) X_1(s) - 4 X_2(s) = 0$$

همه بستگی اکث

$$4s^2 +$$

(22) mesh current : 2

$$(I_1 s^2 + K_1) \theta_1(s) - K_1 \theta_2(s) = T(s) \quad \text{--- (1)}$$

$$-K_1 \theta_1(s) + (B_1 s + K_1) \theta_2(s) - B_1 s \theta_3(s) = 0 \quad \text{--- (2)}$$

$$-B_1 \theta_2(s) + (I_2 s^2 + B_1 s + K_2) \theta_3(s) - K_2 \theta_4(s) = 0 \quad \text{--- (3)}$$

$$-K_2 \theta_3(s) + (B_2 s + K_2 + K_3) \theta_4(s) = 0 \quad \text{--- (4)}$$

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